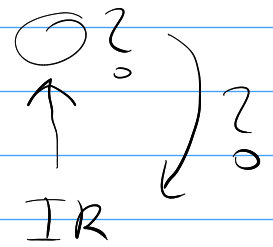
Chiral Lagrangians

Outline

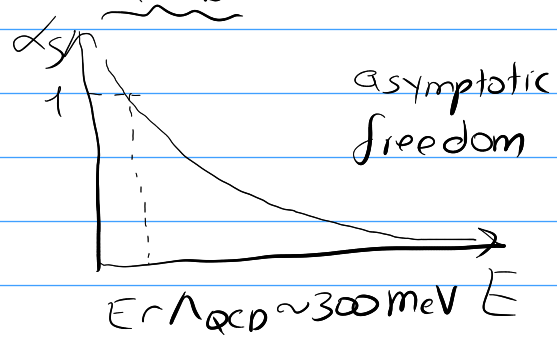
1) Introduction
2) Linear sigma model
3) The CCZW formalism
4) The QCD Chiral Lagrangian

1) Introduction

UV Weak $\rightarrow$ IR Fermi's



QCD



$\mathcal{L}_{EFT}^{QCD}$ $\quad E \ll \Lambda_{QCD}$

$\Rightarrow$ Knowledge of the symmetry breaking.

2) The linear sigma model $\vec{\phi} = \{\phi_i\}_{i=1}^{N}$

$$\mathcal{L} = \frac{1}{2}\partial_\mu \vec{\phi}\cdot\partial^\mu\vec{\phi} - \lambda(\vec{\phi}\cdot\vec{\phi} - v^2)^2$$

$$G = O(N) \qquad \vec{\phi} \to \vec{\phi}' = O(x)\vec{\phi}$$

$$|\vec{\phi}| = v \qquad \mathcal{M}_{vac} = \{\vec{\phi} : \phi_i^2 = v^2\} \sim S^{(N-1)}$$

↑ vacuum manifold

$$\langle\vec{\phi}\rangle = (\underbrace{0, 0, \dots}_{N-1}, v)^T$$

$$\boxed{O(N) \to O(N-1) \subset O(N)}$$

$$O(N) : \langle\vec{\phi}\rangle \neq \langle\vec{\phi}\rangle$$

$$O(N-1) : \langle\vec{\phi}\rangle = \langle\vec{\phi}\rangle$$

$$\frac{N(N-1)}{2} - \frac{(N-1)(N-2)}{2} = \underbrace{N-1}_{\#\text{ of Goldstone bosons}}$$

$\vec{\phi} = N$ d.o.f.

$N-1$ Goldstone

1 radial mode

$$\vec{\phi}(x) = \underbrace{R(x)}_{\text{Goldstone}}\,\vec{\sigma}(x) \qquad \sigma(x) = v + \rho(x)$$

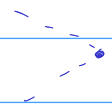
$$\phi_i(x) = R_{iN}\,\sigma(x) \qquad \begin{pmatrix} 0 \\ 0 \\ \vdots \\ \sigma(x) \end{pmatrix} \begin{matrix} \left.\vphantom{\begin{matrix}0\\0\\\vdots\end{matrix}}\right\} N-1 \\ 0 \end{matrix}$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\sigma\,\partial^\mu\sigma + \frac{1}{2}\left[(\partial_\mu R^T)(\partial^\mu R)\right]_{NN} - \lambda(\sigma^2 - v^2)^2$$

$$R = \exp\left\{ i \sum_{s=1}^{N-1} \underbrace{X^s}_{\text{broken generators}} \cdot \underbrace{\pi^s}_{\text{Goldstones}} \right\}$$

$$\mathcal{L} = \frac{1}{2} v^2 \left[\partial_\mu \exp\{\cdots\} \partial^\mu \exp\{\cdots\} \right]_{NN}$$

- Goldstones derivatively coupled.

$\Rightarrow$  $\sim p^\mu$

3) The CCWZ Formalism

Callan, Coleman, Wess, and Zumino

$$\mathcal{L}_{UV} : \quad G \not\to H \subset G, \quad \{\phi_i\}$$

$$hv = v, \quad \forall h \in H$$

$$\left.\begin{array}{l} g \in G : v \longmapsto \phi \\ gh : v \longmapsto \vec{\phi} \end{array}\right\} \Rightarrow g_1 \sim g_2 \text{ if } \exists h \in H : g_1 = g_2 h$$

$$gH = \{ gh : h \in H \} \equiv \text{equivalence class}$$

$$\{gH\} = \text{coset space} \equiv G/H \equiv \begin{array}{c}\text{vacuum} \\ \text{manifold}\end{array}$$

$$\{\ \}_G = \{T^i\}_{i=1}^{\dim H} \cup \{X^a\}_{a=1}^{\dim(G/H) = \dim G - \dim H}$$

$$\left.\begin{array}{l} [T^i, T^j] = i f^{ijk} T^k \\ [T^i, X^a] = i f^{iab} X^b \\ [X^a, X^b] = i f \cdot T + i f \cdot X \end{array}\right\} \text{Cartan decomposition}$$

G/H: $\hat{g}$, $g\,\xi = \hat{g}\,h$ for some $h \in H$.

$$g = \underbrace{\exp\{i\,\xi^a X^a\}}_{\text{Goldstones}}\ \underbrace{\exp\{i\,u^j T^j\}}_{\in H}$$

$$\phi(x) = \underbrace{\gamma(x)}_{\in G}\ \underbrace{\tilde{\phi}(x)}_{\text{no Goldstones}}$$

$$\tilde{\phi}(x) = v \quad , \quad h\,v = v$$

$$\boxed{\gamma(x) = \gamma[\pi] = \exp\{i\,\pi^a(x)\,X^a\}}$$

CCWZ construction ↖ Goldstones

4) The QCD Chiral Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{g_s^2}\,G G + i\,\bar{q}\,\not{D}\,q \quad , \quad N$$

$$G = SU(N)_L \times SU(N)_R$$

$$\downarrow \qquad\qquad \searrow$$

$$q_L(x) \to L\,q_L(x) \qquad\qquad q_R(x) \to R\,q_R(x)$$

$$\langle \bar{q}\,q \rangle \neq 0$$

$$G \not\to H = SU(N)_V \equiv L = R$$

$$2(N^2-1) - (N^2-1) = N^2-1$$

u, d, s, c, b, t : $\Lambda_{QCD} \sim 300$ MeV

$$\frac{m_{u,d}}{\Lambda_{QCD}} \sim \frac{1}{50} \quad , \quad \frac{m_s}{\Lambda_{QCD}} \sim \frac{1}{3} \quad , \quad \frac{m_{c,t,b}}{\Lambda_{QCD}} \gg 1$$

- $m_u = m_d = 0$, $2^2 - 1 = 3 \Rightarrow$ 3 light d.o.f.
- $m_u = m_d = m_s = 0$, $3^2 - 1 = 8 \Rightarrow$ 5 lightish d.o.f.

$m_\pi \sim 140$ MeV #3

$m_K \sim 500$ MeV $\times 4$ $\qquad m_{\eta'} \sim 950$ MeV

$m_\eta \sim 550$ MeV $\times 1$

$SU(2)_L \times SU(2)_R \to SU(2)_V \Rightarrow$ pions

$SU(3)_L \times SU(3)_R \to SU(3)_V \Rightarrow$ pions + kaons + eta

$\mathcal{L} + \bar{q} \mathcal{M} q \qquad m \to$

$H = SU(3)_V \Rightarrow \{T^a\}$

Broken generator $\{T_L^a\}$

$$\underset{\in G}{\gamma\{\pi\}} = \exp\left\{ i \begin{bmatrix} \pi^a T_L^a & 0 \\ 0 & 0 \end{bmatrix} \right\} \equiv \begin{bmatrix} \Sigma(x) & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Sigma(x) = \exp\{ i \pi^a T_L^a \}$$

$$g : \gamma\{\pi\} \longmapsto \gamma\{\pi'\} = g\, \gamma\{\pi\} h^{-1}$$

$$h = \begin{pmatrix} U & 0 \\ 0 & U \end{pmatrix}$$

$$\boxed{g : \Sigma(x) \longrightarrow L \Sigma(x) R^\dagger}$$

$$\mathcal{L}_0 \sim \mathrm{Tr}\, \Sigma\Sigma^\dagger \cdots \Sigma\Sigma^\dagger, \quad \Sigma\Sigma^\dagger = \mathbb{1}$$

$$= \text{constant.}$$

$$\mathcal{L}_2 = \frac{f^2}{4} \mathrm{Tr}\, \partial_\mu \Sigma \partial^\mu \Sigma^\dagger$$

$$\mathcal{L}_4 \sim \mathrm{Tr}\, \partial_\mu \Sigma \partial^\mu \Sigma^\dagger \partial_\nu \Sigma \partial^\nu \Sigma^\dagger$$

$$\mathcal{L}_6 \quad \vdots$$

$$\Sigma(x) = \exp\left\{ \frac{2i}{f} \hat{\pi}^a T^a \right\}, \quad \pi^a T^a = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\mathcal{L}_2 = \mathrm{Tr}\, \partial_\mu \pi \partial^\mu \pi + \frac{1}{3f^2} \mathrm{Tr} \left[\pi, \partial_\mu \pi \right]^2 + \cdots$$

Weinberg's Power Counting

p^4: $\quad A \sim \int (d^4p)^L \frac{1}{(p^2)^I} \prod_k (p^k)^{m_k}$

$$A \sim p^D$$

$$D = 4L - 2I + \sum_k k\, m_k \quad , \quad V - I + L = 1$$

$$D = 2 + 2L + \sum_k (k-2) m_k$$

$$4 = 2 + 0 + 2 \cdot 1 \quad \mathcal{L}_4$$

$$4 = 2 + 2 \cdot 1 + 0 \quad \mathcal{L}_2$$