Gravitational waves from the fragmentation of axion-like particle dark matter

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DESY Theory Seminar, 25/05/20

Outline of the talk

- Axion-like particle dark matter and monodromy

- Nonperturbative dynamics

- Gravitational wave production

- Enhanced GW signal

- Transitions between local minima

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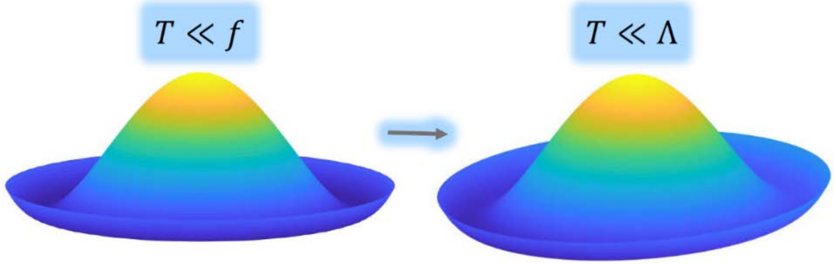
Axion-like particles (ALPs)

- Can arise as
  - pseudo Nambu-Goldstone bosons (e.g. QCD axion)

Weinberg, PRL 40, 223.

Peccei, Quinn, PRL 38, 1440.



  - Kaluza-Klein zero modes of high-dimensional gauge fields: **string theory axions**

Svrcek, Witten, JHEP 06, 051 (2006).

Arvanitaki et al., PRD 81, 123530 (2010).

Cicoli et al., JHEP **10**, 146 (2012).

- Generic feature: approximate shift symmetry, broken by nonperturbative effects
  - Dilute instanton gas approximation:

$$U(\varphi) = \Lambda^4 \left[1 - \cos\left(\frac{\varphi}{f}\right)\right]$$

Callan Curtis et al., PRD 17, 2717 (1978).

ALPs with quasiperiodic potentials

- Breaking of the higher-dimensional symmetry: **monodromy**
  - Can be induced by the presence of background fluxes or branes
- Mixing of several ALPs.

Kim, et al., JCAP 0501, 005 (2005).
Ben-Dayan et al., PRL 113, 261301 (2014).

Marchesano, et al., JHEP 09, 184 (2014).
Blumenhagen et al., PLB 736, 482–487 (2014).
Hebecker et al., PLB 737, 16–22 (2014).

McAllister et al., JHEP 09, 123 (2014).
McAllister et al., PRD 82, 046003 (2010).
Silverstein et al., PRD 78, 106003 (2008).

Importance in cosmology

Large field inflation

Relaxion mechanism

Graham et al., PRL 115 (2015) 221801

Espinosa et al., PRL 115 (2015) 251803

McAllister et al., JHEP **02**, 124 (2018).

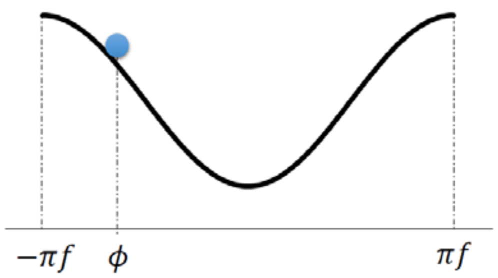
Dark matter

Arias et al., JCAP 1206, 013 (2012).
Preskill, et al., PLB 120, 127–132 (1983).

ALP production via misalignment mechanism

- Principle:
  - Equations of motion: $\ddot{\varphi} + 3H\dot{\varphi} - \frac{\Delta\varphi}{a^2} + \frac{\delta U}{\delta\varphi} = 0$
  - During inflation: strongly overdamped oscillator, $H \gg m_a$.
    - Homogenous initial conditions, $\langle\hat{\varphi}(\mathbf{x})\rangle = \phi_1$, $\langle\delta\hat{\varphi}^2\rangle \sim H_I^2$.
  - At $H \sim m_a$: onset of coherent oscillations.

Standard ALP DM

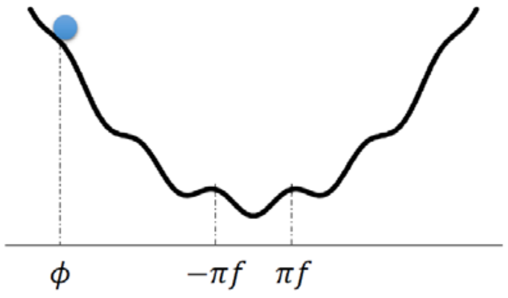


$\frac{\phi_1}{f} < \pi.$

$w = \frac{p}{\rho} \approx 0.$

Monodromy ALP DM

Jaeckel et al., JCAP 1701, 036 (2017).



Growth of fluctuations.

Berges, Chatrchyan, Jaeckel JCAP 1908 (2019) 020

Fonseca et al., 1911.08473

Fonseca et al., JHEP 04, 010 (2020).

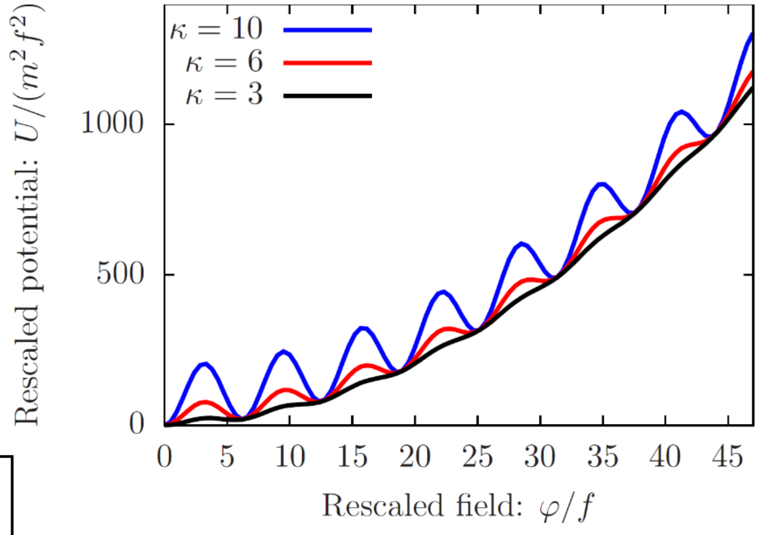
Fragmentation. **Gravity waves**?

- Axion-like particle dark matter and monodromy
- Nonperturbative dynamics
- Gravitational wave production
- Enhanced GW signal
- Transitions between local minima

The considered model and notation

- Considered potential: $U(\varphi) = \frac{1}{2}m^2\varphi^2 + \Lambda^4\left(1 - \cos\frac{\varphi}{f}\right)$
- Strength of the wiggles: $\kappa^2 = \frac{\Lambda^4}{m^2 f^2}$
- Near the minimum: $U(\varphi) = \frac{1}{2}m_a^2\varphi^2 - \frac{\lambda}{4!}\varphi^4 + \ldots$

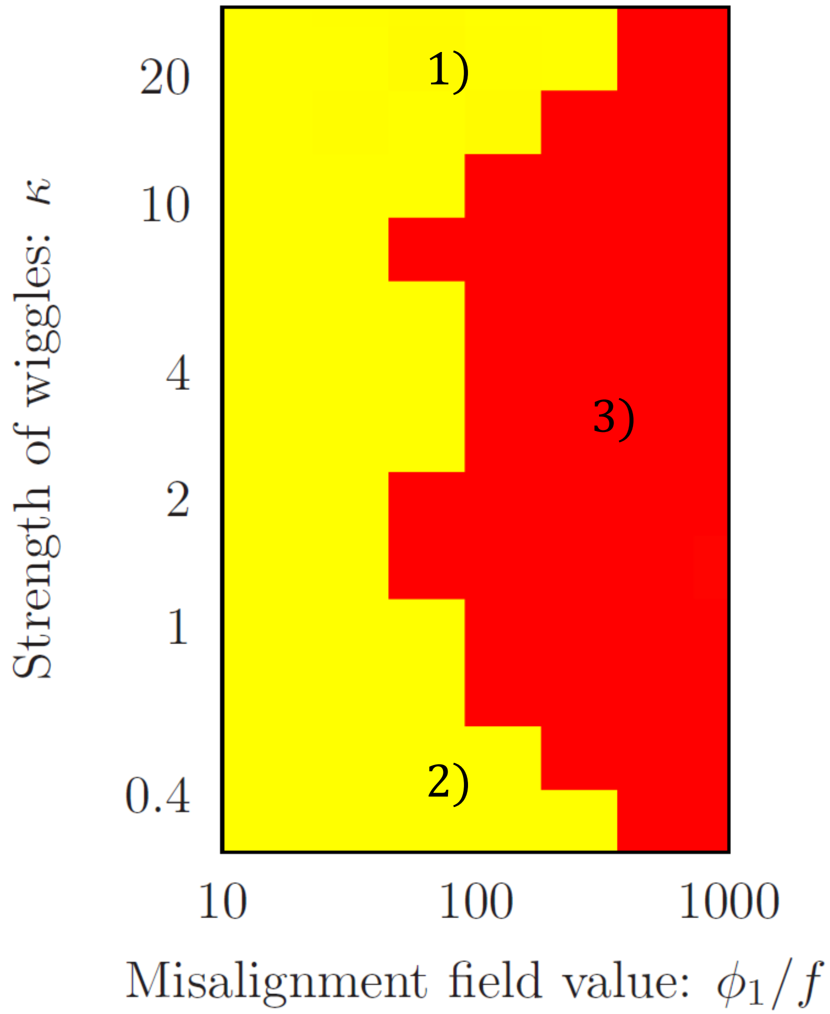
$$\boxed{m_a^2 = m^2(1+\kappa^2)} \qquad \boxed{\lambda = \kappa^2 \frac{m^2}{f^2}}$$



- ALPs start to oscillate in the radiation-dominated universe, $a(t) \propto t^{1/2}$
- Notation
  - Scale factor at $H = \frac{m_a}{3}$: a_{osc}
  - Rescaled comoving momentum: $\eta = \frac{\mathrm{p}}{m_a a_{osc}}$

Nonperturbative dynamics and fragmentation

- Growth of fluctuations unimportant, $\delta\varphi \ll \phi$
- Complete fragmentation of the homogeneous field



Classical(-statistical) approximation

- Classical field evolution

Berges, 1503.02907

- Statistical sampling over initial conditions

$$\varphi_0(\mathbf{x}) = \phi_0 + \int_{\mathbf{p}} \sqrt{\frac{n_0(\mathbf{p})+1/2}{\omega_{\mathbf{p}}}} c_{\mathbf{p}} e^{i\mathbf{p}\mathbf{x}},$$

$$\pi_0(\mathbf{x}) = \pi_0 + \int_{\mathbf{p}} \sqrt{(n_0(\mathbf{p})+1/2)\omega_{\mathbf{p}}} \widetilde{c}_{\mathbf{p}} e^{i\mathbf{p}\mathbf{x}}$$

$$\langle c_{\mathbf{p}} c_{\mathbf{p}'} \rangle = \langle \widetilde{c}_{\mathbf{p}} \widetilde{c}_{\mathbf{p}'} \rangle = (2\pi)^3 \delta(\mathbf{p} - \mathbf{p}')$$

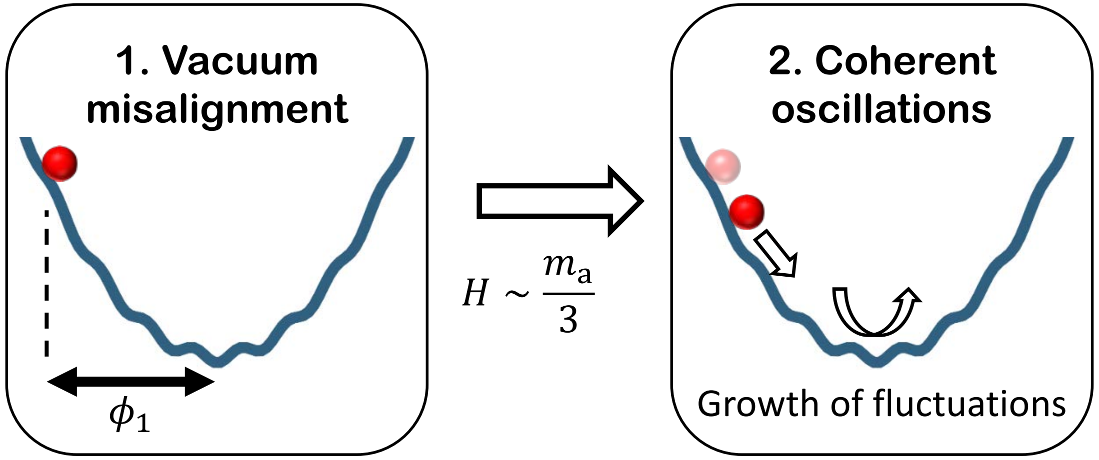
Gaussian random variables

- Range of validity:
  - Weak couplings, $\lambda \ll 1$
  - $\langle \{\hat{\varphi}(x), \hat{\varphi}(y)\} \rangle_c \gg \langle [\hat{\varphi}(x), \hat{\varphi}(y)] \rangle_c$

Large occupation numbers, $n_{\mathbf{p}} \gg 1$

- Linear evolution + lattice simulations

The main stages of the dynamics

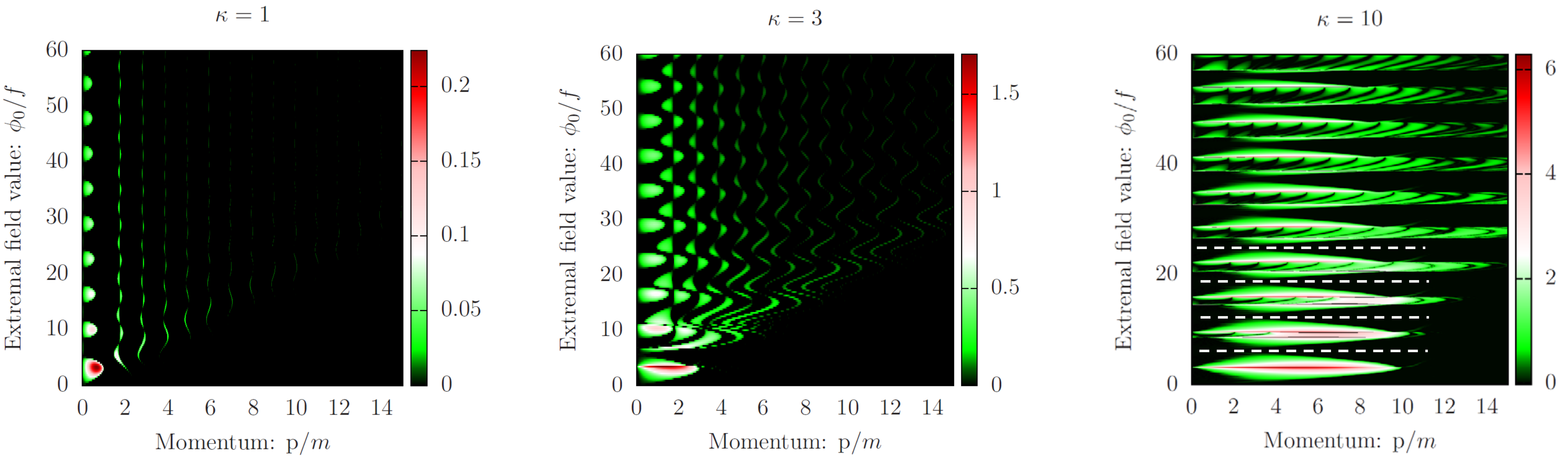


Linear instabilities

Parametric resonance: $\delta\ddot{\varphi}_{\mathrm{p}}(t) + 3H\delta\dot{\varphi}_{\mathrm{p}}(t) + \left[(\mathrm{p}/a)^2 + m^2 + \frac{\Lambda^4}{f^2}\cos\left(\frac{\phi(t)}{f}\right)\right]\delta\varphi_{\mathrm{p}}(t) = 0.$

Jaeckel et al.,
JCAP 1701, 036 (2017).

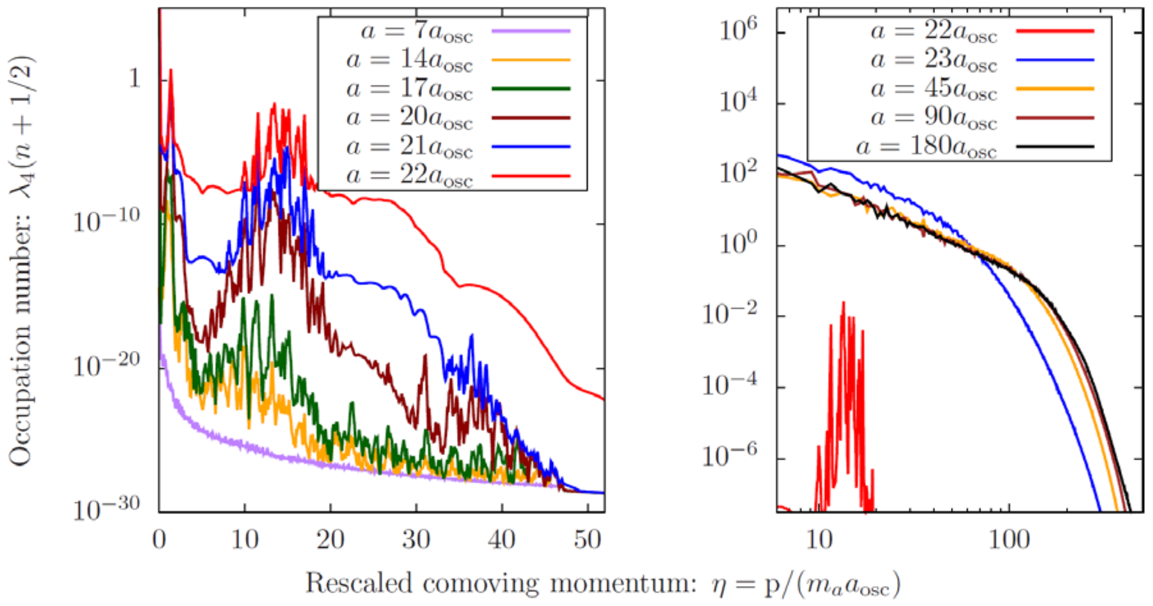
In Minkowski spacetime: $\delta\varphi_{\mathrm{p}}(t) \sim e^{\mu(\mathrm{p})t}$



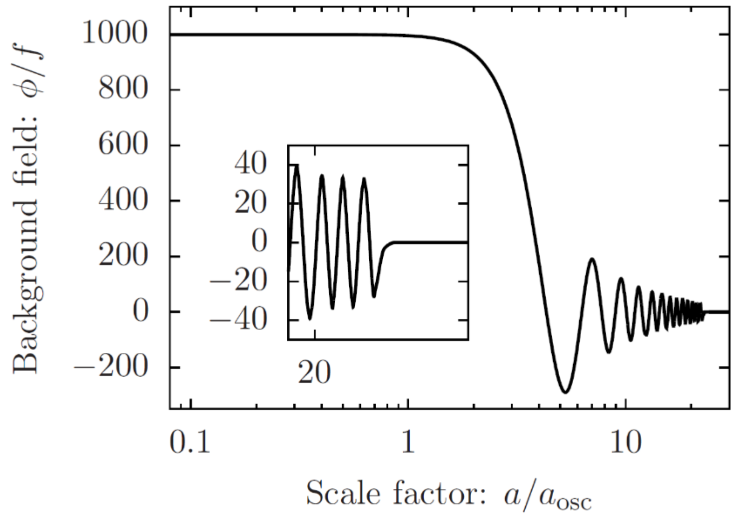
Amplification of fluctuations

Berges, Chatrchyan, Jaeckel
JCAP 1908 (2019) 020

- **Occupation numbers:** $n_{\mathrm{p}}(t) = \sqrt{\langle|\varphi_c(t,\mathbf{p})|^2\rangle\langle|\pi_c(t,\mathbf{p})|^2\rangle}$



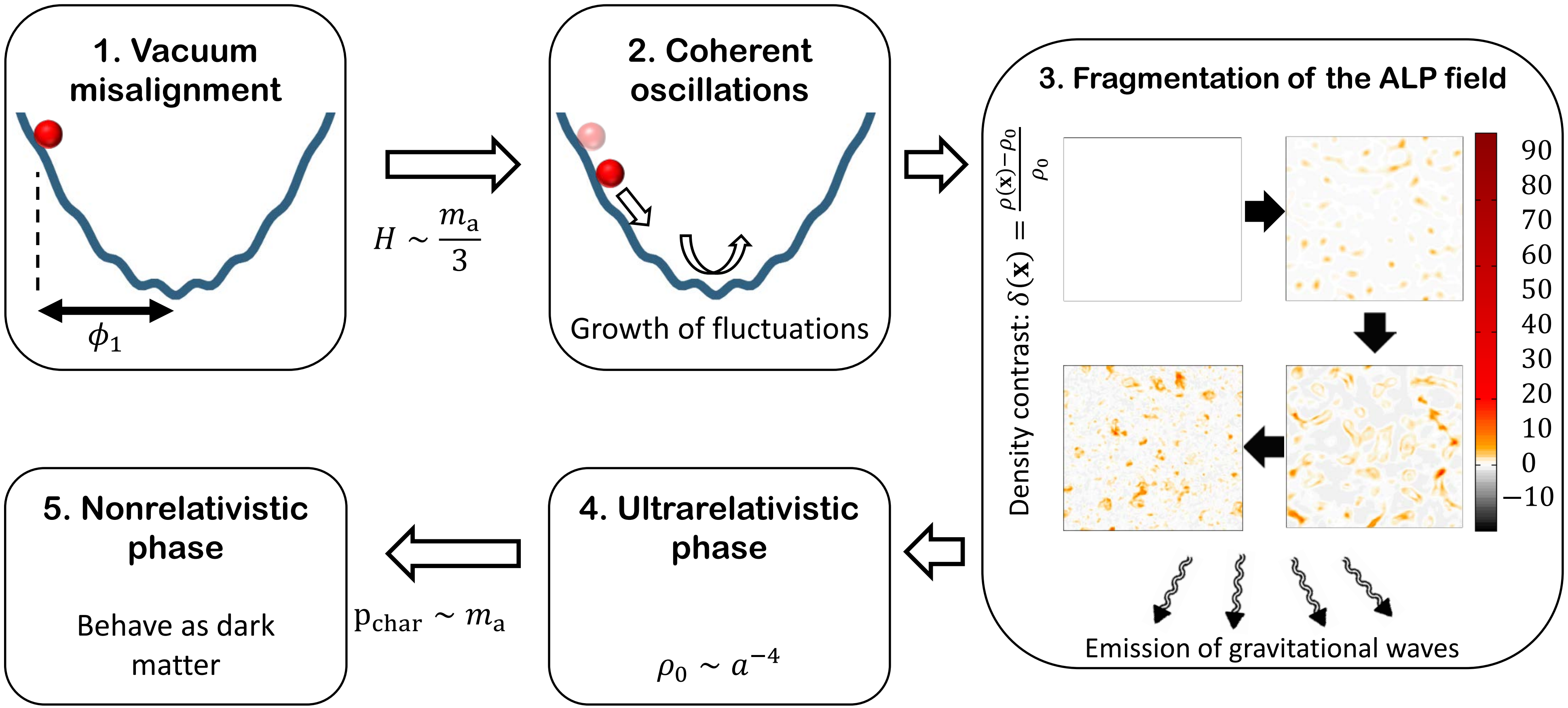
Field expectation value: $\phi(t) = \langle\varphi(t,x)\rangle$



- Nonlinear corrections become important when $\delta\varphi \sim \sqrt{H_I f}$.
- Fully nonperturbative dynamics when $\delta\varphi \sim f$.

Berges and Serreau,
PRL **91**, 111601 (2003).

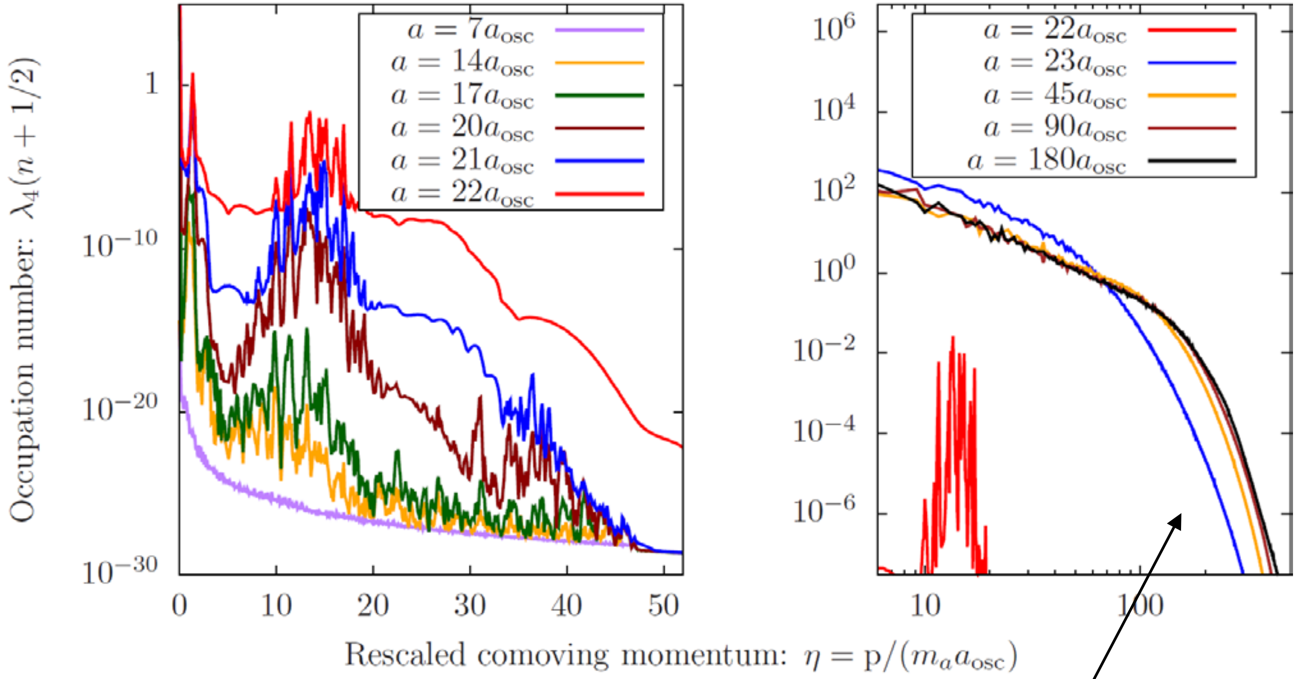
The main stages of the dynamics



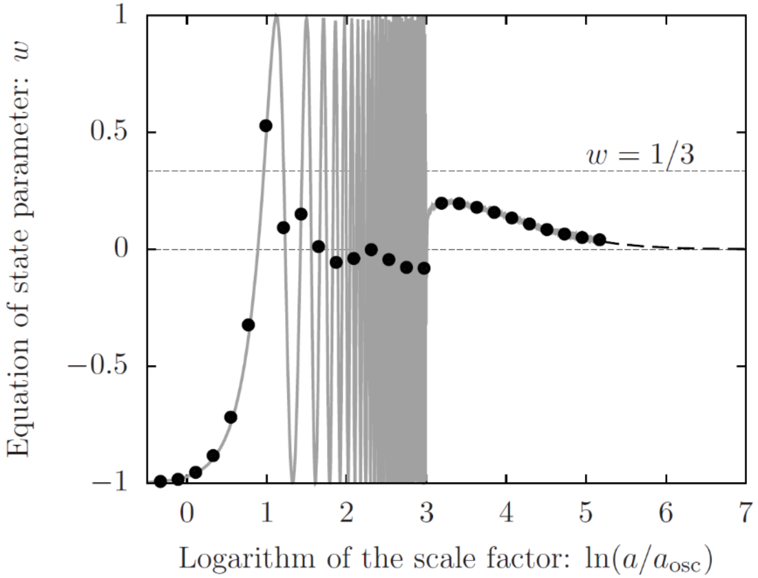
Amin et al, JCAP 1012 (2010) 001,
Kusenko, Mazumdar, PRL 101 (2008) 211301

Equation of state

Occupation numbers:



Equation of state parameter:



- The direct cascade freezes at late times.

- Axion-like particle dark matter and monodromy
- Nonperturbative dynamics
- Gravitational wave production
- Enhanced GW signal
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Linearized theory of gravity

- Linearized metric perturbations in synchronous gauge:

$$g_{\mu\nu}(t,\mathbf{x}) = \begin{pmatrix} 1 & 0 \\ 0 & -a^2(t)\Big(\delta_{ij} + h_{ij}(t,\mathbf{x})\Big) \end{pmatrix}, \quad |h_{ij}| \ll 1.$$

- Gravity waves: transverse-traceless perturbations, $\partial_i h_{ij} = h_{ii} = 0$,

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{\Delta h_{ij}}{a^2} = \frac{16\pi}{M_{\rm Pl}^2}\Pi_{ij}^{TT},$$

Garcia-Bellido et al., PRD 77, 043517 (2008).
Dufaux, et al., PRD 76, 123517 (2007).

with the anisotropic energy-momentum tensor

$$\Pi_{ij}(t,\mathbf{x}) = \frac{1}{a^2}\Big[\partial_i\varphi(t,\mathbf{x})\partial_j\varphi(t,\mathbf{x}) - \delta_{ij}\Big(\mathcal{L}(\varphi(t,\mathbf{x})) - \langle p\rangle\Big)\Big]$$

- Energy density in GWs: $\rho_{\rm GW}(t) = \frac{M_{\rm Pl}^2}{32\pi}\langle \frac{1}{2}\dot{h}_{ij}(t,\mathbf{x})\dot{h}_{ij}(t,\mathbf{x}) + \frac{1}{2}\nabla h_{ij}(t,\mathbf{x})\nabla h_{ij}(t,\mathbf{x})\rangle$

- Evolve h_{ij} numerically, using a modified version of the "HLATTICE" code.

Huang, PRD **83**, 123509 (2011).

Today's observables

Fractional energy density

in GWs today:

$$\frac{\rho_{\rm GW}}{\rho_{\rm c}} = \int d\ln\nu\,\Omega_{\rm GW}(\nu)$$

Today's frequency,

$$\nu = \frac{1}{2\pi}\frac{\mathrm{p}}{a}$$

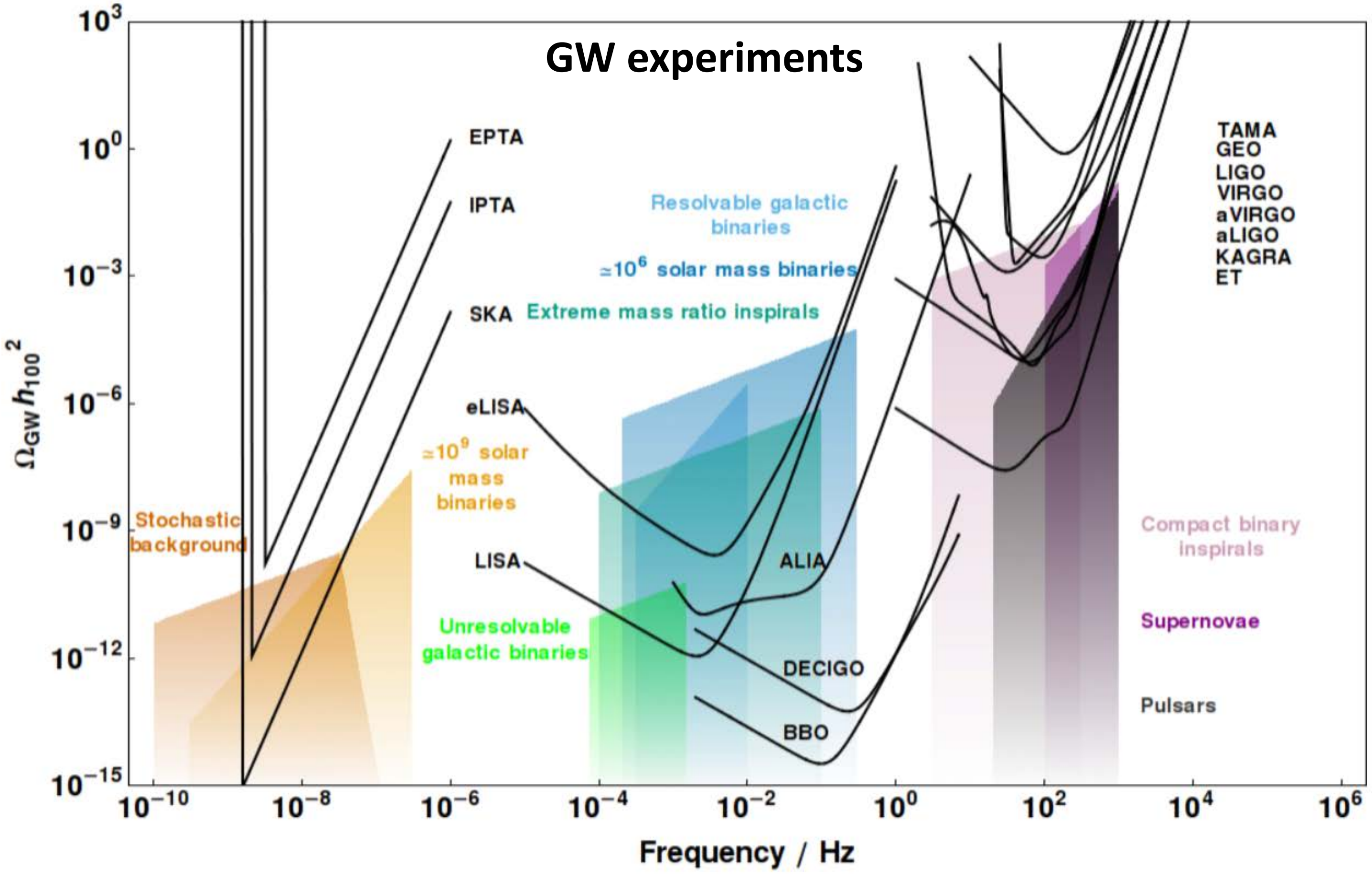
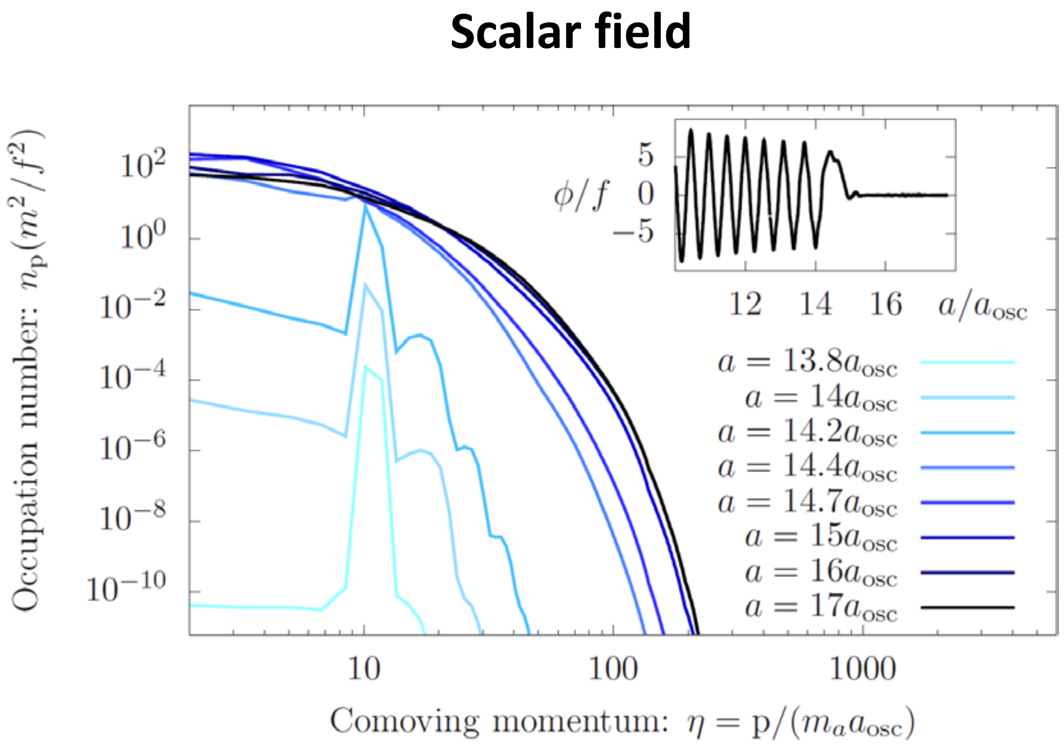
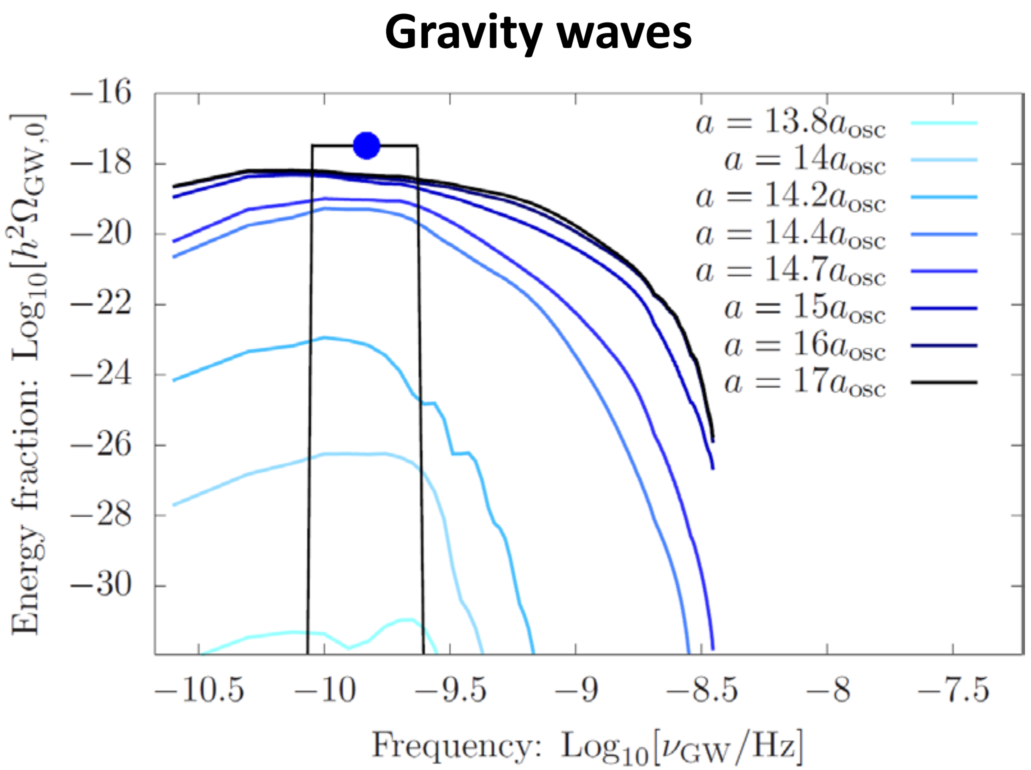


Figure from Moore et al (2015)

Gravity wave production





- Gravity waves produced during the fragmentation and the direct cascade.

GW signal range for ALP DM

- The frequency is determined by the mass, $\nu_\star \propto \eta_\star \sqrt{m_a}$
- The energy fraction in GWs is determined by the ALP energy density, $\rho_\varphi(a_{\rm emit})$.

- For ALP DM,

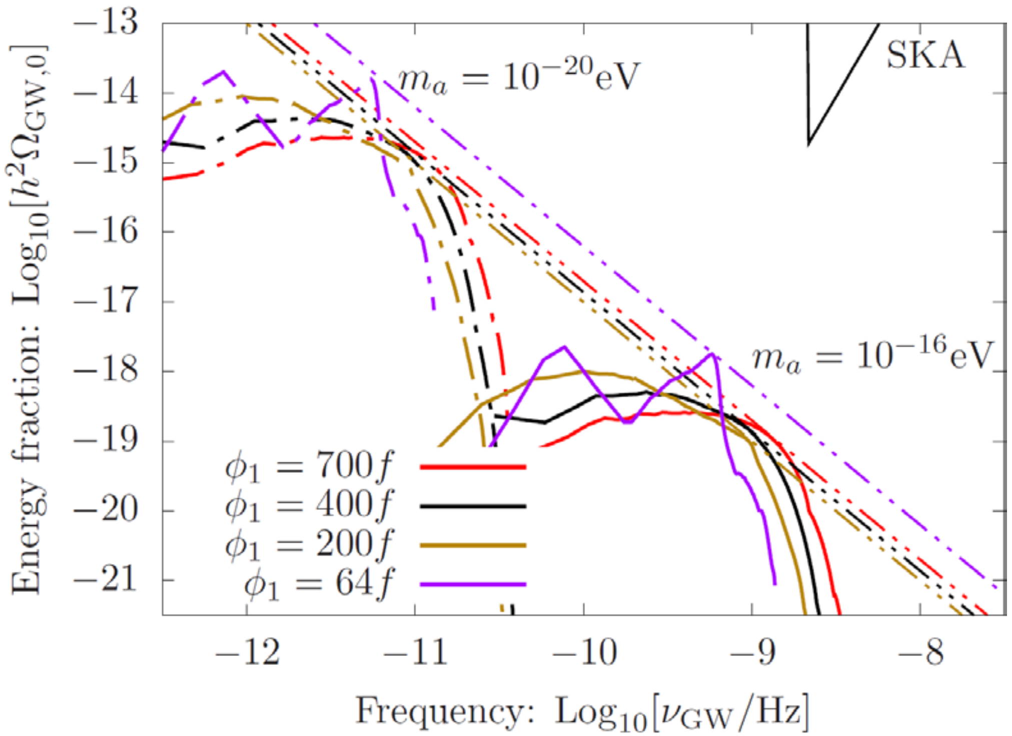
$$\rho_\varphi(a_0) = \rho_\varphi(a_{\rm emit}) \left(\frac{a_{\rm emit}}{a_0}\right)^3 \mathcal{Z}_0^{\rm emit}$$

Fixed

$$\mathcal{Z}_{a_2}^{a_1} = \exp\left(-3\int_{a_1}^{a_2} w_\varphi(\tilde{a})\, d\ln\tilde{a}\right)$$

- Allows to estimate the upper bound on the signal

$$\Omega_{\rm GW,0}(\nu_\star) \sim 2.7 \times 10^{-38} \frac{1}{(\nu_\star/{\rm Hz})^2} \frac{1}{(\mathcal{Z}_0^{\rm emit})^2} \frac{\alpha^2}{\beta}.$$



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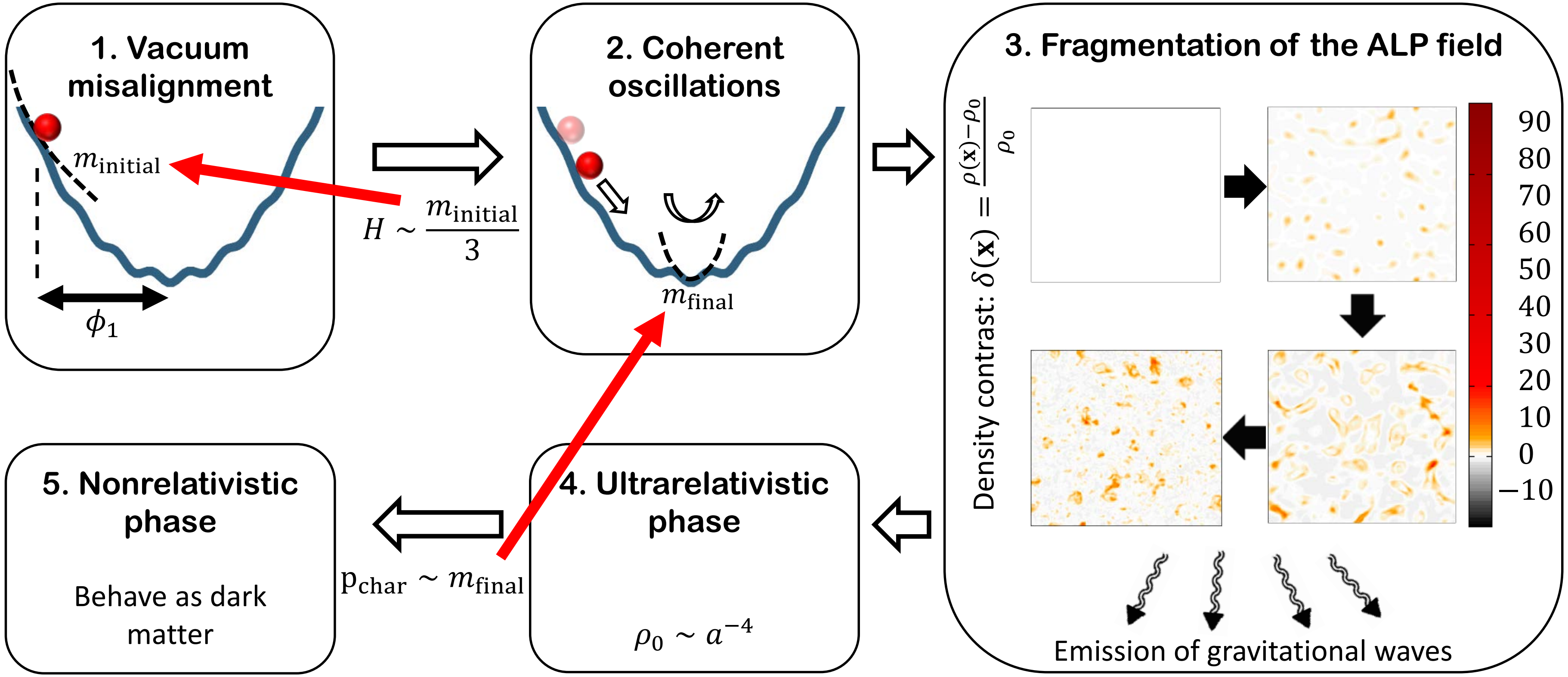
Extended ultrarelativistic phase



- Small mass near the bottom of the potential, $m_{\text{final}} = r m_a$, with $r \ll 1$.

Small mass near the bottom

- How to generate a small mass near the bottom?
  - Add a second "cosine": $$U(\phi) = \frac{1}{2}m^2\phi^2 + \Lambda^4\left(1 - \cos\frac{\phi}{f}\right) + \xi\Lambda^4\left(1 - \cos\frac{2\phi}{f}\right)$$

$$\xi \approx -1/4$$

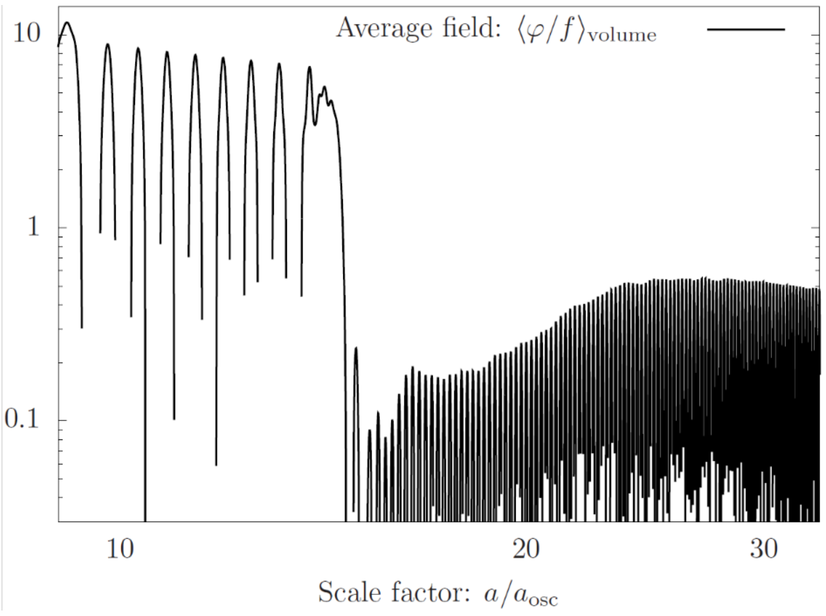
- Early-time dynamics in unaffected
- Transition to the ``mass-dominated'' regime, when $\langle\varphi^2\rangle \sim r^2 f^2$
- After $\langle\varphi^2\rangle \leq f^2$, high-order self interactions unimportant, $U(\varphi) = \frac{1}{2}m_{\rm final}^2\varphi^2 + \frac{\lambda}{4!}\varphi^4 + \ldots$
- Repulsive self-interactions $\Rightarrow$ Formation of a homogeneous condensate

Berges, et al.,
PRD **96**, 076020 (2017).

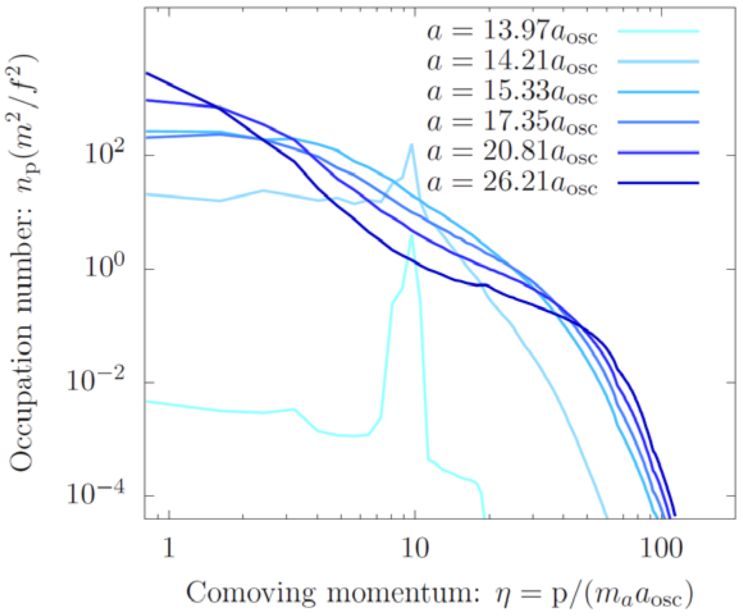
Guth et al.,
PRD **92**, 103513 (2015).

Gravity wave production

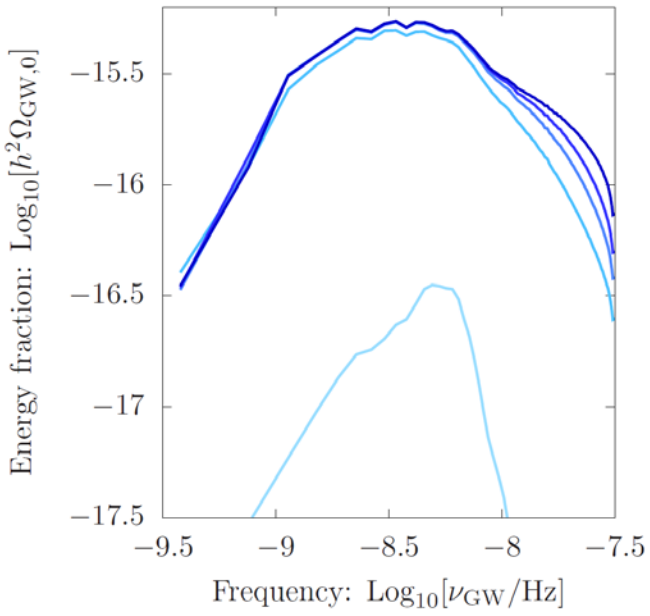
Condensation



Scalar field spectrum



GW spectrum



Problem: Numerical simulations become extremely expensive!

- Way out: self-similar evolution

Self-similar evolution

- The dynamics of the direct cascade becomes self-similar,

$$n(\tau, \mathrm{p}) = \left(\frac{\tau}{\tau_S}\right)^{\alpha} n_S\left(\left(\frac{\tau}{\tau_S}\right)^{\beta} \mathrm{p}\right) \quad \Longrightarrow \quad \bar{\mathrm{p}}(\tau) \propto \tau^{-\beta}$$

with $\beta \approx -1/5$, $\alpha \approx 4\beta$.

Micha and Tkachev,
PRD 70, 043538 (2004).

- Studied in the context of reheating.

- Origin: weak wave turbulence
  - Energy transport to high momenta

Zakharov and Kolmogorov,
Springer-Verlag, Berlin (1992).

Kinetic description

- Description in terms of slowly-changing occupation numbers,

$$\frac{\partial n(\tau,\mathbf{p})}{\partial \tau} = C[n](\tau,\mathbf{p})$$

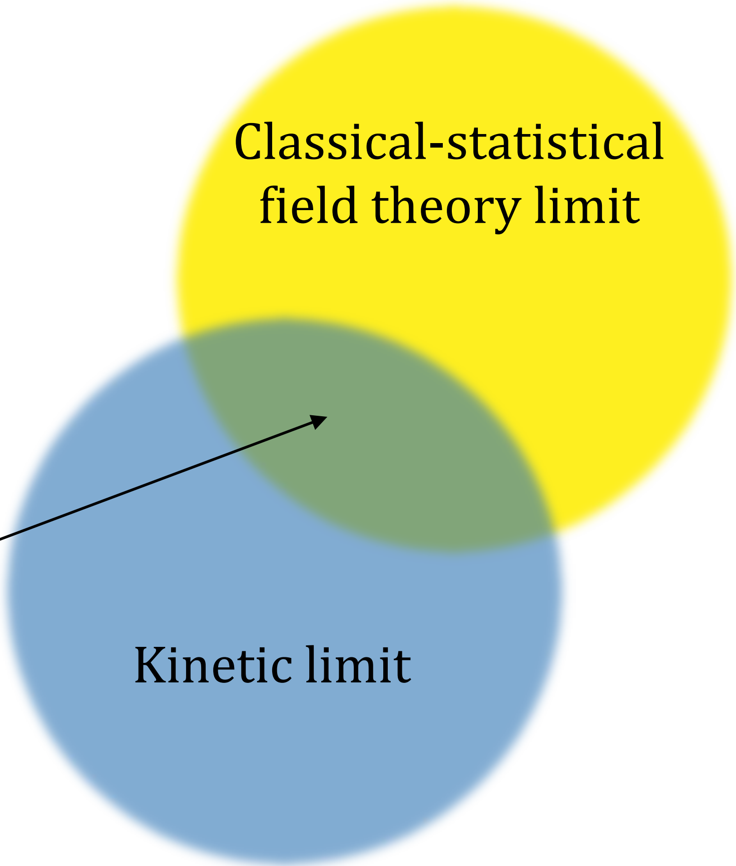
Boltzmann equation

- Can be derived from QFT first principles

Berges, 1503.02907

Berges and Borsanyi, PRD **74**, 045022 (2006).

Quantum field dynamics



Wave kinetic theory

Kinetic description of the cascade

- The value $\beta = -1/5$ can be understood from weak wave turbulence for three-wave interactions,

$$\partial_\tau n(\tau, \mathrm{p}) = C_3[n](\tau, \mathrm{p})$$

with

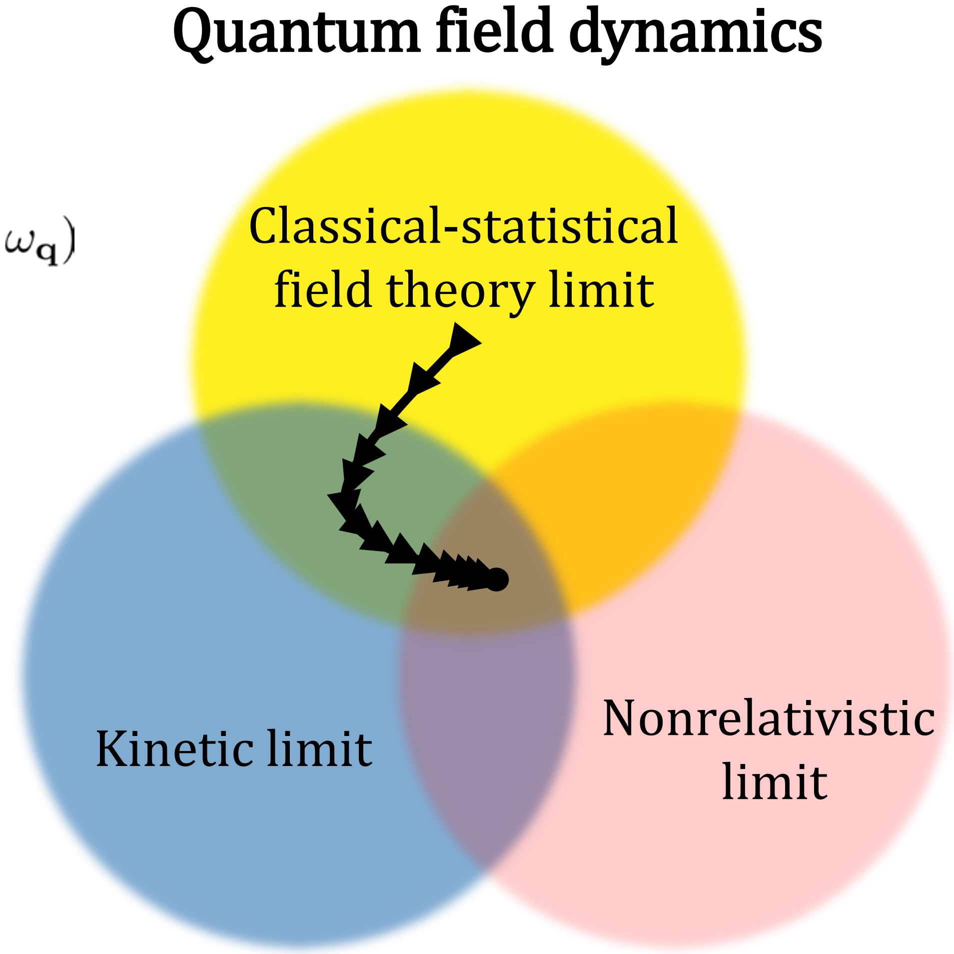
$$C_3 = \frac{\lambda^2 \langle \phi_c^2 \rangle}{2} \int \frac{d^3\mathrm{k} d^3\mathrm{q} \delta^{(3)}(\mathbf{p}-\mathbf{k}-\mathbf{q})}{(2\pi)^2 2\omega_\mathbf{p} 2\omega_\mathbf{k} 2\omega_\mathbf{q}} \delta(\omega_\mathbf{0} + \omega_\mathbf{p} - \omega_\mathbf{k} - \omega_\mathbf{q}) \left[n_\mathbf{k} n_\mathbf{q} - n_\mathbf{p}(n_\mathbf{k} + n_\mathbf{q}) \right] + \ldots$$

Micha and Tkachev,
PRD 70, 043538 (2004).

and $\omega_\mathbf{p}^2 = \mathrm{p}^2 + M^2 a^2 \approx \mathrm{p}^2$.

- $\alpha = 4\beta$ follows from energy conservation.

- Deviations from scaling when $ma \sim \mathrm{p}$,
  - $C_3 \to 0$ (freezing)



Simplified kinetic analysis

- Introduce characteristic quantities $\bar{n}(\tau)$ and $\bar{\mathrm{p}}(\tau)$

- Simplified kinetic equation

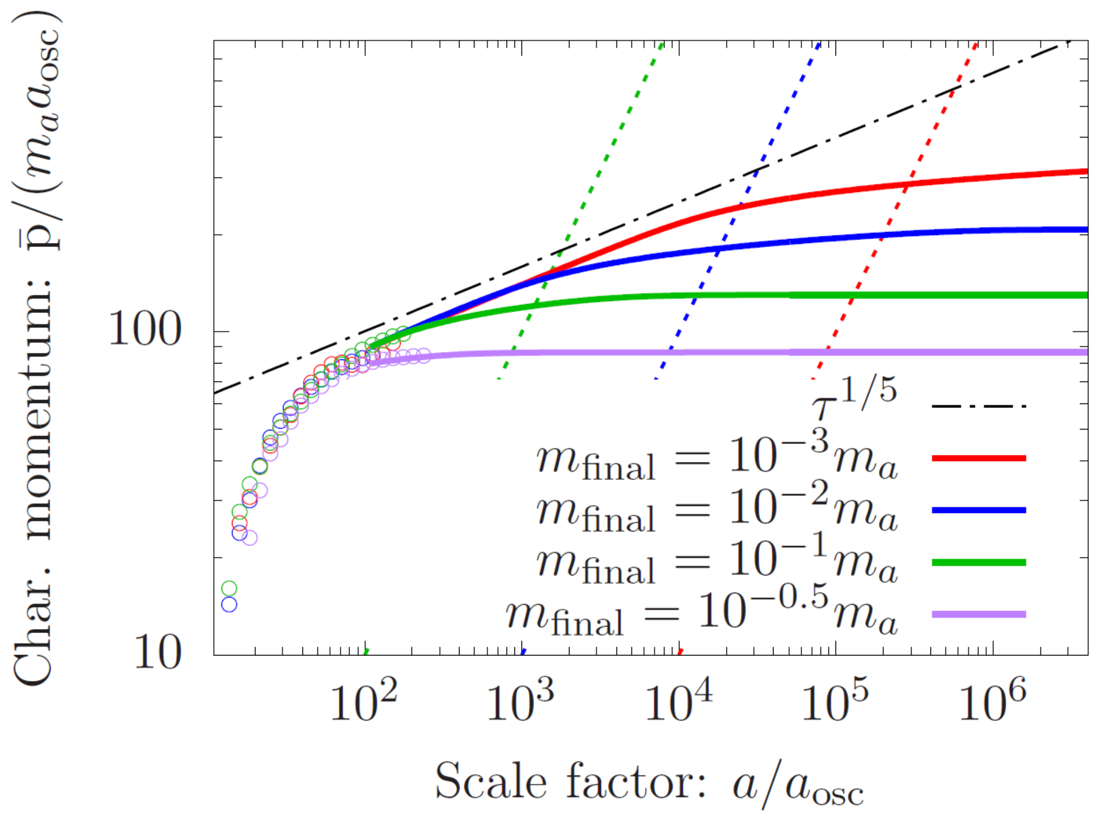
$$\bar{n}' \approx C_{(3)} \frac{\langle \phi_c^2 \rangle \bar{n}^2 \bar{\mathrm{p}}}{\bar{\omega}^2},$$

- Conservation of energy ($\rho \propto a^{-3(1+w)}$)

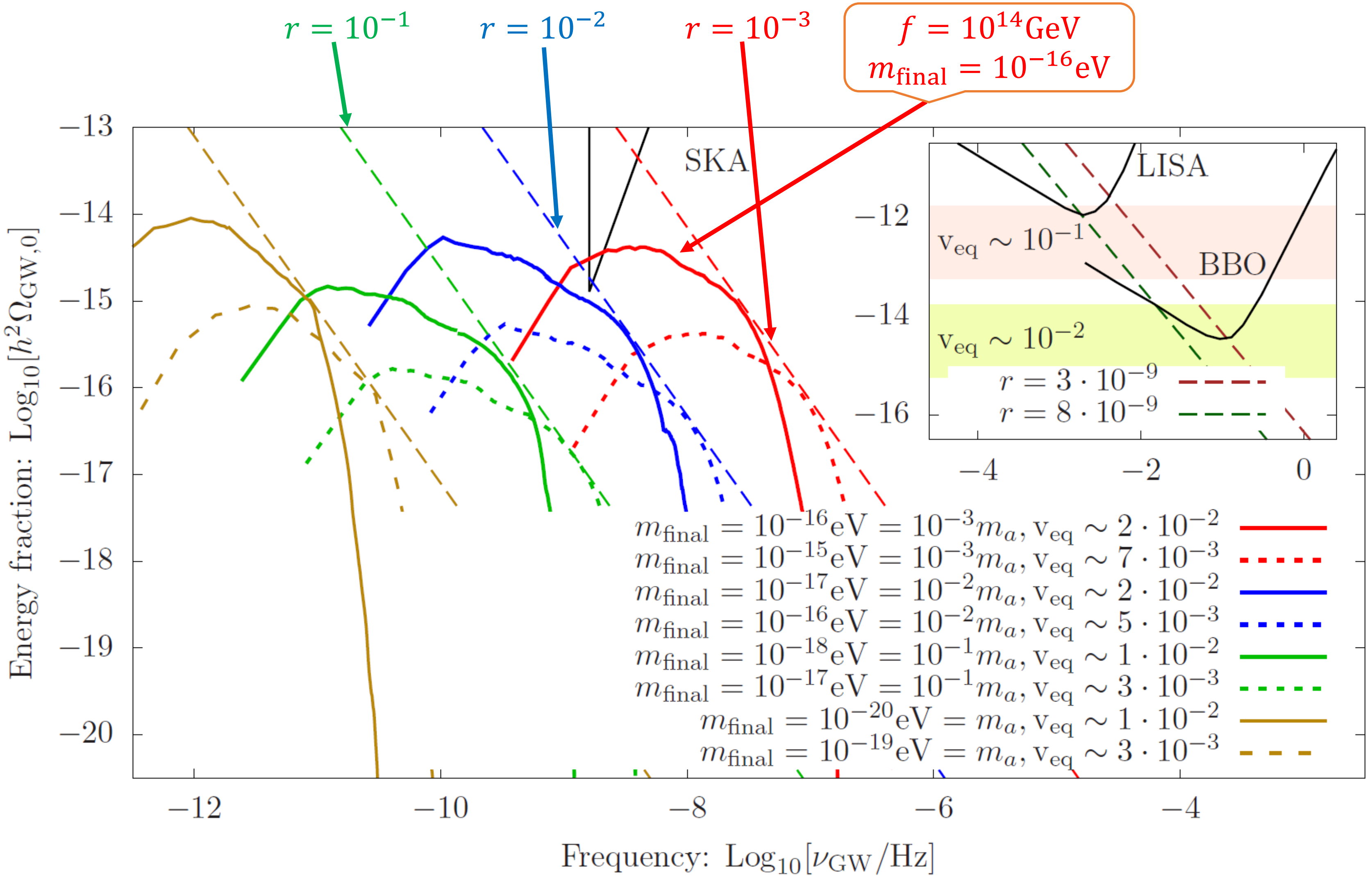
$$3\frac{\bar{\mathrm{p}}'}{\bar{\mathrm{p}}} + \frac{\bar{n}'}{\bar{n}} + \frac{\bar{\omega}'}{\bar{\omega}} = \frac{1-\bar{\mathrm{p}}^2/\bar{\omega}^2}{a}$$

- Dispersion relation

$$\bar{\omega}^2 = \bar{\mathrm{p}}^2 + m_b^2 a^2 + \frac{\lambda \langle \phi_c^2 \rangle}{2}$$



Final gravitational spectra



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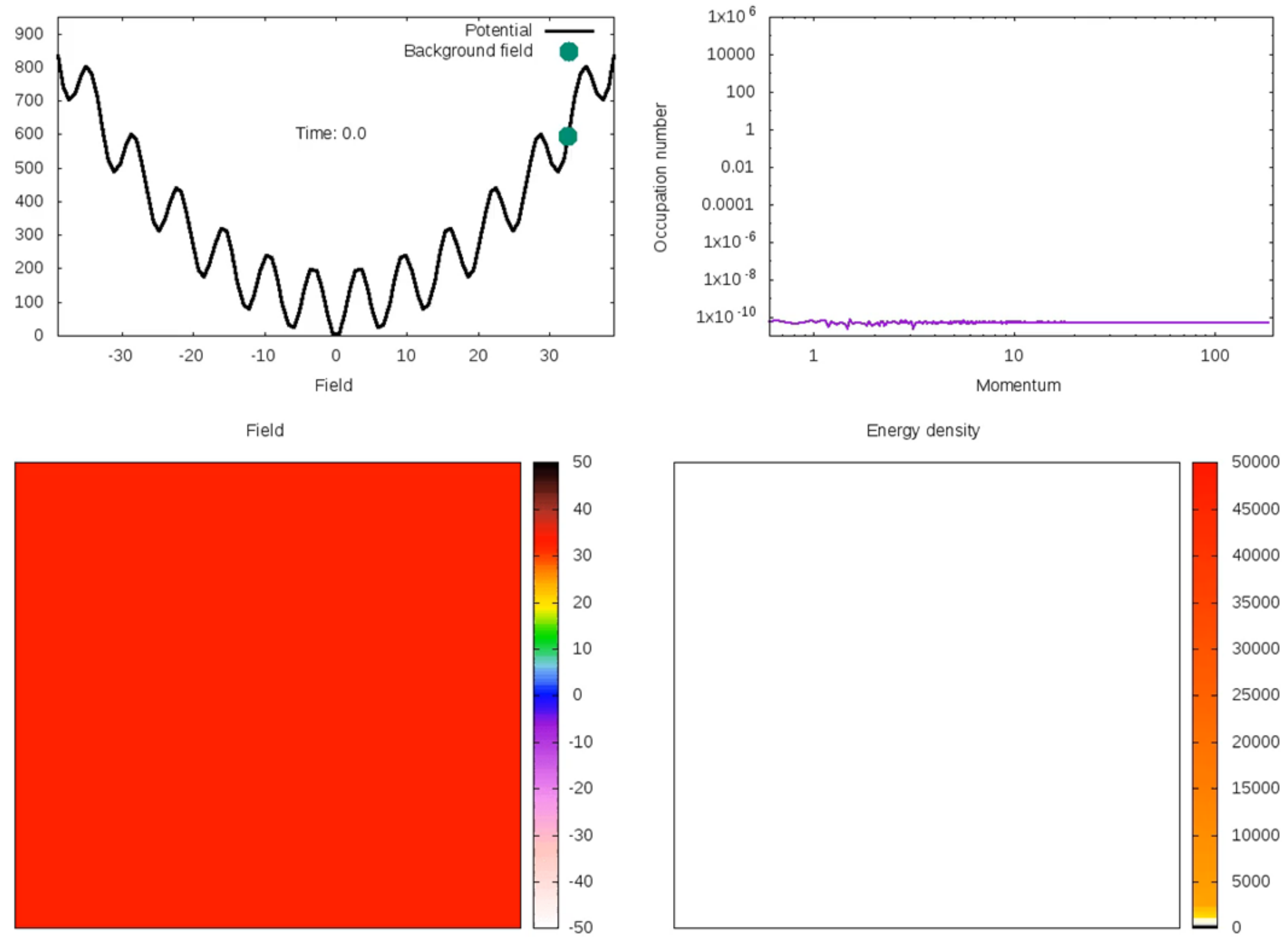
Transitions between local minima

- The field can get trapped

in a local minimum.

⇩

Dynamical bubble nucleation



Summary

- In contrast to standard ALP DM, the evolution over "wiggly" potentials can lead to an amplification of fluctuations and the production of GWs.

- If there is an extended ultrarelativistic phase of the dynamics , fragmentation of ALP DM can produce a GW signal that can be explored with future experiments,

Machado et al., JHEP 01, 053 (2019).

Machado et. al, 1912.01007

$m_{\text{final}} = 10^{-16}\text{eV}, r \sim 10^{-3}$
(SKA)

$m_{\text{final}} = 10^{-8}\text{eV}, r \sim 10^{-10}$
(BBO)

- GWs allow a complementary probe of such ALP DM and its nonperturbative dynamics.

Thanks for your attention!