

Neutrinos in cosmology

Yvonne Y. Y. Wong

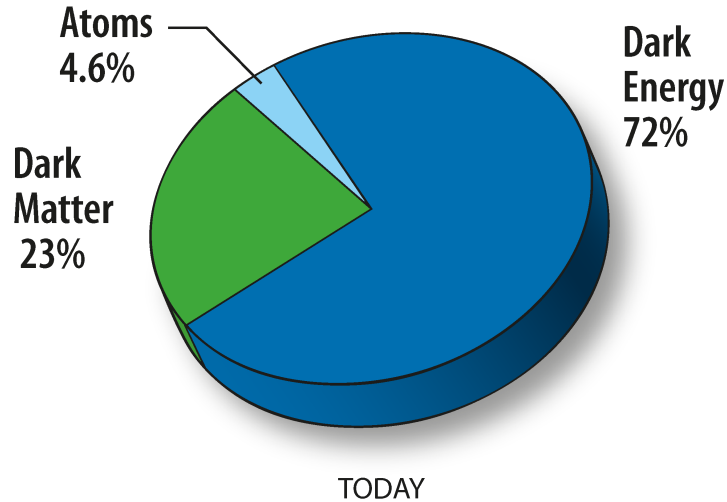
Sydney Consortium for Particle Physics and Cosmology

The University of New South Wales

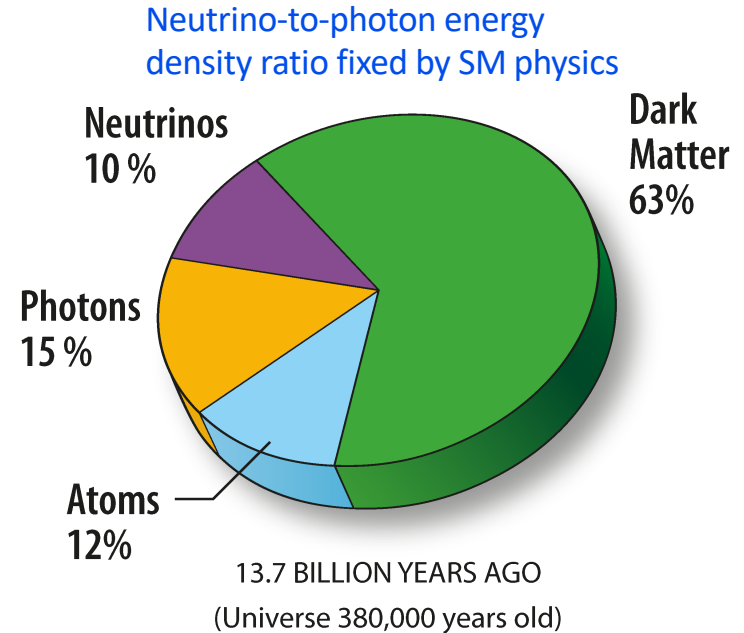
DESY theory workshop “Bright ideas for a dark universe”, September 21 – 24, 2021

Concordance Λ CDM...


- The **simplest** model consistent with most observations.




+ flat spatial geometry and initial conditions consistent with single-field inflation



The cosmic neutrino background...

- Neutrino decoupling at $T \sim O(1)$ MeV.  Fixed by weak interactions

- After e^+e^- annihilation ($T \sim 0.5$ MeV) Assuming $T_{\text{dec}} \gg m_e$

- **FD distribution with temperature:** $T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma$
- **Energy density per flavour:** $\rho_\nu = \frac{7}{8} \frac{\pi^2}{15} T_\nu^4 = \frac{7}{8} \left(\frac{4}{11}\right)^{4/3} \rho_\gamma$  $\frac{3\rho_\nu}{\rho_\gamma} \sim 0.68$

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- If **massive**, then at $T \ll m_\nu$: $\rho_\nu = m_\nu n_\nu$ →

$$\Omega_\nu h^2 = \sum \frac{m_\nu}{94 \text{ eV}}$$

- **Energy density in neutrino dark matter:**

From neutrino oscillations →

$$0.1\% < \Omega_\nu < 7\%$$

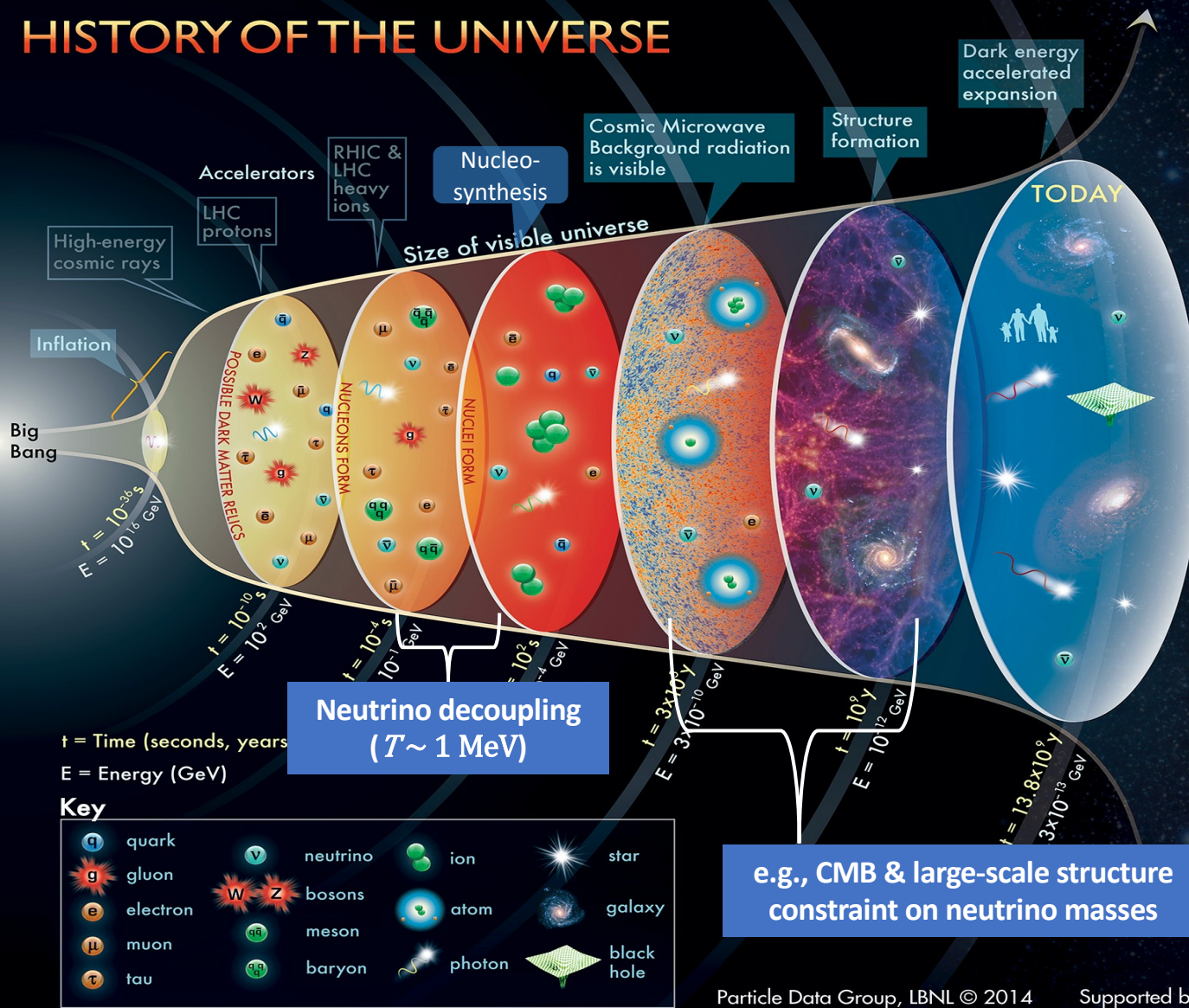
$$\min \sum m_\nu = 0.06 \text{ eV}$$

From KATRIN

$$m_e \equiv \left(\sum_i |U_{ei}|^2 m_i^2 \right)^{1/2} < 1.1 \text{ eV}$$

Aker et al. [KATRIN] 2019

HISTORY OF THE UNIVERSE



This talk...

- **“Standard” neutrinos in cosmology**

- I. New precision calculation of the SM effective number of neutrinos, $N_{\text{eff}}^{\text{SM}}$

- **“Non-standard” neutrinos in cosmology**

- II. Non-standard neutrino interactions

- Revised bound on the neutrino lifetime
 - Neutrino self-interaction and the Hubble tension

- An unashamed advertisement of the works of my students and postdocs (and collaborators) in the past few years.

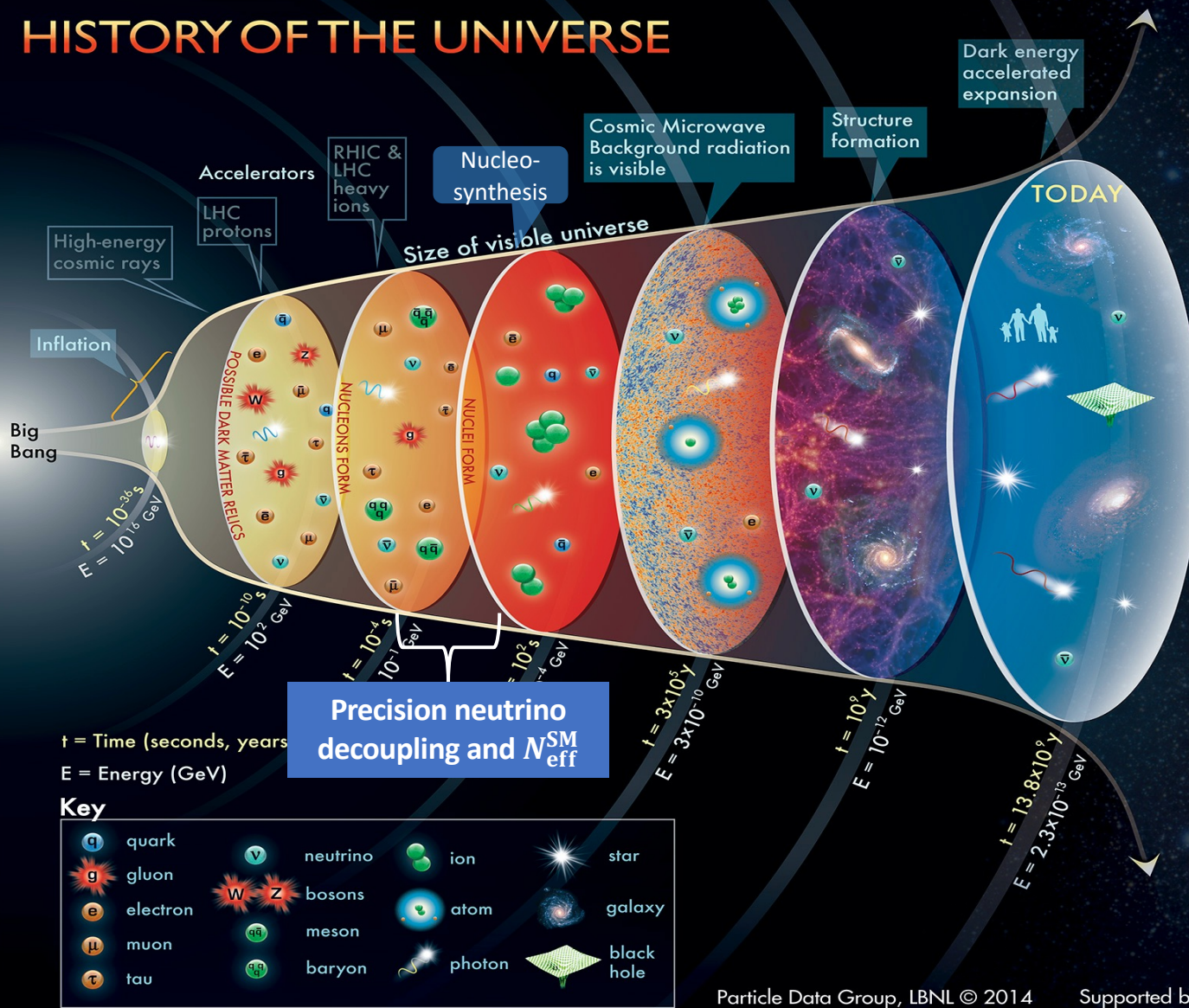
Gabriela Barenboim, Jack Bennett, Celine Boehm, Gilles Buldgen, Joe Chen, Pablo de Salas, Marco Drewes, Stefano Gariazzo, Steen Hannestad, Olga Mena, Isabel Oldengott, Sergio Pastor, Cornelius Rampf, Julia Stadler, Thomas Tram, Amol Upadhye

I. A new precision calculation of the standard model $N_{\text{eff}}^{\text{SM}}$...

Bennett, Buldgen, Drewes & Y³W, *JCAP* 03 (2020) 003, *JCAP* 03 (2021) A01 (addendum)

Bennett, Buldgen, de Salas, Drewes, Gariazzo, Pastor & Y³W, *JCAP* 04 (2021) 073

HISTORY OF THE UNIVERSE



t = Time (seconds, years)
E = Energy (GeV)

Key

	quark		neutrino		ion		star
	gluon		bosons		atom		galaxy
	electron		meson		black hole		
	muon		baryon		photon		
	tau						

The CνB: some small tweaks...

- Neutrino decoupling at $T \sim O(1)$ MeV. ← Fixed by weak interactions

This is not a very good approximation.

- After e^+e^- annihilation ($T \sim 0.5$ MeV)

Assuming $T_{\text{dec}} \gg m_e$

Distortions from non-instantaneous decoupling

- **FD distribution with temperature:**

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma$$

Finite-temperature corrections to the QED equation of state

- **Energy density per flavour:**

$$\rho_\nu = \frac{7\pi^2}{8 \cdot 15} T_\nu^4 = \frac{7}{8} \left(\frac{4}{11}\right)^{4/3} \rho_\gamma$$

ρ_γ

- Lump all corrections into the **effective number of neutrino** parameter:

$$\sum \rho_\nu = N_{\text{eff}} \frac{7}{8} \left(\frac{4}{11}\right)^{4/3} \rho_\gamma = (3 + \delta N_{\text{eff}}) \frac{7}{8} \left(\frac{4}{11}\right)^{4/3} \rho_\gamma$$

3 families

%-level SM corrections

→ Precision $N_{\text{eff}}^{\text{SM}}$

Why bother with precision $N_{\text{eff}}^{\text{SM}}$?

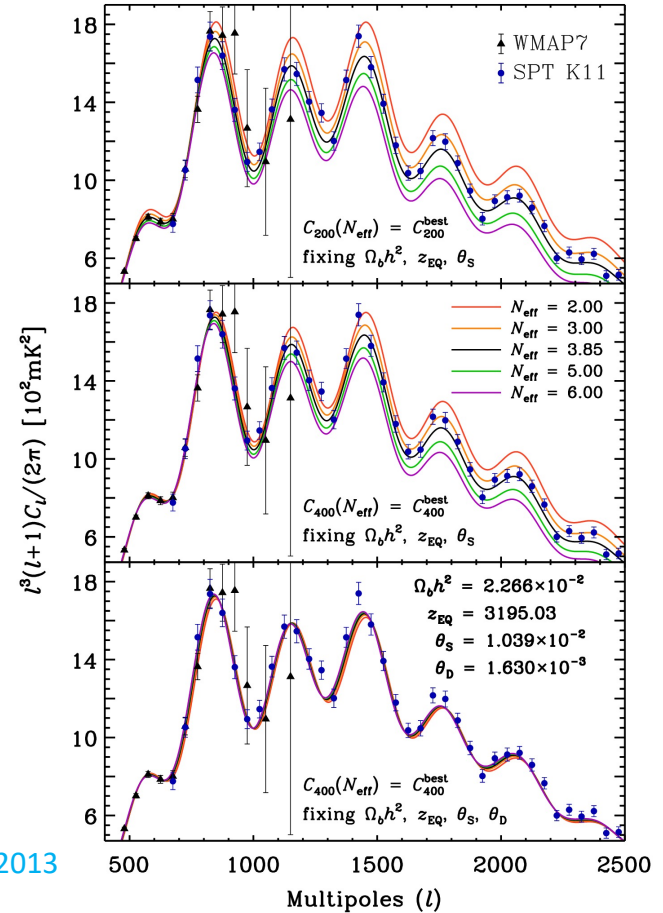
1/2

- Varying N_{eff} impacts directly on matter-radiation equality and the expansion rate at recombination.

→ Observable effects in the CMB anisotropies

- Planck-era signature primarily in the **CMB damping tail**

- An observed $N_{\text{eff}} > N_{\text{eff}}^{\text{SM}}$ indicates an excess of non-photon radiation and could be a **sign of new physics**.



Why bother with precision $N_{\text{eff}}^{\text{SM}}$?

2/2

- **Current cosmological constraints:**

Λ CDM+Neff 7-parameter fit	Planck 2018 (95%)	Planck2015 (95%)
TT+lowE	$3.00^{+0.57}_{-0.53}$	3.13 ± 0.64
+lensing+BAO	$3.11^{+0.44}_{-0.43}$	n/a
TT+lowE+TE+EE	$2.92^{+0.36}_{-0.37}$	2.99 ± 0.40
+lensing+BAO	$2.99^{+0.34}_{-0.33}$	n/a

- **Future 1σ sensitivity (CMB-S4):** $\sigma(N_{\text{eff}}) \sim 0.02 - 0.04$

Aghanim [PLANCK] 2018
Ade [PLANCK] 2015

→ This motivates us to pursue $N_{\text{eff}}^{\text{SM}}$ to the **next significant digit**.

Precision $N_{\text{eff}}^{\text{SM}}$...

Bennett, Buldgen, Drewes & Y³W 2020;
Bennett, Buldgen, de Salas, Drewes, Gariazzo, Pastor & Y³W 2021;
Froustey, Pitrou & Volpe, 2020

See also Akita & Yamaguchi 2020; Hansen, Shalgar & Tamborra
2021; Escudero 2020 for related works

- The **most precise to-date** computation of the standard model N_{eff} :

$$N_{\text{eff}}^{\text{SM}} = 3.0440 \pm 0.0002$$

- **Two independent calculations**: same physics but using **independent numerical implementations** by two independent groups
 - Central values agree to **five significant digits**
 - Broadly consistent uncertainty assessment

Precision $N_{\text{eff}}^{\text{SM}}$...

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- The **most precise to-date** computation of the standard model N_{eff} :

$$N_{\text{eff}}^{\text{SM}} = 3.0440 \pm 0.0002$$

- **What goes into it:**

- **Neutrino decoupling tracked with Quantum Kinetic Equations**, including
 - In-medium 3-flavour oscillations Sigl & Raffelt 1993; McKeller & Thomson 1994
 - All $2 \rightarrow 2$ neutrino-neutrino and neutrino-electron weak interactions
 - Full momentum dependence plus quantum statistics implemented in FortEPiANO
- **Finite-temperature QED correction** to Gariazzo, de Salas & Pastor 2019
 - Equation of state of photon-electron plasma
 - Weak scattering rates (thermal masses only; other corrections yet to be determined although expected to be inconsequential)

Precision $N_{\text{eff}}^{\text{SM}}$...

Bennett, Buldgen, Drewes & Y³W 2020;
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- The **most precise to-date** computation of the standard model N_{eff} :

$$N_{\text{eff}}^{\text{SM}} = 3.0440 \pm 0.0002$$

- **Compare with older values:**

$$N_{\text{eff}}^{\text{SM}} = 3.046 \pm 0.002 \quad \text{Mangano et al. 2005}$$

$$N_{\text{eff}}^{\text{SM}} = 3.045 \pm 0.001 \quad \text{de Salas & Pastor 2015}$$

At face value:

- The new central value, while broadly consistent with older calculations, tends to the low end; **more later**
- Factor of 5 improvement in the uncertainty; primarily due to better numerical implementation

Precision $N_{\text{eff}}^{\text{SM}}$...

Bennett, Buldgen, Drewes & Y³W 2020;
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Beneath the surface, new calculations contain **new elements**:

- NLO finite-temperature QED corrections to the plasma EOS
- EW running of weak couplings
- Full implementation of neutrino-neutrino scattering

Precision $N_{\text{eff}}^{\text{SM}}$...

Bennett, Buldgen, Drewes & Y³W 2020;
Bennett, Buldgen, de Salas, Drewes, Gariazzo, Pastor & Y³W 2021;
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Standard-model corrections to $N_{\text{eff}}^{\text{SM}}$	Leading-digit contribution
m_e/T_d correction	+0.04
$\mathcal{O}(e^2)$ FTQED correction to the QED EoS	+0.01
Non-instantaneous decoupling+spectral distortion	-0.005
* $\mathcal{O}(e^3)$ FTQED correction to the QED EoS	-0.001
Flavour oscillations	+0.0005
Type (a) FTQED corrections to the weak rates	$\lesssim 10^{-4}$

* Previously neglected correction: main cause of the central value shift Bennett et al. 2020

Precision $N_{\text{eff}}^{\text{SM}}$...

Bennett, Buldgen, Drewes & Y³W 2020;
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- The **most precise to-date** computation of the standard model N_{eff} :

$$N_{\text{eff}}^{\text{SM}} = 3.0440 \pm 0.0002$$

- **Uncertainty estimate:**

- ~ 0.0001 from numerical errors (discretisation artefacts in numerical solutions of the QKEs)
- ~ 0.0001 from **experimental uncertainty in the solar mixing angle:**

$$\sin^2 \theta_{12} = (3.18 \pm 0.16) \times 10^{-1}$$

de Salas et al. 2021
Esteban et al. 2020

Take-home message: Part I...

- A most precise to-date determination of the **standard model effective number of neutrinos**:

$$N_{\text{eff}}^{\text{SM}} = 3.0440 \pm 0.0002$$

- Already adopted as **default value** in the latest release of CLASS.
- **Uncertainty**: part numerical errors in the solution of the QKEs, part due to experimental uncertainty in the solar mixing angle $\sin^2 \theta_{12}$
- **Remains to be determined**: corrections from certain types of thermal loop corrections to the weak rates. Watch this space!

II. Cosmological implications of non-standard neutrino interactions...

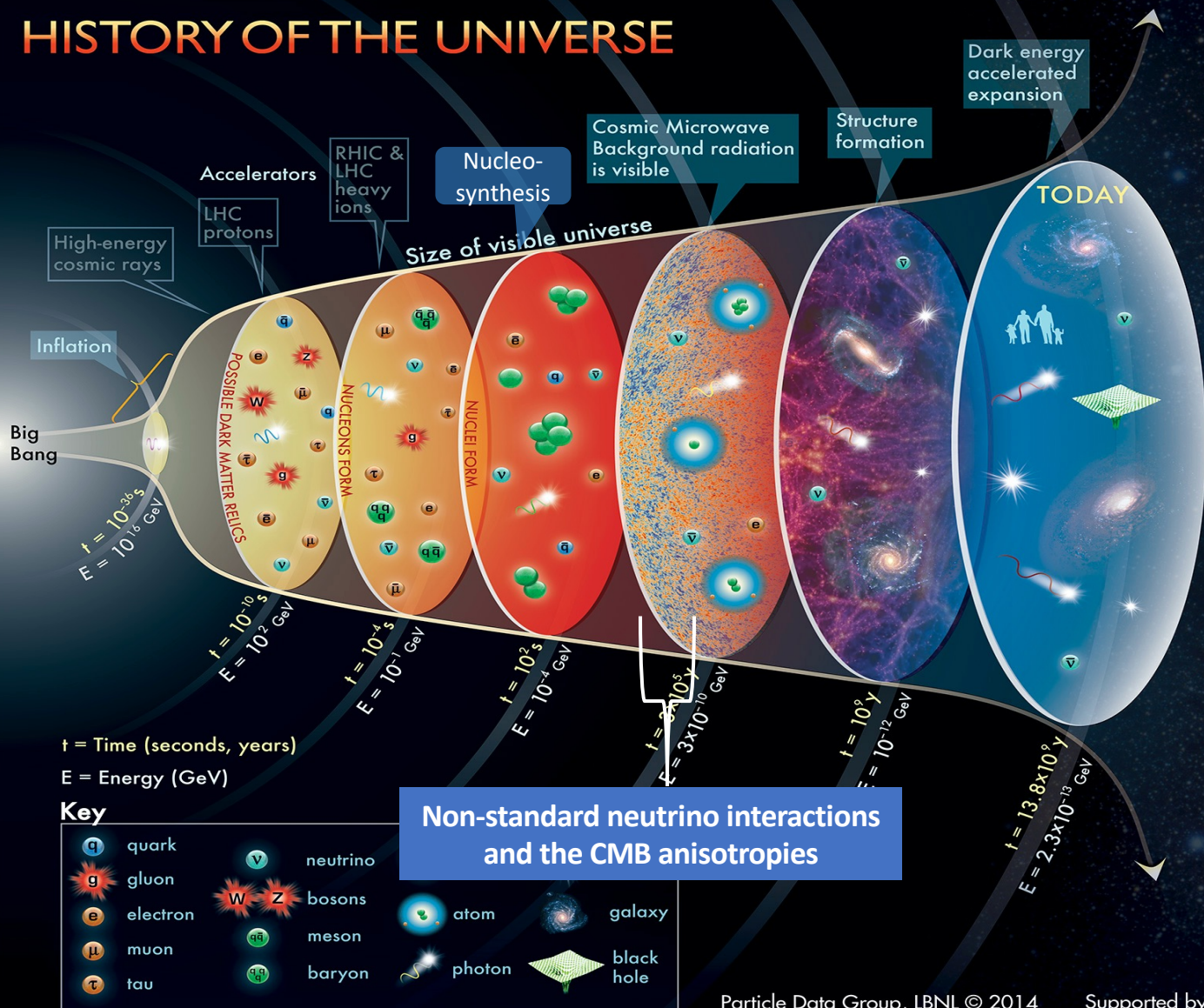
Oldengott, Rampf & Y³W, *JCAP* 04 (2015) 016

Oldengott, Tram, Rampf & Y³W, *JCAP* 11 (2017) 027

Mosbech, Boehm, Hannestad, Mena, Stadler & Y³W, *JCAP* 03 (2021) 066

Barenboim, Chen, Hannestad, Oldengott, Tram & Y³W, *JCAP* 03 (2021) 087

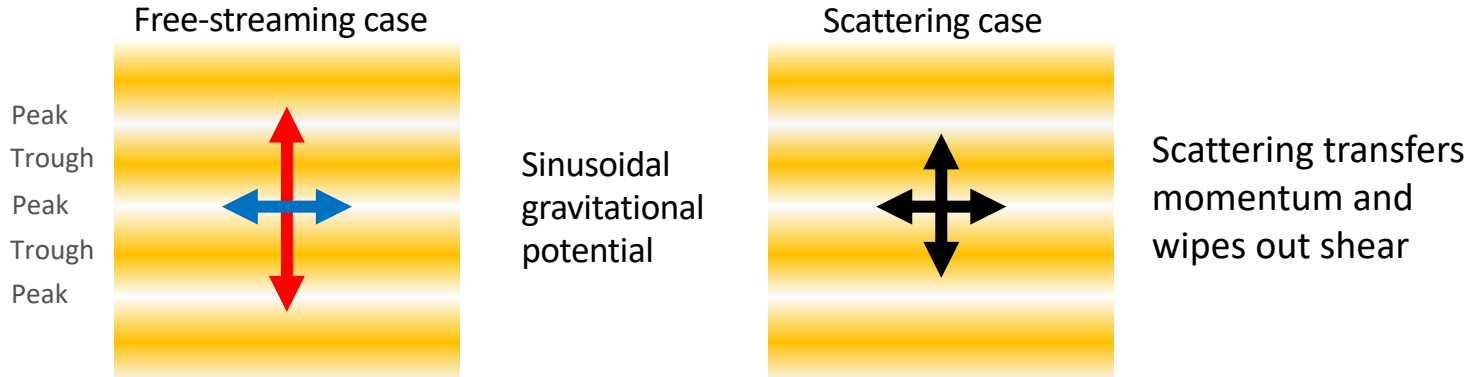
HISTORY OF THE UNIVERSE



Neutrino interactions and the CMB...

1/2

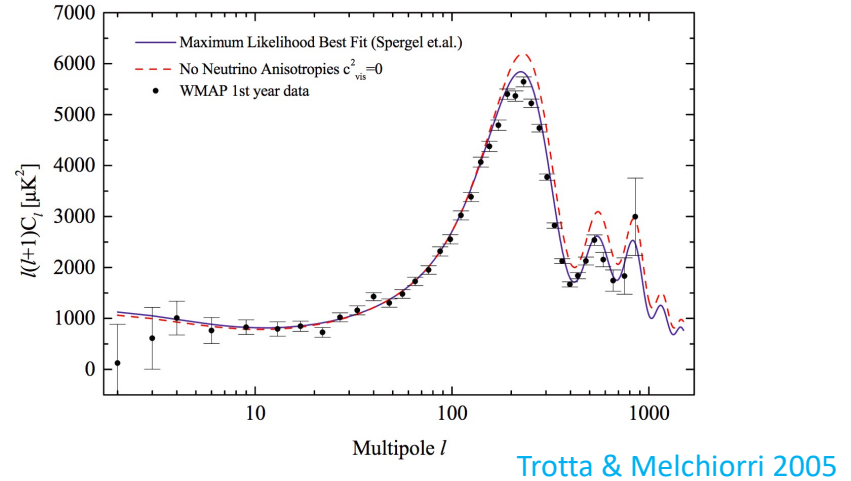
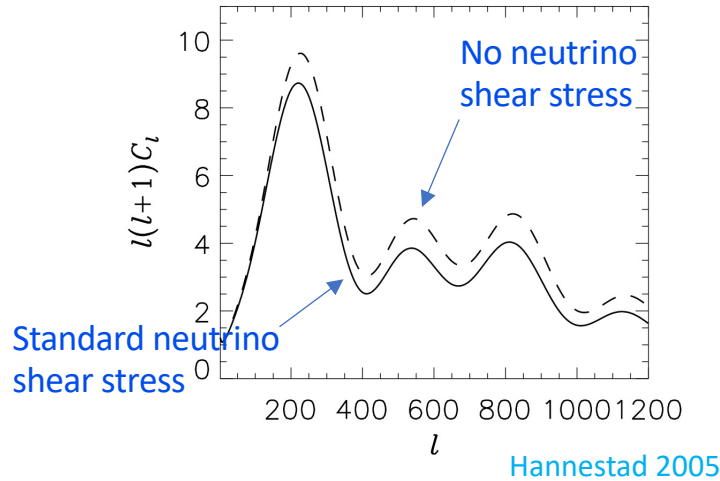
- After neutrino decoupling, standard neutrinos free-stream.
 - **Free-streaming** in a spatially inhomogeneous background induces **shear stress**.
 - Conversely, **scattering tends to isotropise** a phase space element.



- **Observable consequences in the CMB**, when neutrinos are ultra-relativistic and form a substantial fraction of universe's energy density.

Neutrino interactions and the CMB... 2/2

- That **CMB prefers neutrino shear stress to no shear stress** is well known.

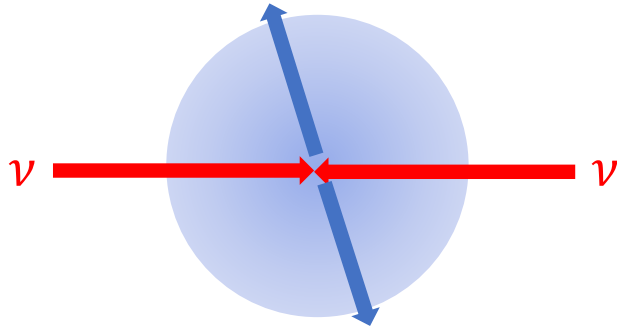


- The tricky part is how to translate this preference to constraints on the **fundamental parameters** of a non-standard neutrino interaction
→ What is the **isotropisation or transport rate**?

Transport rate from scattering...

1/2

- **2 → 2 neutrino-neutrino scattering:** The final state particles are equally likely to be emitted in any direction.
 - It takes one scattering event to transfer momentum by 90 degrees.



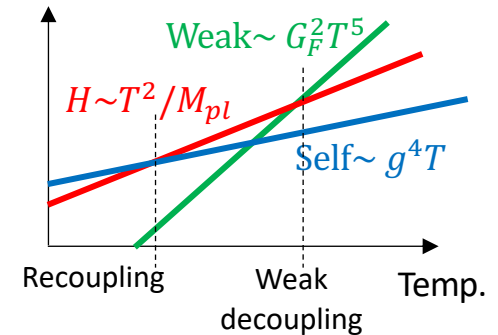
$$\begin{aligned}\Gamma_{\text{transport}} &\sim \Gamma_{\text{scattering}} \\ &\sim G_{\text{eff}}^2 T^5 \\ &\sim g^4 T\end{aligned}$$

- 4-Fermi is a **decoupling scenario**

Cyr-Racine & Sigurdson 2014; Oldengott, Rampf & Y³W 2015; Lancaster, et al. 2017; Oldengott, Tram, Rampf & Y³W 2017; Kreisch, Cyr-Racine & Dore 2019

- Light mediator is a **recoupling scenario**

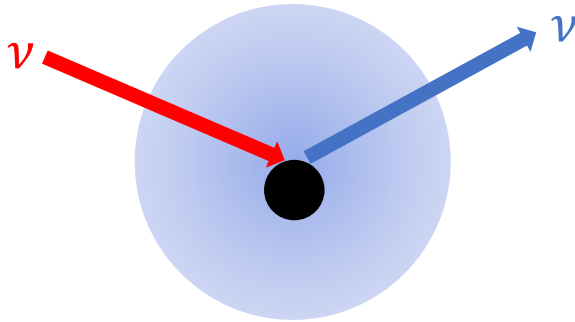
Forastieri, Lattanzi & Natoli 2015



Transport rate from scattering...

2/2

- A variation: **neutrino scattering on cold dark matter** → Again, it takes one scattering event to transfer neutrino momentum by 90 degrees.



$$\Gamma_{\text{transport}} \sim \Gamma_{\text{scattering}}$$

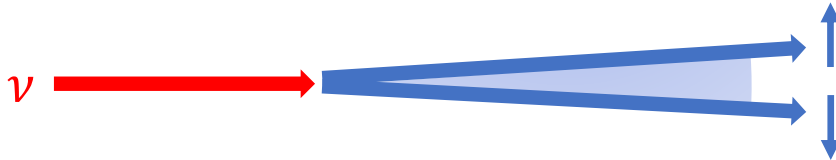
Wilkinson, Boehm & Lesgourgues 2015

Mosbech, Boehm, Hannestad, Mena, Stadler & Y³W 2021

- Also has non-trivial consequences for the dark matter perturbations, particularly on very small scales; so phenomenologically not exactly the same as neutrino self-interaction.

Transport rate from relativistic decay...

- **Relativistic 1 → 2 decay to massless particles** (and inverse decay) has an opening angle $\sim m_\nu/E$ (mass and energy of the decaying particle).



One power from Lorentz boost;
Two powers from opening angle

$$\Gamma_{\text{transport}}^{\text{old}} \sim \left(\frac{m_\nu}{E}\right)^3 \Gamma_{\text{decay}}^{\text{rest}}$$

Hannestad & Raffelt 2005

- Old estimate neglected **momentum conservation**: Only the transverse momentum (suppressed by m_ν/E) can be transported.

$$\Gamma_{\text{transport}}^{\text{new}} \sim \left(\frac{m_\nu}{E}\right)^5 \Gamma_{\text{decay}}^{\text{rest}}$$

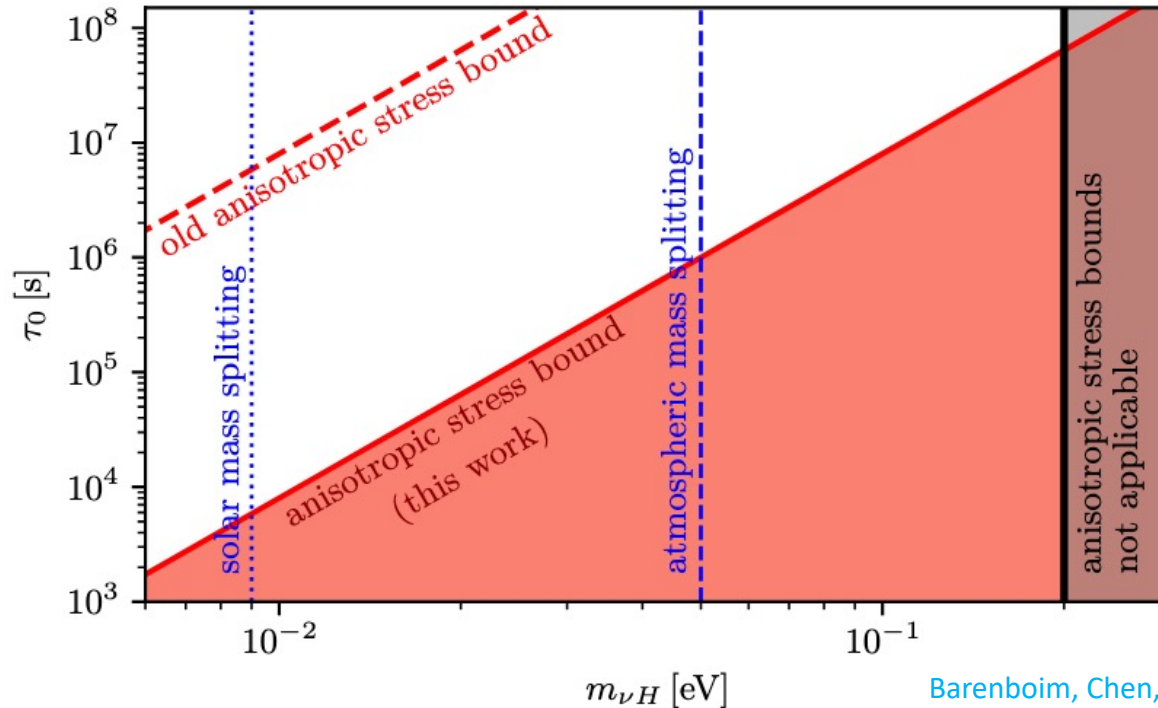
- Revised transport rate as **two extra powers** of m_ν/E .

First-principles derivation in

Barenboim, Chen, Hannestad, Oldengott, Tram & Y³W 2021

New CMB constraint on the neutrino lifetime...

- Revised transport rate → **weaker bound on the neutrino lifetime**



$$\tau_0^{\text{old}} \geq 10^{-9} \text{ s} \left(\frac{m_\nu}{50 \text{ meV}} \right)^3$$

$$\tau_0^{\text{new}} \geq 10^{-6} \text{ s} \left(\frac{m_\nu}{50 \text{ meV}} \right)^5$$

IIb. Shear stress loss and the H_0 tension...

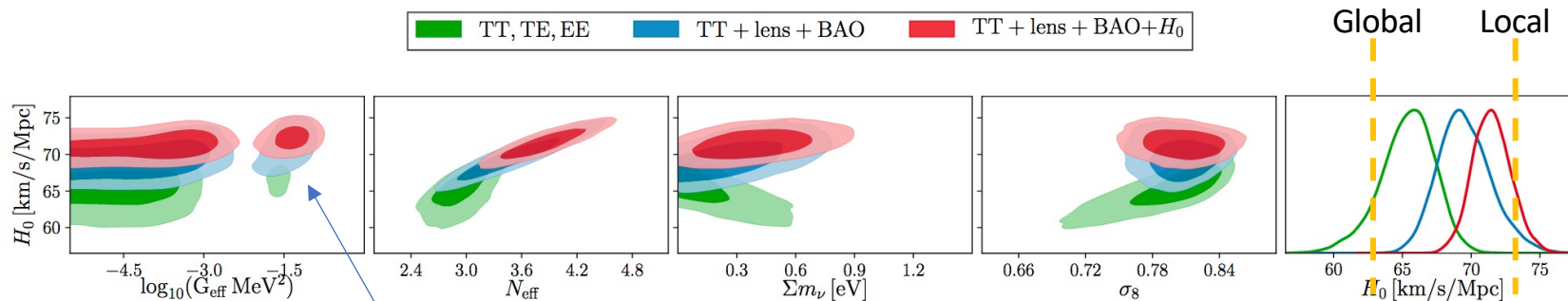
Oldengott, Rampf & Y³W, *JCAP* 04 (2015) 016

Oldengott, Tram, Rampf & Y³W, *JCAP* 11 (2017) 027

Shear stress loss and the H_0 tension... 1/3

Kreisch, Cyr-Racine & Dore 2019

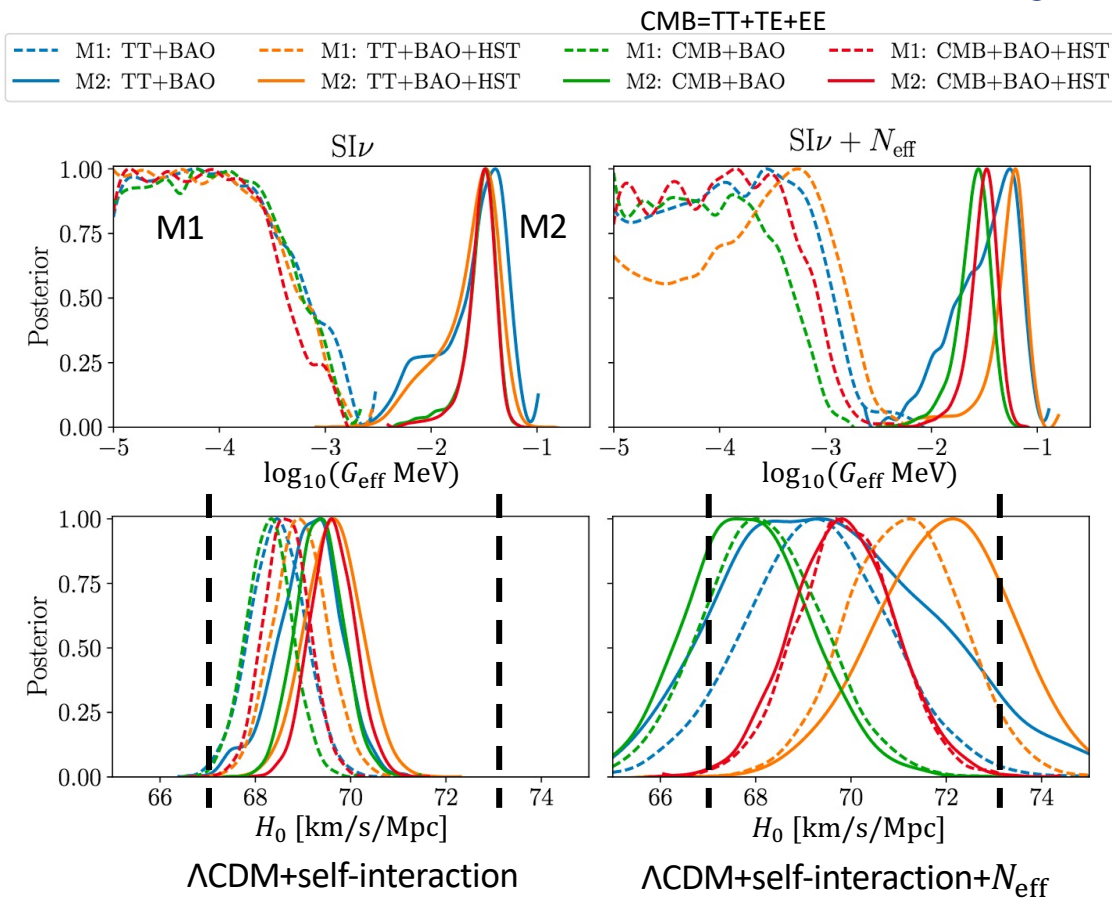
- Recent claim that shear stress loss due to neutrino self-interaction **alleviates the Hubble tension.**
 - **Local/late time:** Cepheid-calibrated SNIa (SHOES) and strong-lensing time delays (H0LiCOW); $H_0 = (73.5 \pm 1.4) \text{ km/s/Mpc}$
 - **Global/early time:** Statistical inference from CMB anisotropies (Planck), weak lensing, BAO; $H_0 = (67.4 \pm 0.5) \text{ km/s/Mpc}$



- Does it?

This island:
 $G_{\text{eff}} \sim 10^{10} G_F$

Shear stress loss and the H_0 tension... 2/3



- Self-interaction alone does shift the inferred H_0 up (**left**).
- But need a large N_{eff} as well to substantially lift H_0 (**right**) because of well-known (N_{eff}, H_0) -degeneracy.
- Even then, TT+BAO and TT+TE+EE+BAO alone do not prefer a large H_0 (**blue, green**)
- TT+BAO only prefers high H_0 when combined with a local measurement HST (**orange**)
- Polarisation “kills” it anyway: TT+TE+EE+BAO fit settles in a **compromise region (red)**.
- M1 (**red dashed**) & M2 (**red solid**) coincide on right \rightarrow all gains in H_0 come from N_{eff} alone; self-interaction adds nothing substantial

Shear stress loss and the H_0 tension... 3/3

Λ CDM+self-int		TT+BAO	TT+BAO+HST	CMB=TT+TE+EE CMB+BAO	CMB+BAO+HST
$\log_{10}(G_{\text{eff}} \text{ MeV})$	<i>M1</i>	< -2.7 (95%)	< -2.8 (95%)	< -2.8 (95%)	< -2.9 (95%)
	<i>M2</i>	$-1.6^{+0.5}_{-0.7}$	$-1.6^{+0.5}_{-0.6}$	$-1.5^{+0.3}_{-0.4}$	$-1.5^{+0.3}_{-0.3}$
H_0 [km/s/Mpc]	<i>M1</i>	$68.5^{+1.2}_{-1.2}$	$68.9^{+1.1}_{-1.1}$	$68.3^{+1.0}_{-1.0}$	$68.7^{+1.0}_{-1.0}$
	<i>M2</i>	$69.1^{+1.3}_{-1.5}$	$69.6^{+1.2}_{-1.2}$	$69.3^{+1.0}_{-1.0}$	$69.6^{+1.0}_{-0.9}$
$\chi^2_{M2} - \chi^2_{M1}$		4.5	2.9	3.4	0.9

Λ CDM+self-int+ N_{eff}		TT+BAO	TT+BAO+HST	CMB+BAO	CMB+BAO+HST
$\log_{10}(G_{\text{eff}} \text{ MeV})$	<i>M1</i>	< -2.3 (95%)	< -2.4 (95%)	< -2.9 (95%)	< -2.6 (95%)
	<i>M2</i>	$-1.5^{+0.5}_{-0.6}$	$-1.3^{+0.3}_{-0.6}$	$-1.6^{+0.3}_{-0.4}$	$-1.5^{+0.3}_{-0.3}$
N_{eff}	<i>M1</i>	$3.2^{+0.5}_{-0.5}$	$3.4^{+0.4}_{-0.4}$	$3.0^{+0.4}_{-0.4}$	$3.3^{+0.3}_{-0.3}$
	<i>M2</i>	$3.1^{+0.8}_{-0.7}$	$3.5^{+0.5}_{-0.6}$	$2.8^{+0.4}_{-0.4}$	$3.1^{+0.3}_{-0.3}$
H_0 [km/s/Mpc]	<i>M1</i>	$69.3^{+3.1}_{-3.1}$	$71.1^{+2.4}_{-2.4}$	$68.2^{+2.6}_{-2.5}$	$69.9^{+2.1}_{-2.1}$
	<i>M2</i>	$69.6^{+4.5}_{-4.2}$	$71.9^{+2.9}_{-3.1}$	$67.9^{+2.6}_{-2.6}$	$69.8^{+2.2}_{-2.2}$
$\chi^2_{M2} - \chi^2_{M1}$		4.9	3.5	2.6	1.8

Compare with
 Λ CDM+ N_{eff} fit:

$$N_{\text{eff}} = 3.27 \pm 0.3$$

$$H_0 = 69.32 \pm 1.9$$

km/s/Mpc

TT+TE+EE+lowE+lensing
+BAO+R18

[Aghanim et al. 2018](#)



It's N_{eff} doing the leg work; adding self-interaction too has no statistically significant effect.

Summary...

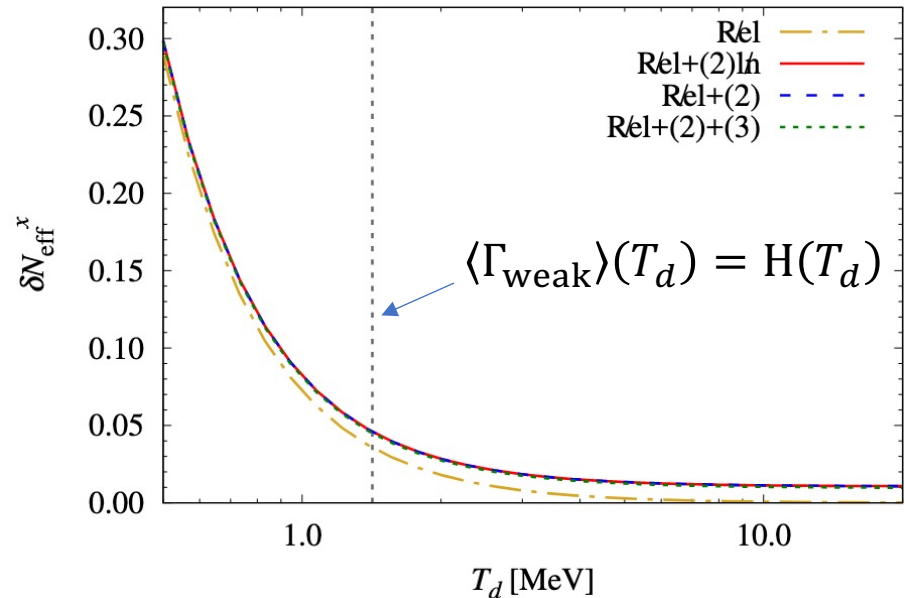
- I have presented:
 - A new precise calculation of the **standard model N_{eff}** ; the new value has already been implemented in the latest release of CLASS.
 - A revised cosmological limit on the neutrino lifetime based on a revised transport rate.
- **Neutrino self-interaction as a solution to the H_0 tension is doubtful.**
 - Superficial agreement between local measurement and global inference of H_0 can be achieved only if you cherry-pick your data sets.
 - Check the free parameters of the fit: if N_{eff} is allowed to vary, then it's just the (N_{eff}, H_0) - degeneracy doing the legwork; self-interaction is a red herring.

Extra slides...

Standard-model corrections to $N_{\text{eff}}^{\text{SM}}$	Leading-digit contribution
m_e/T_d correction	+0.04
$\mathcal{O}(e^2)$ FTQED correction to the QED EoS	+0.01
Non-instantaneous decoupling+spectral distortion	-0.005
$\mathcal{O}(e^3)$ FTQED correction to the QED EoS	-0.001
Flavour oscillations	+0.0005
Type (a) FTQED corrections to the weak rates	$\lesssim 10^{-4}$

Non-relativistic correction

- Dominant contribution
- Its size can be estimated from standard textbook entropy conservation arguments.

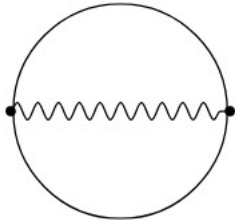


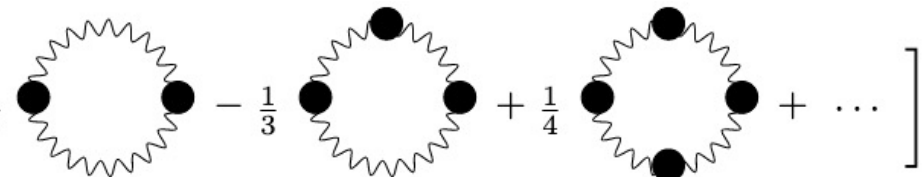
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Type (a) FTQED corrections to the weak rates	$\lesssim 10^{-4}$

Finite-temperature corrections to the QED equation of state

- At $T \geq m_e$, the $\gamma e^+ e^-$ plasma is **no longer an ideal gas**.

- Evolution of plasma energy density and pressure modified by FTQED corrections to the partition function.

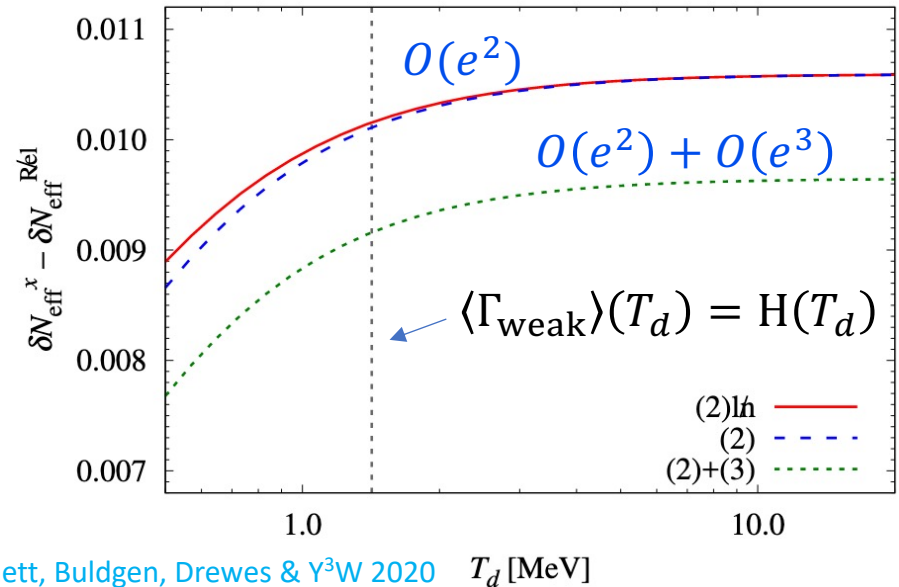
$$\ln Z^{(2)} = -\frac{1}{2} \text{ (diagram) } \mathcal{O}(e^2)$$


$$\mathcal{O}(e^3) \ln Z^{(3)} = \frac{1}{2} \left[\frac{1}{2} \text{ (diagram 1) } - \frac{1}{3} \text{ (diagram 2) } + \frac{1}{4} \text{ (diagram 3) } + \dots \right]$$


Standard-model corrections to $N_{\text{eff}}^{\text{SM}}$	Leading-digit contribution
m_e/T_d correction	+0.04
$\mathcal{O}(e^2)$ FTQED correction to the QED EoS	+0.01
Non-instantaneous decoupling+spectral distortion	-0.005
$\mathcal{O}(e^3)$ FTQED correction to the QED EoS	-0.001
Flavour oscillations	+0.0005
Type (a) FTQED corrections to the weak rates	$\lesssim 10^{-4}$

Finite-temperature corrections to the QED equation of state

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- Evolution of plasma energy density and pressure modified by FTQED corrections to the partition function.



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Non-instantaneous decoupling + neutrino oscillations

- Higher-energy neutrinos stay coupled to the QED plasma for longer.
- Oscillations mix ν_e and $\nu_{\mu,\tau}$ (the latter has no CC coupling to e^+e^-).
- Tracked by the **quantum kinetic equations (QKEs)**: [Sigl & Raffelt 1993](#); [McKellar & Thomson 1994](#)

$$\frac{\partial \rho}{\partial t} = -i[H, \rho] + I[\rho]$$

Neutrino density matrix

Oscillation Hamiltonian incl. matter effects

Weak collisions (non-unitary evolution) from $\nu_e \leftrightarrow \nu_e$ and $\nu\nu \leftrightarrow \nu\nu$ scattering

- **Publicly available** solver package: FortEPiano

[Gariazzo, de Salas & Pastor 2019](#)

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Oscillations not a big effect on N_{eff} when $\nu\nu \leftrightarrow \nu\nu$ scattering is switched on.



Non-instantaneous decoupling + neutrino oscillations

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Neutrino density matrix

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Oscillation Hamiltonian incl. matter effects

Weak collisions (non-unitary evolution) from $\nu e \leftrightarrow \nu e$ and $\nu\nu \leftrightarrow \nu\nu$ scattering

New in 2020/2021
precision calculations

- **Publicly available** solver package: FortEPiano

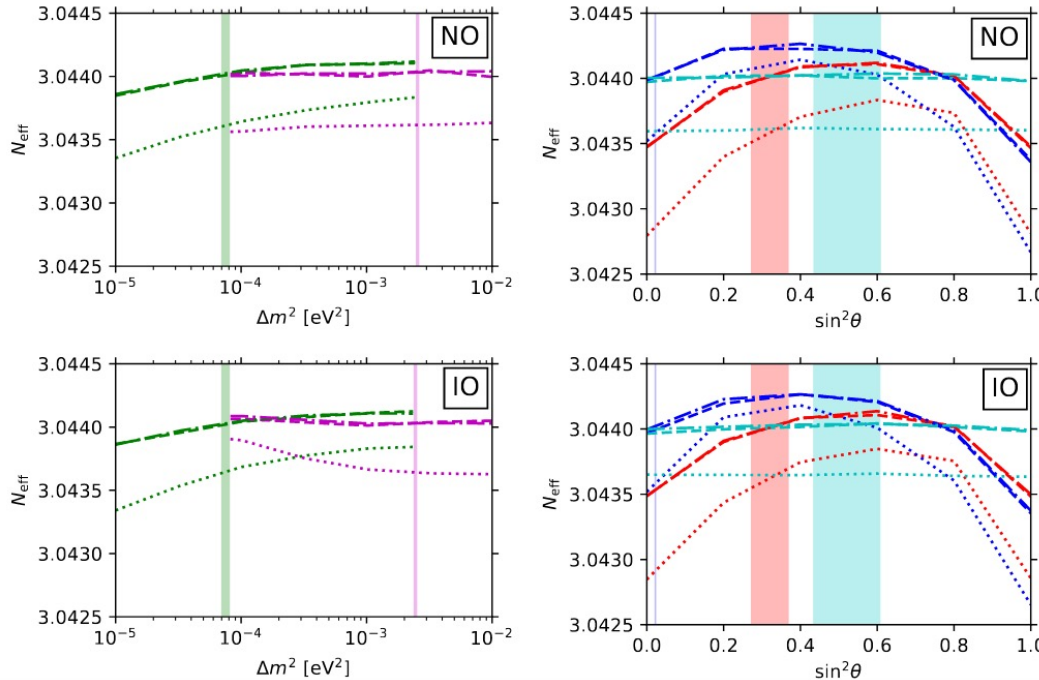
[Gariazzo, de Salas & Pastor 2019](#)

- However, **uncertainties in the neutrino oscillation parameters** do account for a **large chunk of the theoretical uncertainty** in $N_{\text{eff}}^{\text{SM}}$.

Shaded blue regions = 3σ region favoured by osc. experiments

..... no $\nu\nu$ — $\sin^2\theta_{12}$ — Δm_{21}^2
 - - - $\nu\nu$, GL — $\sin^2\theta_{13}$ — Δm_{31}^2
 - · - $\nu\nu$, NC — $\sin^2\theta_{23}$

$$N_{\text{eff}}^{\text{SM}} = 3.0440 \pm 0.0002$$



- Negligible effects from varying Δm_{21}^2 and Δm_{31}^2 .
- Of the mixing angles, only $\sin^2\theta_{12}$ leads to an $O(0.0001)$ change in $N_{\text{eff}}^{\text{SM}}$ within the 3σ exp. allowed region.
- Remaining $O(0.0001)$ = error from numerical solution of QKEs.

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Type (a) FTQED corrections to the weak rates	$\lesssim 10^{-4}$

Smaller than current uncertainty due to numerical solution of the QKEs.



FTQED corrections to the weak rates

- At $T \geq m_e$, expect in-medium QED corrections to the weak interaction rates.
 - Type (a) = thermal mass electron mass; included in calculation.
 - Type (b) to (d) corrections currently under investigation.

