EVADING THE GROSSMAN-NIR BOUND (INSPIRED BY RECENT KOTO “ANOMLY”)

DESY Jounal Club, June 11th 2020

Zhuoni Qian

arXiv: 2005.00451

Three Exceptions to the Grossman-Nir Bound

Robert Ziegler,[1] **Jure Zupan,**[2] **Roman Zwicky**[3]

[1] *Institute for Theoretical Particle Physics (TTP), Karlsruhe Institute of Technology, Engesserstrasse 7, D-76128 Karlsruhe, Germany*

[2] *Department of Physics, University of Cincinnati, Cincinnati, Ohio 45221,USA*

[3] *Higgs Centre for Theoretical Physics, School of Physics and Astronomy, University of Edinburgh, Edinburgh EH9 3JZ, Scotland*

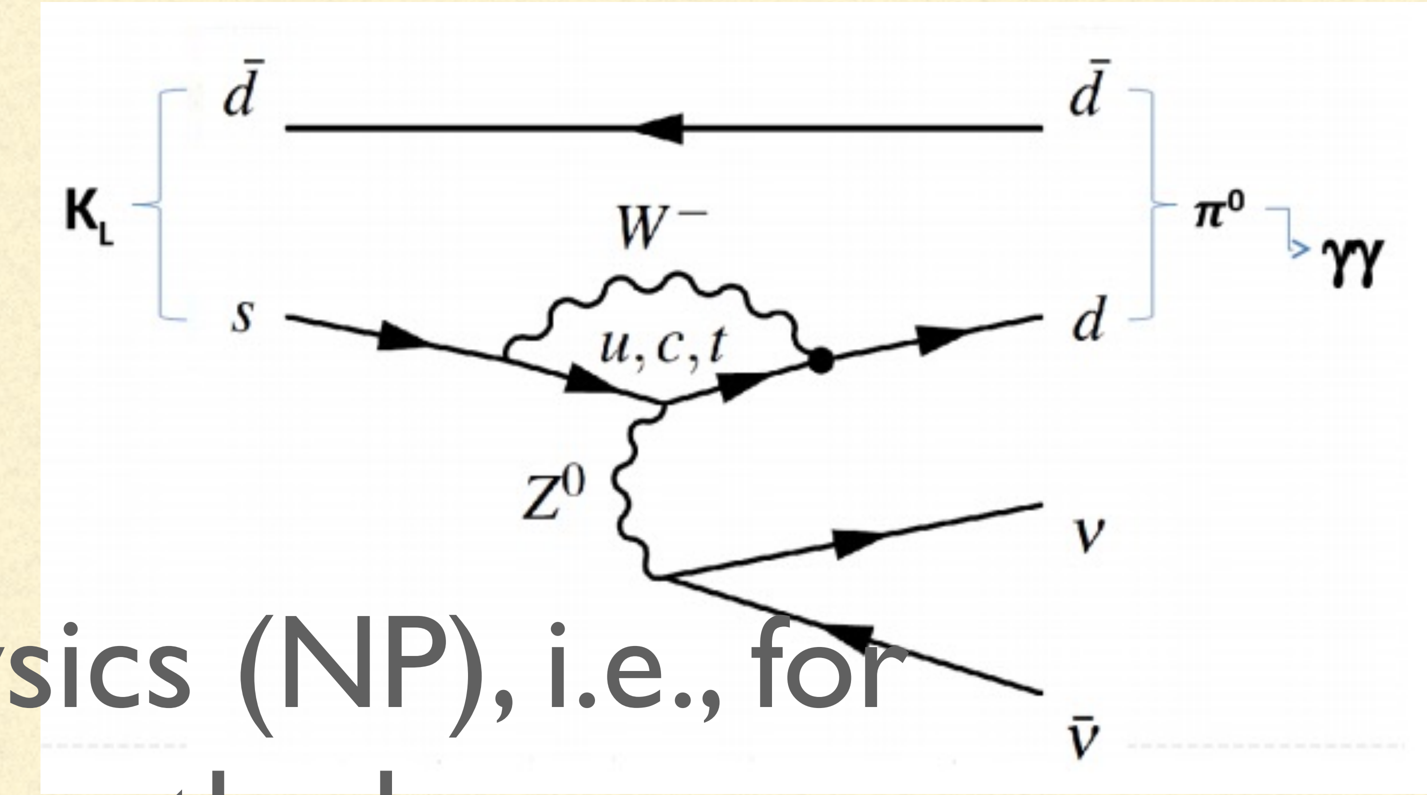
Recent arXiv:
1911.03755, 1911.10203,
1911.12334, 1912.10433,
2001.06522, 2001.06572,
2002.05467, 2005.00753,

and more…

Grossman-Nir Bound

$$\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu}) \leq 4.3\,\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu}).$$

Grossman, Nir (1997)



The bound remains valid in the presence of heavy New Physics (NP), i.e., for NP modification due to new particles with masses well above the kaon mass. The bound is saturated for the case of maximal CP violation.

$(s \to d\nu\bar{\nu})$ C× $[\bar{s}\gamma^\mu(1-\gamma_5)d]\,[\bar{\nu}\gamma_\mu(1-\gamma_5)\nu]\,.$

{K⁺ ,K$_L$}, Related with iso-spin symmetry {u,d}

$$\frac{\mathrm{Br}\,K_L}{\mathrm{Br}\,K^+} = \frac{\Gamma_+^{tot}}{\Gamma_L^{tot}} \cdot \frac{|M_L|^2}{|M_+|^2} = \underbrace{\frac{\tau_L}{\tau_+}}_{\sim 4.1} \; \underbrace{\frac{A_+}{A_L}}_{\sim 1} \; \underbrace{\frac{\mathrm{Im}\,C}{C}}_{\sim \eta^2}$$

$\eta \sim 0.35$. CP-angle in Wolfenstein param.

- Br(K+ → π+νν̄)SM = (8.4±1.0)×10^{-11}
- Br(K+ → π+νν̄)exp < 1.85 × 10^{-10} [NA62, 2020]

- Br(K_L → π^0νν̄)SM = (3.4 ± 0.6) × 10^{-11}
- Br(K_L → π^0νν̄)exp < 3.0 × 10^{-9} [KOTO, 2019]
- Br(KL → π^0 + inv) = 2.1+2.0 × 10–9 (S. Shinohara, Talk at KAON201909, arXiv 1909.11111)

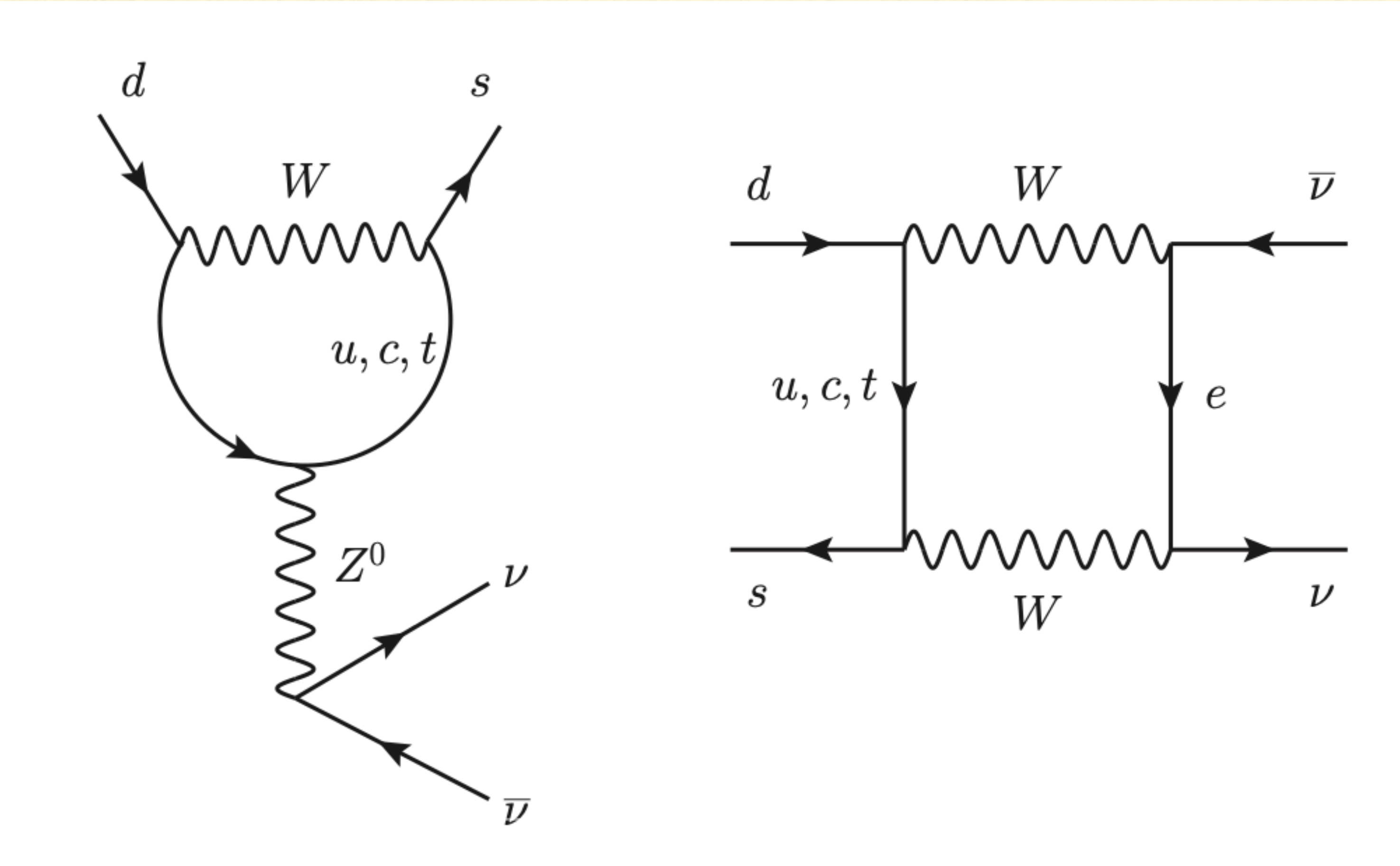
Grossman-Nir Bound

Grossman, Nir (1997)

$$\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu}) \leq 4.3\, \mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu}).$$

K⁺	**K⁻**	$u\bar{s}$	493.677 ±0.016
K⁰	**$\bar{\text{K}}^0$**	$d\bar{s}$	497.611 ±0.013
K^0_S	Self	$\frac{d\bar{s}-s\bar{d}}{\sqrt{2}}$ [†]	497.611 ±0.013[‡]
K^0_L	Self	$\frac{d\bar{s}+s\bar{d}}{\sqrt{2}}$ [†]	497.611 ±0.013[‡]

SM:

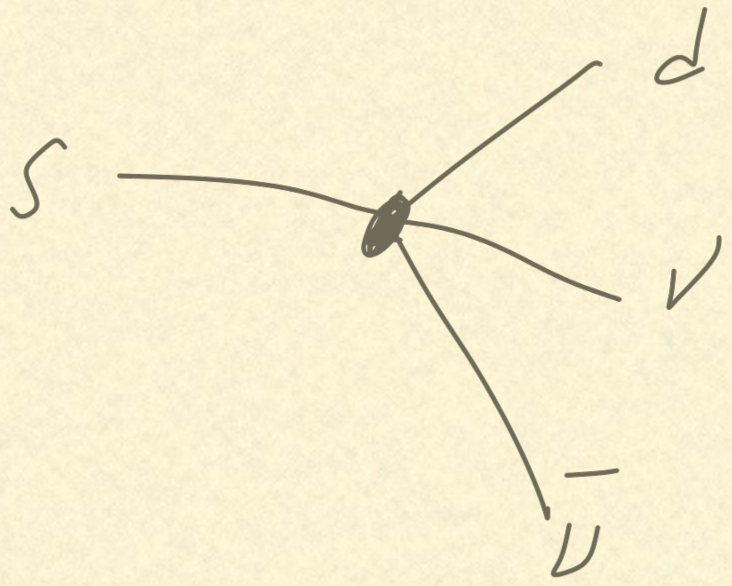


$$A(s \to d\nu\bar{\nu}) \sim \sum_{q=u,c,t} \lambda_q X_{\mathrm{SM}}(x_q)$$

$$\sim \frac{m_t^2}{M_W^2}\lambda_t + \frac{m_c^2}{M_W^2}\ln\frac{M_W}{m_c}\lambda_c + \frac{\Lambda_{\mathrm{QCD}}^2}{M_W^2}\lambda_u,$$

where $\lambda_q = V_{qd}V_{qs}^*$ and $x_q = m_q^2/M_W^2$. The

arXiv: 1107.6001



Dominated by top loop -> described well by LEFT with single operator:

$$[\bar{s}\gamma^\mu(1-\gamma_5)d]\,[\bar{\nu}\gamma_\mu(1-\gamma_5)\nu]\,.$$

Grossman-Nir Bound

Valid in EFT,
>>> Assuming no new degrees of freedom (d.o.f) below Kaon mass
>>> Assuming no/negligible lepton flavour violation (LFV)
>>> Bound saturated with maximal CP violation

A violation of the bound → indicate New d.o.f. below $m_{K^\pm}$ scale,
Although

A large violation of the bound indicate new d.o.f below mK scale, though
Additional constraints
>>> Meson decay, beam-dump experiments
>>> Astro, Cosmology

Grossman-Nir Bound Dilemma:

EFT $$\mathcal{L}_{\text{eff}} = c^{(4)}(\bar{s}d)\varphi + \sum_i \frac{c_i^{(7)}}{\Lambda^3}(\bar{s}\Gamma_i d)(\bar{d}\Gamma_i' d)\varphi + \cdots$$

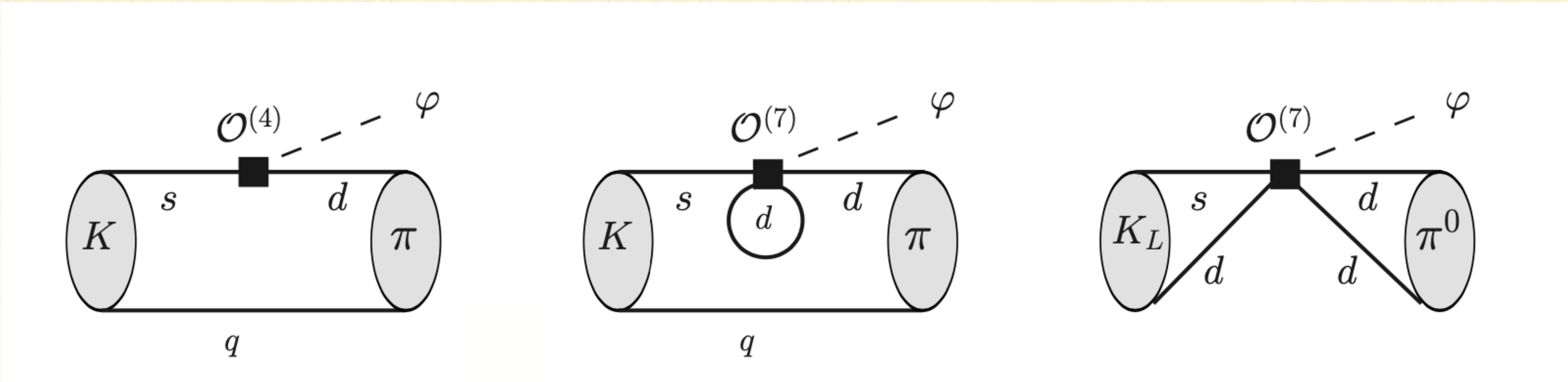
$$\{\mathcal{M}^{(4)}(K_L \to \pi^0\varphi), \mathcal{M}^{(4)}(K^+ \to \pi^+\varphi)\} = \frac{m_K^2 - m_\pi^2}{m_s - m_d} f_+(0)\{\mathrm{Im}\, c^{(4)}, c^{(4)}\}.$$

$$|\mathcal{M}^{(4)}(K_L \to \pi^0\varphi)| \le |\mathcal{M}^{(4)}(K^+ \to \pi^+\varphi)|.$$

$$\{\mathcal{M}^{(7)}(K_L \to \pi^0\varphi), \mathcal{M}^{(7)}(K^+ \to \pi^+\varphi)\} \propto \frac{m_K^3}{\Lambda^3}\left\{\mathrm{Im}\, c_i^{(7)}, \frac{1}{(4\pi)^2}\left(\frac{\alpha_s}{4\pi}\right)^n c_i^{(7)}\right\}$$

$$f \cdot \frac{m_K^2 - m_\pi^2}{m_s - m_d} \underbrace{\frac{m_s}{\Lambda}}_{c_4} \ll \frac{m_K^3}{\Lambda^3}\underbrace{(\sim 1)}_{c_7}$$

$\Rightarrow \Lambda \ll 3$ GeV



Even c4 << c7: (Quark loop type: QL) << (Weak: Annihilation: WA) difficult. (Lattice QCD)

$\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu})_{\text{exp}}$ $\lesssim$ few $\mathrm{Br}(K^+ \to \pi^+ + \text{inv})$

KOTO Excess?

New d.o.f:
>>> Phase space difference $m_K - m_{\pi 0} > m_{K+} - m_{\pi+}$
Forbidding $K^+ \to \pi^+ X_{inv}$, or with a fine tuned mX_{inv} : 1911.03755, 2001.06572

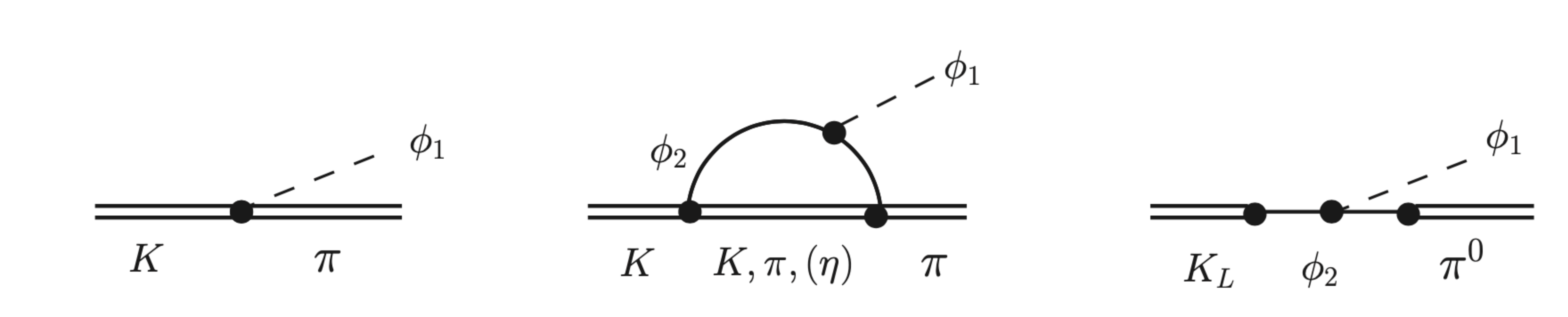
>>> $mX_{inv} \approx m_\pi$ [11,14-21] where NA62 veto/blind 2 pion events: 2001.06572

Long life time, X_{inv} decay outside KOTO detector : 1911.10203 , 1911.12334, 2001.06522, 2005.00753

EFT analysis: 1912.10433 (exp %), 2002.05467 (dim9 GN bound violation?)

M1. Two-body decay

Evading EFT description by introducing light mediator: $m_S \sim m_{\phi_2} \sim m_K$ $\quad m_{\phi_2} > m_K - m_\pi$



$$\mathcal{L} \supset g^{(i)}_{qq'} (\bar{q}_L q'_R)\phi_i + \text{h.c.} + \lambda m_S \phi_2^2 \phi_1$$

$\phi 2$ mix with K and π, enabling sizeable Fig.3 for K_L.

Suppressing Fig. 1, large violation of "GN" bound by loop factor ≈ $16\pi^2$ (NDA).

$$\mathcal{M}(K_L \to \pi^0 \phi_1) \propto \text{Im}\; g^{(1)}_{sd} + \mathcal{O}(1) \times \lambda\, \text{Im}\, g^{(2)}_{sd}\; \text{Im}\, g^{(2)}_{dd}$$
$$\mathcal{M}(K^+ \to \pi^+ \phi_1) \propto g^{(1)}_{sd} + \mathcal{O}(1) \times \frac{1}{16\pi^2} \lambda\, g^{(2)}_{sd}\, g^{(2)}_{dd}\;,$$

$$g^{(1)}_{sd} \ll g^{(2)}_{sd} \ll g^{(2)}_{dd}$$

$\phi 2$ mixes, $\phi 1$ couple mostly only through $\phi 2$.

M1. Two-body decay

$$\mathcal{L}_{\mathrm{QCD}+\phi} = \bar{q}(i\partial\!\!\!/ + g_s G\!\!\!/^a T^a)q - \bar{q}\mathcal{M}_q q - \sum_i \phi_i\, \bar{q}(\chi_S^{(i)} - i\chi_P^{(i)}\gamma_5)q,$$

$$[\chi_S^{(i)}]_{qq'} = -\frac{1}{2}(g_{qq'}^{(i)} + g_{q'q}^{(i)*}), \qquad [\chi_P^{(i)}]_{qq'} = -\frac{i}{2}(g_{qq'}^{(i)} - g_{q'q}^{(i)*})$$

Matching to ChPT+scalars for form factors:

$$s + ip \to g_R(s+ip)g_L^\dagger$$

$$s = \mathcal{M}_q + \sum_i \chi_S^{(i)}\phi_i, \qquad p = \sum_i \chi_P^{(i)}\phi_i,$$

$$\mathcal{L}^{(2)}_{\mathrm{ChPT}+\phi} = \frac{f^2}{4}\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + B_0\frac{f^2}{2}\,\mathrm{Tr}\left[(s-ip)U + (s+ip)U^\dagger\right]$$
$$+\frac{1}{2}\partial_\mu\phi_i\partial^\mu\phi_i - \frac{m_{\phi_i}^2}{2}\phi_i^2 + \lambda m_S \phi_2^2\phi_1 + \cdots\ ,$$

$$U(x) = \exp(i\lambda^a\pi^a/f)$$

$$B_0(\mu = 2\ \mathrm{GeV}) = 2.666(57)\ \mathrm{GeV}$$

$$f \simeq f_\pi/\sqrt{2} = 92.2(1)\ \mathrm{MeV}$$

M1. Two-body decay

$$\mathcal{L}^{(2)}_{\mathrm{ChPT}+\phi} \supset B_0 f \sum_i \phi_i \Big(\sqrt{2}\hat{g}^{(i)}_{ds} \bar{K}^0 + \sqrt{2}\hat{g}^{(i)}_{sd} K^0 - \mathrm{Im}\, g^{(i)}_{dd} \pi^0 + \tfrac{1}{\sqrt{3}} \mathrm{Im}\, (g^{(i)}_{dd} - 2 g^{(2)}_{ss}) \eta \Big)$$

$$+ B_0 \sum_i \phi_i \Big\{ \mathrm{Re}(g^{(i)}_{dd} + g^{(i)}_{ss}) K^0 \bar{K}^0 + \mathrm{Re}(g^{(i)}_{dd}) \big(\tfrac{1}{2} (\pi^0)^2 - \tfrac{1}{\sqrt{3}} \eta \pi^0 \big) +$$

$$+ \mathrm{Re}\, g^{(i)}_{ss} K^+ K^- + \mathrm{Re}\, g^{(i)}_{dd} \pi^+ \pi^- +$$

$$+ \Big[\bar{g}^{(i)}_{sd} \big(- \tfrac{1}{\sqrt{2}} K^0 \pi^0 + K^+ \pi^- - \tfrac{1}{\sqrt{6}} K^0 \eta \big) + \mathrm{h.c.} \Big] + \cdots \Big\} ,$$

$$\mathcal{M}(K_L \to \pi^0 \phi_1)_{\mathrm{NP}} = \Big\{ 2\, \mathrm{Im}\, \hat{g}^{(2)}_{sd}\, \mathrm{Im}\, g^{(2)}_{dd} \Delta_{\phi_2}(m_K^2) \Delta_{\phi_2}(m_\pi^2) \lambda m_S B_0 f_K f_\pi$$

← (WA)

$$+ \mathrm{Im}\, \bar{g}^{(1)}_{sd} - \frac{\mathrm{Im}\, \bar{g}^{(2)}_{sd}}{8\pi^2} \lambda m_S B_0 \mathcal{F}^{(2)}_L(I) \Big\} B_0 ,$$

← (QL)

$$\mathcal{M}(K^+ \to \pi^+ \phi_1)_{\mathrm{NP}} = - \Big\{ \bar{g}^{(1)}_{sd} - \frac{\bar{g}^{(2)}_{sd}}{8\pi^2} \lambda m_S B_0 \mathcal{F}^{(2)}_+(I) \Big\} B_0 ,$$

$$\Delta_X(k^2) \equiv 1/(k^2 - m_X^2)$$

$$I(m_X) = C_0(m_K^2, m_{\phi_1}^2, m_\pi^2, m_X^2, m_{\phi_2}^2, m_{\phi_2}^2)$$

M1. Two-body decay

Reproducing KOTO anomaly:

$$\mathrm{Br}(K_L \to \pi^0 + \mathrm{inv}) = \underbrace{(3.4 \pm 0.6) \times 10^{-11}}_{\mathrm{SM}} \\ \underbrace{+\, 6.0 \times 10^{-9} \left(\frac{\mathrm{Im}\,\hat{g}_{sd}^{(2)}}{5 \cdot 10^{-9}}\right)^2 \left(\frac{\mathrm{Im}\,g_{dd}^{(2)}}{10^{-3}}\right)^2 \left(\frac{\lambda m_S}{1\ \mathrm{GeV}}\right)^2 \left(\frac{1\ \mathrm{GeV}}{m_{\phi_2}}\right)^8}_{\mathrm{NP}},$$

Satisfying K+ measurement:

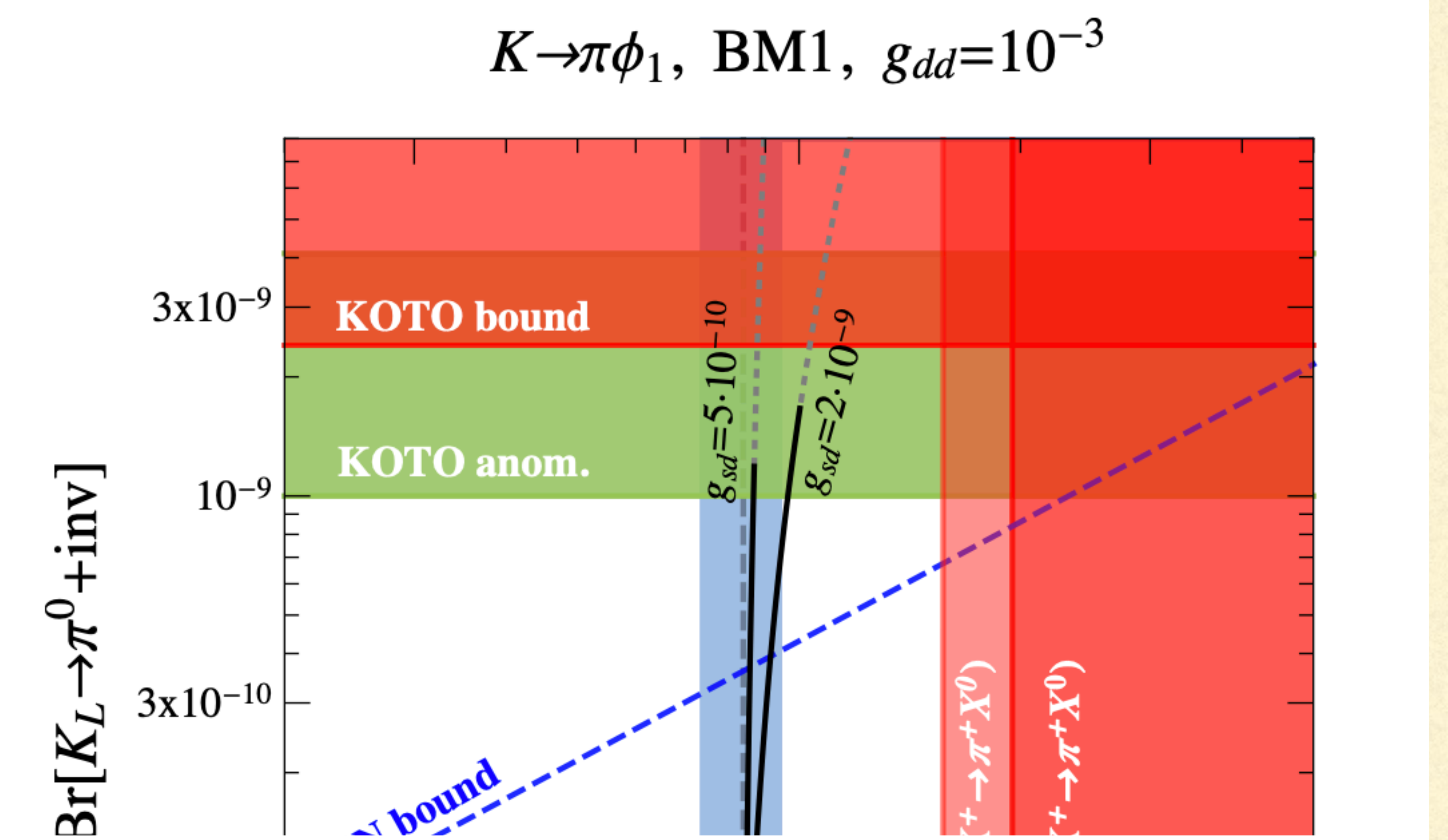
$$\mathrm{Br}(K^+ \to \pi^+ + \mathrm{inv}) = \underbrace{(8.4 \pm 1.0) \times 10^{-11}}_{\mathrm{SM}} + \underbrace{5.0 \times 10^{-11} \left|\frac{\bar{g}_{sd}^{(1)}}{10^{-13}}\right|^2}_{\mathrm{NP}},$$

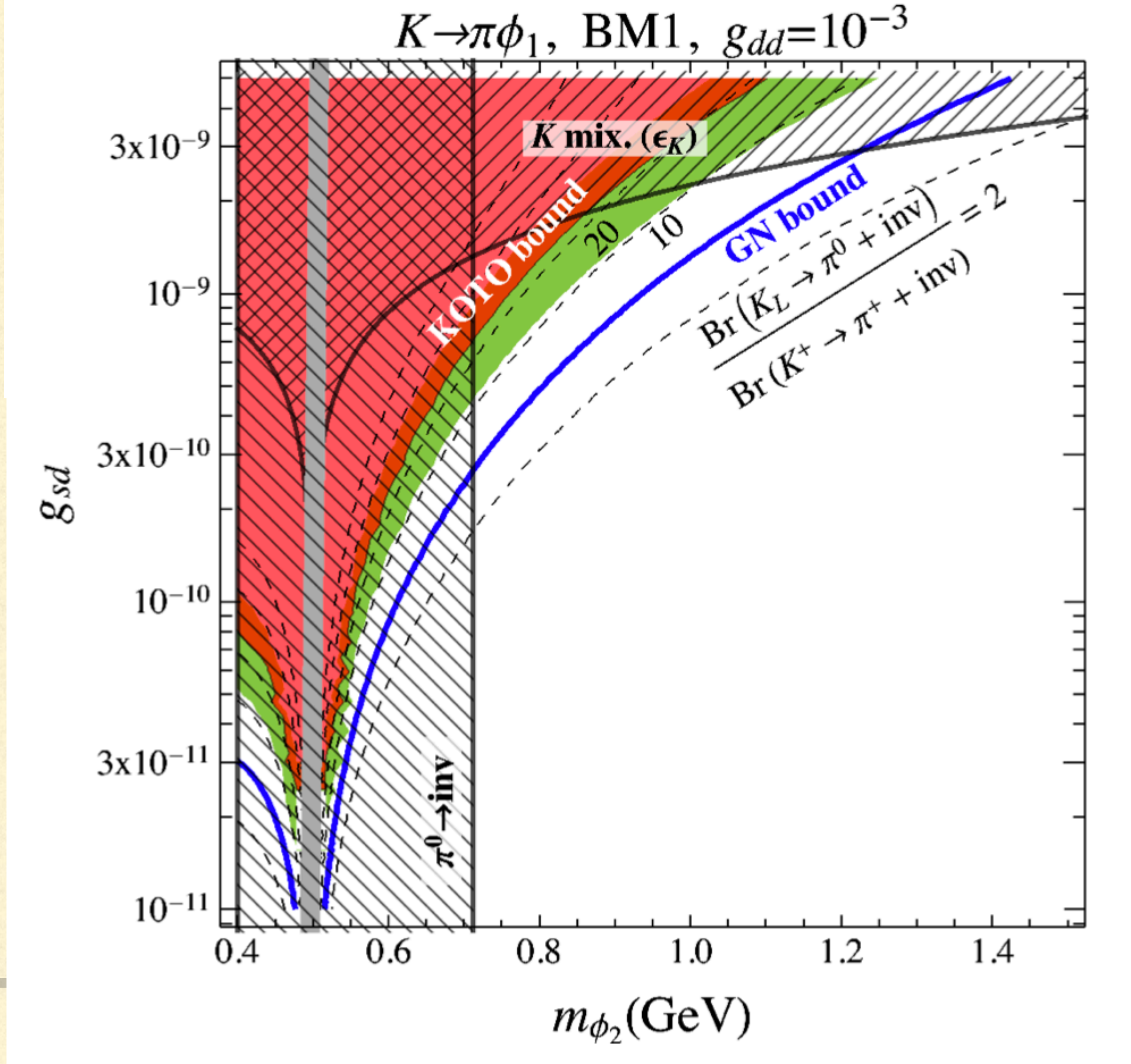
$$\begin{aligned} &\mathbf{BM\ 1:} \quad g_{dd}^{(2)} = \tfrac{(1+i)}{\sqrt{2}} g_{dd}\,, \quad \bar{g}_{sd}^{(2)} = \hat{g}_{sd}^{(2)} = \tfrac{(1+i)}{\sqrt{2}} g_{sd}\,, \\ &\mathbf{BM\ 2:} \quad g_{dd}^{(2)} = i g_{dd}\,, \quad \bar{g}_{sd}^{(2)} = 0\,, \quad \hat{g}_{sd}^{(2)} = i g_{sd}\,. \end{aligned}$$

$\lambda_S m_S = 1\ \mathrm{GeV}$

$m_{\phi_1} = 1\ \mathrm{MeV}$

(*BM parameters chosen for maximum GN bound violation)

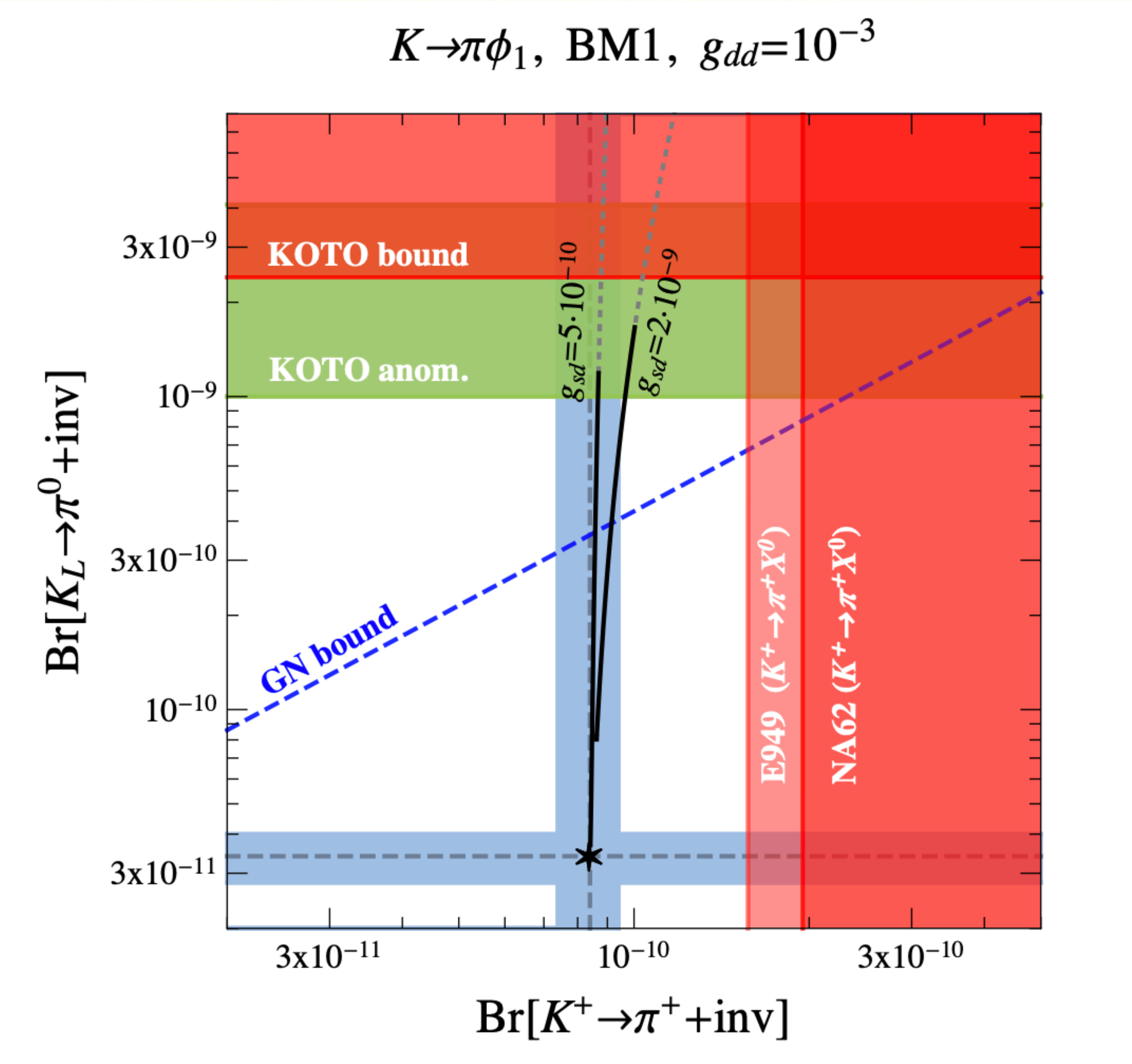


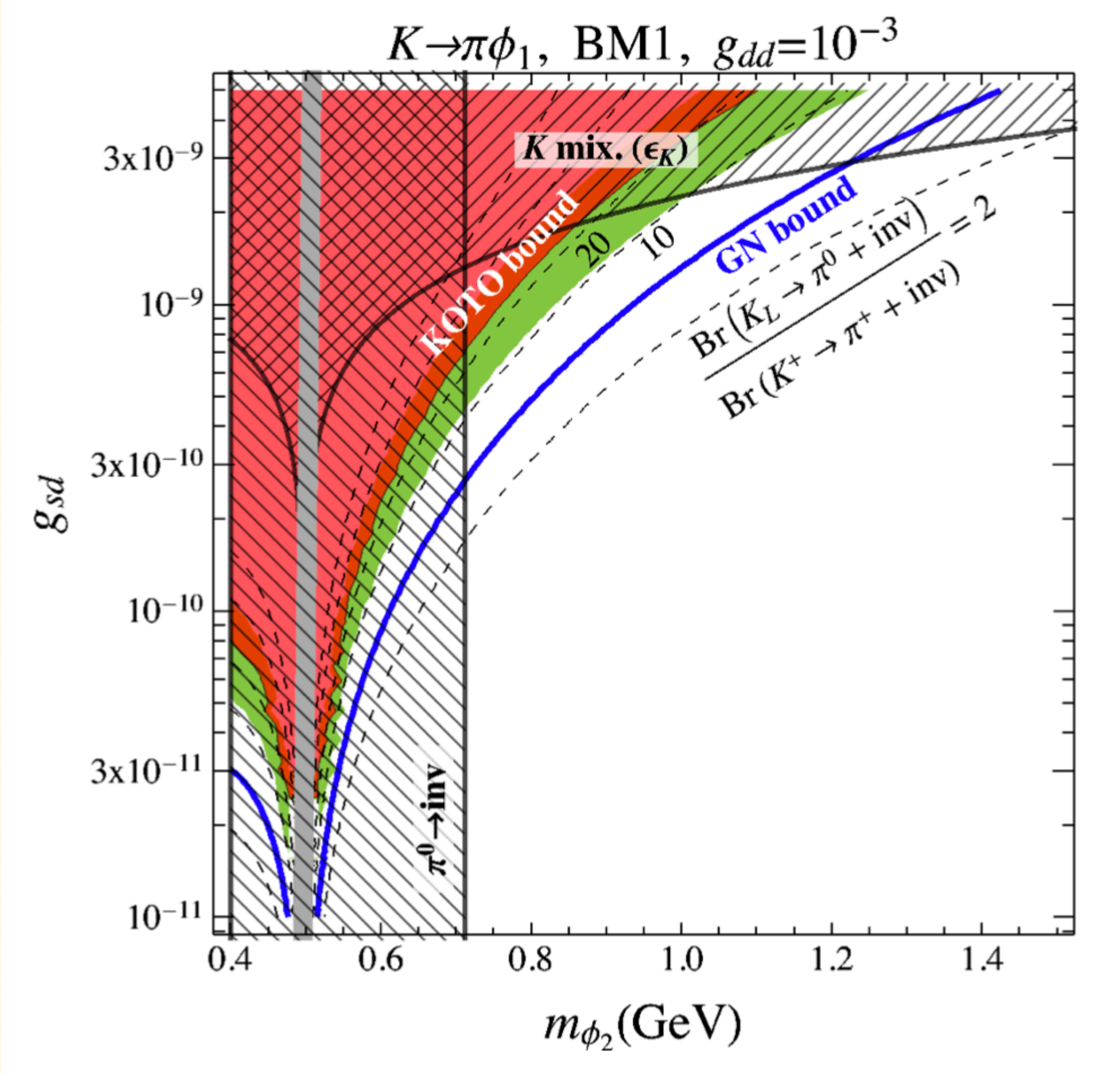


M1. Two-body decay

$$\textbf{BM 1}: \quad g^{(2)}_{dd} = \frac{(1+i)}{\sqrt{2}} g_{dd}\,, \quad \bar{g}^{(2)}_{sd} = \hat{g}^{(2)}_{sd} = \frac{(1+i)}{\sqrt{2}} g_{sd}\,,$$

$\lambda_S m_S = 1$ GeV $m_{\phi_1} = 1$ MeV





M1. Two-body decay Experimental constraints

More constraints from:

KKbar-mixing;

π>ϕ1,ϕ1 (invisible π decay);

ϕ1>**γγ**-decay (BBN);

ϕ1-N coupling (SN cooling)

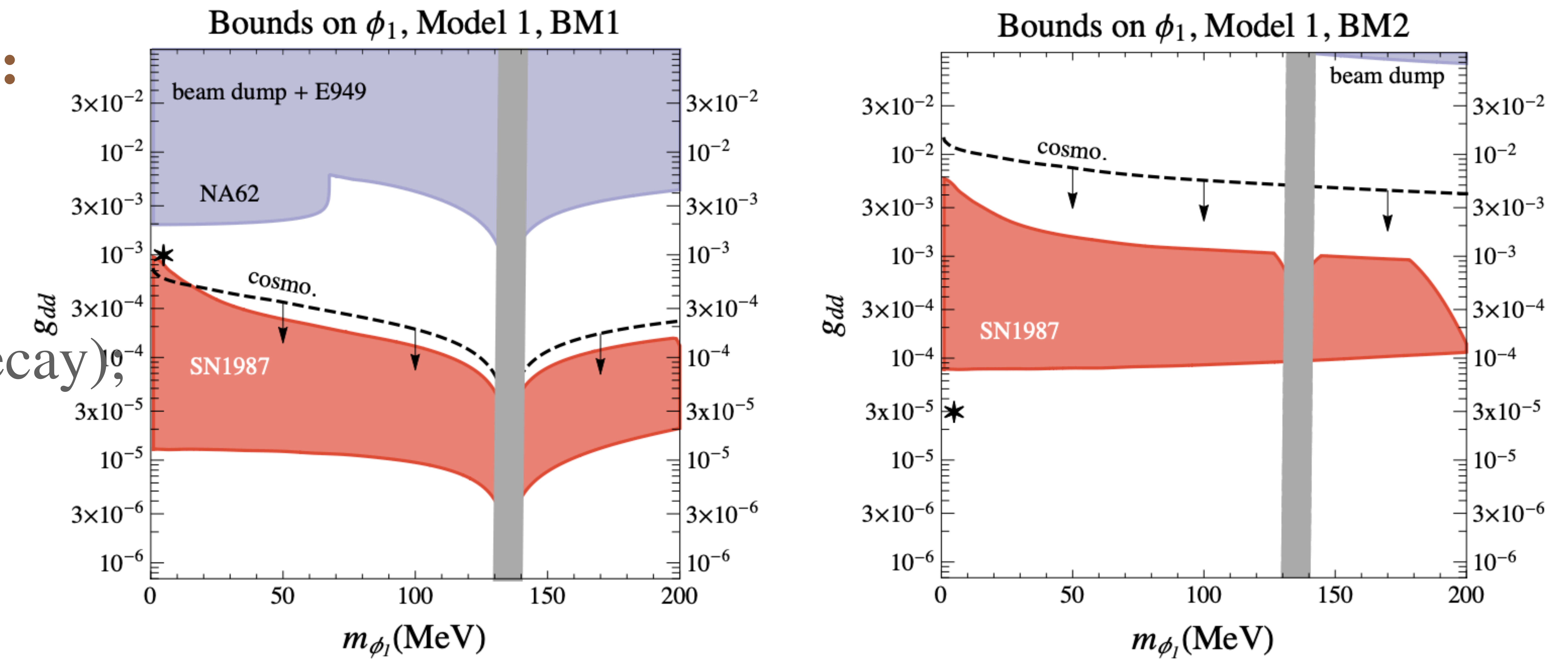
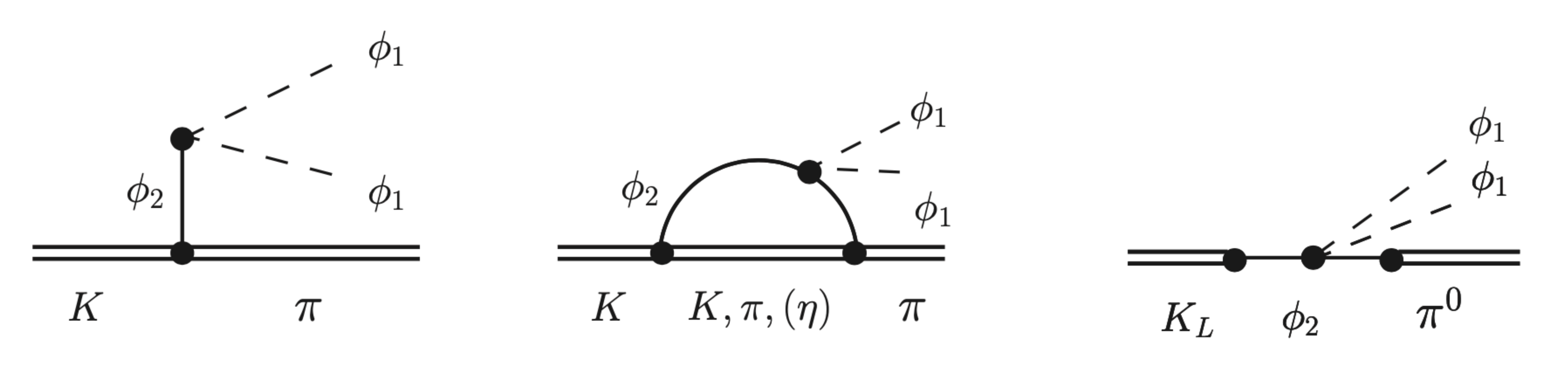


Figure 13. The constraints on the g_{dd} coupling in BM1 (left) and BM2 (right) due to couplings of ϕ_1 to photons and nucleons as a function of the ϕ_1 mass. The purple regions are excluded by beam dump searches, E949 ($K^+ \to \pi^+ X$) and NA62($\pi^0 \to$ inv), the red region by SN1987, while the dashed line shows the upper bound from cosmology in the absence of any other light states or ϕ_1-couplings. The star denotes the values of g_{dd} and m_{ϕ_1} in Fig. 5 (Fig. 6) for BM1 (BM2). The region around $m_{\phi_1} \simeq m_{\pi^0}$ is masked out (gray region).

M2. Three-body decay (light scalar), M1 with Z2: ϕ1 → −ϕ1.

$$\mathcal{L} \supset g^{(2)}_{qq'}(\bar{q}_L q'_R)\phi_2 + \text{h.c.} + \lambda_4 \phi_2^2 \phi_1^2 + \lambda' \mathrm{m_S} \phi_2 \phi_1^2 + \lambda'' \mathrm{m_S} \phi_2^3 + \cdots .$$

With $\lambda', \lambda'' \ll \lambda_4$, $m_\pi \leq m_{\phi 1}, \psi_1 \leq (m_K - m_\pi)/2$



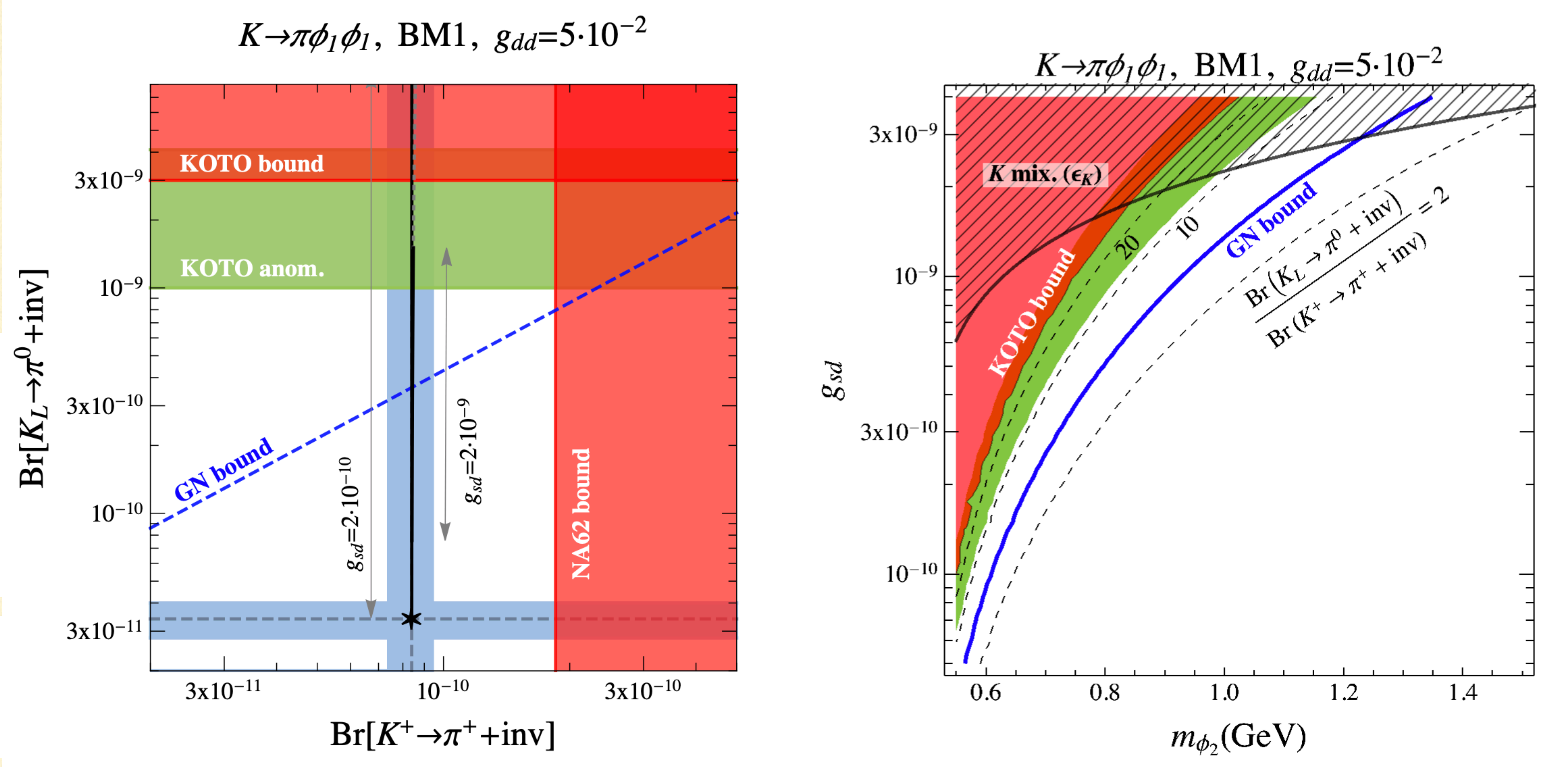
$$\mathcal{M}(K_L \to \pi^0 \phi_1 \phi_1)_{\text{NP}} = i\Big\{ 4\,\text{Im}\,\hat{g}^{(2)}_{sd}\,\text{Im}\,g^{(2)}_{dd}\lambda_4 \Delta_{\phi_2}(m_K^2)\Delta_{\phi_2}(m_\pi^2) B_0 f_K f_\pi - 2\,\text{Im}\,\bar{g}^{(2)}_{sd}\lambda' m_S \Delta_{\phi_2}(q^2) - \frac{\text{Im}\,\bar{g}^{(2)}_{sd}}{4\pi^2}\lambda_4 \mathcal{F}^{(2)}_L(\tilde{I}) B_0 \Big\} B_0 \,,$$

$$\mathcal{M}(K^+ \to \pi^+ \phi_1 \phi_1)_{\text{NP}} = \Big\{ 2\bar{g}^{(2)}_{sd}\lambda' m_S \Delta_{\phi_2}(q^2) + \frac{\bar{g}^{(2)}_{sd}}{4\pi^2}\lambda_4 B_0 \mathcal{F}^{(2)}_+(\tilde{I}) \Big\} B_0$$

M2. Three-body decay (light scalar), M1 with Z2: ϕ1 → −ϕ1.

$$\mathcal{L} \supset g^{(2)}_{qq'} (\bar{q}_L q'_R) \phi_2 + \mathrm{h.c.} + \lambda_4 \phi_2^2 \phi_1^2 + \lambda' \mathrm{m_S} \phi_2 \phi_1^2 + \lambda'' \mathrm{m_S} \phi_2^3 + \cdots .$$

With $\lambda', \lambda'' \ll \lambda_4$, $m_\pi \le m_{\phi_1}, \psi_1 \le (m_K - m_\pi)/2$. BM: $m_{\phi_1} = 100$ MeV$, \lambda_4 = 1,\ \lambda' = \lambda'' = 0$

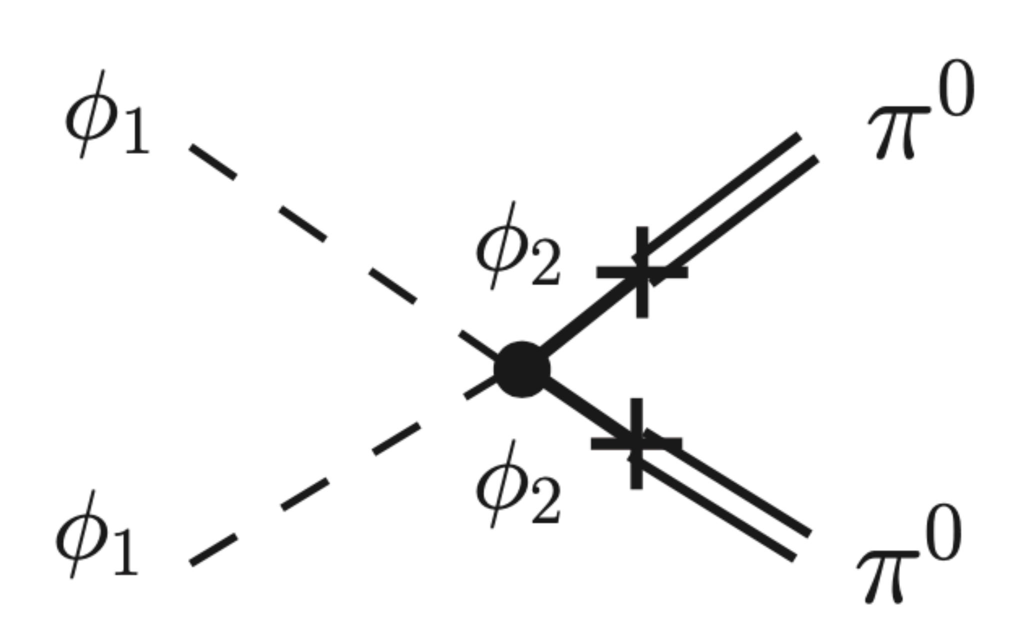


M2. Three-body decay (light scalar), M1 with Z2: ϕ1 → −ϕ1.

$$\mathcal{L} \supset g_{qq'}^{(2)}(\bar{q}_L q'_R)\phi_2 + \text{h.c.} + \lambda_4 \phi_2^2 \phi_1^2 + \lambda' \mathrm{m_S} \phi_2 \phi_1^2 + \lambda'' \mathrm{m_S} \phi_2^3 + \cdots .$$

ϕ1 as a viable DM candidate with: $m_{\pi^0} \leq m_{\phi_1} \leq (m_{K_L} - m_{\pi^0})/2$

(135 MeV~181 MeV)



$$\langle \sigma v \rangle = \frac{1}{16\pi} \lambda_4^2 \left(\mathrm{Im}\, g_{dd}^{(2)}\right)^4 \left(\frac{B_0 f_\pi}{m_{\phi_2}^2}\right)^4 \frac{p_\pi}{m_{\phi_1}^3}$$

where in this approximation $p_\pi = (m_{\psi_1}^2 - m_\pi^2)^{1/2}$. Taking $m_{\phi_1} = 160$ MeV as a representative value gives

$$\langle \sigma v \rangle \simeq 3 \cdot 10^{-26} \frac{\mathrm{cm}^3}{\mathrm{s}} \times \lambda_4^2 \left(\frac{\mathrm{Im}\, g_{dd}^{(2)}}{2.56 \times 10^{-2}}\right)^4 \left(\frac{1\ \mathrm{GeV}}{m_{\phi_2}}\right)^4 , \tag{4.11}$$

which is of the right size to get the correct DM relic abundance ($3 \cdot 10^{-26} \mathrm{cm}^3/\mathrm{s} \approx 1\,\mathrm{pb}$).

Conclusion:

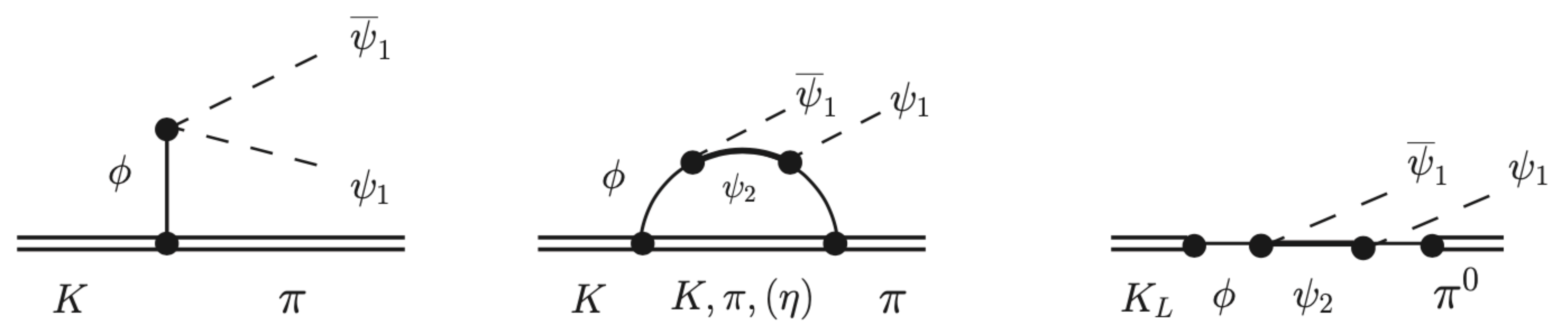
>>> ~GeV scale light mediator(s) that enhance the rare but clean decay mode {K>π+inv} of KL but keeping K0 approx SM

>>> Generic coupling strength and contents of a light dark sector

>>> Possible DM candidates

>>> Link to SM Flavor Puzzle?

M3. Three-body decay (light fermion)



$$\mathcal{L} \supset g^{(\phi)}_{qq'}(\bar q_L q'_R)\phi + y_{ij}\phi\bar\psi_{L,i}\psi_{R,j} + \text{h.c.}\ .$$

$$g^{(\phi)}_{sd,ds} \ll g^{(\phi)}_{dd,ss}\ , \qquad y_{11,22} \ll y_{12,21}\ .$$

$$\textbf{Model 3, BM 1}:\quad g^{(\phi)}_{dd} = \frac{(1+i)}{\sqrt2}g_{dd}\ ,\quad \bar g^{(\phi)}_{sd} = \hat g^{(\phi)}_{sd} = \frac{(1+i)}{\sqrt2}g_{sd},\quad y_{12}=y_{21}=1\ ,$$

$$\textbf{Model 3, BM 2}:\quad g^{(\phi)}_{dd} = i g_{dd}\ ,\quad \bar g^{(\phi)}_{sd} = 0\ ,\quad \hat g^{(\phi)}_{sd} = i g_{sd},\quad y_{12}=y_{21}=1\ ,$$

Heavy ψ2: mψ2+mψ1 > mK-mπ allowing only K>π ψ1 ψ1