Virtual photon velocities and the uncertainty of localization of a quark in the proton in DIS at HERA. Part I

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Plan

At a next ZAF, Part II

- Virtual photon velocities

►Today, Part I (a simpler part):

the accuracy of localization of a quark

- **General remarks and motivation**
- **A model, the mathematical framework (a tool)**
- **Results**
- **Conclusions**

Motivation:

2016 : The ZEUS remarkable paper
Limits on the effective quark radius from inclusive ep scattering at HERA
DESY-16-035 (February 2016) / Phys. Lett. B 757 (2016) 468

For general public nicely introduced by Jon Butterworth in April 2016:
How big is a quark?
(www.theguardian.com/science/life-and-physics/2016/apr/07/how-big-is-a-quark)

2016, January, February: discussions with Filip and Iris

Boris**: With Rq and Q in hands, even possible to check how well works an estimation of Rq based on the Heisenberg's uncertainty relation,**

$$R_q \sim h/Q \text{ ?}$$

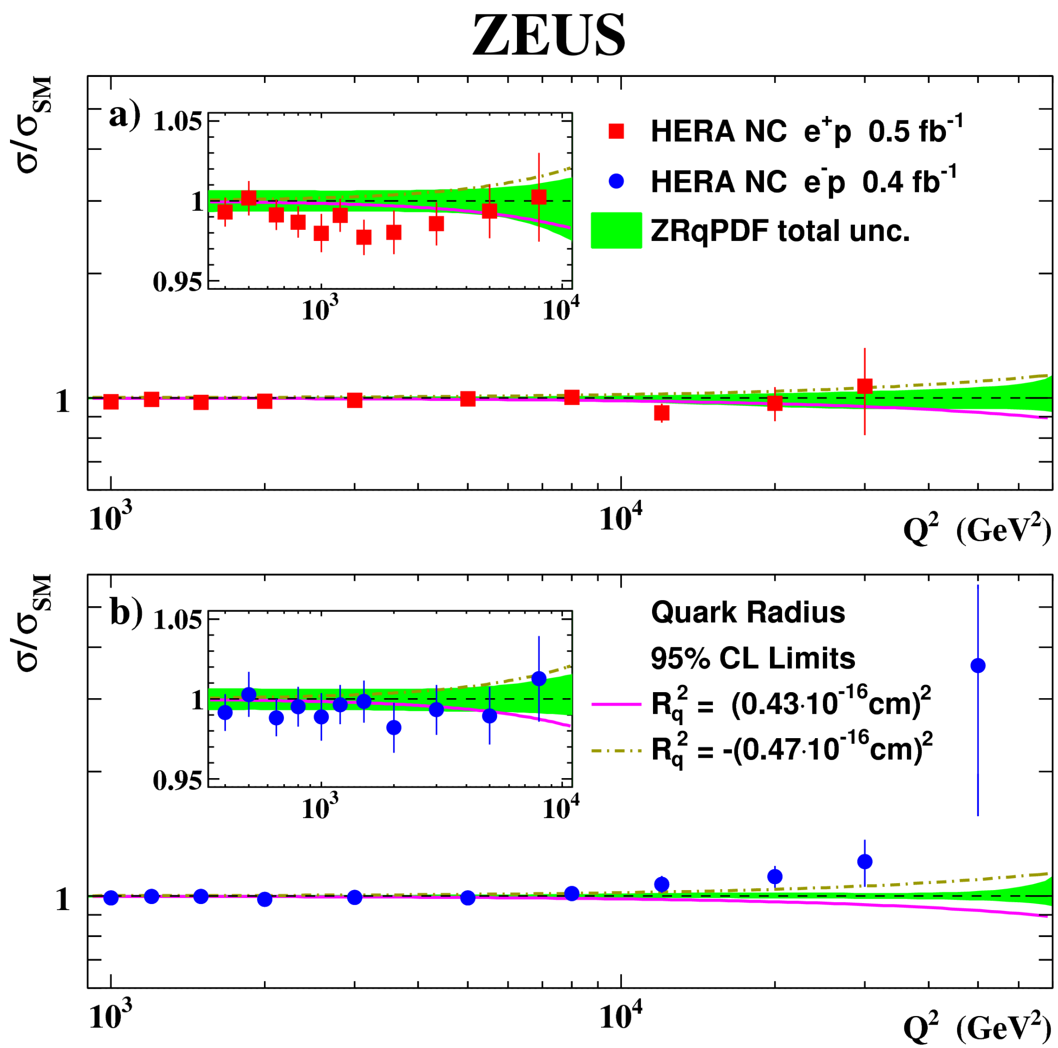
The effective «quark radius» limits

Phys. Lett. B757 (2016) 468, arXiv:1604.01280

A model

$$\frac{d\sigma}{dQ^2} = \frac{d\sigma^{\rm SM}}{dQ^2}\left(1 - \frac{R_q^2}{6}Q^2\right)^2$$

$$\bar{r_q} = \sqrt{|\langle R_q^2 \rangle|} < 0.43 \cdot 10^{-3}\ \mathrm{fm}$$



The term "quark radius" is only one possible interpretation of BSM effects parameterized as form factors.

$$-(0.47 \cdot 10^{-16}\mathrm{cm})^2 < R_q^2 < (0.43 \cdot 10^{-16}\mathrm{cm})^2$$

How big is a quark?

In the macro-world, to measure the size of the simplest geometric entity, 1D line segment, it is necessary to measure the coordinates of the ends of the segment and compile their difference:

$$\boldsymbol{L} = \sqrt{(z_2 - z_1)^2}$$

A minimum of two coordinate measurements are required to define a geometric size.

In the microworld, we do not have a similar method for directly measuring within a particle two coordinates simultaneously.

However, we have the method that allows us to estimate an accuracy of this type (hypothetical) measurement.

The Heisenberg's uncertainty relations (HUR, 1925)

connect the uncertainty in the particle position (x,y,z) with the uncertainty of the conjugated momentum of the same particle. For instance for the z-component

$$[\hat{z}, \hat{p_z}] = i\hbar \implies \sqrt{\langle(\Delta z)^2\rangle\langle(\Delta p_z)^2\rangle} \geq \frac{\hbar}{2}$$

In the relativistic domain HUR is also valid (Landau, Peierls,1931)

In our case, the particle is a quark within the proton with which a virtual boson interacts.

Since the transfered 4-momentum q is some indirect way related to the quark momentum after a collision, we need to specify a model how the proton momentum is distributed between quarks before a collision.

The parton model (Feynman)

Variables $x_{Bj}, \; Q^2, \; x_{Bj} y s = Q^2, \; E_e, \; E_p, \; s = 4 E_e E_p$

$$p_z = x_{Bj} \cdot P \implies \sqrt{\langle (\Delta p_z)^2 \rangle} = \sqrt{\langle (\Delta x_{Bj})^2 \rangle} P$$

With a virtual photon we «see directly» only quarks

Sharpen your tool before tackling a task
(Chinese saying)

The tool : Indirect measurements

Bases of the theory of errors
T.A. Agekyan, 1972
Measurement Errors and Uncertainties
S. G. Rabinovich, 2005

Very often it happens that the $x_1, x_2, \ldots, x_m$ values to be determined cannot be directly measured.

However, one can measure $y_1, y_2, \ldots, y_k$ variables that are known functions of $x_1, x_2, \ldots, x_m$ variables,

$$f(x_1, x_2, \ldots, x_m) = y_i$$

Example, 1D: $z = \phi(x), \;\; z \approx \phi(\bar{x}) + \phi'(x - \bar{x})$

$$\langle(\Delta z)^2\rangle = \langle z^2\rangle - \langle z\rangle^2 = \langle(\phi(\bar{x}) + \phi' \cdot (x - \bar{x}))^2\rangle - \phi(\bar{x})^2 = (\phi')^2 \cdot \sigma_x^2$$

$$\sigma_x^2 = \langle(\Delta x)^2\rangle = \langle x^2\rangle - \langle x\rangle^2$$

The standard or dispersion for several variables

$$z = \mathcal{F}(x,y), \;\; \langle(\Delta z)^2\rangle = \left(\frac{\partial \mathcal{F}}{\partial x}\right)^2 \langle(\Delta x)^2\rangle \; + \; \left(\frac{\partial \mathcal{F}}{\partial y}\right)^2 \langle(\Delta y)^2\rangle$$

For the selected model, the parton model

$$\langle(\Delta x_{Bj})^2\rangle \;=\; \left(\frac{\partial x_{Bj}}{\partial Q^2}\right)^2 \langle(\Delta Q^2)^2\rangle + \left(\frac{\partial x_{Bj}}{\partial y}\right)^2 \langle(\Delta y)^2\rangle$$

$$\Rightarrow \; \langle(\Delta x_{Bj})^2\rangle \;=\; \left(\frac{x_{Bj}}{Q^2}\right)^2 \langle(\Delta Q^2)^2\rangle + \left(\frac{x_{Bj}}{y}\right)^2 \langle(\Delta y)^2\rangle$$

$$\langle(\Delta Q^2)^2\rangle \;=\; \frac{4Q^2}{1-y_e}\langle(c\Delta p_t)^2\rangle + \left(\frac{Q^2}{1-y_e}\right)^2 \langle(\Delta y)^2\rangle$$

That is

$$\langle(\Delta x_{Bj})^2\rangle = \frac{4x_{Bj}^2}{(1-y)Q^2}\langle(c\Delta p_t)^2\rangle + x_{Bj}^2\left[\frac{1}{(1-y)^2}+\frac{1}{y^2}\right]\langle(\Delta y)^2\rangle$$

$\Uparrow$ $\Uparrow$

As you can see, it is not enough to know only the $Q^2 \wedge x_{Bj}$ variables.

You must also know the accuracy with which the ZEUS detector works.

So,

$$\Delta p_t = \left(\frac{\sigma(p_t)}{p_t}\right)p_t = p_t\sqrt{(0.0058p_t)^2+(0.0065)^2+(0.0014/p_t)^2}$$

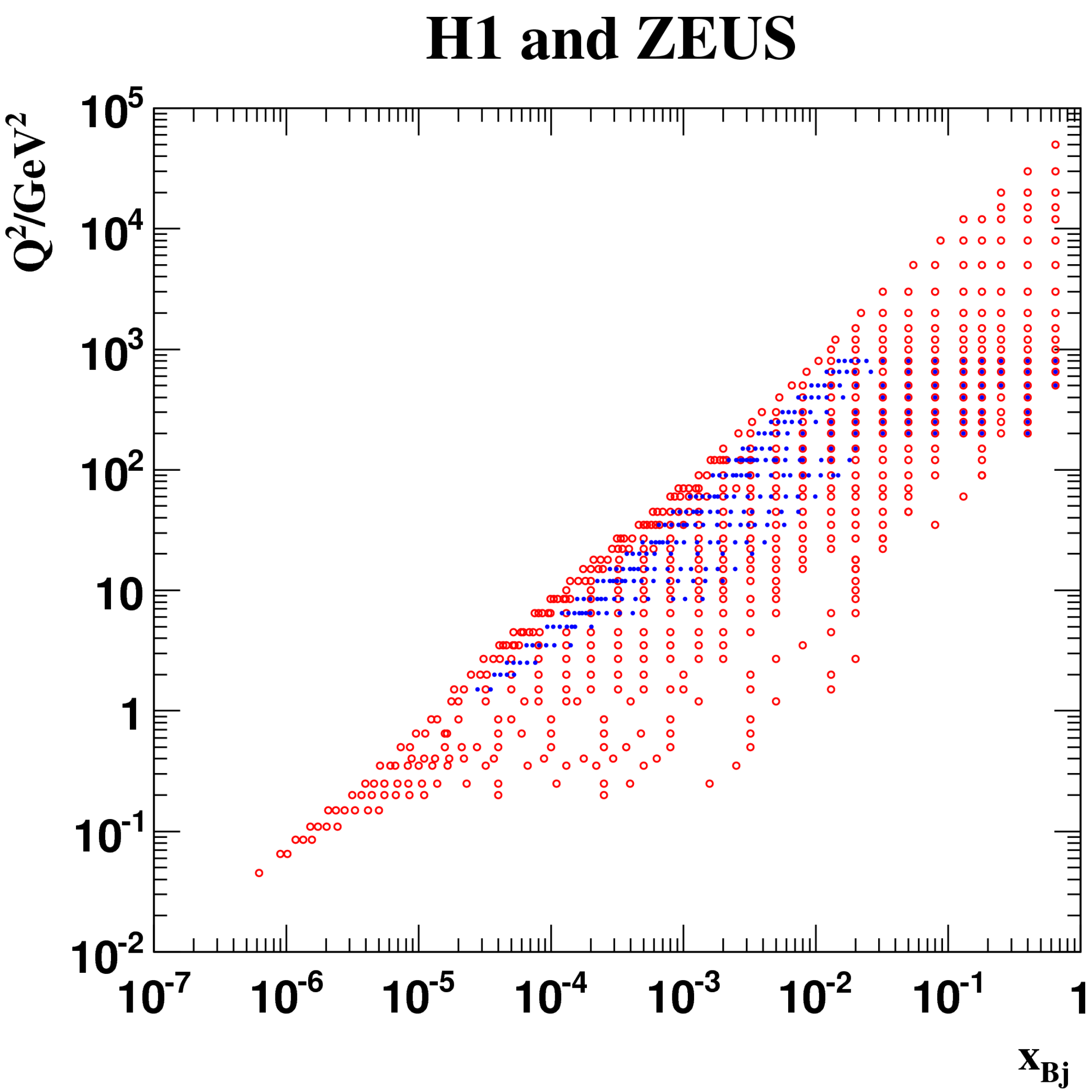
and

$$\sigma_\Sigma/\Sigma_e = [0.18/\sqrt{\Sigma_e}],$$

$$\Delta\Sigma_e = \sigma_\Sigma = \left[\frac{0.18}{\sqrt{\Sigma_e}}\right]\Sigma_e = 0.18\sqrt{2(1-y)k_e} \Rightarrow \Delta y = 0.18\sqrt{\frac{1-y}{2ck_e}}$$

Combination of H1 and ZEUS data

Data range



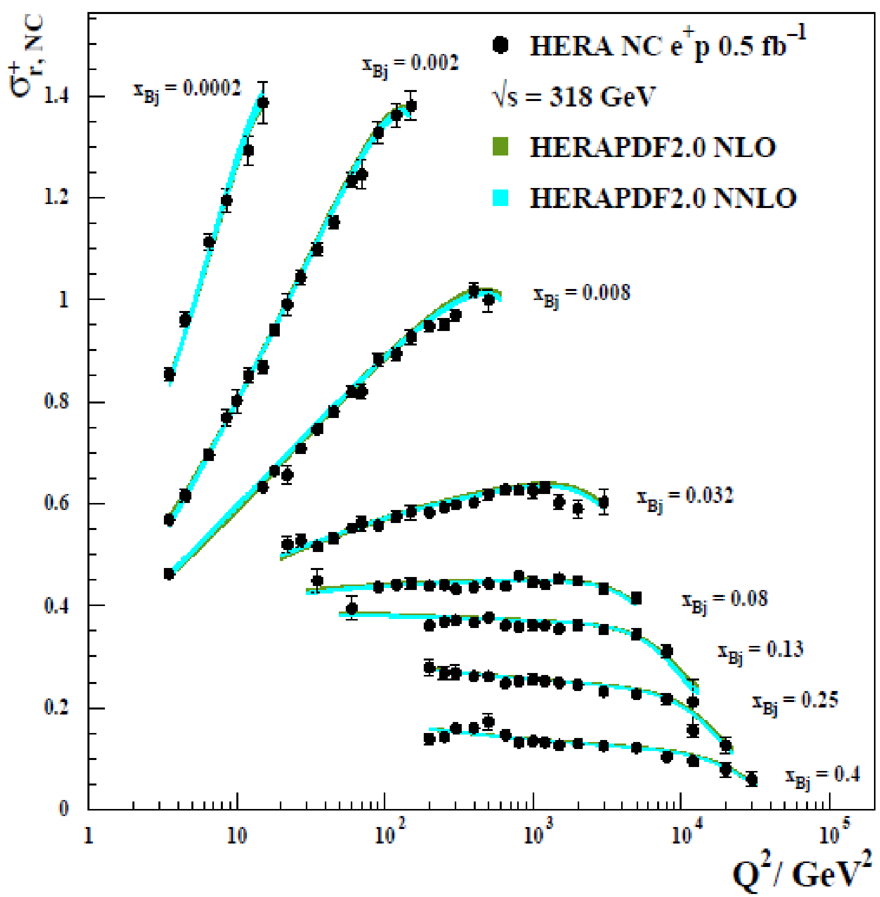
Data on Q2 and X_Bj in the tabulated form

Combination of Measurements of Inclusive Deep Inelastic e+- p Scattering Cross Sections and QCD Analysis of HERA Data

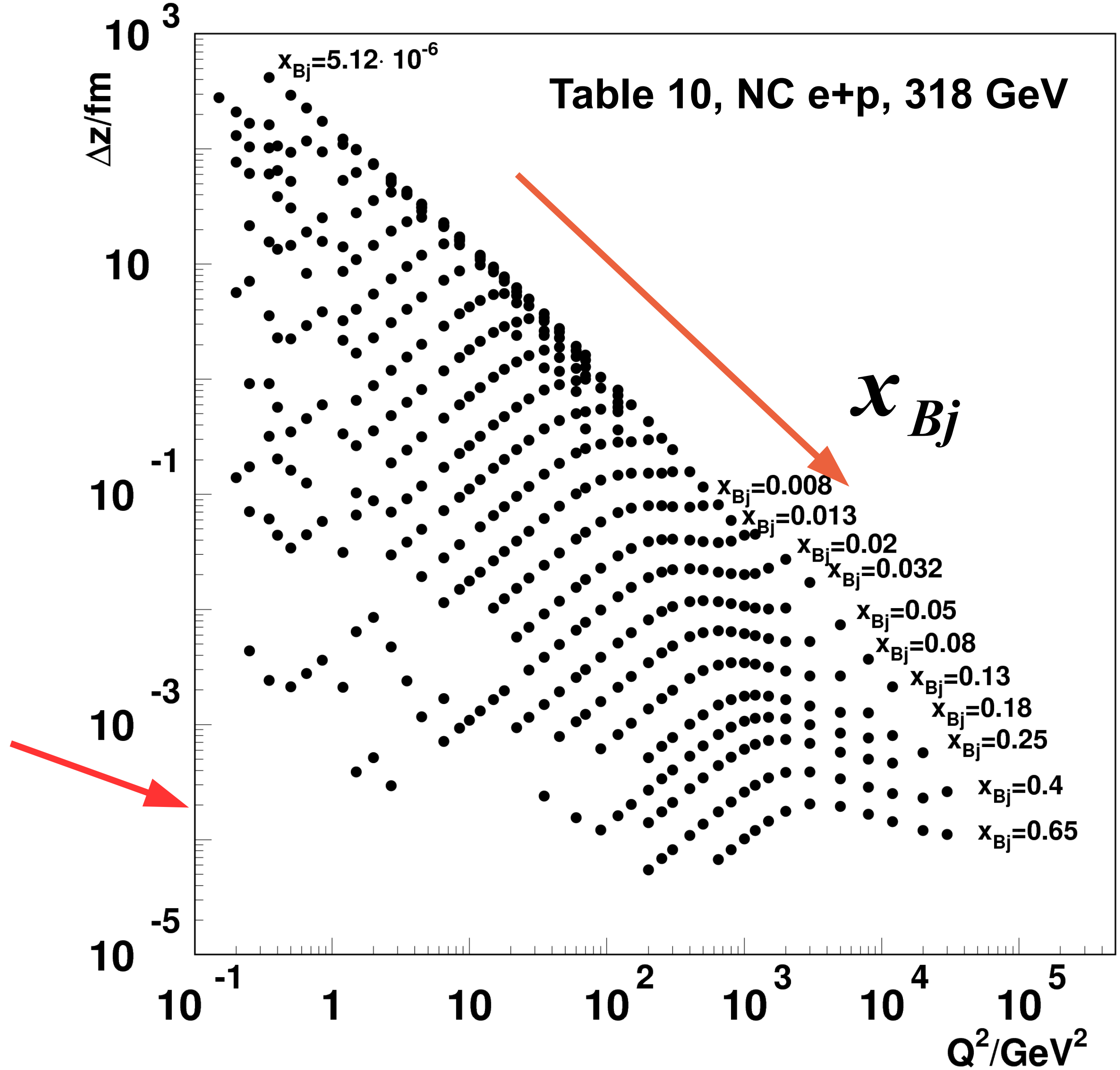
Eur. Phys. J. C 75 (2015) 580

Table 10, NC e+p, 318 GeV

Q^2 GeV^2	x_{Bj}	$\sigma^+_{r,NC}$	δ_{stat} %	δ_{uncor} %	δ_{cor} %	δ_{rel} %
0.15	0.502×10^{-5}	0.185	3.79	1.50	3.62	1.39
0.2	0.669×10^{-5}	0.227	1.65	0.78	1.70	0.86
0.2	0.849×10^{-5}	0.223	1.61	0.61	2.19	1.06
0.2	0.110×10^{-4}	0.208	2.79	1.50	2.83	1.01
0.2	0.398×10^{-4}	0.211	14.93	11.96	5.18	0.33
0.2	0.251×10^{-3}	0.180	13.49	6.17	3.00	0.32
0.25	0.836×10^{-5}	0.265	1.46	0.73	1.92	1.17
0.25	0.106×10^{-4}	0.260	1.29	0.66	1.84	1.11
0.25	0.138×10^{-4}	0.249	1.27	0.72	1.85	1.24
0.25	0.230×10^{-4}	0.243	1.41	1.50	2.37	2.23
0.25	0.398×10^{-4}	0.236	3.32	1.54	2.79	0.50



Results



$$\Delta z \geq \frac{\hbar}{2}\frac{1}{\sqrt{\langle(\Delta x_{Bj})^2\rangle P}}$$

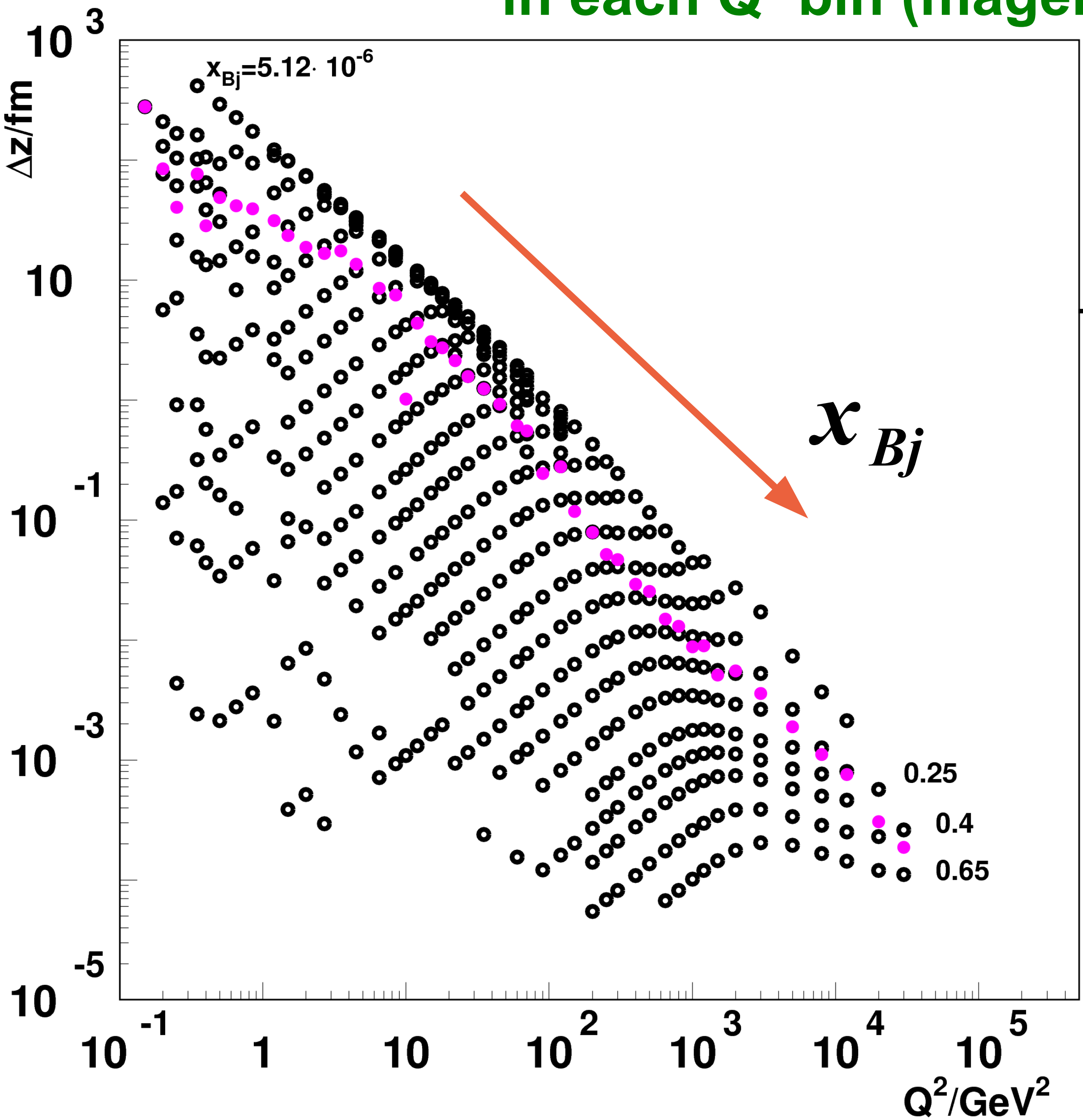
$$Q^2 = \frac{(cp_t)^2}{1-y_e}$$

At fixed x_{Bj} the best resolution is at smallest Q^2 .

See the set with $x_{Bj} = 0.002$

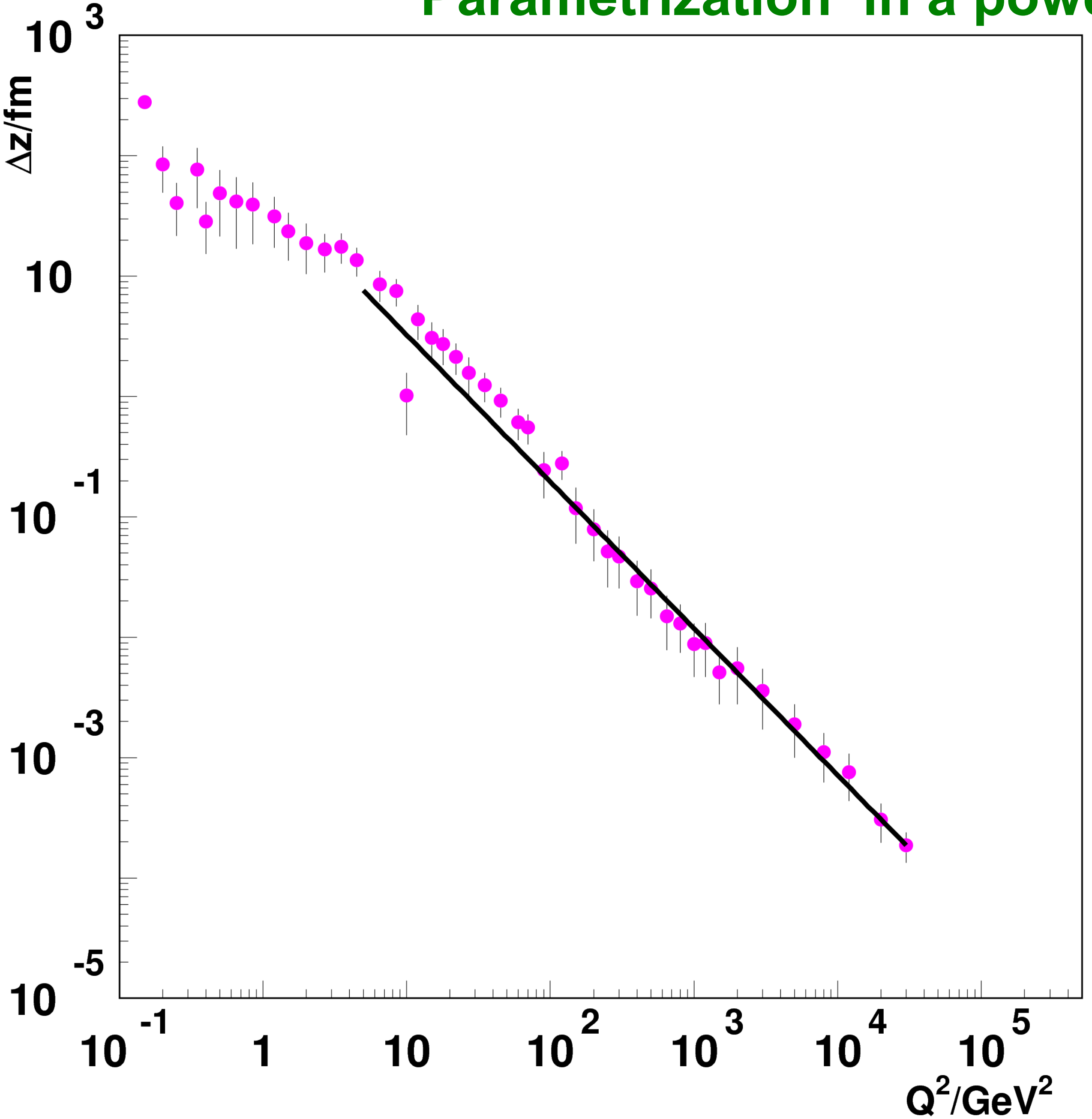
$$\bar{r_q} = \sqrt{|\langle R_q^2\rangle|} < 0.43 \cdot 10^{-3}\,\mathrm{fm}$$

After calculating the arithmetic mean in each Q^2 bin (magenta points)



The resolution getting worse

Parametrization in a power like form



$$\Delta z = A/(Q^2)^B$$

A = 53.97± 8.81

B = 1.22± 0.27

In the Q^2 range
5 - $3 \cdot 10^4$ GeV^2

Which does not coincide with a naive estimate

$$\Delta z \sim 1/Q$$

Conclusions

- With the use of the method of indirect measurements and the Heisenberg uncertainty principle, the accuracy of quark localization in the proton as a function of x_{Bj} and Q^2 is estimated.

- So far, I used only published H1 and ZEUS data with common (x_{Bj} , Q^2) grid (NC, e+p, 318 Gev, Table 10, DESY-15-039).

- This analysis indicates that the best localization Δz , for a given x_{Bj} is achieved at the lowest Q^2 and therefore with a larger data set.

- At a next ZAF I'm going to present an update of Δz vs (x_{Bj}, Q^2) with the ZEUS CN data and the Part II.

- Your comments and feedbacks are welcome!