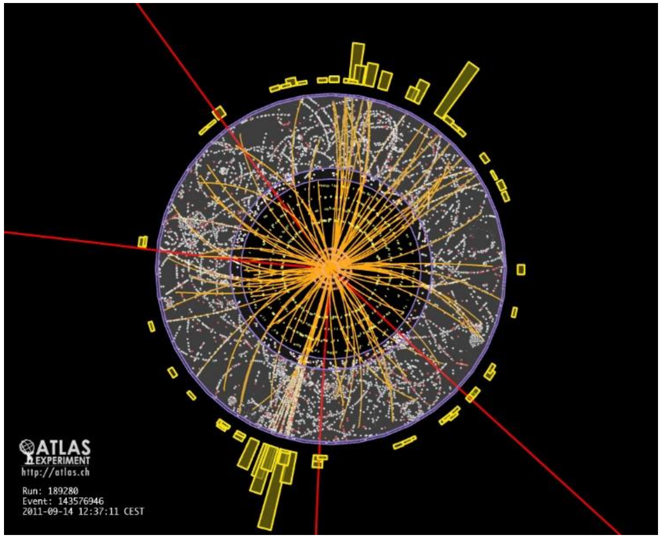
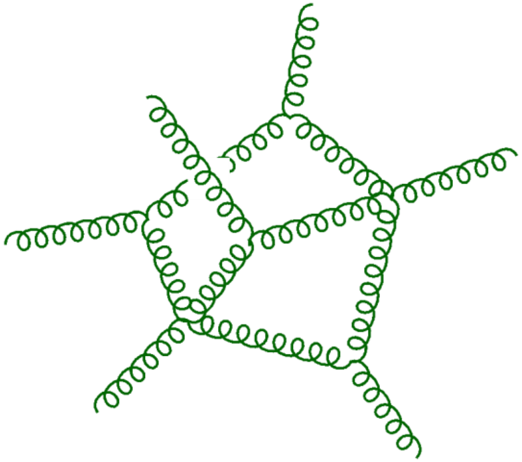
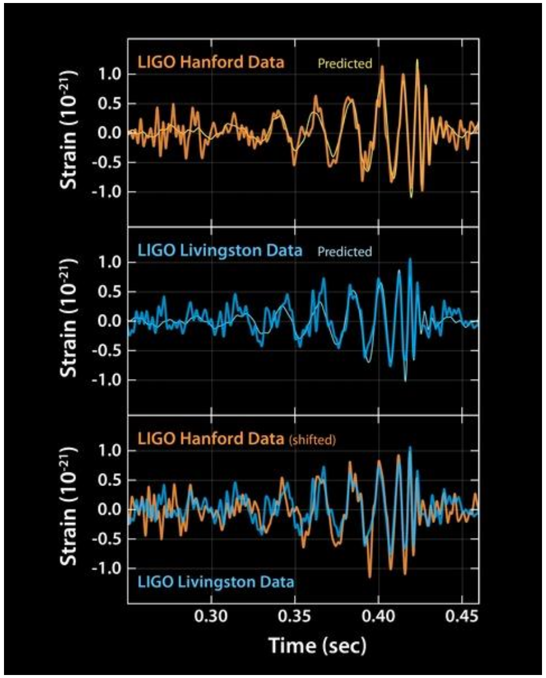
Scattering Amplitudes from LHC to LIGO and Beyond





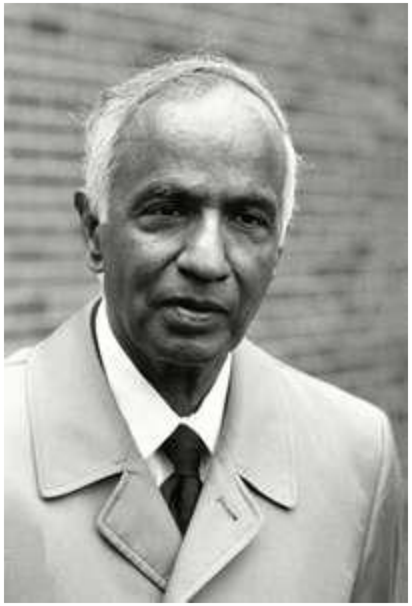


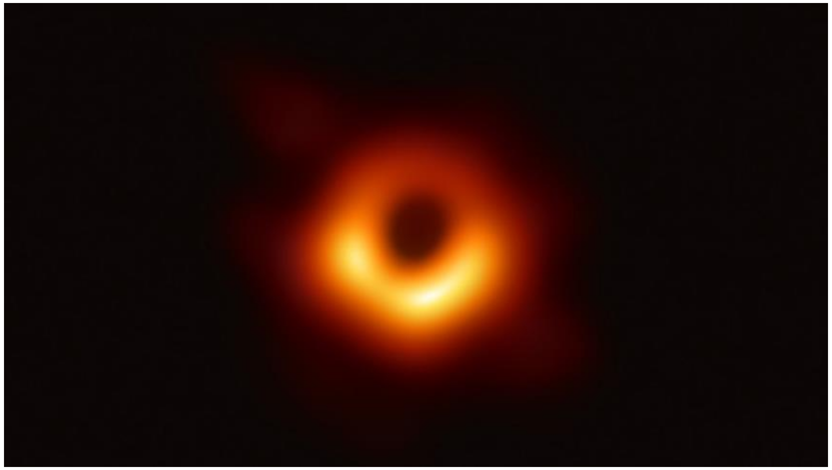
Lance Dixon

"Quantum Universe" Colloquium
DESY and University of Hamburg
23 June 2020

“**The black holes** of nature are
the most perfect macroscopic objects there are in the universe:
the only elements in their construction are our concepts of space and time.”

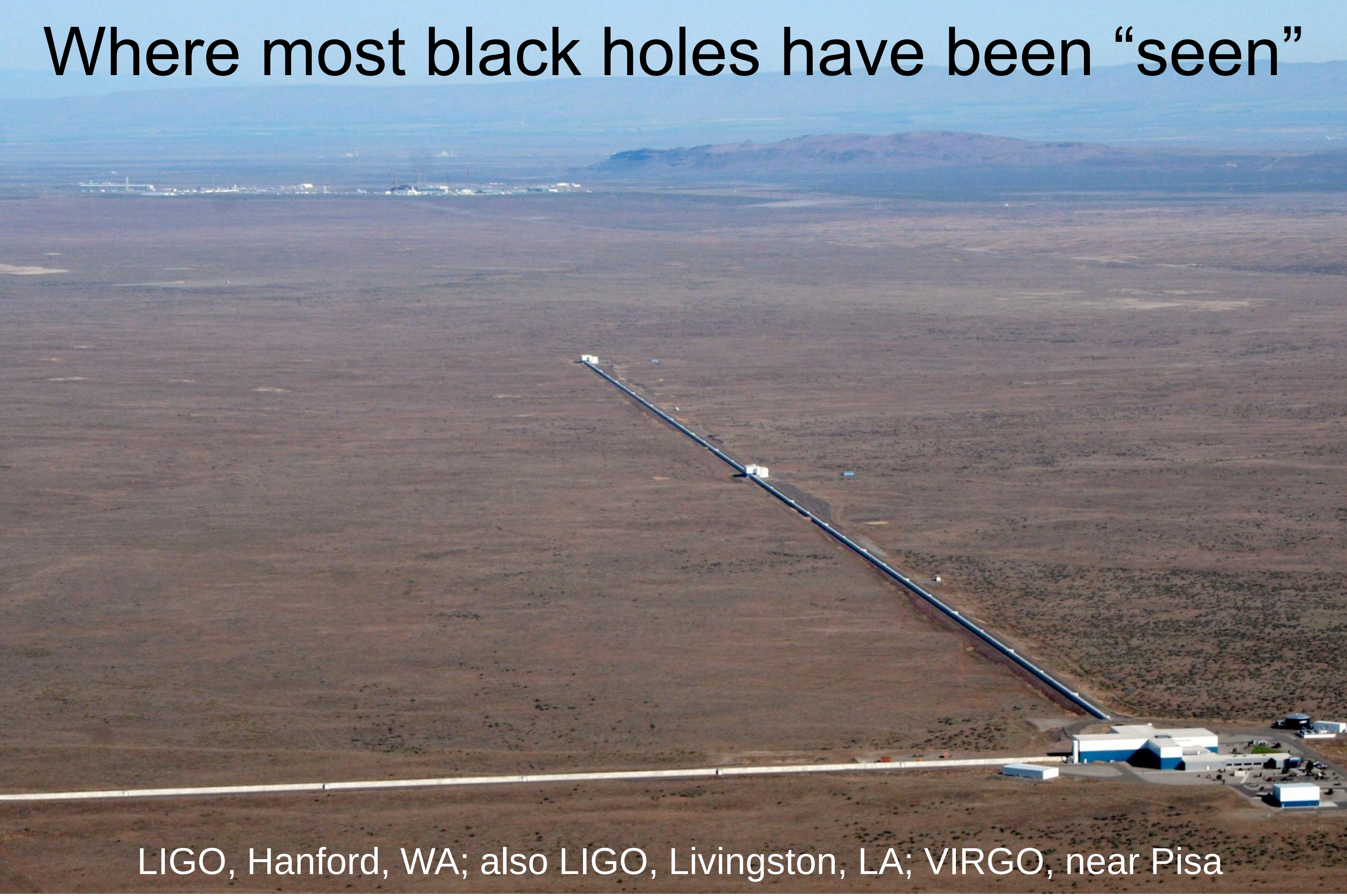
- S. Chandrasekhar (1979)





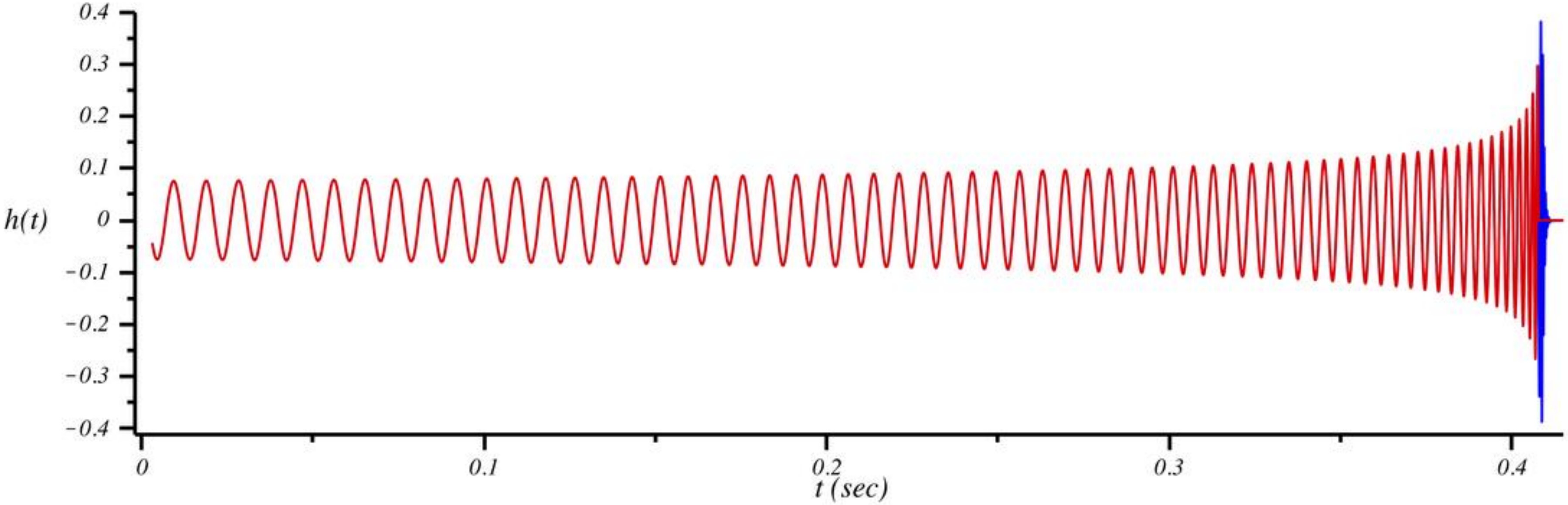
Event Horizon Telescope (2019)

Where most black holes have been “seen”

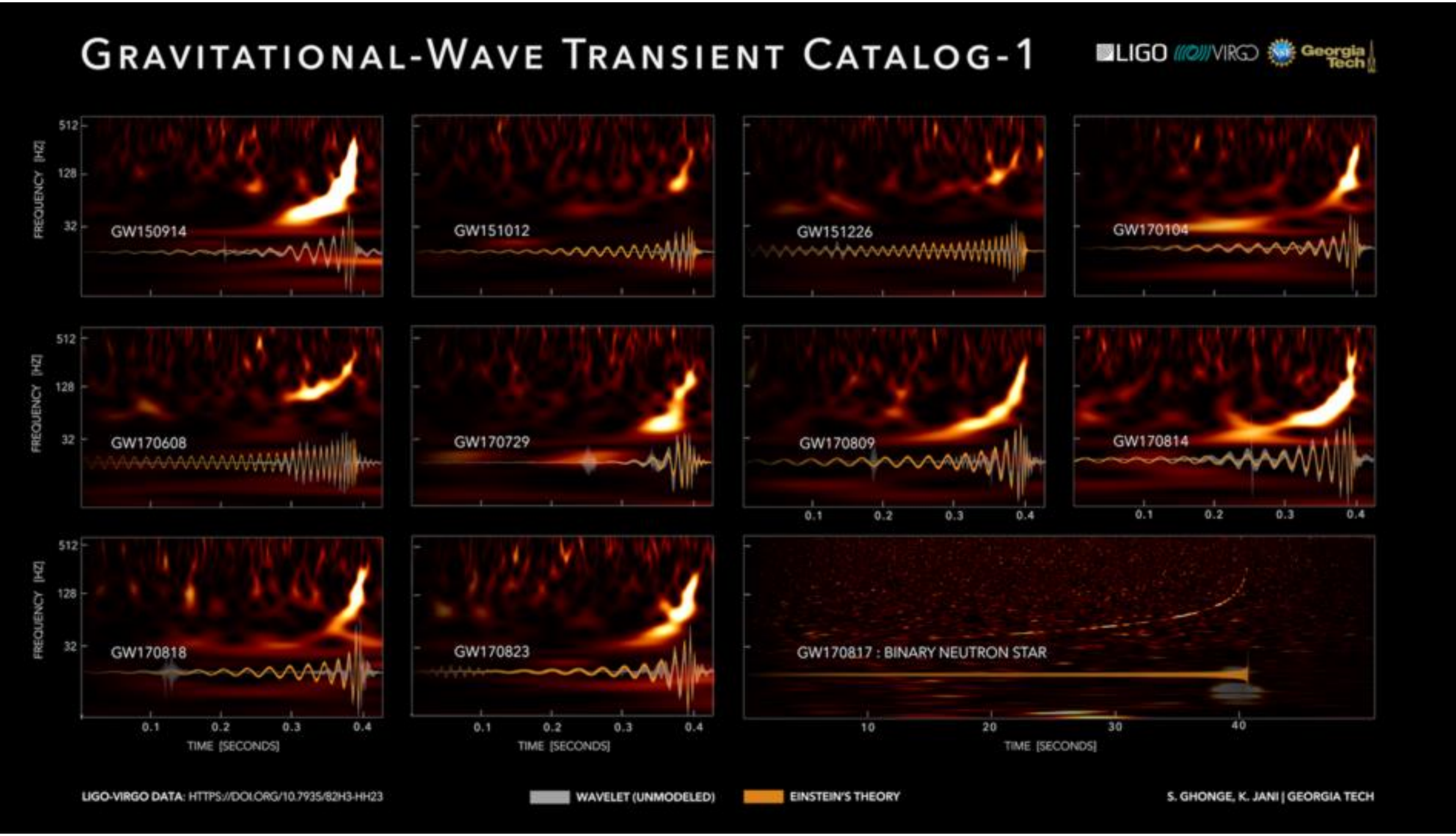


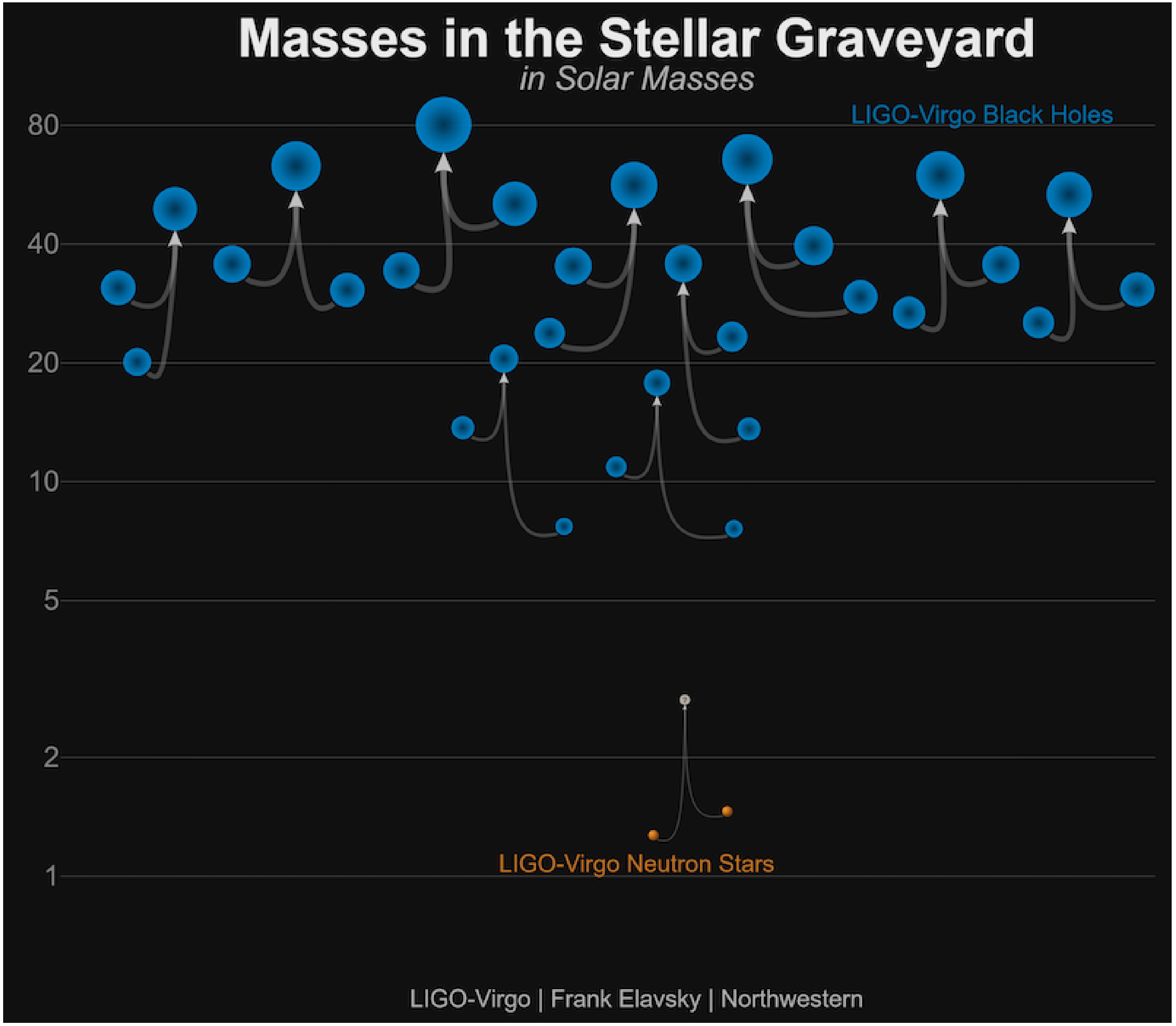
LIGO, Hanford, WA; also LIGO, Livingston, LA; VIRGO, near Pisa

Binary inspiral

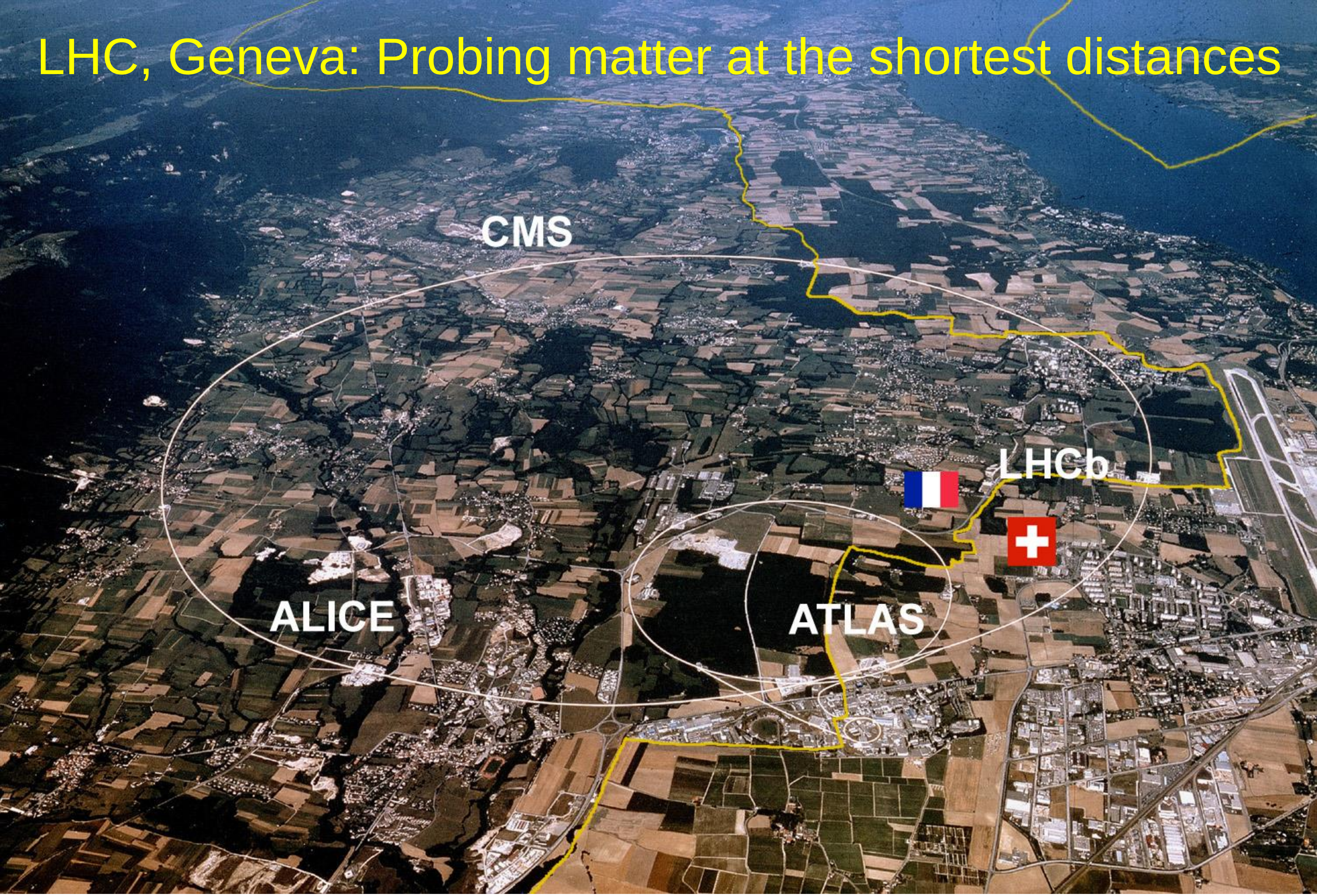


Some events (many more on tape)

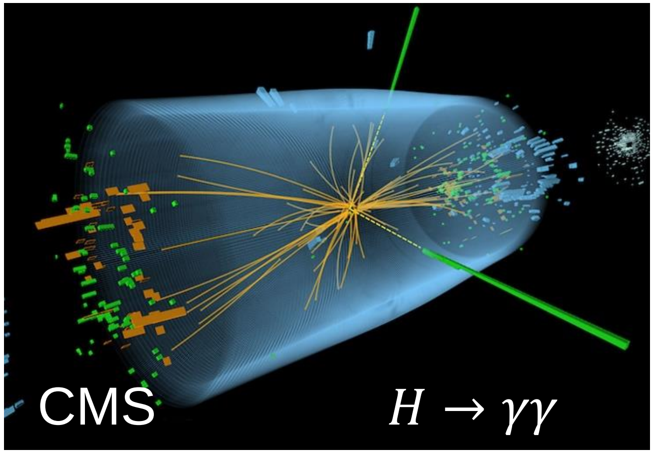


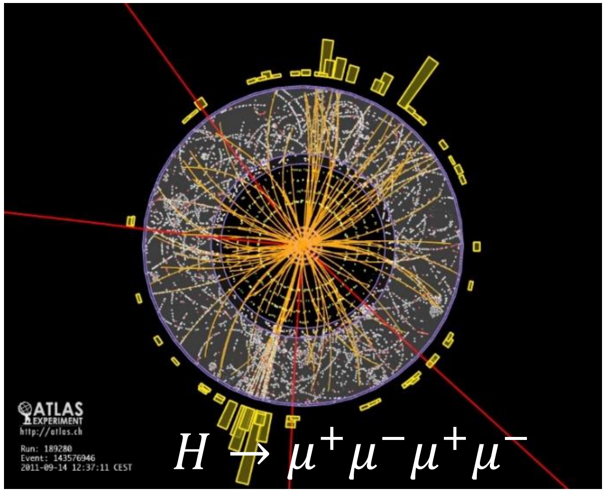

Masses in the Stellar Graveyard
in Solar Masses
LIGO-Virgo Black Holes
80
40
20
10
5
2
1
LIGO-Virgo Neutron Stars
LIGO-Virgo | Frank Elavsky | Northwestern

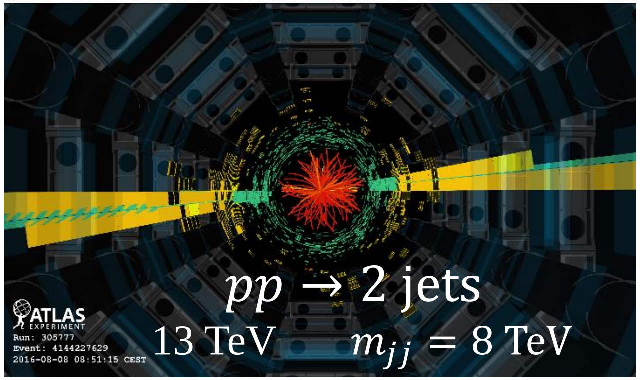
LHC, Geneva: Probing matter at the shortest distances

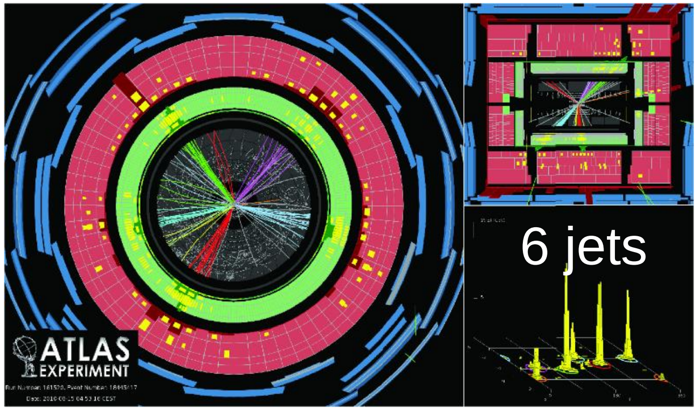


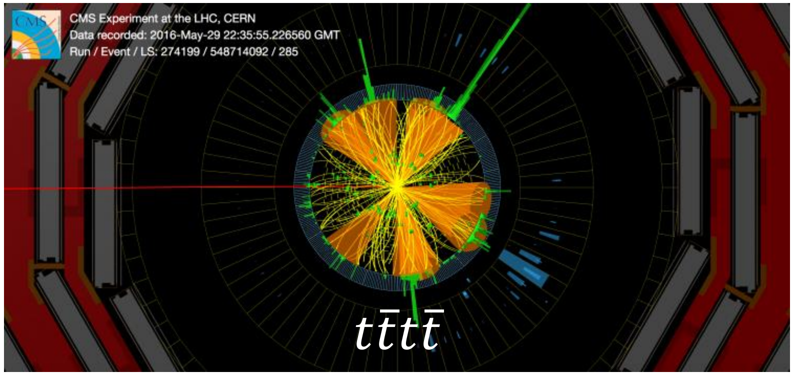
A few events, out of trillions





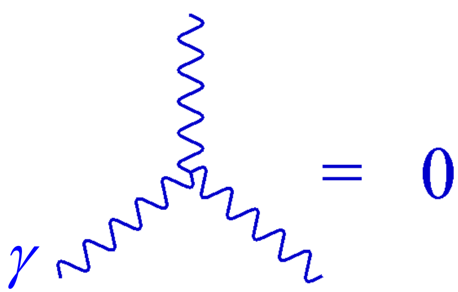


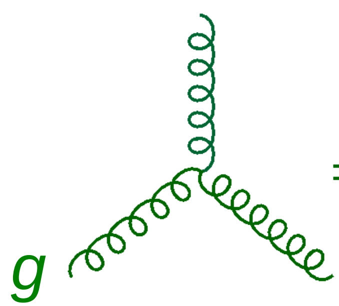




At the heart of the LHC

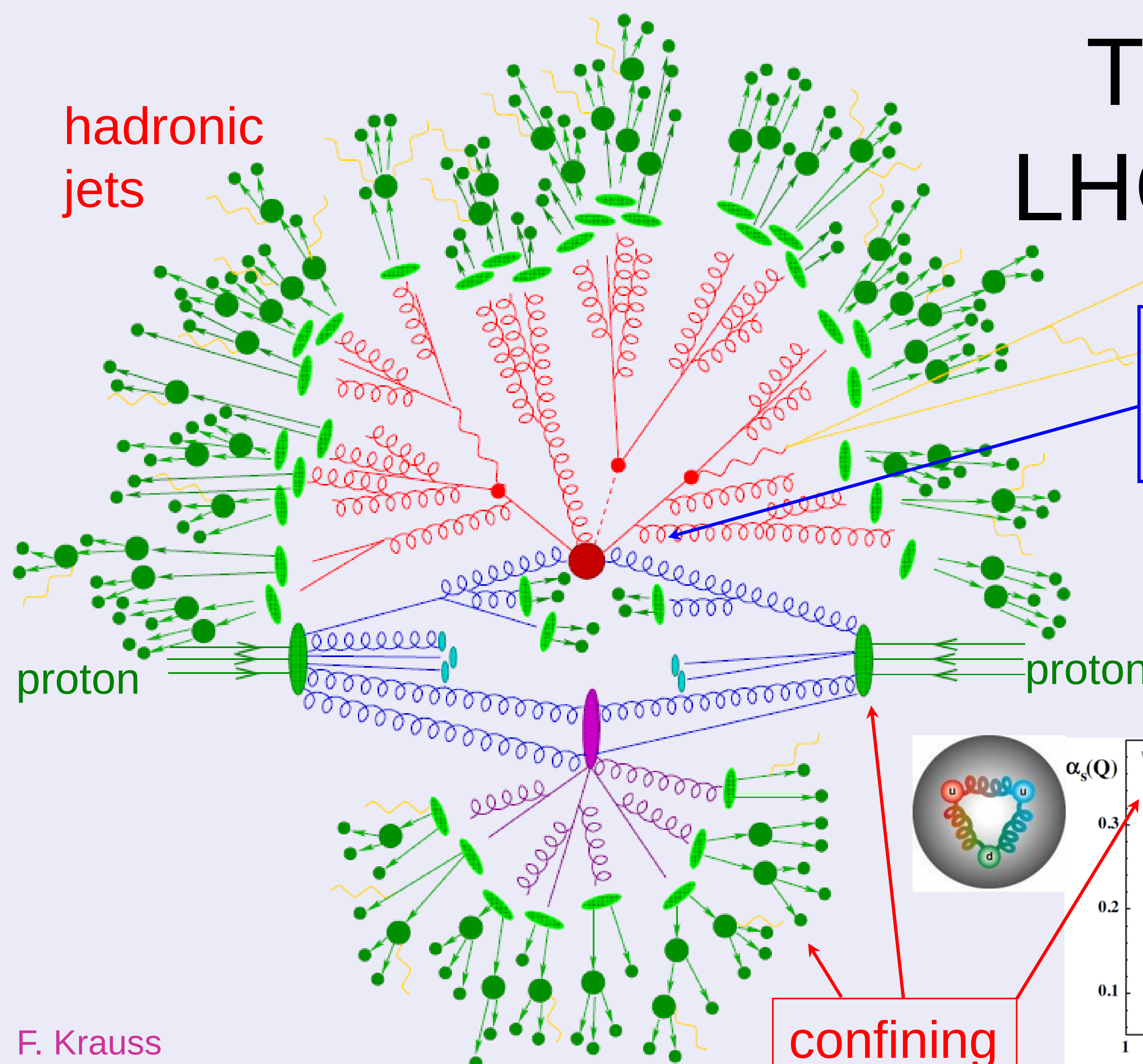
- The most copious events at short distances (large momentum transfer p_T) are from quantum chromodynamics (QCD), also known as the strong interactions.
- Force carried by gluons (g), a nonlinear cousin of the photon (γ)





$$= g_s = \sqrt{\alpha_s}$$

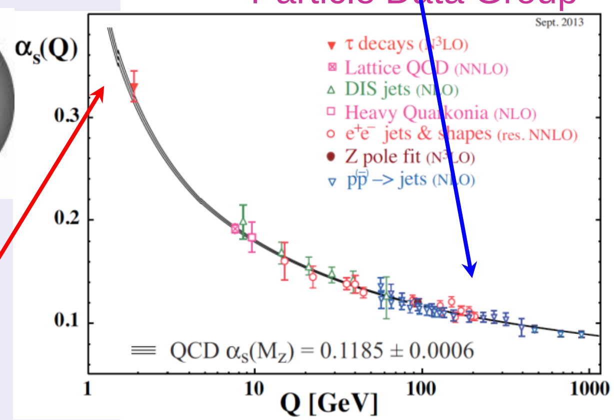
Typical LHC event



nonlinearities small, perturbatively calculable

confining

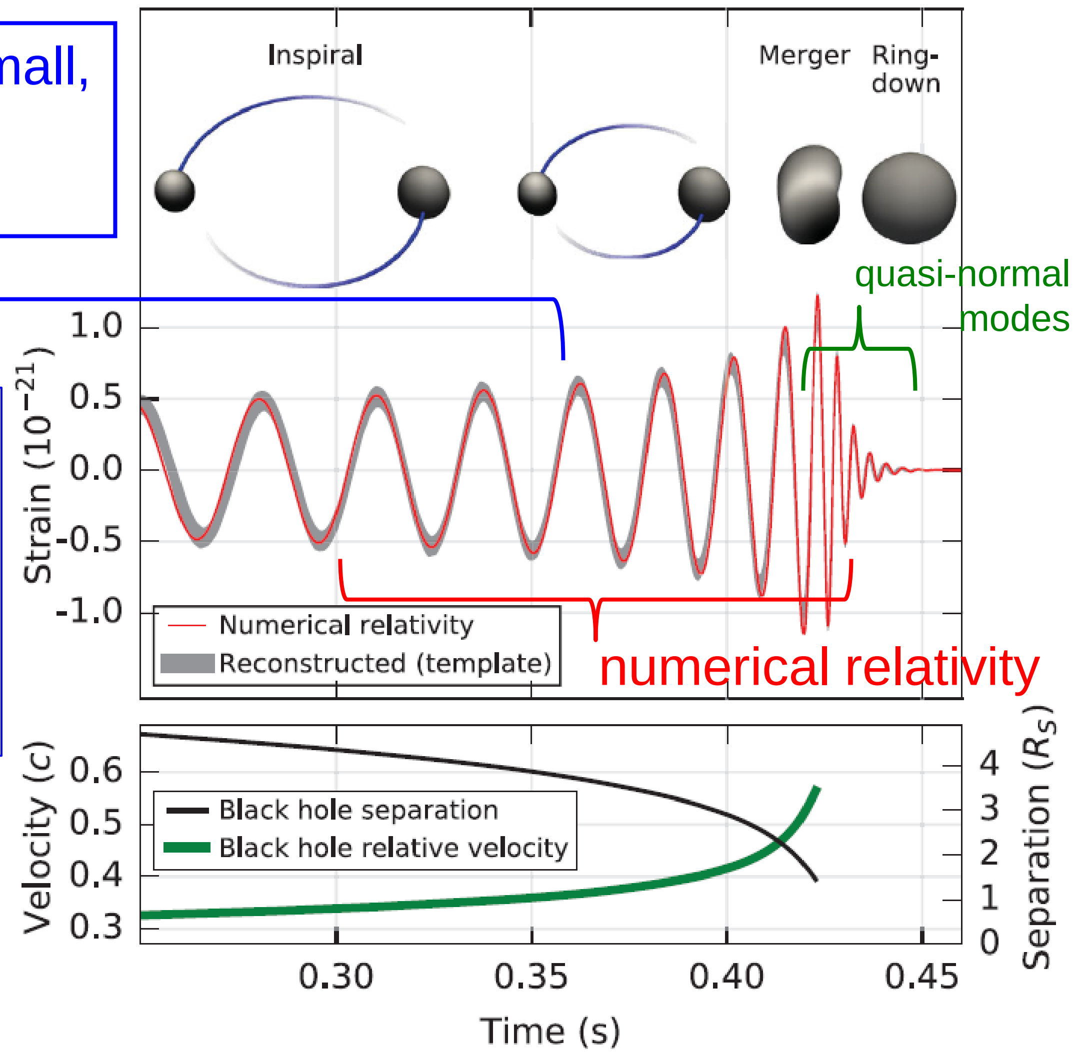
Particle Data Group



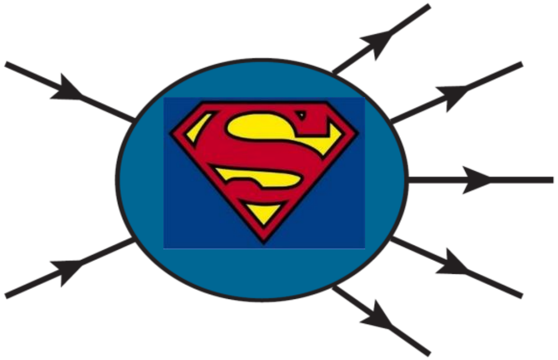
Typical LIGO Event

nonlinearities small, perturbatively calculable

NS-NS mergers @ LIGO/VIRGO/...
or BH's @ LISA
→ many, many cycles
in perturbative regime
→ phase of orbit can be measured
very precisely



Scattering amplitudes



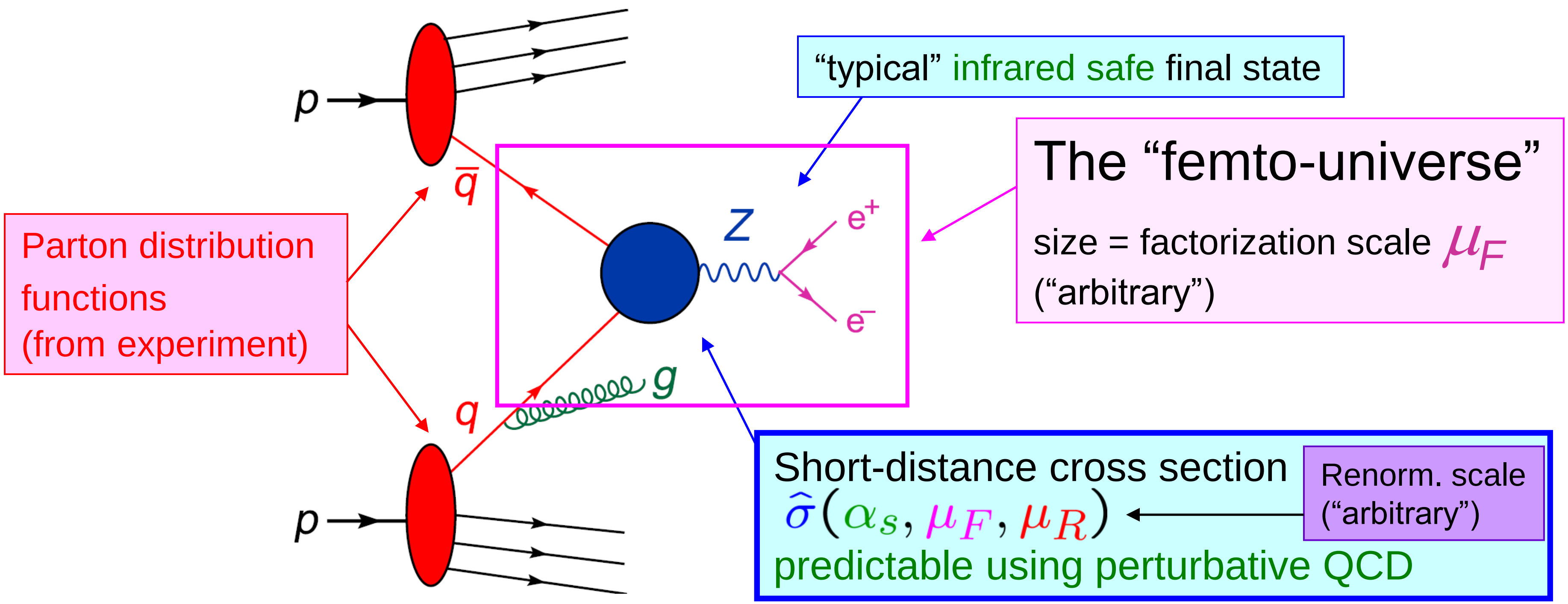
- “The most perfect microscopic structures in the universe”
- Where the “rubber meets the road” between quantum field theory and experiment
- Amplitudes for quantum chromodynamics (QCD) dominate collisions at LHC
- Smooth functions of the external kinematic variables and essentially nothing else

Both shrouded in mystery

- Black holes often shrouded in gas and dust (which makes it possible to “see” them electromagnetically)
- Quarks and gluons scattering in QCD are shrouded by confinement: bound within the initial protons, and emerge as collimated jets of hadrons in the final state.

QCD Factorization & Parton Model

At short distances, quarks and gluons (partons) in proton are almost free. Sampled “one at a time”

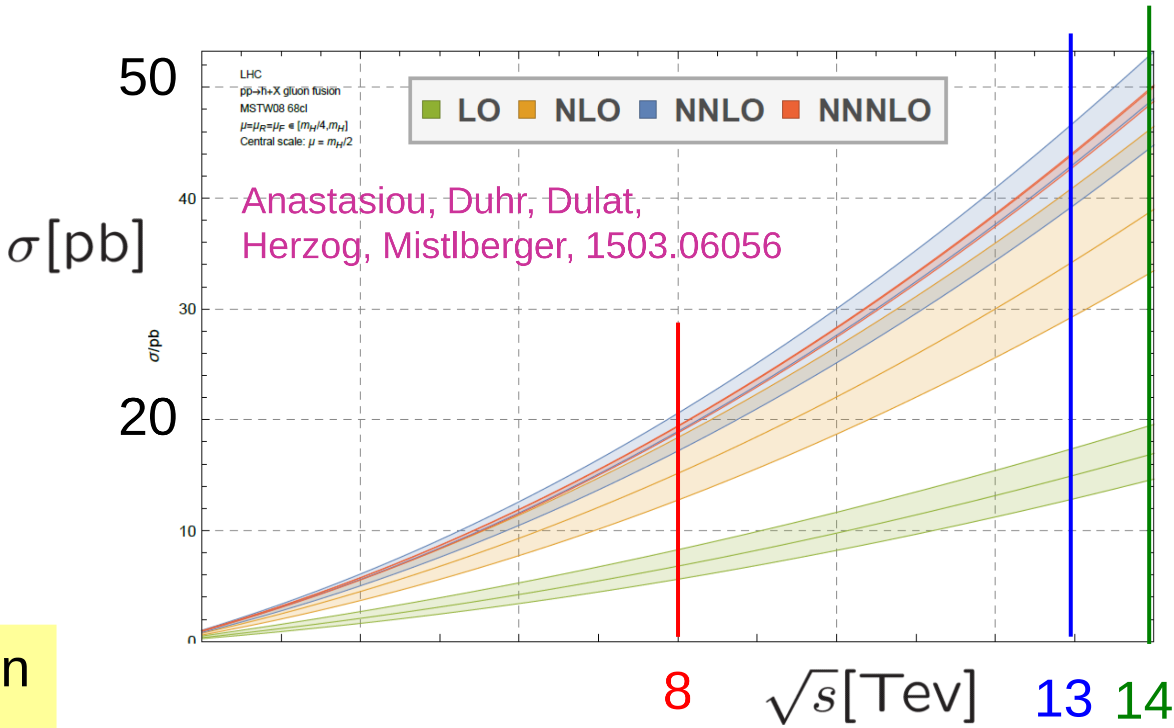


Perturbative Short-Distance Cross Section

$$\hat{\sigma}(\alpha_s,\mu_F,\mu_R) = [\alpha_s(\mu_R)]^{n_\alpha}\left[\hat{\sigma}^{(0)} + \frac{\alpha_s}{2\pi}\hat{\sigma}^{(1)}(\mu_F,\mu_R) + \left(\frac{\alpha_s}{2\pi}\right)^2\hat{\sigma}^{(2)}(\mu_F,\mu_R) + \cdots\right]$$

LO NLO NNLO

Leading-order (LO) predictions qualitative:
poor convergence
of expansion in $\alpha_s(\mu)$
Uncertainty bands from varying $\mu_R = \mu_F = \mu$

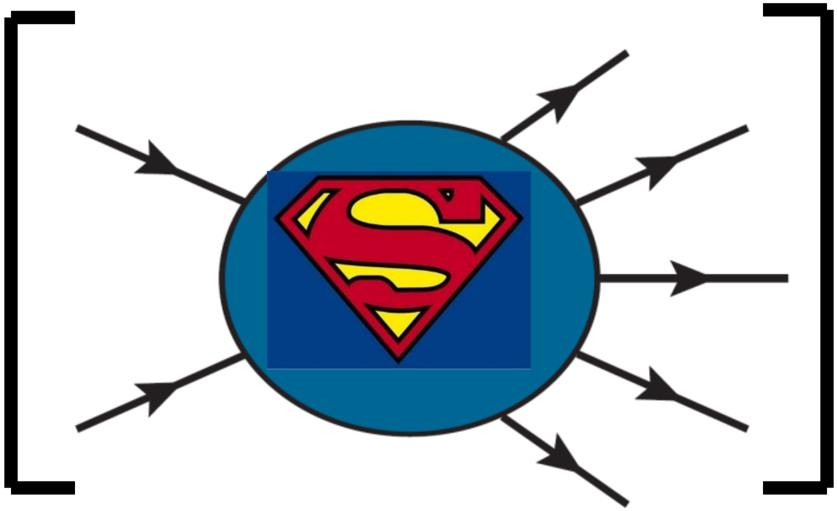


Example: Higgs gluon fusion cross section vs. LHC CM energy $\sqrt{s}$

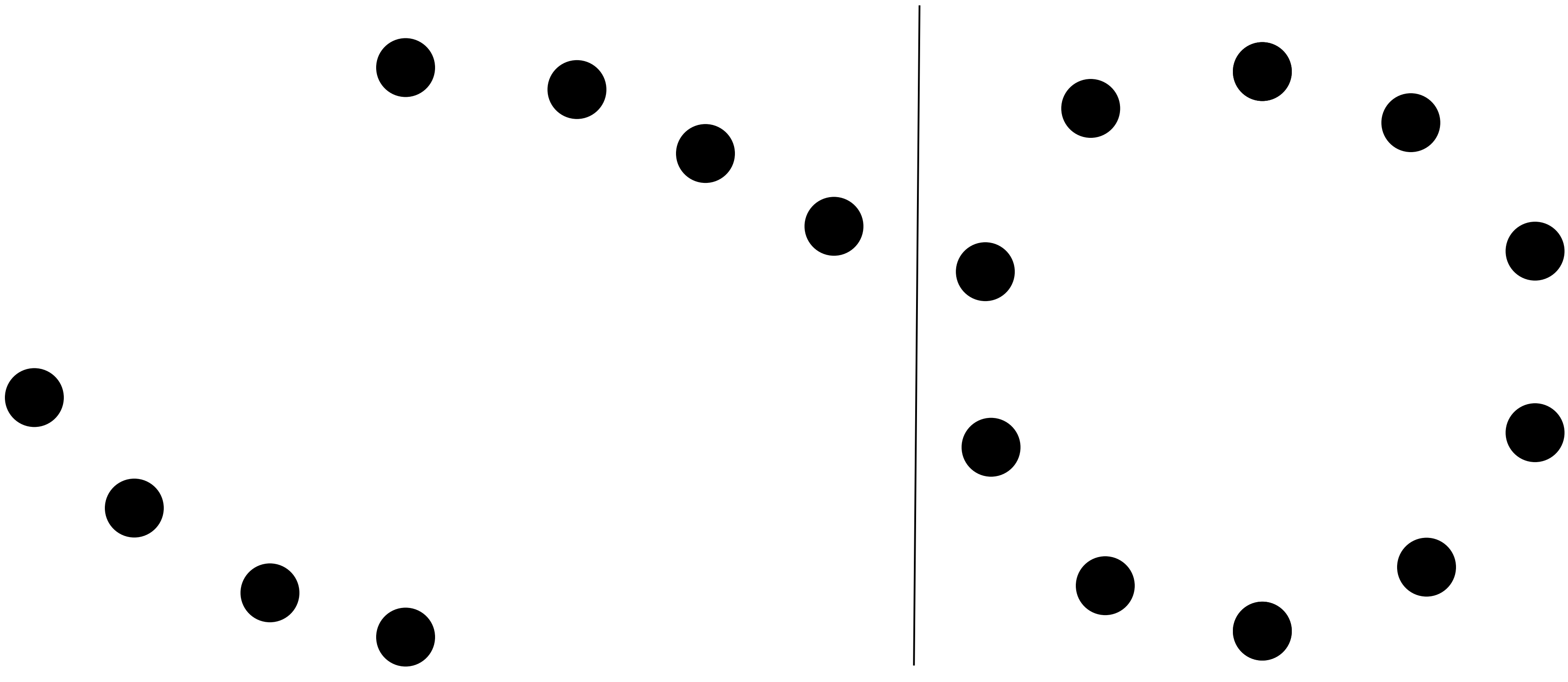
LO → NNLO
→ factor of 2.7 increase!

Short-distance cross sections built out of scattering amplitudes, ***S***-matrix elements

$$\sigma = \int$$



Black hole scattering vs. inspiral



- Related by “analytic continuation around $r = \infty$”
- Accomplish with effective Hamiltonian, e.g. Cheung, Rothstein, Solon, 1808.02489
- Or more directly in terms of trajectories Kälin, Porto, 1910.03008, 1911.09130

Spinless black hole example

- Scattering depends on both relative velocity v and strength of potential $G_N M_1 M_2/r \equiv G/(\frac{r}{r_{Schw}})$ (deviation from Minkowski metric)
- In bound state, locked together by virial theorem:
- Kinetic energy ~ potential energy

$$v^2 \sim G$$

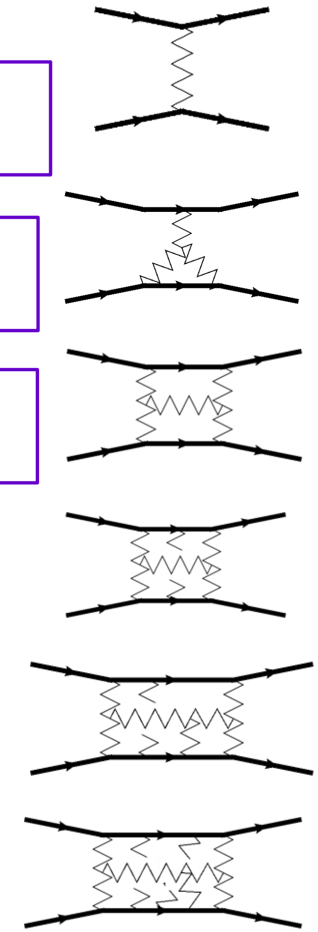
- Common parameter controls perturbative post-Newtonian approximation relevant for inspiral accuracy
- But in scattering one can compute separate orders in v^2 (or p^2) and G

$$H^{(0)}(r^2, p^2) \;=\; \sqrt{\boldsymbol{p}^2 + m_1^2} + \sqrt{\boldsymbol{p}^2 + m_2^2} + \frac{G}{r} c_1^{(0)}(\boldsymbol{p}^2) + \left(\frac{G}{r}\right)^2 c_2^{(0)}(\boldsymbol{p}^2) + \mathcal{O}(G^3)$$

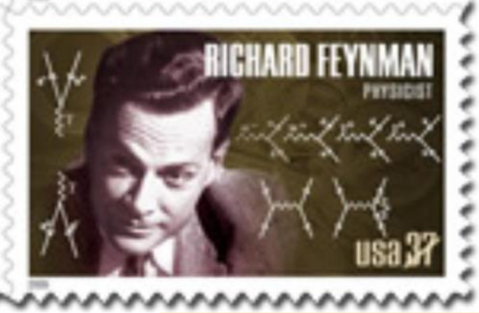
- Powers of G alone referred to as post-Minkowskian

Double expansion of spinless conservative Hamiltonian

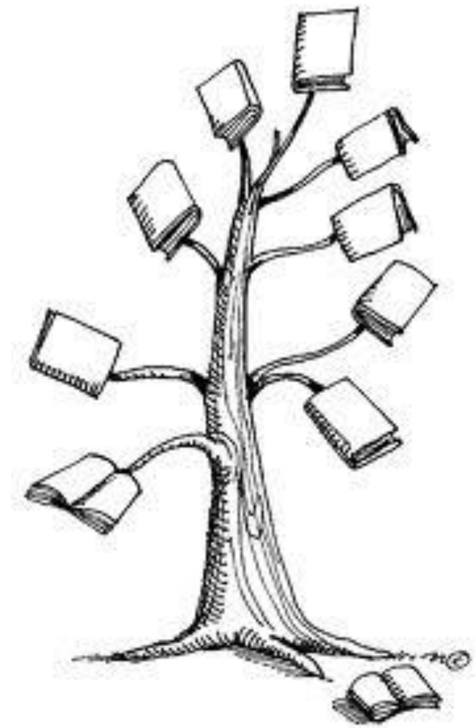
	0PN (1687)	1PN (1938)	2PN (1972 -1985)	3PN (2001)	4PN (2014)	5PN		
1PM (1956)	$(1$	$+ v^2$	$+ v^4$	$+ v^6$	$+ v^8$	$+ v^{10}$	$+ v^{12} + \cdots)$	G
2PM (1960)		$(1$	$+ v^2$	$+ v^4$	$+ v^6$	$+ v^8$	$+ v^{10} + \cdots)$	G^2
3PM (2019)			$(1$	$+ v^2$	$+ v^4$	$+ v^6$	$+ v^8 + \cdots)$	G^3
4PM				$(1$	$+ v^2$	$+ v^4$ (2019-2020)	$+ v^6 + \cdots)$	G^4
5PM					$(1$	$+ v^2$ (almost)	$+ v^4 + \cdots)$	G^5
6PM						$(1$ (2019)	$+ v^2 + \cdots)$	G^6



Many contributed to these advances, for PN notably T. Damour and collaborators



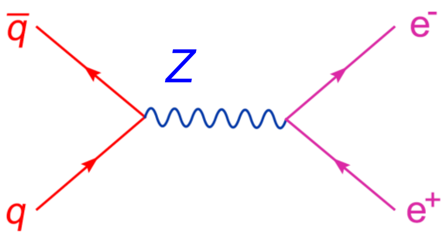
QCD at LHC:
LO = Trees



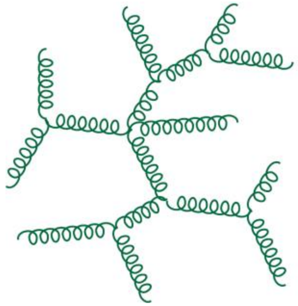
LO cross section uses only Feynman diagrams with no closed loops – tree diagrams.
Here's a very simple one:

$\hat{\sigma}^{(0)}$

LO



Although there are many kinds of trees, some harder than others, "textbook" methods often suffice



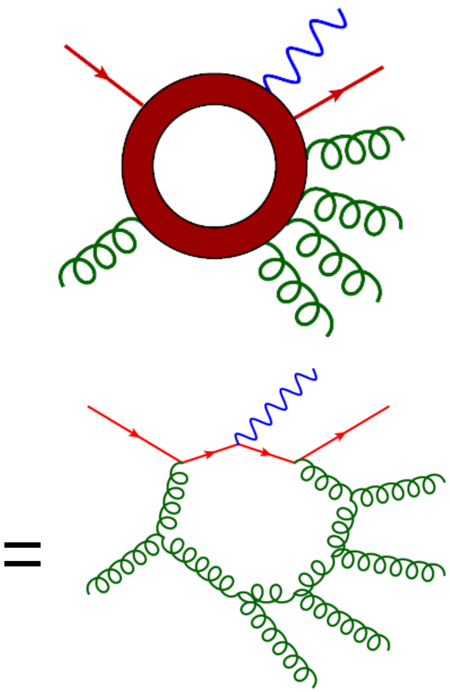
+ …

NLO needs 1 loop

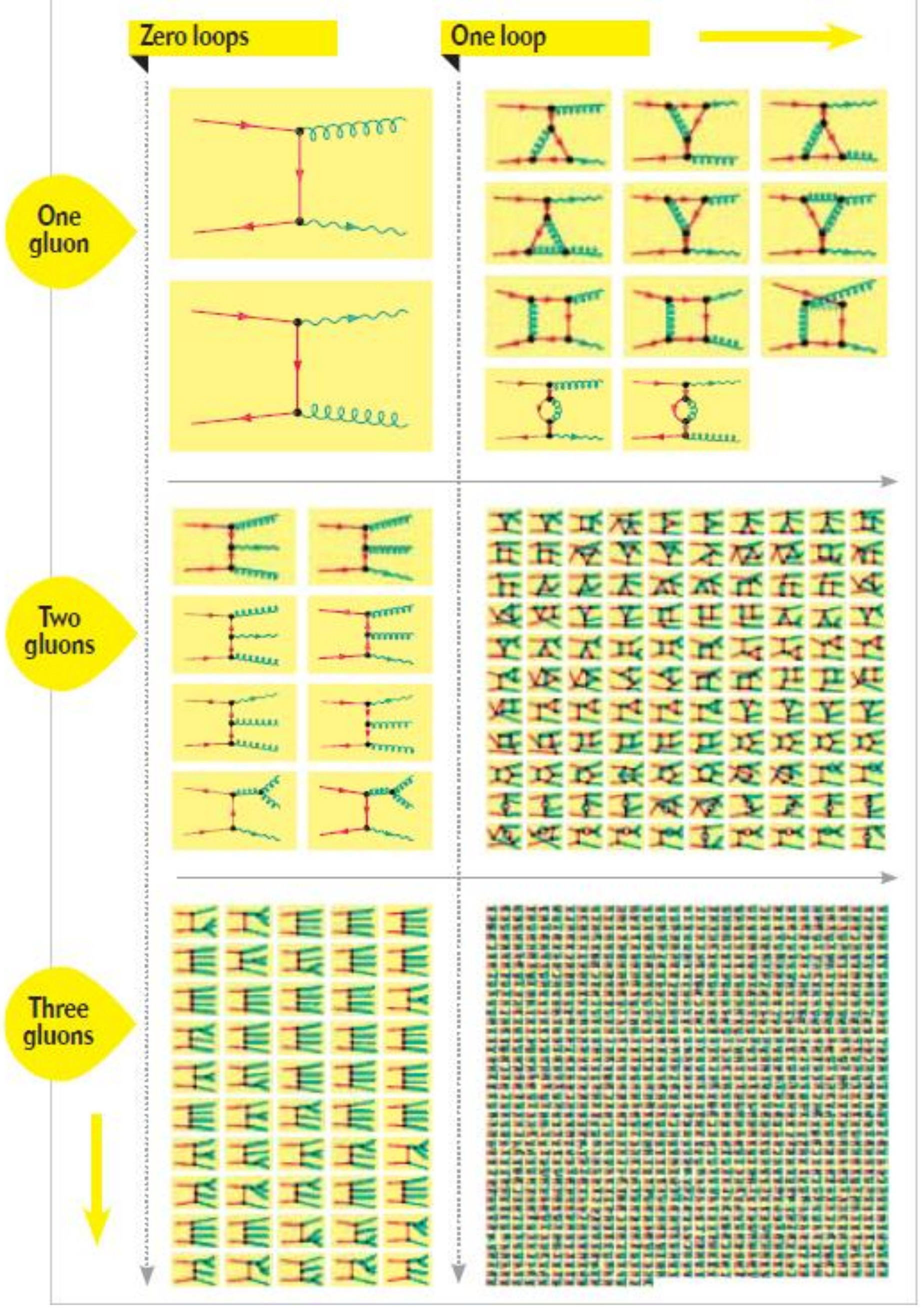
first quantum corrections

Challenging in QCD if many legs – depends on many variables

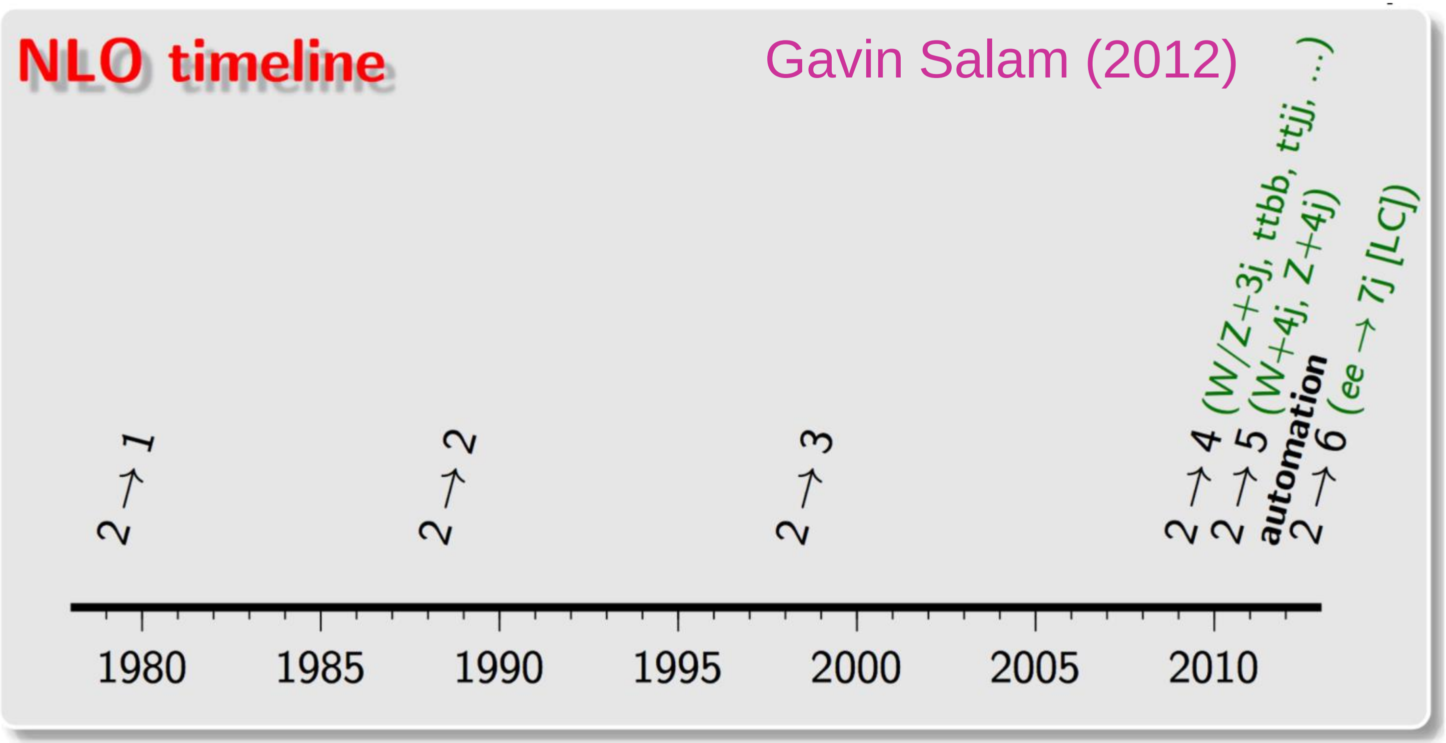
$q\bar{q} \rightarrow W + n$ gluons



= + 256,264 more

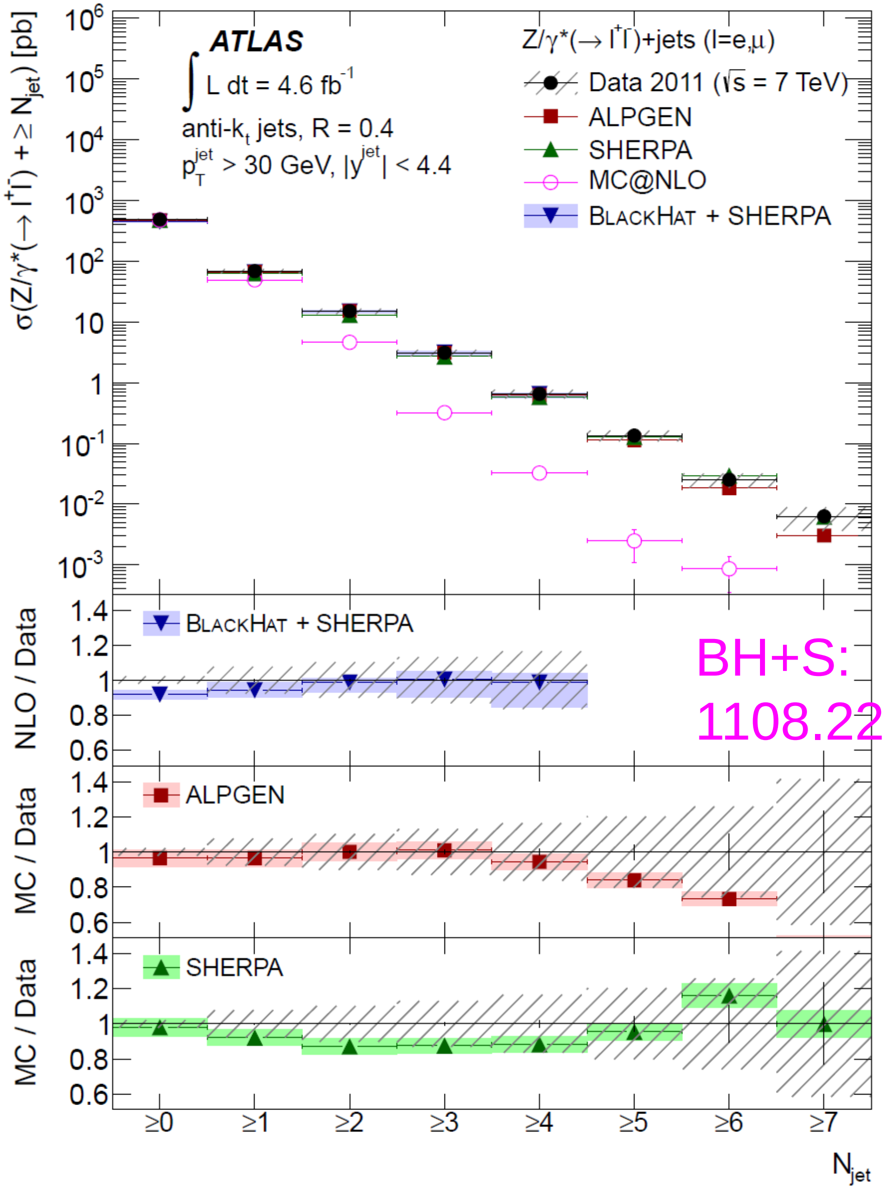


“NLO QCD revolution”



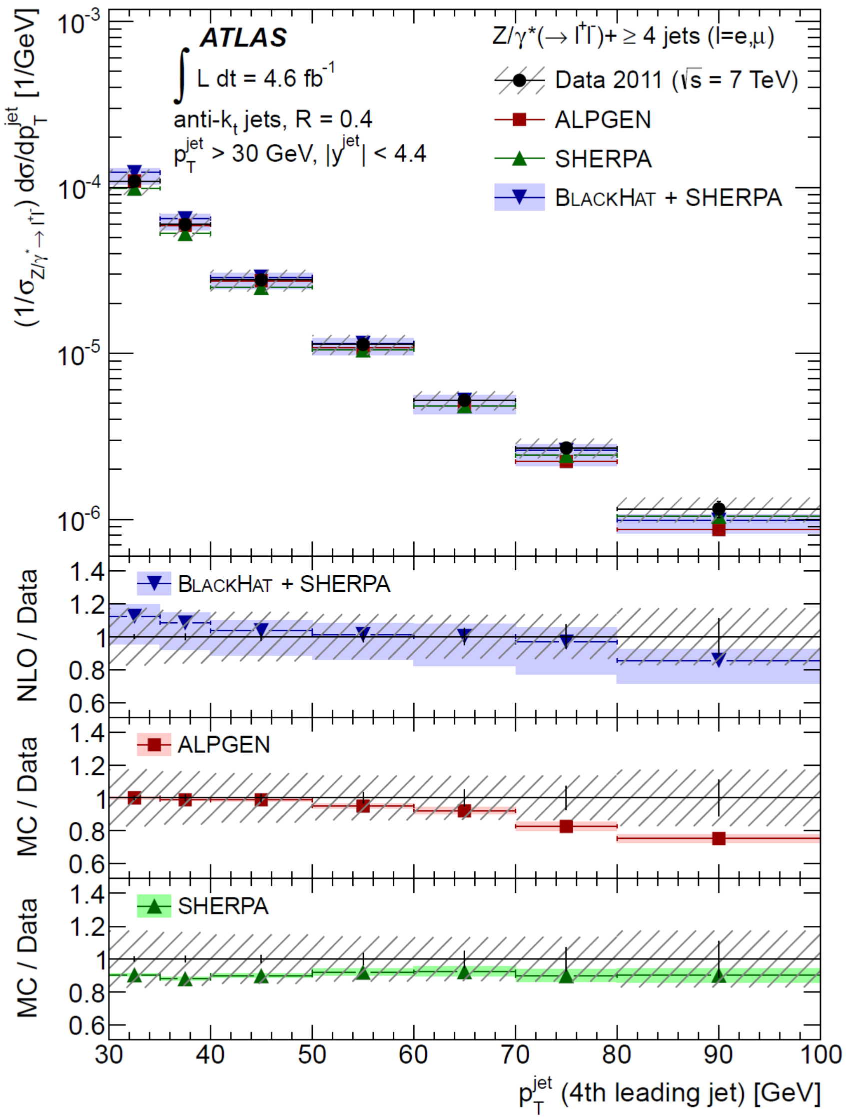
2010: NLO $W+4j$ [BlackHat+Sherpa: Berger et al] [unitarity]
2011: NLO $WWjj$ [Rocket: Melia et al] [unitarity]
2011: NLO $Z+4j$ [BlackHat+Sherpa: Ita et al] [unitarity]
2011: NLO $4j$ [BlackHat+Sherpa: Bern et al] [unitarity]
2011: first automation [MadNLO: Hirschi et al] [unitarity + feyn.diags]
2011: first automation [Helac NLO: Bevilacqua et al] [unitarity]
2011: first automation [GoSam: Cullen et al] [feyn.diags(+unitarity)]
2011: $e^+e^- \to 7j$ [Becker et al, leading colour] [numerical loops]

NLO $pp \to Z + 1,2,3,4$ jets vs. ATLAS 2011 data

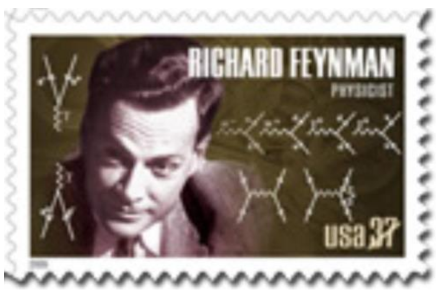


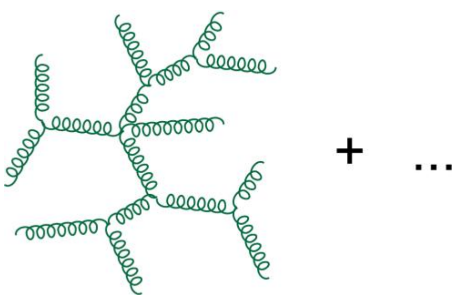
BH+S: 1108.2229

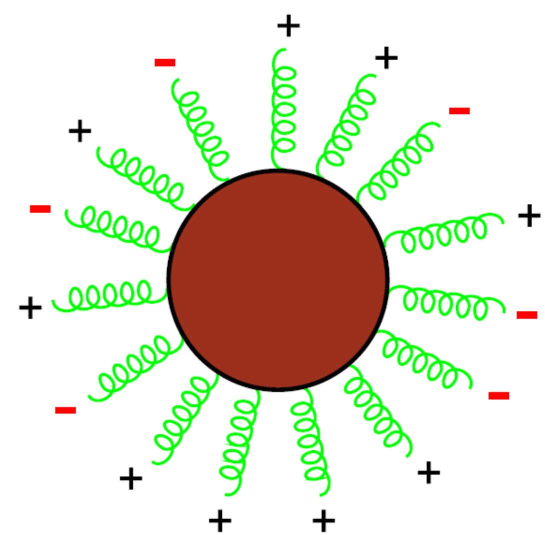
ATLAS 1304.7098



Revolution made possible by new perspective on particle scattering: “On-shell” Methods, or Granularity vs. Fluidity

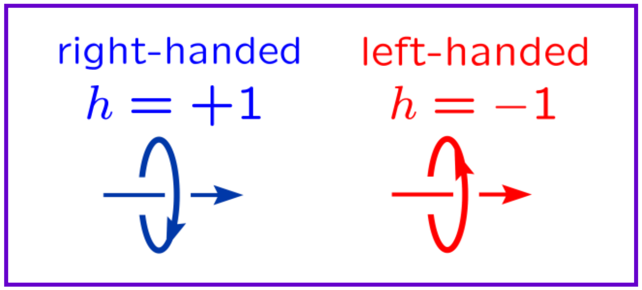


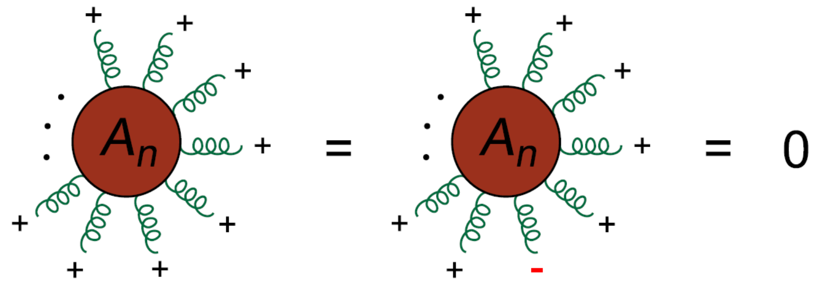




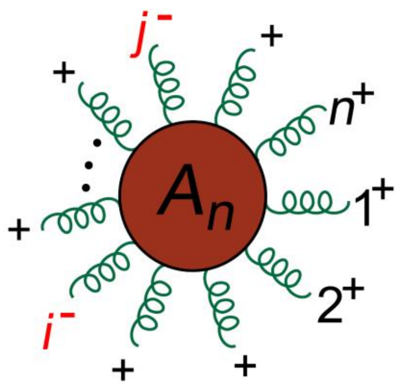
QCD Tree Amplitudes

Many tree-level helicity amplitudes vanish or are very simple.
Much simpler than individual Feynman diagrams!





Analyticity/unitarity makes it possible to recycle this simplicity into loop amplitudes



$$= \frac{\langle i\, j\rangle^4}{\langle 1\, 2\rangle\langle 2\, 3\rangle\cdots\langle n\, 1\rangle}$$

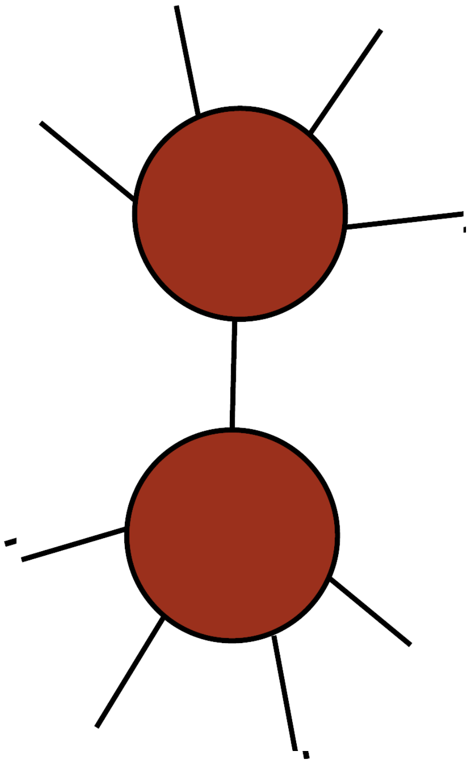
Parke-Taylor formula (1986)

Recycling “Fluid” Amplitudes

Amplitudes fall apart into simpler ones in special limits – pole information

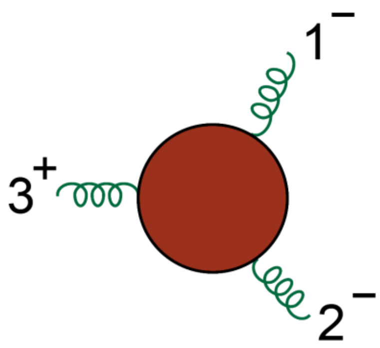
Picture leads directly to
BCFW (on-shell) recursion relations:
Reconstruct amplitude from poles in complex plane, where intermediate particle is on shell, and amplitude factorizes into product of 2 simpler amplitudes with fewer external legs

Britto, Cachazo, Feng, Witten, hep-th/0501052



Trees recycled into trees

All Gluon Tree Amplitudes Built From:



$$= \frac{\langle 1\,2\rangle^4}{\langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,1\rangle}$$

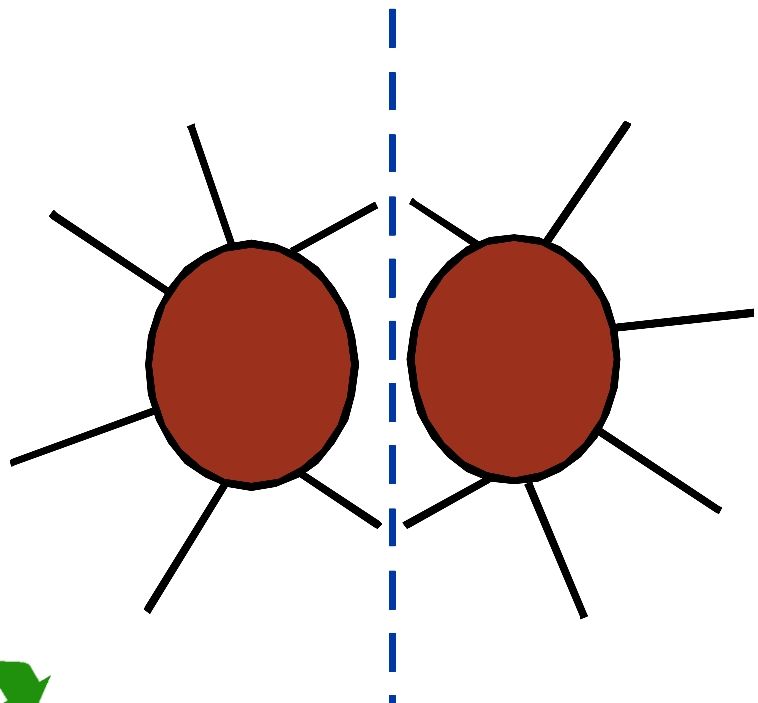
In contrast to Feynman vertices, it's on-shell, completely physical

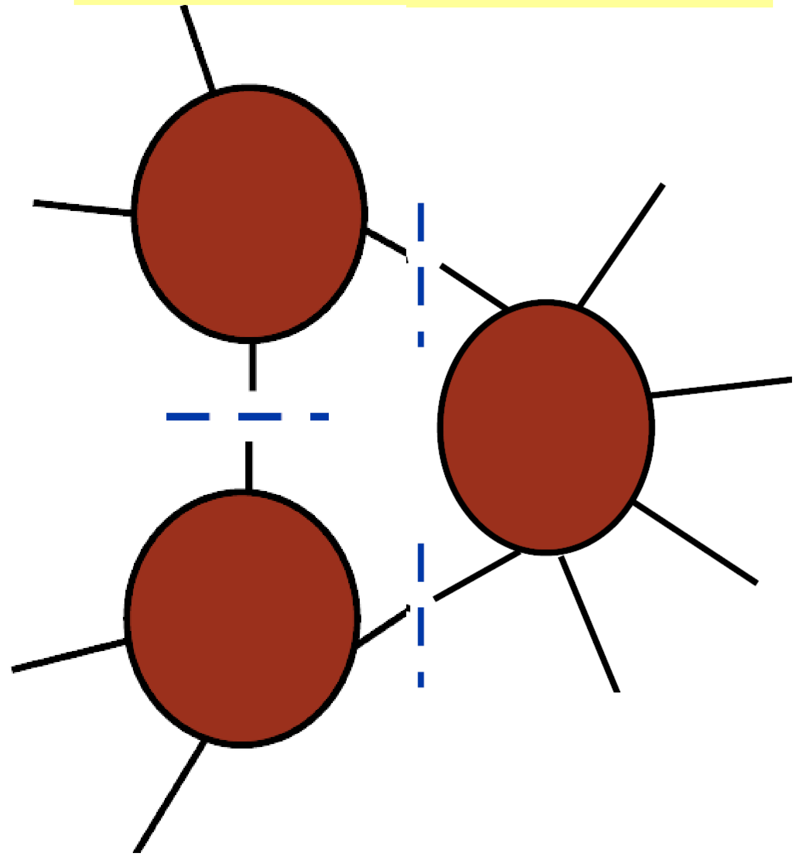
- On-shell recursion → very compact analytic formulae, fast numerical implementation.
- Can do same sort of thing at loop level.

Branch cut information → Generalized Unitarity (One-loop Fluidity)

Ordinary unitarity:
put 2 particles on shell,
with real momenta

Generalized unitarity:
put 3 or 4 particles on shell,
complex momenta



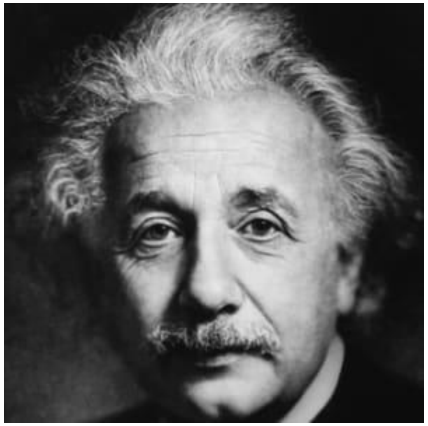


Trees recycled into loops!

What does all this have to do with black holes?

Orbiting black holes obey General Relativity:

$$S_{EH} = \int d^4x \sqrt{g}\ R$$



Gluons scatter according to

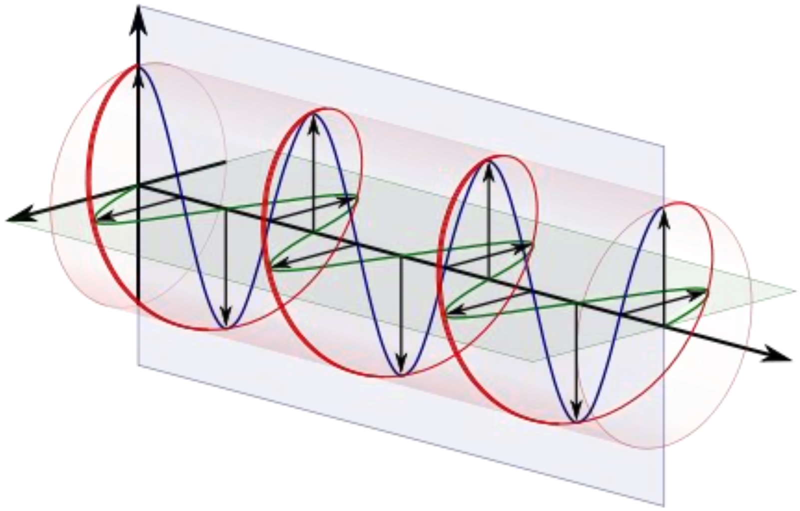
Yang-Mills theory

$$S_{YM} = \int d^4x\ F_{\mu\nu}^a F^{\mu\nu,a}$$

Both theories contain massless particles

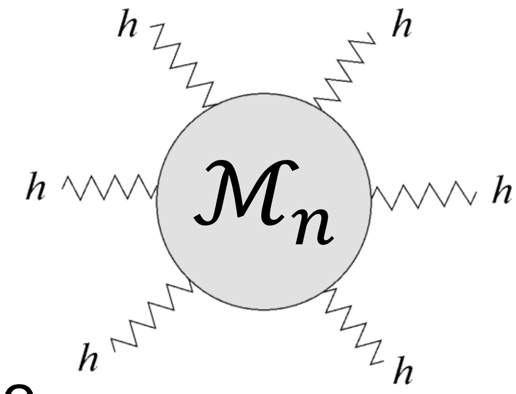
- Graviton is a traceless symmetric tensor
  → helicity ± 2

- Gluon helicity ± 1
  like photon



Gedanken calculation

- **Graviton scattering amplitudes $\mathcal{M}_n$**
- Could compute Feynman rules from

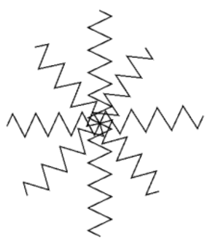


$$S_{EH} = -\frac{2}{\kappa^2}\int d^4x \sqrt{g}\; R(g_{\mu\nu})$$

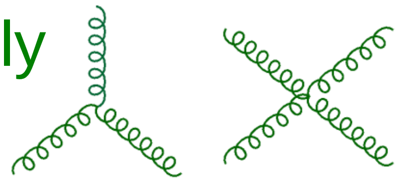
by expanding metric around Minkowski space,

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}, \quad \kappa = \sqrt{32\pi G_N}$$

- But action contains inverse metric, leading to incredibly complicated Feynman vertices of arbitrary multiplicity



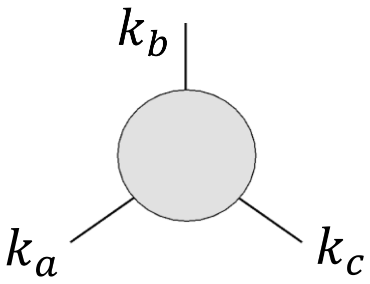
- While Yang-Mills → 3- and 4-gluon vertices only



- Feynman rules not the way to go

“On-shell” methods

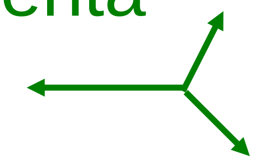
- Ignore Lagrangians and Feynman rules, construct on-shell scattering amplitudes using causality and analyticity



- Basic on-shell amplitude, exposing

theory’s nonlinearity, is 3-point amplitude

- For all massless particles, complex momenta needed to realize it nonsingularly:



$$s_{ab} \equiv (k_a + k_b)^2 = k_c^2 = 0$$

→ All 3 vectors parallel, if $k_{a,b,c}$ all real

3-point amplitudes (cont.)

- Complex 3-point kinematics: no free parameters, like scattering angles
- 3-point amplitudes **completely dictated** by

1. External helicities
2. Overall energy scaling (dimension of coupling)

- For YM, g_s dimensionless and helicity ± 1, find:

$$\mathcal{A}_3^{\mathrm{YM}}(1^+,2^+,3^+) = \mathcal{A}_3^{\mathrm{YM}}(1^-,2^-,3^-) = 0$$

$$\mathcal{A}_3^{\mathrm{YM}}(1^-,2^-,3^+) = g_s \frac{<12>^3}{<23><31>} f^{a_1 a_2 a_3}$$

color factor

- $<ab>$ are inner products of Weyl spinors, would be $\sqrt{s_{ab}}$ if momenta were real

3-point graviton amplitude

- For gravity, $\kappa \propto \frac{1}{M_{\text{Planck}}}$ has dimension -1 and graviton helicity ± 2, together dictate:

$$\mathcal{M}_3^{\text{grav}}(1^{++}, 2^{++}, 3^{++}) = \mathcal{M}_3^{\text{grav}}(1^{--}, 2^{--}, 3^{--}) = 0$$

$$\mathcal{M}_3^{\text{grav}}(1^{--}, 2^{--}, 3^{++}) = \frac{\kappa}{2}\left[\frac{<12>^3}{<23><31>}\right]^2 \propto \left[\mathcal{A}_3^{\text{YM}}\right]^2$$

Note: $< ab > \propto energy$. 1 more $< ab >$ compensates κ

- In summary: 1+1=2, Lorentz symmetry, and dimension of 3-point couplings

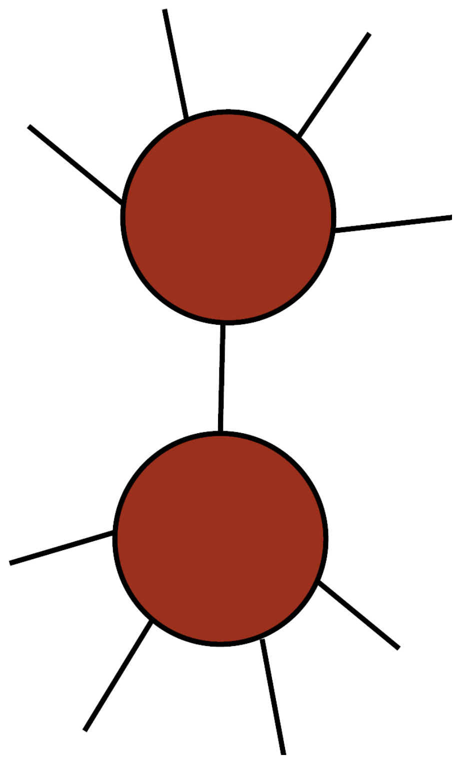
→ **Gravity = YM²**

Beyond 3-point tree

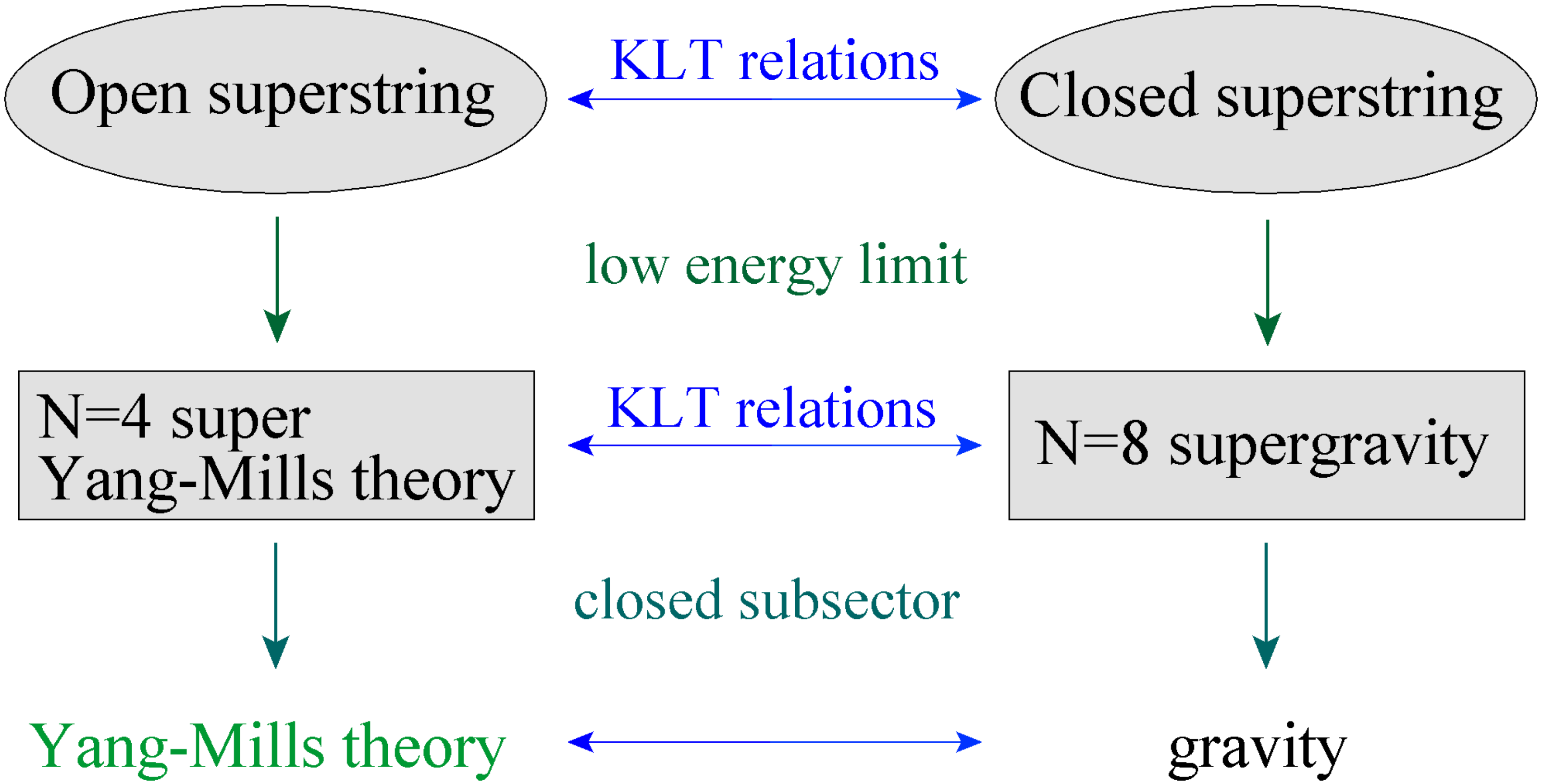
- BCFW recursion relations work for gravity as well as gauge theory.
- Uniquely determine n-point tree amplitude $\mathcal{M}_n$, given $\mathcal{M}_3$. (Higher-point Feynman vertices all linked to 3-point vertex by gauge symmetry.)
- Suggests that in general

$$\mathcal{M}_n \sim A_n \otimes A'_n$$

- Relations actually known much earlier from string theory (KLT)



Other theories lurking in background

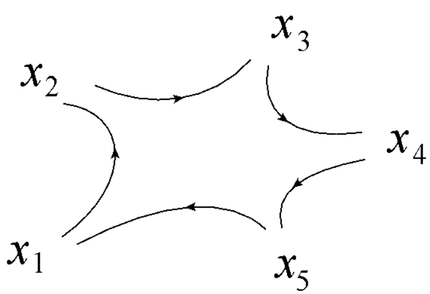


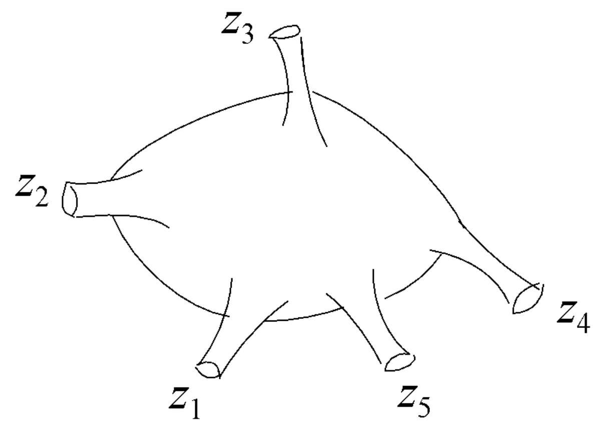
KLT relations

Kawai, Lewellen, Tye (1985)

1-dimensional string sweeps out a 2-dimensional world-sheet

open → with boundary (disk) closed → no boundary (sphere)





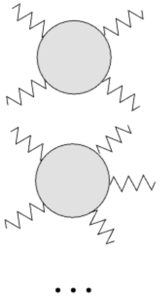
$$A_n^{\text{open}} \sim \int dx_a\, f(x_b,k_b) \qquad \mathcal{M}_n^{closed} \sim \iint dz_a d\bar{z}_a\, |f(z_b,k_b)|^2$$

deform integral contours, take low energy limit, ignore couplings and color factors

$$M_4^{\text{tree}}(1,2,3,4) = -is_{12} A_4^{\text{tree}}(1,2,3,4)\, A_4^{\text{tree}}(1,2,4,3)\,,$$

$$M_5^{\text{tree}}(1,2,3,4,5) = is_{12}s_{34} A_5^{\text{tree}}(1,2,3,4,5) A_5^{\text{tree}}(2,1,4,3,5)$$
$$+ is_{13}s_{24} A_5^{\text{tree}}(1,3,2,4,5)\, A_5^{\text{tree}}(3,1,4,2,5)\,,$$

…

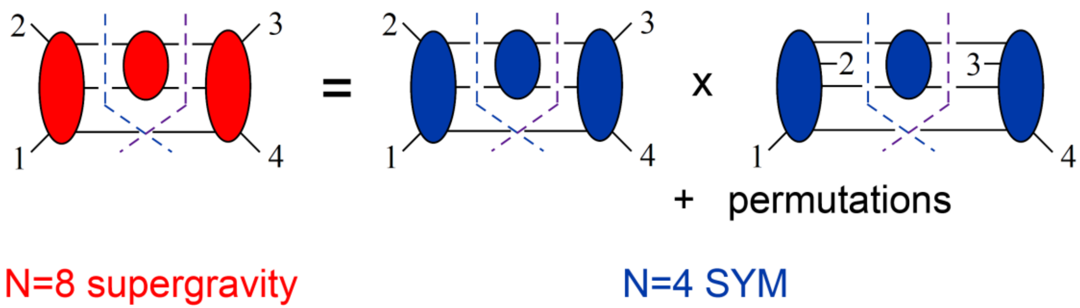


KLT relations (cont.)

- Machine for building any **graviton** tree amplitude from simpler gauge theory ones.
- Can also get massless external scalars, by dimensional reduction or supersymmetry, make them massive by giving them momentum in a 5th dimension (Kaluza-Klein):
- $k^\mu = (E, \vec{k}, k_5)$
- $k^2 = 0 = E^2 - \vec{k}^2 - k_5^2 \quad \rightarrow \quad mass = k_5$
- Massive scalar ~ spinless black hole!

Quantum supergravity

- Unitarity → construct loop amplitudes from tree amplitudes
- KLT relations → (super)gravity trees from (super)YM trees
- Three-loop example:



→ Determine at what loop (if ever?) various supergravities have ultraviolet divergences

→ For example, find that N=8 SUGRA is finite through 5 loops!

Z. Bern et al., 9802162, 0702112, 0808.4112, 0905.2326, 1201.5366, 1708.06807, 1804.09311

- These calculations would be impossible using Feynman diagrams

Another approach – the double copy

Bern, Carrasco, Johansson (2008,2010)

- Can always write a gauge theory amplitude in terms of cubic graphs:

$$\mathcal{A}_m^{(L)} = i^L g^{m-2+2L} \sum_{i\in\Gamma} \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_i} \frac{n_i C_i}{\prod_{\alpha_i} p_{\alpha_i}^2}$$

kinematic factor

color factor

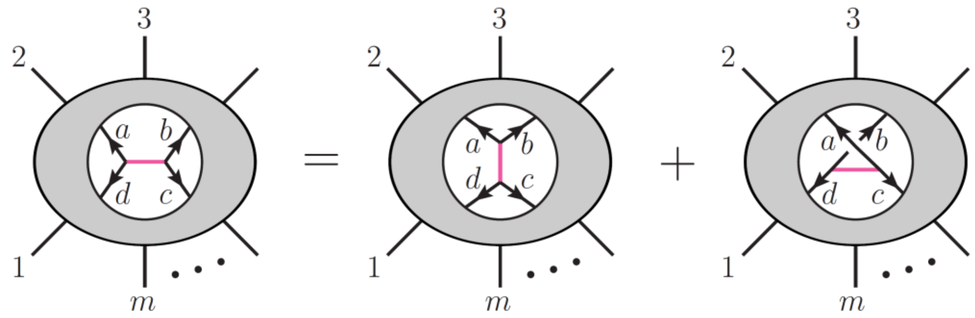
symmetry factor

scalar propagator for each edge in Γ

n_i are not unique, can be shifted around. Remarkably, can (usually) be chosen to obey a set of 3-term identities

Double copy (cont.)

- Color factors C_i for these triplets of graphs,



obey a Jacobi identity ($f^{\cdots} f^{\cdots} = f^{\cdots} f^{\cdots} + f^{\cdots} f^{\cdots}$)

$$C_i = C_j + C_k$$

Also require $$n_i = n_j + n_k$$

for every such triplet.

Double copy (cont.)

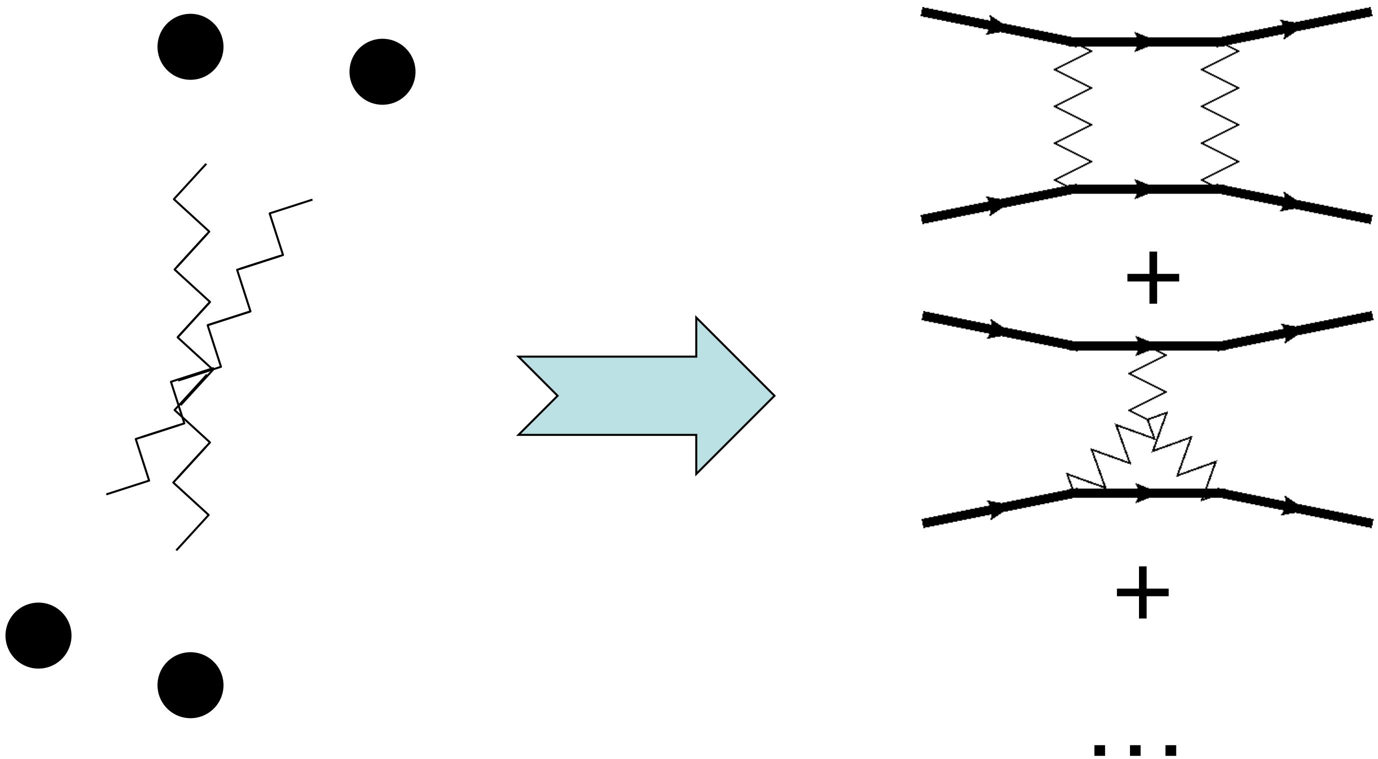
- Then the gravity amplitude is the literal square of the YM amplitude:

$$\mathcal{M}_m^{(L)} = \left(\frac{\kappa}{2}\right)^{m-2+2L} \sum_{i\in\Gamma} \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_i} \frac{(n_i)^2}{\prod_{\alpha_i} p_{\alpha_i}^2}$$

- Machine for building graviton loop amplitudes from simpler gauge theory ones

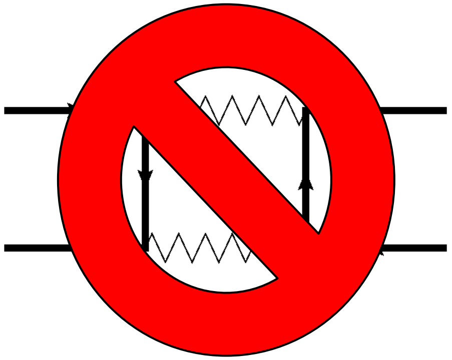
Loops contain classical pieces

- Especially if particles move slowly, lots of time for multiple exchanges of virtual gravitons, to build up smooth classical trajectory.



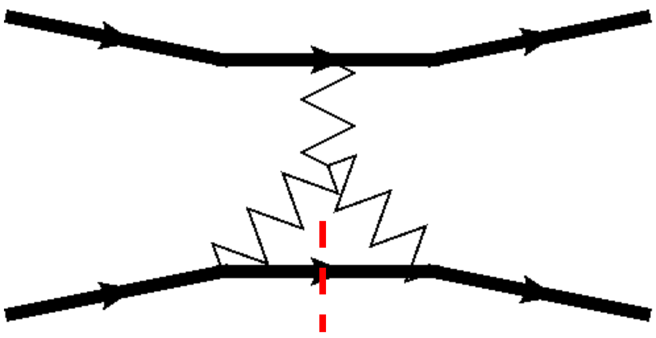
But not all loop diagrams are classical

- E.g. black holes don't annihilate in this regime



- graviton vacuum is pure quantum

- Black holes never get very far off shell



Classical restrictions compatible with on-shell methods

3PM computation

Bern, Cheung, Roiban, Shen, Solon, 1901.04424, 1908.01493

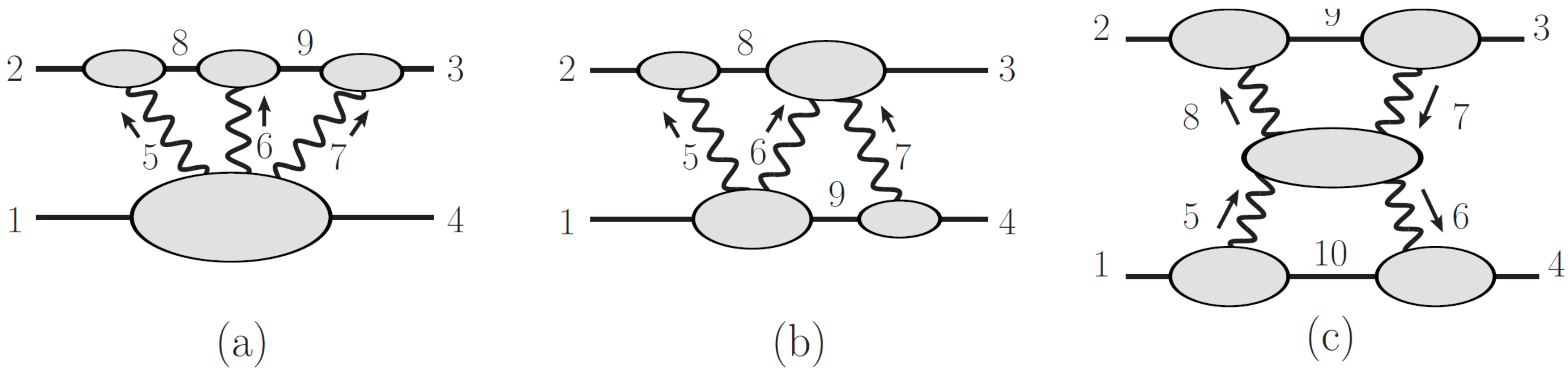
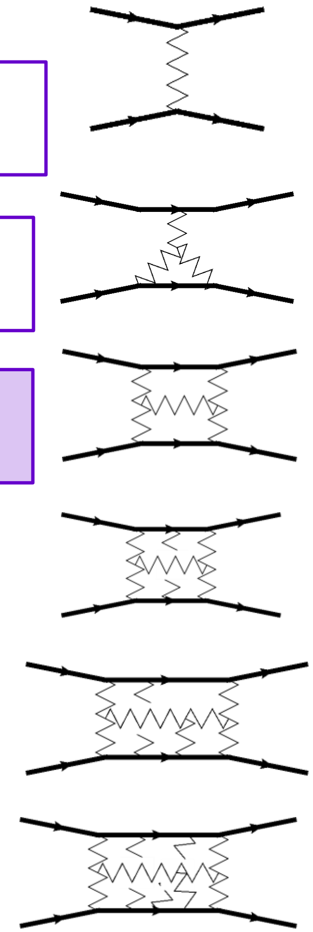


Figure 12: The independent generalized cuts needed at two loops for the classical potential. The remaining contributing cuts are given by simple relabeling of external legs. Here the straight lines represent on-shell scalars and the wiggly lines correspond to on-shell gravitons or gluons.

One of first contributions of “amplitudes” to LIGO physics

	0PN (1687)	1PN (1938)	2PN (1972-1985)	3PN (2001)	4PN (2014)	5PN			
1PM 1956	$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+v^{12}$	$+\cdots)$	G
2PM 1960		$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+\cdots)$	G^2
3PM 2019			$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+\cdots)$	G^3
4PM				$(1$	$+v^2$	$+v^4$ (2019-2020)	$+v^6$	$+\cdots)$	G^4
5PM					$(1$	$+v^2$ (almost)	$+v^4$	$+\cdots)$	G^5
6PM						$(1$ (2019)	$+v^2$	$+\cdots)$	G^6



Many contributed to these advances, for PN notably T. Damour and collaborators

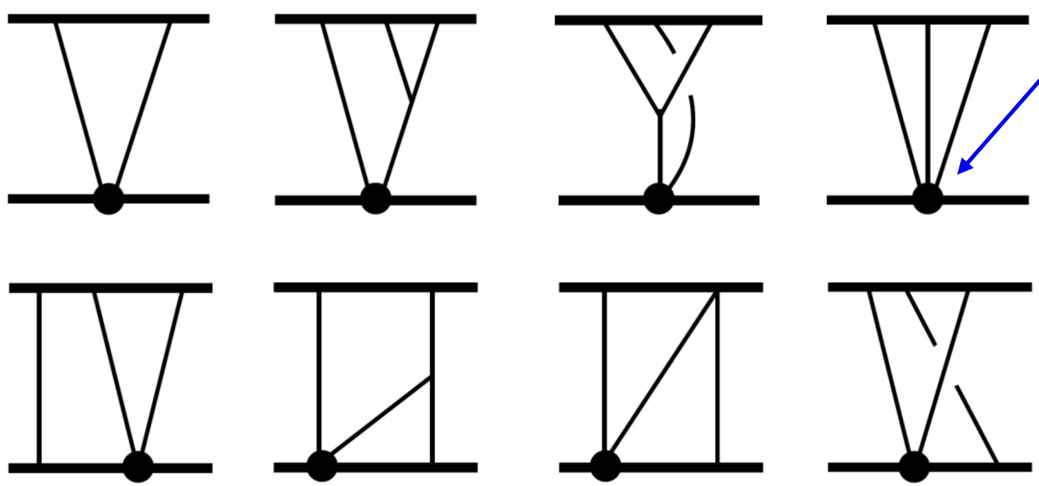
Spin and tidal effects also computable within similar framework

Bern, Luna, Roiban, Shen, Zeng, 2005.03071

Spinning Black Hole Binary Dynamics, Scattering Amplitudes and Effective Field Theory

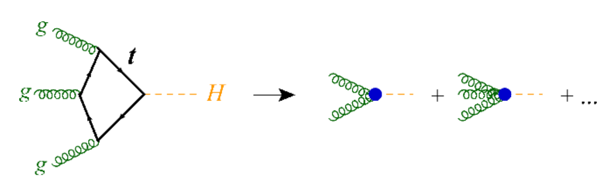
Cheung, Solon, 2006.06665

Tidal Effects in the Post-Minkowskian Expansion



operator(s) encoding multipole moments of neutron star

analogous to $HF_{\mu\nu}F^{\mu\nu} + \cdots$ operator(s) encoding couplings of gluons to Higgs boson at LHC



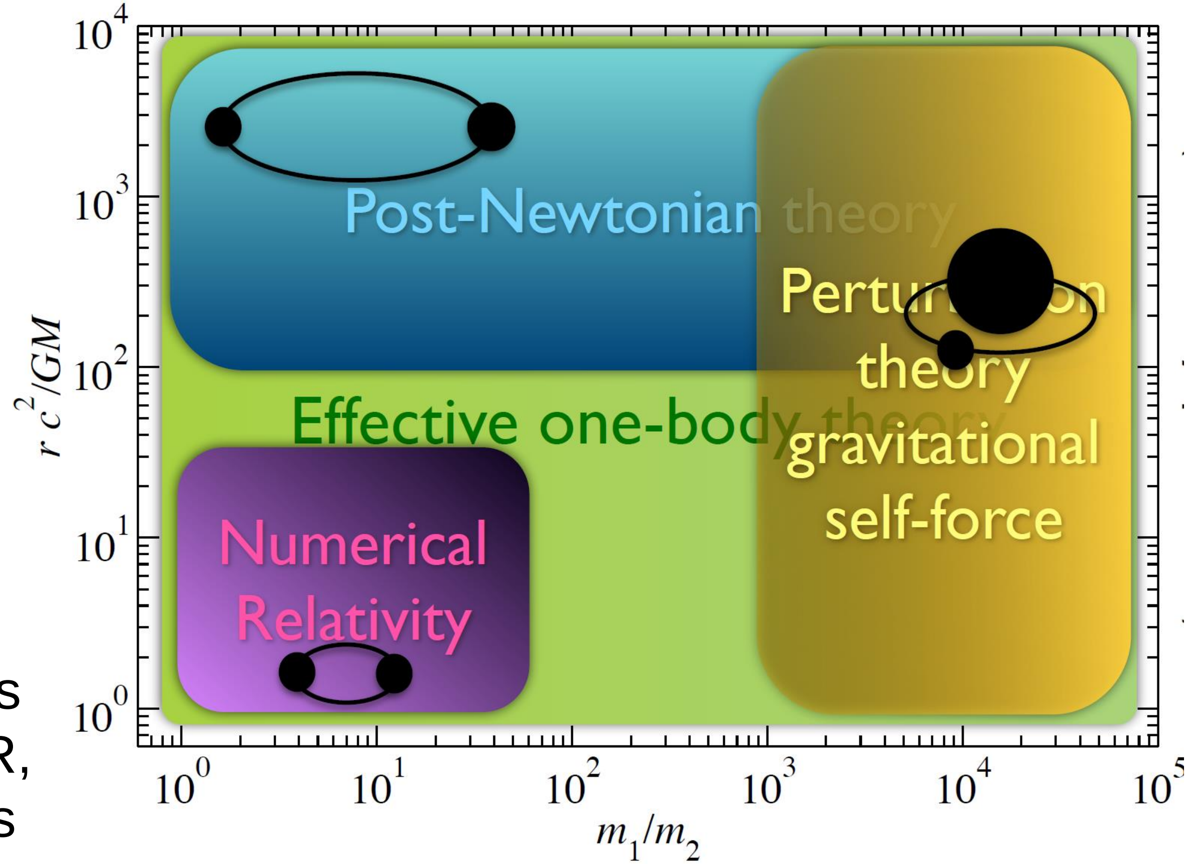
Effective one-body approach

Inspired by properties of bound states in QFT.

Interpolates information from various sources, including PN and PM expansions, test particle limit $m_1 \ll m_2$, and numerical relativity results.

Provides accurate gravitational wave templates very close in, faster than NR, allowing many combinations of initial masses and spins.

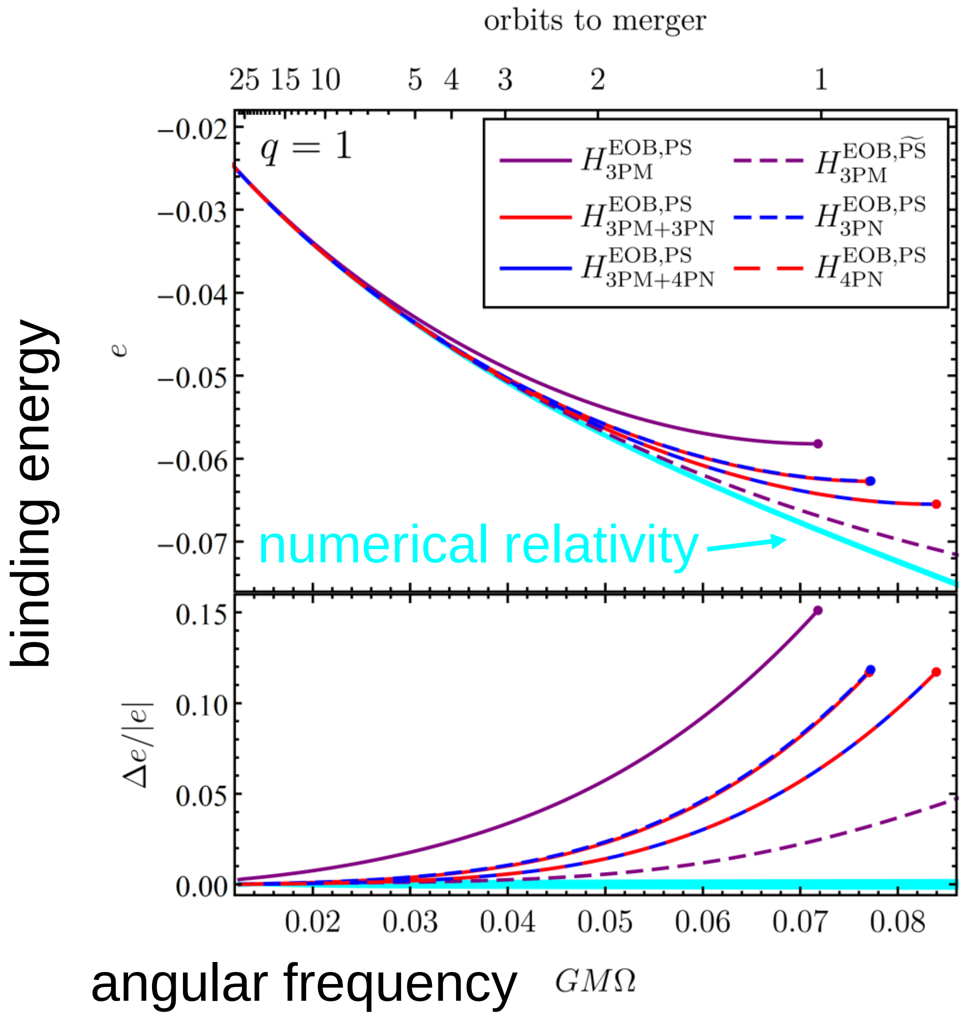
Buonanno, Damour, 9811091



Buonanno, Satyaprakash, 1410.7832

EOB matched to different PN/PM orders

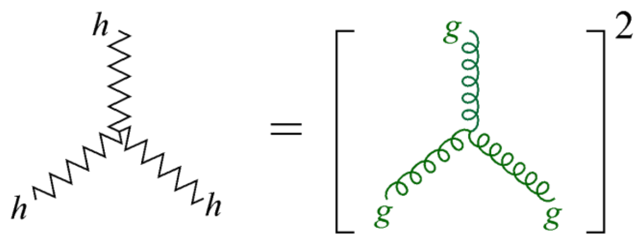
Antonelli, Buonanno, Steinhoff, van de Meent, Vines, 1901.07102



Adding more PN and PM orders improves EOB performance, but it also depends strongly on matching scheme, PS vs. $\widetilde{\rm PS}$

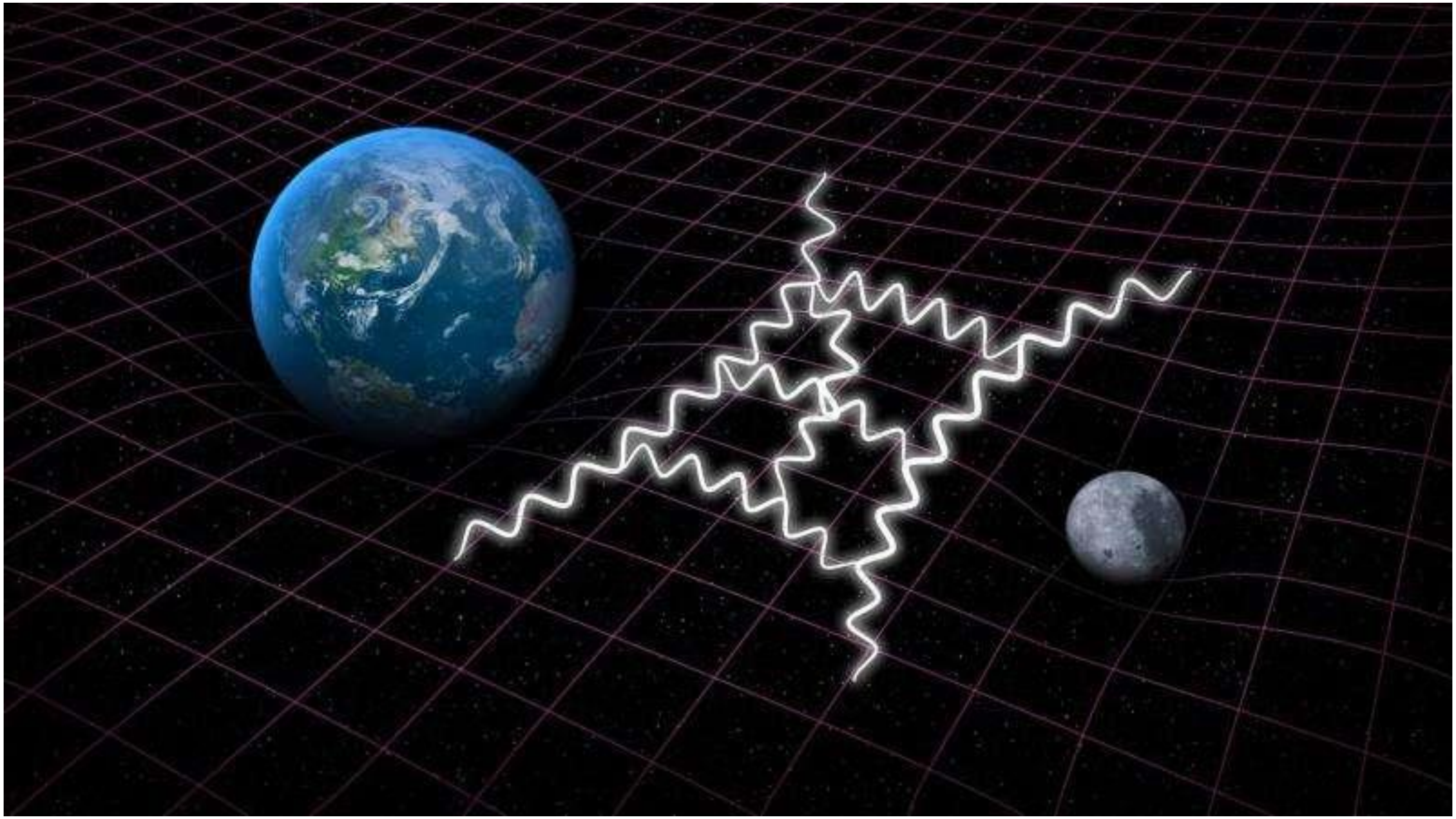
Conclusions

- At the heart of the LHC is the "square root" of the fundamental nonlinearity of General Relativity



- One can combine this information with on-shell methods, originally developed to improve QCD predictions for LHC, and study supergravities to high quantum loop order
- Similar methods just now beginning to bear fruit for the classical problem of binary inspiral, relevant for LIGO, VIRGO, and future gravitational wave detectors!
- Bright future ahead for higher precision waveforms, thanks to modern understanding of particle scattering in gauge theory and gravity.

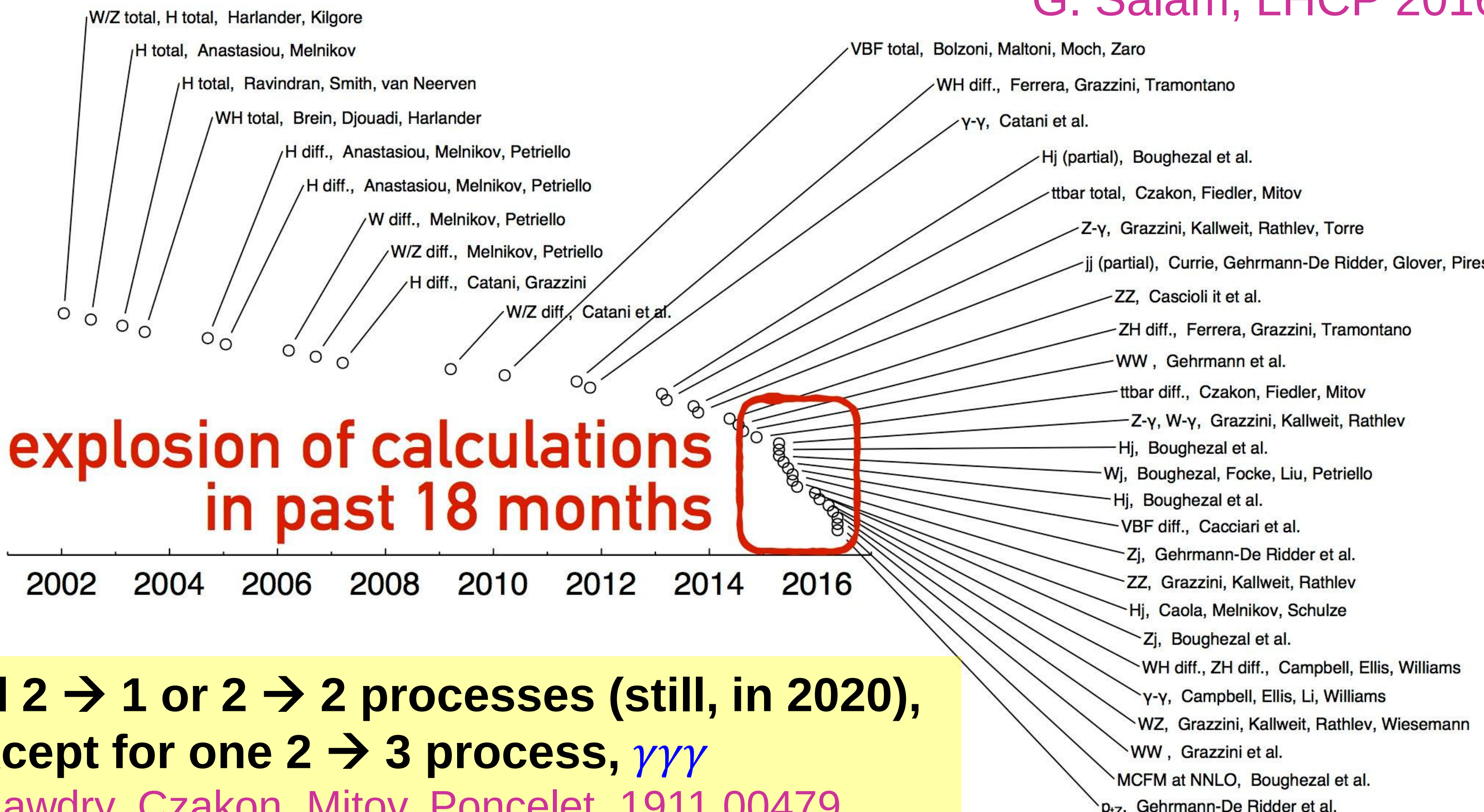
Thanks for your attention!



Extra slides

NNLO QCD revolution still in progress

G. Salam, LHCP 2016

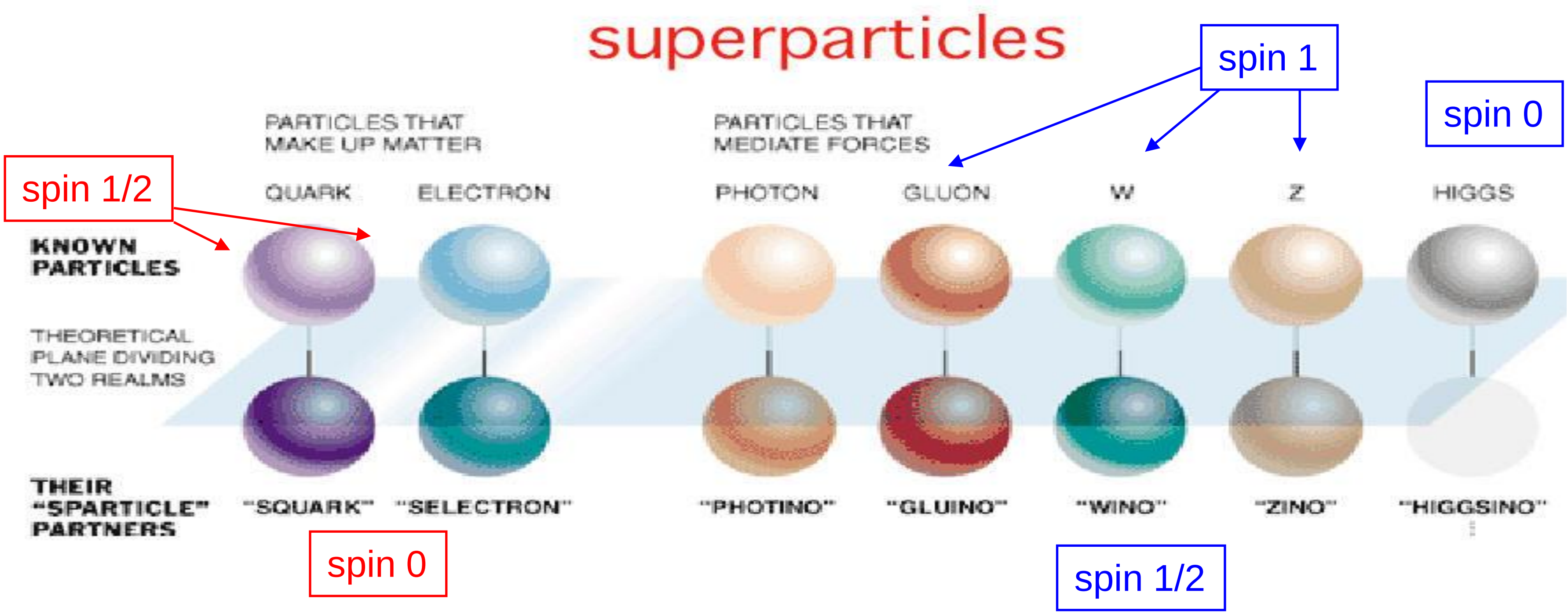


All 2 → 1 or 2 → 2 processes (still, in 2020), except for one 2 → 3 process, $\gamma\gamma\gamma$

Chawdry, Czakon, Mitov, Poncelet, 1911.00479

New Physics Example: Supersymmetry

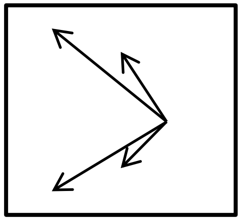
- Symmetry between fermions (matter) and bosons (forces)
- Very elegant, solves hierarchy problem
- Lightest supersymmetric particle (LSP) can be **dark matter**
- **Cornucopia** of new elementary particles at LHC.

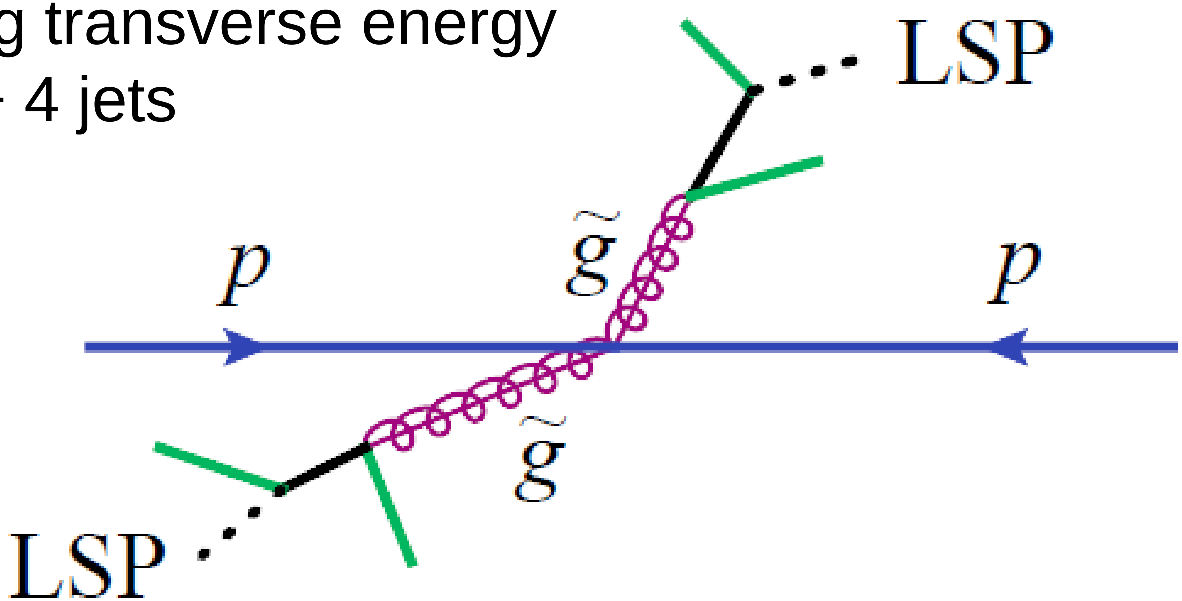


Classic SUSY dark matter signature

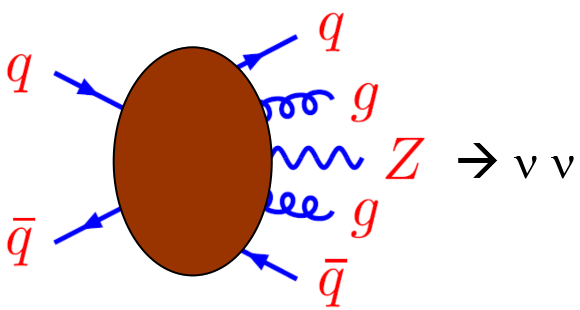
Heavy colored particles decay rapidly to stable Weakly Interacting Massive Particle (WIMP = LSP) plus jets

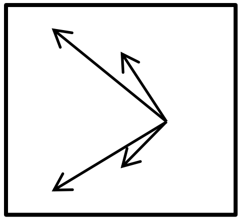
→ Missing transverse energy
MET + 4 jets





Not background free: happens in Standard Model too





MET + 4 jets from pp → Z + 4 jets,
Z → neutrinos

Neutrinos escape detector.

Irreducible background.

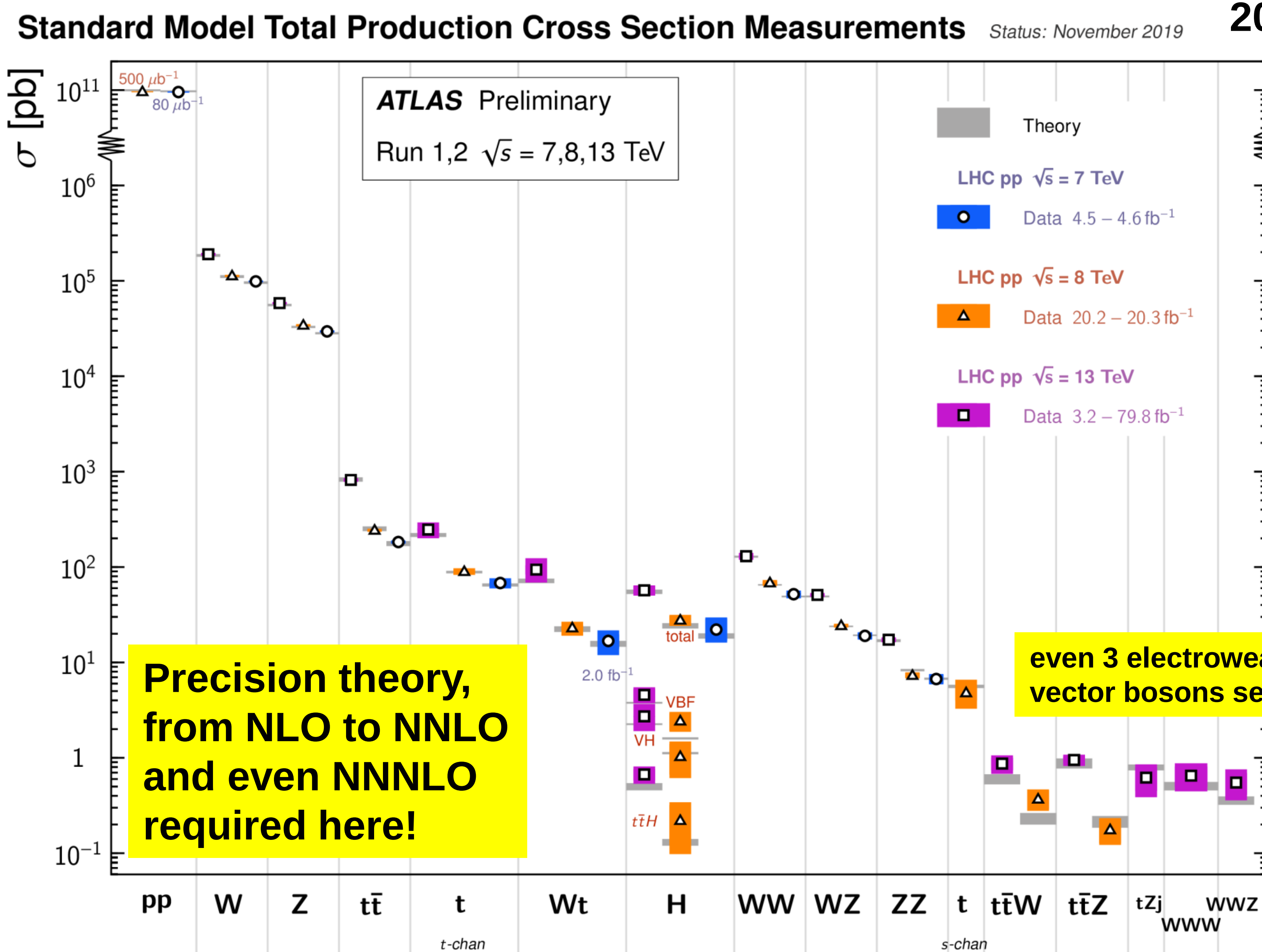
Plus there are many reducible backgrounds from W + jets, tt + jets, ...

Precision theory (typically NLO) can help with this, usually when embedded in parton shower Monte Carlos

From searches to measurements

- No convincing evidence for SUSY, or any other direct production of new particles.
- Also look for deviations in rates for Standard Model processes, especially involving the brand-new Higgs boson.
- Measurements are hard, take a while to perform.
- More precise theory typically needed.

Standard Model Total Production Cross Section Measurements



2019

Precision theory, from NLO to NNLO and even NNNLO required here!

even 3 electroweak vector bosons seen

Many Automated Programs for One-Loop QCD

Blackhat: Berger, Bern, LD, Diana, Febres Cordero, Forde, Gleisberg, Höche, Ita, Kosower, Maître, Ozeren, 0803.4180, 0808.0941, 0907.1984, 1004.1659, 1009.2338…
+ **Sherpa** → NLO W,Z + 3,4,5 jets pure QCD 4 jets

CutTools: Ossola, Papadopolous, Pittau, 0711.3596
NLO WWW, WWZ, ... Binoth+OPP, 0804.0350
NLO $t\bar{t}b\bar{b}$, $t\bar{t}$ + 2 jets,…
Bevilacqua, Czakon, Papadopoulos, Pittau, Worek, 0907.4723; 1002.4009
MadLoop: Hirschi, Frederix, Frixione, Garzelli, Maltoni, Pittau 1103.0621
HELAC-NLO: Bevilacqua et al, 1110.1499

Rocket: Giele, Zanderighi, 0805.2152
Ellis, Giele, Kunszt, Melnikov, Zanderighi, 0810.2762
NLO W + 3 jets Ellis, Melnikov, Zanderighi, 0901.4101, 0906.1445
$W^+W^\pm$ + 2 jets Melia, Melnikov, Rontsch, Zanderighi, 1007.5313, 1104.2327

SAMURAI → GoSAM: Mastrolia, Ossola, Reiter, Tramontano, 1006.0710,…

NGluon: Badger, Biedermann, Uwer, 1011.2900,…

OpenLoops: Cascioli, Maierhofer, Pozzorini, 1111.5206,...

On to two loops

- State-of-art currently stuck at 2 → 2

– with a couple of 2 → 3 exceptions

- Why? In part because 2 loop multiscale integrals are typically very hard
- All 1 loop integrals with external legs in D=4 are reducible to scalar box integrals + simpler

→ combinations of

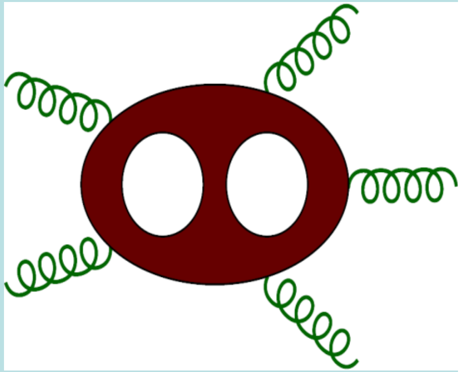
$$\mathrm{Li}_2(x) = -\int_0^x \frac{dt}{t} \ln(1-t)$$

+ simpler

Brown-Feynman (1952), Melrose (1965), Passarino-Veltman (1979), van Neerven-Vermaseren (1984), Bern, LD, Kosower (1992)

First 2 loop 2 → 3 amplitudes

- All massless partons (or photons) in large N_c (planar) limit for QCD gauge group SU(3) → SU(N_c):



$$gg \to ggg,\ qg \to qgg,\ q\bar{q} \to q\bar{q}g, \ldots$$

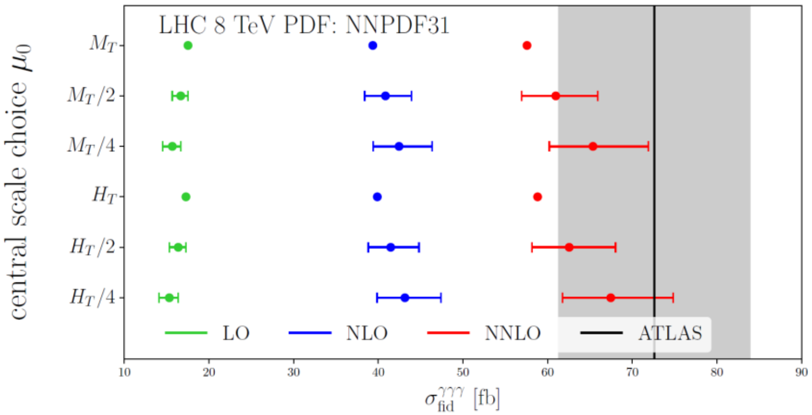
Gehrmann, Henn, Lo Presti, 1511.05409;
Badger, Brønnum-Hansen, Hartanto, Peraro, 1712.02229, 1811.11699;
Abreu, Dormans, Febres Cordero, Ita, Page, Zeng, Sotnikov, 1712.03946, 1812.04586, 1904.00945

- More work needed to compute NNLO cross section for pp → 3 jets

- And $q\bar{q} \to \gamma\gamma\gamma$

– already with NNLO cross section for pp → $\gamma\gamma\gamma$!

Chawdhry, Czakon, Mitov, Poncelet, 1911.00479



The “QCD for LHC” revolution

- Many important hadron collider processes have been computed at NLO and NNLO in the past decade (even 2 → 1 at NNNLO), well beyond what was previously thought possible
- Required a new understanding of scattering amplitudes, at a formal level, as well as efficient, stable implementation
- Many people contributed to this progress
- Parallel progress in understanding supersymmetric gauge & gravity theories
- Revolution far from over; e.g. NNLO 2 → 3+ awaits!