Standard Model Effective Field Theory (SMEFT)

- Often: NP described by particles heavier than NP $\Lambda \gg$ exp. accesible energies

$\xrightarrow[\text{out}]{\text{integrate}}$ EFT

- assumptions: • no new light particles
  • NP respects SM gauge symmetry $SU(3)_c \times SU(2)_L \times U(1)_Y$

$\rightarrow$ SMEFT

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \frac{1}{\Lambda} \sum_k C_k^{(5)} Q_k^{(5)} + \frac{1}{\Lambda^2} \sum_k C_k^{(6)} Q_k^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right)$$

- advantages: model independent description of NP
  $\rightsquigarrow$ only have to derive bounds on SMEFT parameters
  later recast into bounds on specific models
  $\rightsquigarrow$ don't have to specify BSM model to parameterize
  deviations of exp. from SM

<u>Outline</u>:
1. Field redefinitions and equations of motion (EOM)
2. How to construct a basis SMEFT?
3. Operator mixing and EOM-vanishing operators

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1. Field redefinitions and EOM

- path integral picture:

$$Z[j_i] = \int \prod_i \mathcal{D}\varphi_i \, \exp\left(i \int d^4x \left(\mathcal{L} + \sum_i j_i \varphi_i \right)\right)$$

$\rightsquigarrow$ observables invariant under field redefinitions

- for EFT: small shift of field φ_i of order $\sim 1/\Lambda^2$

  $\longleftrightarrow$ addition of EOM-vanishing operator to the dim 6 Lagrangian (in the calculation of observables)

- sketch of proof for a scalar field ϕ:

  Write Lagrangian as:

$$\mathcal{L} = \sum_{n=0}^{\infty} \eta^n \mathcal{L}_n$$

redefinition: $\phi^+ = (\phi')^+ + \eta\, T[\varphi]$, $T[\varphi]$: any local function of all the fields and their derivatives

$$Z[j_i] = \int \prod_i \mathcal{D}\varphi_i' \left|\frac{\delta\phi^+}{\delta(\phi')^+}\right| \exp\Big(i\int d^4x \Big[\mathcal{L}_0' + \delta\mathcal{L}_0' + \eta(\mathcal{L}_1' + \delta\mathcal{L}_1') + \sum_i j_i \varphi_i + j_{\phi^+}\eta T + \mathcal{O}(\eta^2)\Big]\Big)$$

$$\mathcal{L}_i' \equiv \mathcal{L}_i\left((\phi')^+, \partial_\mu(\phi')^+\right)$$

$$\delta\mathcal{L}_i' \equiv \frac{\partial\mathcal{L}_i'}{\partial(\phi')^+}\delta\phi^+ - \frac{\partial\mathcal{L}_i'}{\partial\partial_\mu(\phi')^+}\,\delta\partial_\mu\phi^+$$

$$\delta\phi^+ \equiv \phi^+ - (\phi')^+ = \eta T, \quad \delta\partial_\mu\phi^+ \equiv \partial_\mu\phi^+ - \partial_\mu(\phi')^+$$

$\xrightarrow[\text{Gauss}]{\text{partial integration}}$

$$\delta\mathcal{L}_i' = \left(\frac{\delta\mathcal{L}_i'}{\delta(\phi')^+} - \partial_\mu\frac{\delta\mathcal{L}_i'}{\delta\partial_\mu(\phi')^+}\right)\underbrace{\delta\phi^+}_{\eta T}$$

$$Z[j_i] = \int \prod_i \mathcal{D}\varphi_i' \left|\frac{\delta\phi^+}{\delta(\phi')^+}\right| \exp\Big(i\int d^4x \Big[\mathcal{L}_0' + \left(\frac{\delta\mathcal{L}_0'}{\delta(\phi')^+} - \partial_\mu\frac{\delta\mathcal{L}_0'}{\delta\partial_\mu(\phi')^+}\right)\eta T + \eta\mathcal{L}_1' + \sum_i j_i\varphi_i + j_{\phi^+}\eta T + \mathcal{O}(\eta^2)\Big]\Big)$$

- functional determinant can be neglected at $\mathcal{O}(\eta)$
- new source term has no influence on observables

details: [hep-ph/9304230]

2. How to construct a basis for SMEFT?

- consider only gauge covariant objects as building blocks:
  - fermion spinors
  - higgs doublet φ^j
  - gauge field tensors $X_{\mu\nu} \in \{ G^A_{\mu\nu}, W^I_{\mu\nu}, B_{\mu\nu} \}$
  - and covariant derivatives thereof

$$D_\mu = \partial_\mu - i g_s \frac{1}{2} \lambda^A G^A_\mu - i g \frac{1}{2} T^I W^I - i g' Y B$$

- only dim 5 term is the Majorana mass term of the neutrinos, violates L

$\boxed{\text{dim } 6}$

- mass dimensions $d=4$: $[\varphi]=1$, $[X_{\mu\nu}]=2$, $[\text{fermions}]=3/2$, $[D]=1$
  - $\rightsquigarrow$ constrains possible combinations at given mass dim
  - $\rightsquigarrow$ have to have even number of fermions
    - $\rightsquigarrow$ can have 4 fermions $(4 \cdot 3/2 = 6)$
    - $\rightsquigarrow$ 2 fermions + combination of X, φ, D with total dim 3
    - $\rightsquigarrow$ purely bosonic operators!

↑ now

- only higgs boson has half-integer weak-isospin → have to have an even # of φ's
- must have an even number of D because they are the only objects with odd # of Lorentz indices
- possibilities: $X^2\varphi^2$, X^2D^2, $X\varphi^4$, XD^4, $X\varphi^2D^2$, φ^6, φ^4D^2, φ^2D^4
- $X_{\mu\nu}$ antisymmetric ⇝ can't be contracted with anything in a non-zero way

$\Rightarrow X\varphi^4 = \emptyset$

- XD^4 can only contracted such that it contains $[D_\mu, D_\nu] \sim X_{\mu\nu}$ ⇝ "lower" class

⌈ strategy: • order operators from high # of derivatives to low number, for equal # of D: from low number of X to high number

• first use EOM + other relations to shift operators from higher → lower class whenever possible

• then list operators based on rep. theory ⌋

- $\underbrace{\varphi^2D^4}_{\to\text{ now}}$, $X\varphi^2D^2$, X^2D^2 can be "eliminated" by EOM

- $(D_\mu D^\mu \varphi)^j = m^2\varphi^j - \lambda(\varphi^\dagger\varphi)\varphi^j - \bar{e}\,\Gamma_e^\dagger \ell^j + \varepsilon_{jk}\,\bar{q}^k\,\Gamma_u u - \bar{d}\,\Gamma_d^\dagger q^j$

Yukawa matrices

- To satisfy $SU(2)_L \times U(1)_Y$, we need one φ, one $\varphi^\dagger$

- $D_\mu = \partial_\mu - i\underbrace{M_\mu}_{\text{hermitian}}$

$$(D_\mu\varphi)^\dagger\varphi + \varphi^\dagger D_\mu\varphi = (\partial_\mu\varphi^\dagger)\varphi + \cancel{i(M_\mu\varphi)^\dagger} + \varphi^\dagger\partial_\mu\varphi - \cancel{i\varphi^\dagger M_\mu\varphi}$$
$$= \partial_\mu(\varphi^\dagger\varphi)$$

generalize : $(D^n\varphi)^\dagger(D^m\varphi) = -(D^{n+1}\varphi)^\dagger(D^{m-1}\varphi) + \partial\left[(D^n\varphi)^\dagger(D^{m-1}\varphi)\right]$

$$\Rightarrow (D^4\varphi)^\dagger\varphi + \underbrace{\partial(\dots)}_{\text{doesn't contribute}}$$

- all possible contractions $\epsilon_{\mu\nu\rho\sigma} \rightsquigarrow [D_\mu, D_\nu] \sim X^{\mu\nu} \to$ lower class
- can reorder derivatives as we want (reordering just introduces additional "lower class" operators

$\rightsquigarrow D^\mu D_\mu\varphi \xrightarrow{EOM} \underbrace{\varphi^4 D^2}, \varphi^2\varphi D^2, \underline{D^2\varphi^2}$

$\boxed{\varphi^4 D^2}$

- need exactly $2\varphi^\dagger$ $(Y=0)$
- both D's acting on same field $\overset{EOM}{\rightsquigarrow}$ lower class

$\rightsquigarrow$ only 2 independent $SU(2)_L$-singlets left:

$$(\varphi^\dagger\tau^I\varphi)\left[(D_\mu\varphi)^\dagger\tau^I(D^\mu\varphi)\right], \quad (\varphi^\dagger\varphi)\left[(D_\mu\varphi)^\dagger(D^\mu\varphi)\right]$$

other classes : similarly

3. Operator mixing and EOM-vanishing operators

• Mixing and renormalization:

$$C_j^{(n),\text{bare}} Q_j^{(n)} = \sum_i C_i \underbrace{Z_{ij}}_{\text{ren. constants}} Q_j^{(n)}$$

consist of renormalized SM fields

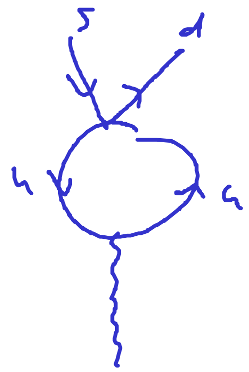
• $$0 = \frac{d C_j^{(n),\text{bare}}}{d\mu} = \sum_i \left[\frac{d C_i^{(n)}}{d\mu} Z_{ij} + C_i^{(n)} \frac{d Z_{ij}}{d\mu} \right]$$

$$\Leftrightarrow \quad \frac{d C_j^{(n)}}{d \log\mu} = C_i^{(n)} \gamma_{ij} \quad , \quad \gamma_{ik} = \sum_j -(Z^{-1})_{kj} \frac{d Z_{ij}}{d \log \mu}$$

Anomalous dim. matrix (ADM)

• e.g. effective Lagrangian for weak decays: can chose basis of 4-quark operators

$$O_q = (\bar u \gamma^\mu P_L s)(\bar d \gamma_\mu P_L u)$$



generates divergence

$$\propto O_p = (\bar d T^A \gamma^\mu P_L s)\, g_s [D^\nu, G_{\nu\mu}]^A$$

$$\xrightarrow{\text{EOM}} (\bar d T^A \gamma^\mu P_L s) \sum_q g_s^2 \left(\bar q T^A \gamma_\mu P_L q + \bar q T^A \gamma_\mu P_R q \right)$$