CCFM-based equations

June 18, 2020

Why?

- CCFM connection DGLAP and BFKL (small-x)
- Interested in effects of TMD splitting functions
- TMD splitting functions are calculated in small-x framework

- CCFM is equation for gluon distribution, we use features of CCFM in PB equations for gluons and quarks

Adaptation of the equation

PB equation (with angular ordering):

$$\tilde{\mathcal{A}}_a(x, k_\perp, \mu^2) = \Delta_a(\mu^2, \mu_0^2)\tilde{\mathcal{A}}_a(x, k_\perp, \mu_0^2) + \sum_b \int \frac{d^2\boldsymbol{\mu}'_\perp}{\pi\mu'^2} \int_x^{z_M} dz \times$$

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- In PB: approximate AO: $\frac{p_{i+1}}{1-z_{i+1}} > \frac{p_i}{1-z_i} \to \mu_i = \frac{p_i}{1-z_i}$
  Full AO: $\frac{p_{i+1}}{1-z_{i+1}} > \boldsymbol{z_i} \frac{p_i}{1-z_i}$ (p: emitted transverse momentum, μ evolutions scale)

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- Include Non-Sudakov form factor, resums logarithms of z

Non-Sudakov Form Factor

- Original CCFM kernel:
  $Pgg = 2C_A\bar{\alpha}_s\left(\frac{1}{z}\Delta_{ns}(z,\mu',k_\perp) + \frac{1}{1-z}\right)$

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$$P_{gg}^{DGLAP} = 2C_A\bar{\alpha}_s\left(\frac{1}{z} + \frac{1}{1-z} + z(1-z) - 2\right)$$

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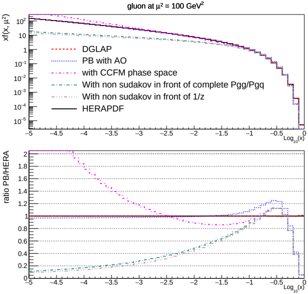
- Option 3: $Pgg =$
  $2C_A\bar{\alpha}_s\left((\frac{1-z}{z} + \frac{1}{2}z(1-z))\Delta_{ns}(z,\mu',k_\perp) + \frac{z}{1-z} + \frac{1}{2}z(1-z)\right)$,
  but what with P_{gq}?

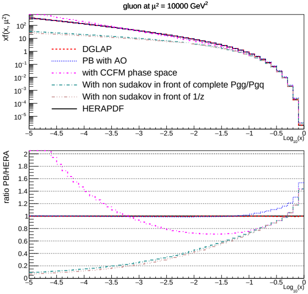
Different steps

- DGLAP limit: $\alpha_s(\mu)$, fixed $z_M = 1 - \epsilon$
- PB with AO: $\alpha_s(p)$, $z_M = 1 - q_0/\mu'$
- CCFM phase space: after branching the evolution continues from scale $z_i \mu_i$
- Partial inclusion of the non sudakov form factor
  - In front of complete Pgg/Pgq
  - In front of 1/z

RESULTS



gluon at μ² = 100 GeV²
xf(x, μ²)
DGLAP
PB with AO
with CCFM phase space
With non sudakov in front of complete Pgg/Pgq
With non sudakov in front of 1/z
HERAPDF
Log10(x)
ratio PB/HERA



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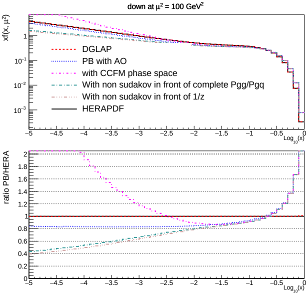

down at μ² = 100 GeV²
xf(x, μ²)
DGLAP
PB with AO
with CCFM phase space
With non sudakov in front of complete Pgg/Pgq
With non sudakov in front of 1/z
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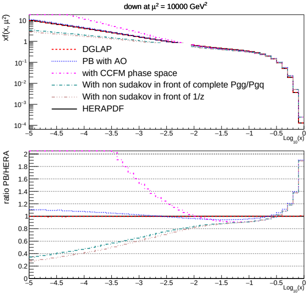

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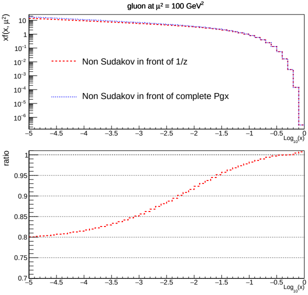
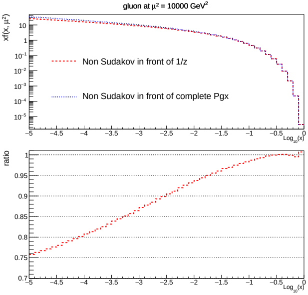
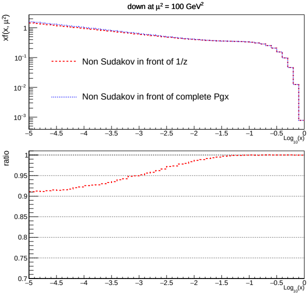
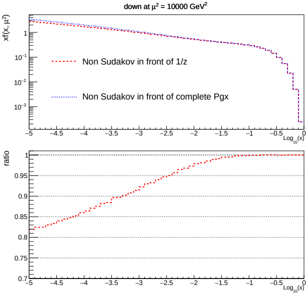
RESULTS placement Non-Sudakov form factor

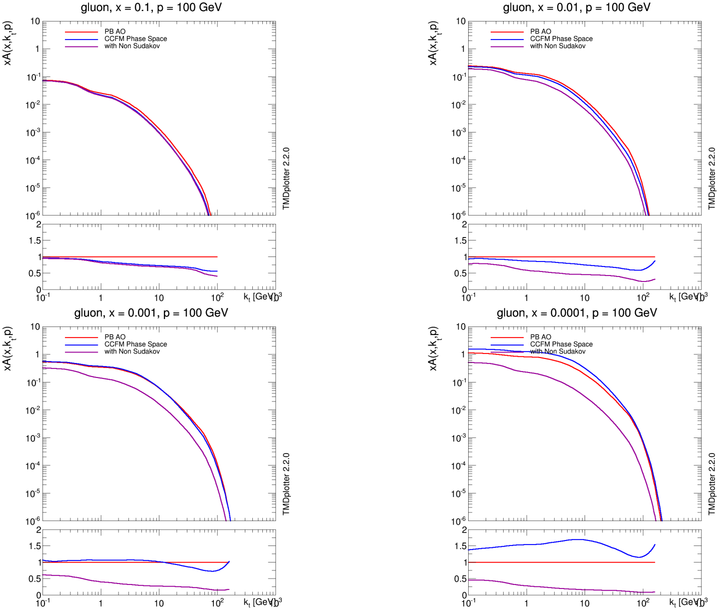








TMDs: gluon



gluon, x = 0.1, p = 100 GeV
gluon, x = 0.01, p = 100 GeV
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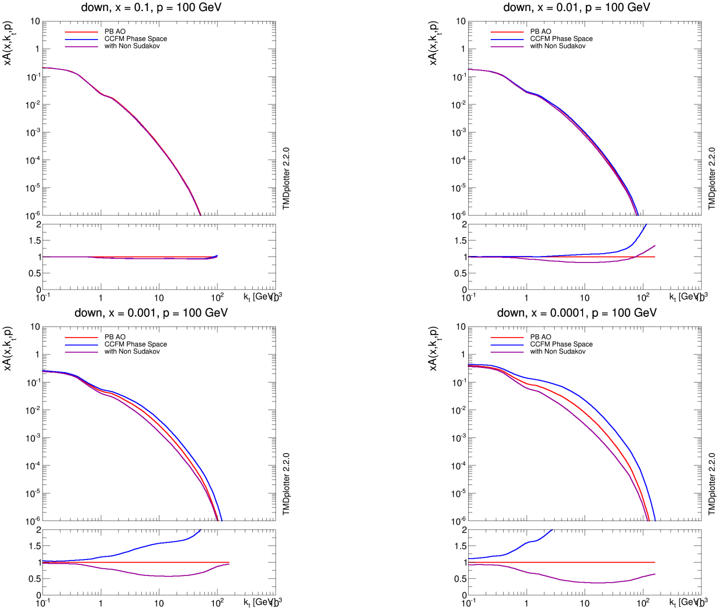
xA(x,k_t,p)
PB AO
CCFM Phase Space
with Non Sudakov
k_t [GeV]
TMDplotter 2.2.0

TMDs: down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Note: integrals over z and angle, since Non-Sudakov depends on outgoing (not emitted) $k_\perp$

Conclusions

- Introduction of CCFM phase space $\rightarrow$ large increase in PDFs for small x
- Introduction of Non-Sudakov $\rightarrow$ very large suppresion in small x
- Different choices of placement of the Non-Sudakov gives differences of up to 25%

Back-up

Start from:

$$\widetilde{f}_a(x,\mu^2) = \Delta_a(\mu^2)\ \widetilde{f}_a(x,\mu_0^2) + \sum_b \int_{\ln\mu_0^2}^{\ln\mu^2} d\ln\mu_1^2 \frac{\Delta_a(\mu^2)}{\Delta_a(\mu_1^2)} \int \mathrm{d}x_1 \int_x^{z_M} \mathrm{d}z_1 P_{ab}^R(\mu_1^2, z_1) x f_b(x_1,\mu_1^2)\delta(z_1 x_1 - x)$$

Insert:

$$\frac{\mathrm{d}\Delta_b(\mu_1^2)}{\mathrm{d}\ln(\mu_1^2)} = \Delta_b(\mu_1^2)\left(-\sum_c \int_0^{z_M} \mathrm{d}z\ z P_{cb}^R\left(\mu_1^2, z\right)\right)$$

New form of equation:

$$\begin{aligned}\widetilde{f}_a(x,\mu^2) &= \Delta_a(\mu^2)\ \widetilde{f}_a(x,\mu_0^2) \\ &+ \sum_b \int_{\ln\mu_0^2}^{\ln\mu^2} \mathrm{d}\left(-\Delta_b(\mu_1^2)\right) \frac{\Delta_a(\mu^2)}{\Delta_a(\mu_1^2)} \int \mathrm{d}x_1 \frac{\int_x^{z_M} \mathrm{d}z_1 P_{ab}^R(\mu_1^2, z_1)}{\sum_c \int_0^{z_M} \mathrm{d}z z P_{cb}^R(\mu_1^2, z)} x f_b(x_1,\mu_0^2)\delta(z_1 x_1 - x) \\ &+ \ \ldots\ .\end{aligned}$$

When the splitting functions in the Sudakov and equation are not the same, the denominator has different splitting functions.

Multiply by $\frac{\sum_c \int_0^{z_M} dz\ \ z\bar{P}_{cb}}{\sum_c \int_0^{z_M} dz\ \ zP_{cb}^{sud}}$