Written by Philip Sørensen
philip.soerensen@desy.de

Heavy Quark Effective Theory

A top-down effective field theory of QCD.

Strong sector:

$u \quad c \quad t \quad m_{cbt} \gg \Lambda_{QCD} \qquad \sim Q$

$d \quad s \quad b$

$\hookrightarrow m_{uds} \ll \Lambda_{QCD} \qquad \sim q$

Chiral Lagrangians

- $m_q \to 0 \Rightarrow$ pNGB's
- $m_Q \to \infty \Rightarrow$ decouple

Heavy Quark Effective Theory

- $\lim\limits_{m_Q \to \infty} \mathcal{L}_{QCD} = \lim\limits_{m_Q \to \infty} \bar{Q}(i\cancel{D} - m_Q)Q$

- without decoupling the heavy quark

-> allow us to study heavy d.o.f.

Litterature:

-Heavy Quark Physics, Manohar and Wise

-Heavy Quark Symmetry, Neubert

-MIT OCW Course on Effective Field Theory, Stewart

The Idea

QCD bound state with a heavy quark

light junk $\sim \Lambda_{QCD}$

$m_Q \gg \Lambda_{QCD}$

$\Rightarrow$ Q is almost <u>on-shell</u>

Motivation

$$p^\mu = \underbrace{m_Q v^\mu}_{\substack{\text{on-shell} \\ \gg \Lambda_{QCD}}} + \underbrace{k^\mu}_{\sim \Lambda_{QCD}}$$

v^μ const.

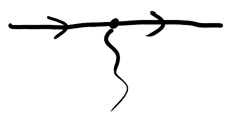
$v^2 = 1$

Star with QCD propagator

$$\frac{i(\not p + m_Q)}{p^2 - m_Q^2 + i\epsilon} \simeq \frac{i}{v \cdot k + i\epsilon} \underbrace{\frac{1+\not v}{2}}_{} + \mathcal{O}\left(\frac{1}{m_Q}\right)$$

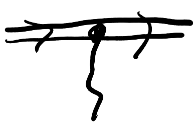
$$P_\pm = \frac{1 \pm \not v}{2} \qquad P_\pm^2 = P_\pm \qquad P_+ P_- = 0$$

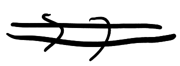
QCD vertex


$= -i g \gamma^\mu T^a$

$$P_+ \gamma^\mu P_+ = P_+ v^\mu P_+$$

HQET Feynman rules


$= -i g v^\mu T^a$


$= \frac{i}{v k} \frac{1+\not v}{2}$

Identify a Lagrangian that leads to these rules:

$$\mathcal{L}_{HQET} = \bar{Q}_v \, i \, v \cdot D \, Q_v + O\left(\frac{1}{m_Q}\right)$$

- m_Q does not appear

$$\lim_{m_Q \to \infty} \mathcal{L}_{QCD} \stackrel{?}{=} \mathcal{L}_{HQET}$$

_ Q does not decouple

Proper top-down derivation

Start with

$$\mathcal{L}_{QCD} = \bar{Q} \left(i \not{D} - m_Q \right) Q$$

Define

$$Q_v(x) = e^{i m_Q v \cdot x} P_+ Q$$

$$B_v(x) = e^{i m_Q v \cdot x} P_- Q$$

Such that

$$Q(x) = e^{-i m_Q v \cdot x} \left[Q_v(x) + B_v(x) \right]$$

$$\mathcal{L}_{QCD} \Rightarrow \mathcal{L}_{HQET} = \bar{Q}_v \, i v D \, Q_v - \bar{B}_v \left(i v D + 2 m_Q \right) B_v$$

$$+ \bar{Q}_v \, i \not{D}_\perp B_v + \bar{B}_v \, i \not{D} Q_v \quad \text{where } D_\perp^\mu = D^\mu - v^\mu v \cdot D$$

$$\text{s.t. } v \cdot D_\perp = 0$$

In m_Q -> inf:

- B_v decouples
- Q_v remains
- Valid at $\mathcal{O}(1/M_Q)$ and $\alpha_s(M_Q)$

In rest frame:

$$\frac{1+\cancel{v}}{2} = \frac{1+\gamma_0}{2}$$

$$P_+ \, \psi_{dirac} = \begin{pmatrix} \psi_u \\ 0 \end{pmatrix} \begin{matrix} \text{particle} \\ \text{anti-particle} \end{matrix}$$

Q_v is heavy quark particle
B_V is heavy quark anti-particle

-Anti-particles are split by 2 m_Q
-> They decouple
-> Particle number becomes conserved

Heavy Quark Symmetry

- No anti-matter -> perserved particle numbers -> U(1)

- No dirac matricies in the Lagrangian
  - -> Heavy quark spin is preserved
  - -> SU(2) symmetry

- No masses appear in the Lagrangian (after m_Q -> inf)
  - -> U(N) flavour symmetry
  - -> SU(2N) spin symmetry
  - -> Overall U(2N) Heavy Quark Symmetry

Meson spectroscopy

Conserved: $J, S_Q, S_L = J - S_Q$

$\Rightarrow j, s_Q, s_L$

Meson doublets

$j_\pm = s_L \pm \frac{1}{2}$

- doublets must be degenerate in the m_Q -> inf limit
- we expect splitting of order O(1/m_Q)

Covariant representation

Define fields that transform covariantly under HQS rotations. We will represent the heavy meson by

$$H_v^Q \sim P_v^Q \,,\; P_{\mu v}^{*Q}$$

Simplest combination

$$H_v^Q = \frac{1+\not{v}}{2}\left[\not{P}_v^{*Q} + i\,P_v^Q \gamma_5\right]$$

Under HQS rotation R,

$$Q_v \to D(R)\,Q_v$$

$$H_v^Q \to D(R)\,H_v^Q$$

Allows us to perform computations based on symmetry.

Decay constants

In QCD:

meson state

Pseudoscalar $\langle 0 | \bar{q} \gamma^\mu \gamma_5 Q | P(p) \rangle = -i f_P \, p^\mu$

Vector $\langle 0 | \bar{q} \gamma^\mu Q | P^*(p) \rangle = f_{P^*} \, \varepsilon^\mu$

QCD has two parameters: $f_P, \; f_{P^*}$

In HQET:

$|P(v)\rangle$ or $|P^*(v)\rangle$

$$\langle 0 | \bar{q} \Gamma^\mu Q_v | H(v) \rangle$$

$\gamma^\mu \gamma^5$ or γ^μ

We want to express the current

$$\bar{q} \Gamma^\mu Q_v$$

In terms of the meson field H

To do this, we will operators with correct.

Helping trick:

- Pretend $\Gamma^\mu \to \Gamma^\mu D(R)^{-1}$
- Construct invariant operators
- Restore normal, constant Γ^μ

-> Operator with correct transformation

Because of:

a) H_v^Q must appear once

b) Gamma^mu must only appear as $\Gamma^\mu H_v^Q$

c) Lorentz covariance

the current must have the form

$$\mathrm{Tr}\left[X\,\Gamma^\mu H_v^Q\right]$$ where X is an unkown bispinor

Because x should not depend on spin + Lorentz

$X = \frac{a}{2}$ where we also used $v^2 = 1$, $H_v^Q \cancel{v} = -H_v^Q$

The current is then

$$\bar{q}\,\Gamma^\mu Q_v = \frac{a}{2}\mathrm{Tr}\left[\Gamma^\mu H_v^Q\right]$$

This trace can be taken explicitly, which yields

$$\langle 0|\bar{q}\,\gamma^\mu \gamma_5 Q_v|P(v)\rangle = -i\,a\,v^\mu$$

$$\langle 0|\bar{q}\,\gamma^\mu Q_v|P^*(v)\rangle = a\,\varepsilon^\mu$$

$\dim a = 3/2$

$a \sim \Lambda_{QCD}^{3/2}$

Note that: To make HQET state normalization independt of m_Q, they are differently than the QCD states.

$$|H(p)\rangle_{QCD} = \sqrt{m_H}\left(|H(v)\rangle + \mathcal{O}\left(1/m_Q\right)\right)$$

We pick up a factor of $\sqrt{m}$:

$$\Rightarrow f_P = \frac{a}{\sqrt{m_P}} \quad \text{and} \quad f_{P^*} = a\sqrt{m_{P^*}}$$

- HQET has related the two states in the meson doublet
- Made possible by the covariant representation

Example HQET predictions:

$$f_B = \frac{\Lambda_{QCD}^{3/2}}{\sqrt{m_B}} \sim 180\ \text{MeV} \quad \text{vs.} \quad 173^{\pm 4}\ \text{MeV}$$

(lattice)

$$\frac{f_B}{f_D} \sim \frac{\sqrt{m_D}}{\sqrt{m_B}} \sim 0.6 \quad \text{vs} \quad 0.88$$

Another area where HQET is powerful is semileptonic decay:
- QCD: 6 form factors
- HQET: 1 form factors, normalized
  -> Isgur-Wise function, covered Manohar and Wise sec. 2.9

Summary:

- HQET is an EFT which allows us to study heavy particles at low energies without integrating them out.

- Features Heavy Quark Symmetry, which in QCD but emerges in the kinematic regime of HQET.

- Can be used as a powerful tool to study heavy bound states

- As an appoach it can be applied to other heavy particles outside of QCD.

Topics I unfortunately did not have time to discuss:

- Radiative corrections and power matching

- Reparametrization invariance
  (The statement that the v does not matter.
  It is still the full momentum of the heavy quark
  that is physical. Does not matter how we dice it up)