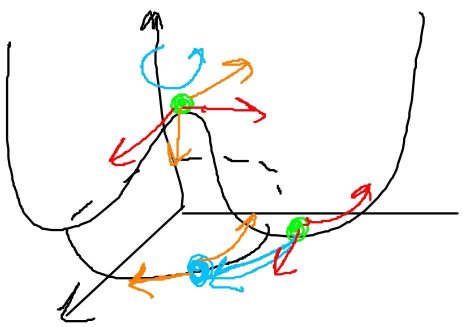
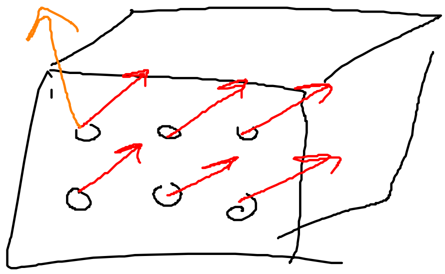
SSB of global sym.

- Broken or hidden?
- GBs
- CCWZ
- Discrete / spacetime syms

- Charges, currents..
- GBs
- Pion physics

• <u>B or H?</u>





UBS : linearly realized

SBS : non- ________

• <u>Goldstone bosons</u>

$$V(\phi_i) \qquad\qquad \delta\phi_i = \alpha^a T^{a\,j}_{i}\,\phi_j$$

$$V(\phi_i) = V(\phi_i') \simeq V(\phi_i) + \alpha^a \underbrace{T^{a\,j}_{i}\,\phi_j \frac{\partial V}{\partial \phi_i}}_{=0}$$

$$T^{a}_{i}{}^{j}\left(\underbrace{\cancel{\frac{\partial V}{\partial \phi_i}}}_{=0 \text{ at } \phi_0}+\langle\phi_j\rangle\underbrace{\frac{\partial^2 V}{\partial\phi_i\partial\phi_j}}_{\mathcal{M}^2_{ij}}\right)=0$$

$T^{a}_{i}{}^{j}\langle\phi_j\rangle\delta\phi_i$

has mass $0 \longrightarrow$ GBs

- CCWZ construction

$$\phi=\frac{v+h(x)}{\sqrt{2}}e^{i\frac{a(x)}{v}}$$

$$\phi'=e^{i\alpha q_\phi}\phi \longrightarrow \begin{cases}\delta a= \alpha q_\phi v\\ \delta h=0\end{cases}$$

$$\psi'=e^{i\alpha q_\psi}\psi \longrightarrow \begin{cases}\tilde{\psi}=e^{-i\frac{q_\psi}{q_\phi}\frac{a}{v}}\psi\\ \delta\tilde{\psi}=0\end{cases}$$

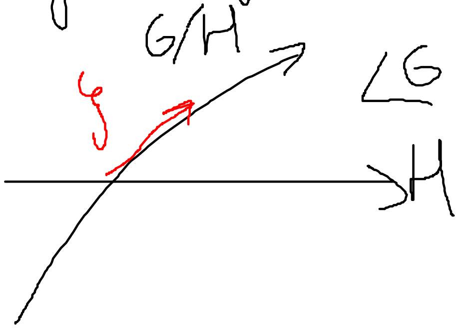
$$e^{i\alpha}e^{i\frac{a}{v}}\times\cdots=e^{i\left(\frac{a}{v}+\alpha\right)}\times\cdots$$

$$\tilde{\phi}=e^{-i\frac{q}{q}a}\phi$$

$\tilde{\phi} \equiv h$

G/H $\qquad$ SSB: $G \to H$

$g \sim g'$ if $g = g'h$, $h \in H$



$$\gamma(\xi) = e^{i\xi^a T^a}$$

$$g \times \gamma(\xi) = \gamma(\xi')h$$

$$\xi \xrightarrow{g} \xi'$$

ψ charged under H

$$\psi \xrightarrow{g} \psi' = h(g,\xi)\psi$$

$$\gamma^{-1}\partial_\mu \gamma = i D_{\mu a} T^a + i E_{\mu i} T^i$$

$$\partial_\mu \psi \to \mathcal{D}_\mu \psi = \partial_\mu \psi + i E_{\mu i} T_H^i \psi$$

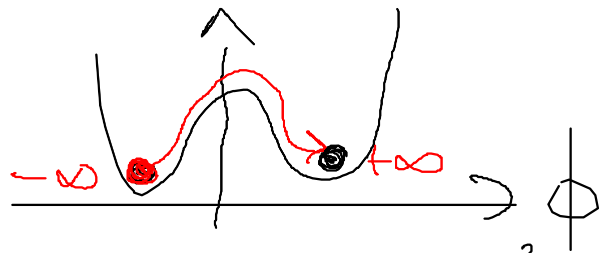
$\mathcal{L}(D_{\mu a}, \psi, D_\mu \psi)$ inv. under H

$\longrightarrow$ under G too

$$\mathcal{L}(\xi, \partial_\mu \xi, \psi, \partial_\mu \psi) \xrightarrow{g=\gamma^{-1}} \mathcal{L}(0, D_{\mu a}, \psi, D_\mu \psi) + \dots$$

- Discrete / spacetime syms

Discrete: no GB

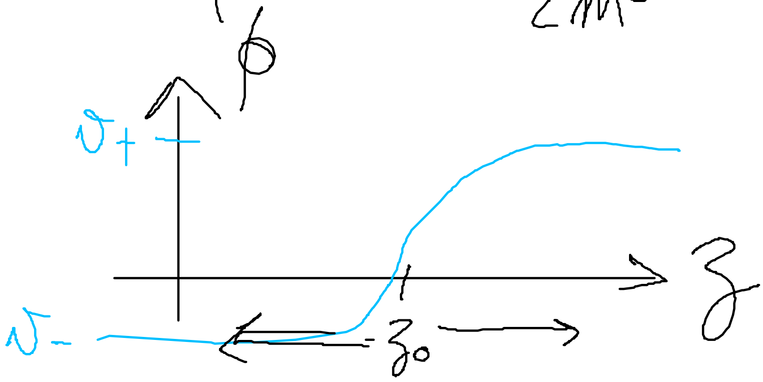


$$V = \frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4$$

$$\phi = \langle \phi \rangle + \delta\phi$$

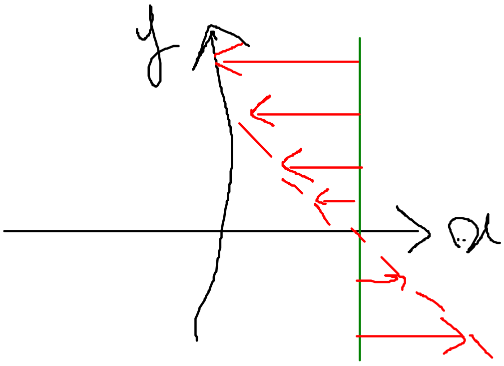
$$\text{quartic} = -\frac{\text{cubic}^2}{2m^2}$$

+ Domain walls



$\rightarrow$ Spacetime: how many GBs?

SUSY : goldstino
Scale : dilaton



$D_\mu a = 0$

Inverse Higgs Constraints

Quantum aspects

- Fabri-Picasso theorem

  Q does not exist

$$\langle 0 \, Q \, Q \, 0 \rangle = \| Q|0\rangle \|^2$$

$$\| $$

$$\int d^3x \langle 0 \, \underbrace{J^0(x)}_{e^{iP\cdot x} J^0(0) e^{-iP\cdot x}} \, Q \, 0 \rangle$$

$$= \int d^3x \, \underbrace{\langle 0 \, J^0(0) \, Q \, 0 \rangle}$$

$$= V_{space} \times$$

$$\underbrace{\| Q|0\rangle \|^2} \begin{cases} \to 0 \text{ , unbr. sym} \\ \to \infty \end{cases}$$

- $[Q, \phi] = i \delta \phi$

$$\partial_\mu J^\mu = 0 + \cdots$$

Quantum GBs

↳ effective action

↳ complete basis of states

$$\eta = \langle 0 [Q(t), \phi(0)] 0 \rangle = i \delta \langle \phi \rangle$$

$$\eta \neq 0 \ , \quad \dot{\eta} = 0$$

$$|n\rangle \ / \quad \sum_n |n\rangle\langle n| = \mathbb{1}$$

$$\eta = \sum_n \langle 0\, Q^a |n\rangle \langle n\, \phi(0)\, 0 \rangle - \langle 0\, \phi(0) n\rangle \langle n\, Q\, 0 \rangle$$

$$= \sum_n \int d^3x \Big(\langle 0\, J^0(0)\, n \rangle \langle n\, \phi(0)\, 0 \rangle \times e^{-i p_n \cdot x} - e^{i p_n \cdot x} \times \cdots \Big)$$

$$= \sum_n \delta^{(3)}(\vec{p}_n) \Big[e^{-iE_n t} \langle 0\, J^{a0}(0)\, n \rangle \langle n\, \phi(0)\, 0 \rangle - h.c.$$

$$\dot{\eta} = -i \sum_n \delta^{(3)}(\vec{p}_n) \underbrace{E_n}_{\to 0} \Big[e^{-iE_n t} \langle \cdot \rangle \langle \cdot \rangle + h.c. \Big]$$

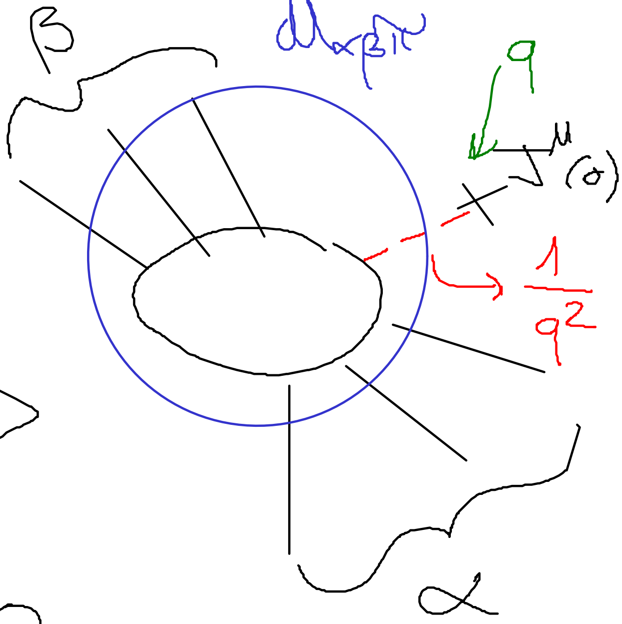
$$|n_0\rangle \ / \quad E_{n_0} \xrightarrow[\vec{p}_{n_0}]{} 0 \implies m^{\alpha}_{|n_0\rangle} = 0 \quad \text{GB}$$

$\langle \pi_0 | \phi(0) | 0 \rangle \neq 0 \to |\pi_0\rangle$ has spin 0

$\langle 0 | J_0^a(0) | \pi_0 \rangle \neq 0 \to |\pi_0^a\rangle$, one GB for each T^a

• GB interactions with matter (pions)

$$A^\mu = \langle \beta | J^\mu(0) | \alpha \rangle$$



$$q_\mu = p_\beta - p_\alpha$$

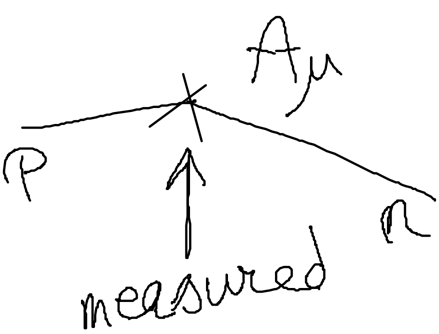
$$q_\mu \langle \beta | J^\mu(0) | \alpha \rangle = 0$$

$$\langle 0 | J^\mu(0) | \pi \rangle \neq 0$$

$$A^\mu \xrightarrow[q^2 \to 0]{} \frac{q^\mu}{q^2} \mathcal{M}_{\alpha\beta\pi}, \quad A^\mu = N^\mu_{\alpha\beta} + \frac{q^\mu}{q^2} \mathcal{M}$$

$$q_\mu A^\mu = 0 \Rightarrow \mathcal{M}_{\alpha\beta\pi} = - q_\mu N^\mu_{\alpha\beta}$$

• for pions: $\langle p | A_{\mu+} | n \rangle$



$\mathcal{L} \supset G_{\pi N} \vec{\pi} \bar{N} \gamma_5 \vec{\tau} N$

can be rewritten as

$$G_{\pi N} = \frac{2 m_N g_A}{f_\pi}$$

Goldberger-Treiman relation

$$\partial_\mu \vec{\pi} \cdot \bar{N} \gamma^\mu \gamma_5 \vec{t} N$$

$(\to 0 \text{ as } q \to 0)$

$\hookrightarrow$ Adler zero