Black holes

Vítor Cardoso

[Técnico & Niels Bohr Institute]

Image: Einstein and Snoop Dogg in Lisbon
Photograph by Ana Carvalho (2018)

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Uniqueness: the Kerr solution

Theorem (Carter 1971; Robinson 1975; Chrusciel & Costa 2012):
A stationary, asymptotically flat, vacuum BH solution must be Kerr

$$
\begin{aligned}
ds^2 \quad = \quad & \frac{\Delta - a^2 \sin^2\theta}{\Sigma} dt^2 + \frac{2a(r^2+a^2-\Delta)\sin^2\theta}{\Sigma} dt d\phi \\
& - \frac{(r^2+a^2)^2 - \Delta a^2 \sin^2\theta}{\Sigma} \sin^2\theta d\phi^2 - \frac{\Sigma}{\Delta} dr^2 - \Sigma d\theta^2
\end{aligned}
$$

$$
\Sigma = r^2 + a^2\cos^2\theta\,, \quad \Delta = r^2 + a^2 - 2Mr
$$

Describes a rotating BH with mass M and angular momentum J=aM, iff a<M

"In my entire scientific life, extending over forty-five years, the most shattering experience has been the realization that an exact solution of Einstein's equations of general relativity provides the *absolutely exact representation* of untold numbers of black holes that populate the universe."

S. Chandrasekhar, The Nora and Edward Ryerson lecture, Chicago April 22 1975

Singularities & Censorship

Theorem (Penrose 1965; 1969):

For “reasonable” matter, trapped surface formation results in “singularity,” where at least one of the following holds:

a. Negative local energy occurs.
b. Einstein's equations are violated.
c. The space-time manifold is incomplete.
d. The concept of space-time loses its meaning at very high curvatures – possibly because of quantum phenomena.

Conjecture (Penrose 1969):

No singularity is visible from future null infinity (weak CCC)
General Relativity is deterministic (strong CCC)

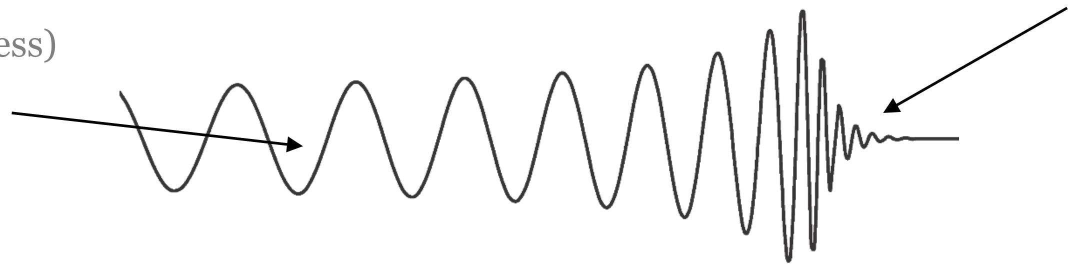
The simplicity slide: BHs as perfect laboratories

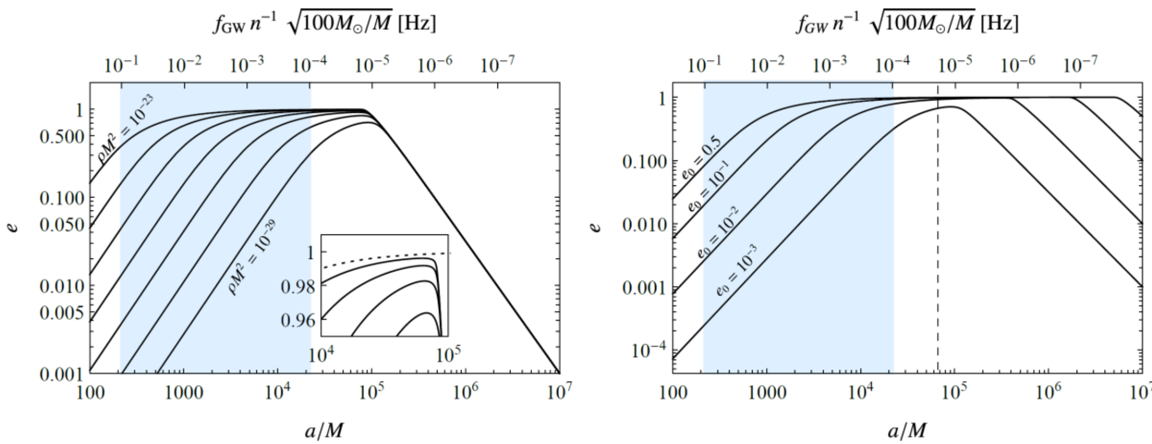
Simple spin-induced multipoles (uniqueness)

Tidal heating

Tidal Love number (BHs don't polarize)

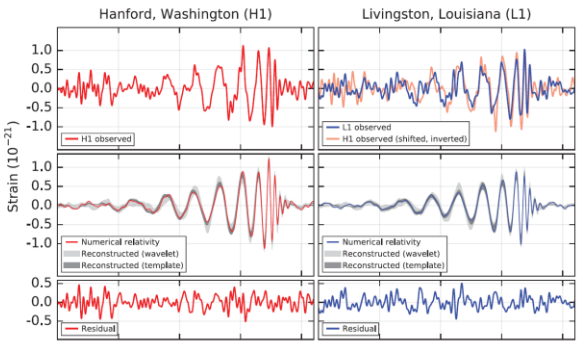
Simple relaxation of final state (uniqueness)



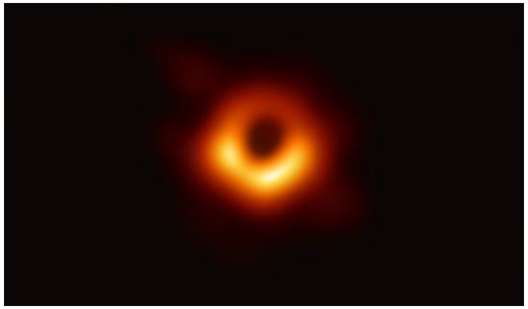


Cardoso & Pani, Nature Astronomy 1: 586 (2017); Living Reviews in Relativity 22: 1 (2019)
Peters PR136: B1224 (1964); Cardoso, Macedo & Vicente PRD103: 023015 (2021)

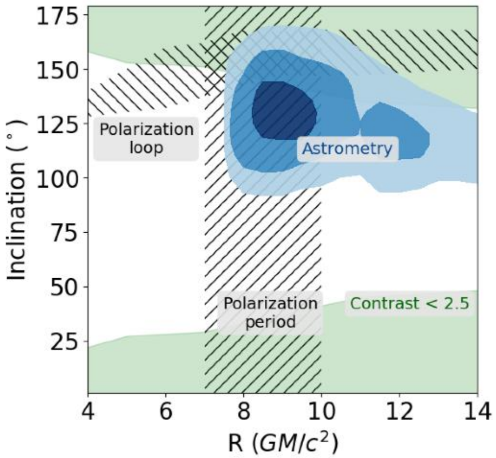
They are out there



LIGO/Virgo Collaboration PRL116:061102 (2016)



EHT Collaboration ApJL 875: 1 (2019)



GRAVITY Collaboration AA 635: A143 (2020)

Fundamental questions

a. BH seeds, BH demography, galaxy co-evolution (how many, where, how?)
See review Barack+ CQG 36: 143001 (2019); arXiv:1806.05195

b. Is cosmic censorship preserved?
Sperhake+ PRL103:131102 (2009); Cardoso+ PRL120:031103 (2018)

c. Is it a Kerr black hole? Can we constrain alternatives?
Cardoso+ CQG33:174001 (2016); Isi+PRL123:111102 (2019); Jaramillo+ PRX11: 031003 (2021)

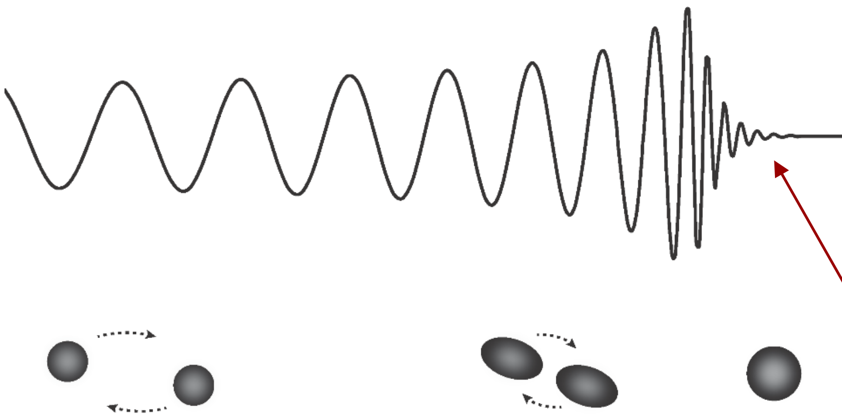
d. Is the final - or initial - object really a black hole?
Cardoso+ PRL116: 171101 (2016); Nature Astronomy 1: 586 (2017)

e. Can we do galaxy tomography, or constrain DM?
Barausse+ PRD89:104059 (2014); Cardoso+ AA644:147 (2020); Cardoso+ arXiv:2109.00005

f. Can GWs from BHs inform us on new fundamental fields?
Arvanitaki+ PRD95: 043001 (2016); Brito+ PRL119:131101 (2017)

Answer requires understanding of theoretical framework, PDEs, precise modelling, challenging simulations & challenging data analysis techniques

BH spectroscopy: testing the Kerr nature



Ringdown

$$\frac{\partial^2\Psi}{\partial r_*^2} - \frac{\partial^2\Psi}{\partial t^2} - V(r_*)\Psi = \mathrm{S} \qquad \sim e^{-i\omega t}$$

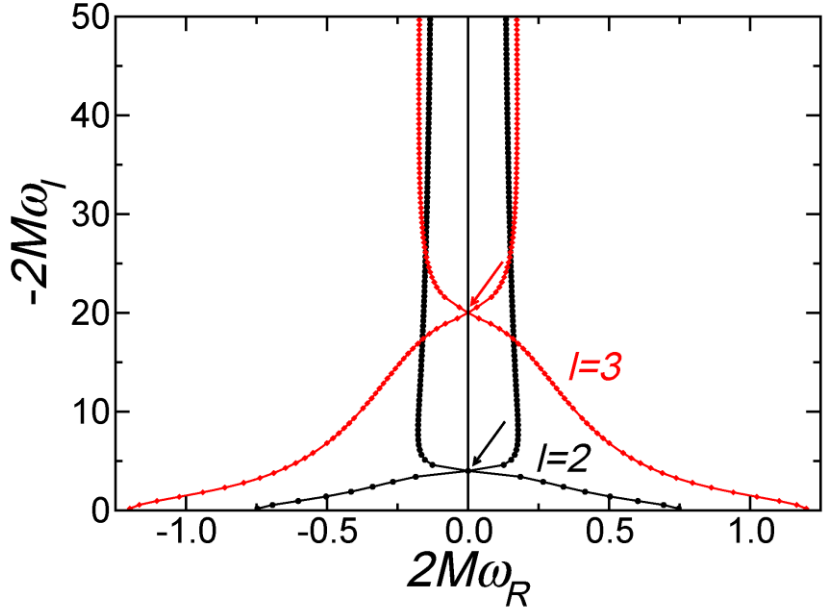
$$\left(\omega^2 - \mathcal{L}\right)\psi = s \rightarrow \psi = \left(\omega^2 - \mathcal{L}\right)^{-1} s$$

Resolvent

$$h = \sum_{n\ell m} A_{n\ell m} e^{-t/\tau_{n\ell m}} \sin(2\pi f_{n\ell m} t) Y_{\ell m}(\theta, \phi)$$

Leaver PRD34:384 (1986)

Berti + PRD73: 064030 (2006); PRL 117:101102 (2016); Cardoso & Gualtieri CQG33: 174001 (2016)



$$f = \omega_R/2\pi = 1.207 \left(\frac{10\ M_\odot}{M}\right) \mathrm{kHz}$$

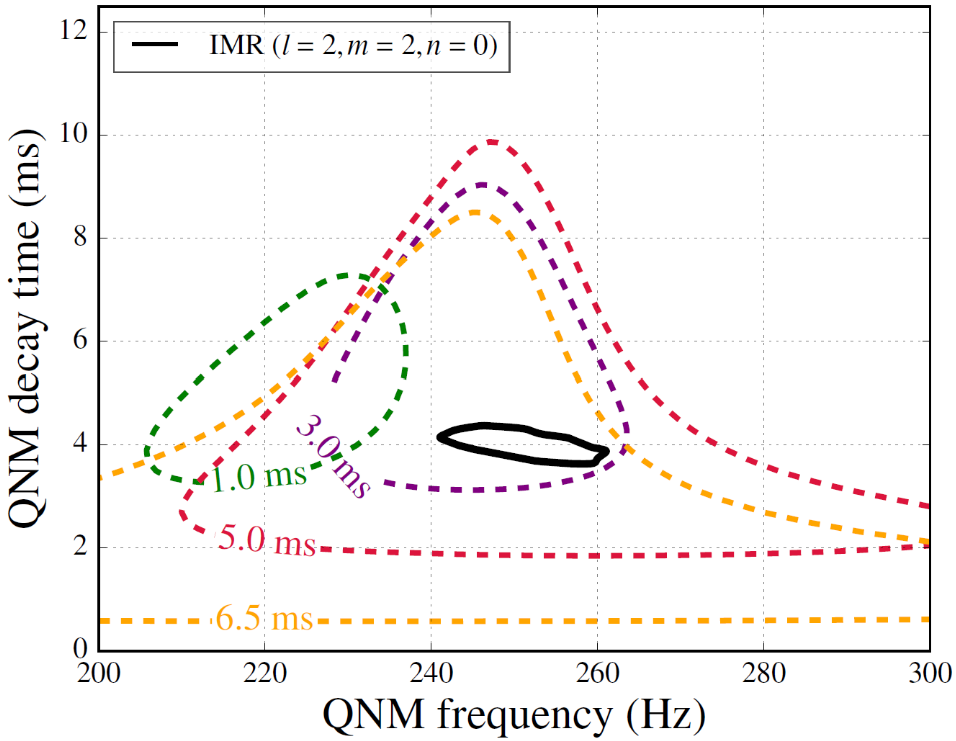
$$\tau = 1/|\omega_I| = 0.5537 \left(\frac{M}{10\ M_\odot}\right) \mathrm{ms}$$

Berti+ PRD73: 064030 (2006); Berti + CQG 26: 163001 (2009)

"*After the advent of gravitational wave astronomy, the observation of these resonant frequencies might finally provide direct evidence of BHs with the same certainty as, say, the 21 cm line identifies interstellar hydrogen*" (S. Detweiler ApJ239:292 1980)

Black hole spectroscopy

Berti, Cardoso & Will PRD73:064030 (2006)



90% posterior distributions.

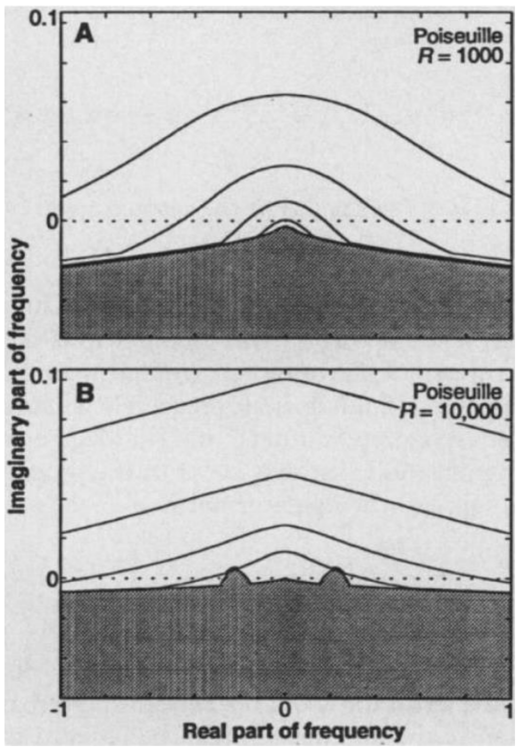
Black solid is 90% posterior of QNM as derived from the posterior mass and spin of remnant

LSC PRL116:221101 (2016); see Bhaibav+PRD97:044048 (2019); Isi+ PRL123:111102 (2019)
See also recent LIGO/Virgo analysis arXiv:2010.14529

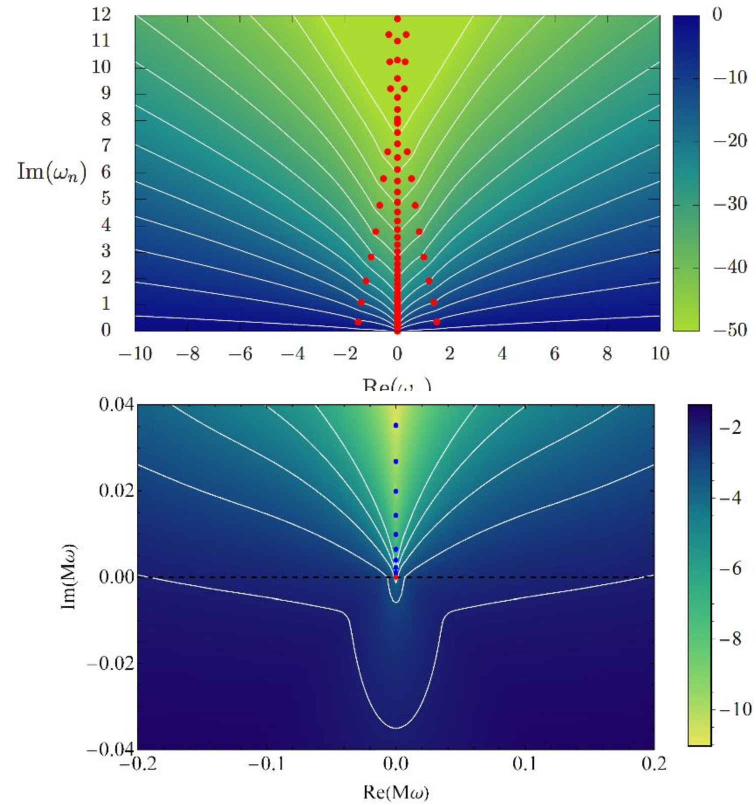
Pseudospectra

Trefethen+ Science 261: 578 (1993); Jaramillo+ PRX11:031003 (2021)

$$\left\| \left(\omega^2 - \mathcal{L}\right)^{-1} \right\| \geq \epsilon^{-1}$$



(plane) Poiseuille: parabolic velocity profile between static infinite plates



Pseudospectrum of extremal BHs

Destounis+ arXiv:2107.09673

Quantifying the evidence for black holes

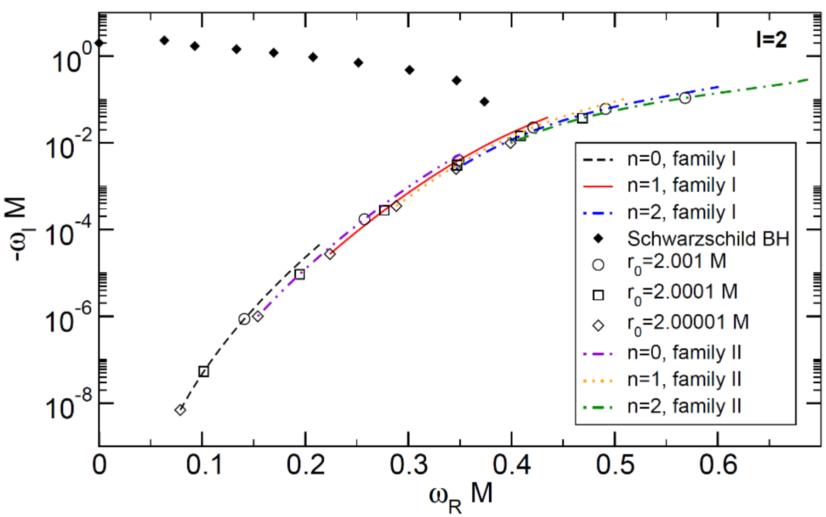
Cardoso & Pani, Living Reviews in Relativity (2019)

BH exterior is pathology-free, interior is not.

Quantum effects not fully understood. Non-locality to solve information paradox? BH area quantization? (Mathur 2005; Bekenstein & Mukhanov 1995; etc); Planck scale could be significantly lower etc (Arkani-Hamed+ 1998; Giddings & Thomas 2002).

Dark matter exists and interacts gravitationally. Are there compact DM clumps?

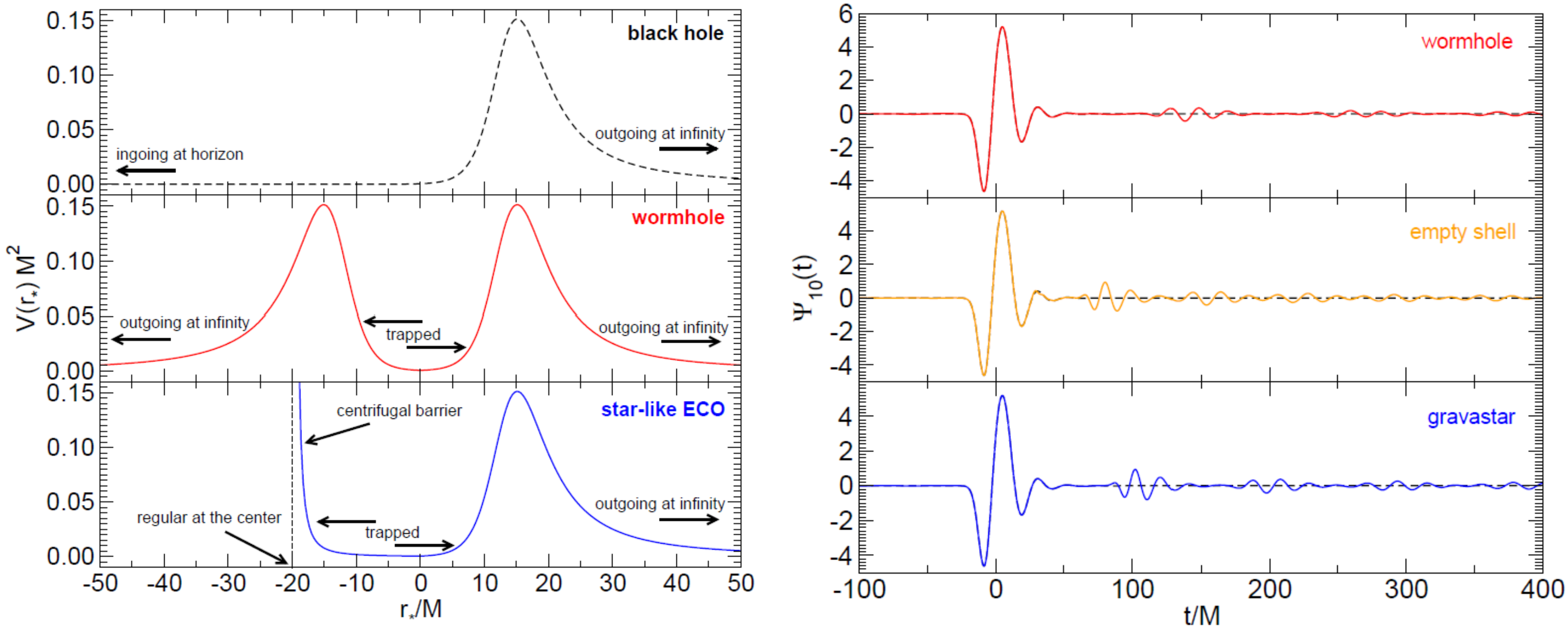
Physics is experimental science. We can test exterior. Aim to quantify evidence for horizons. Similar to quantifying equivalence principle.



QNMs of a Schwarzschild-like wormhole

Cardoso+ PRL116:171101 (2016); Maggio+PRD99:064007 (2020); Barausse+PRD89:104059 (2014)

Echoes: point particles



Cardoso + PRL116:171101 (2016); Nature Astronomy 1 (2017)
Cardoso and Pani, Living Reviews in Relativity 22:1 (2019)
Searches for echoes were conducted by the LIGO/Virgo Collaboration arXiv:2010:14529

The evidence for black holes

Cardoso and Pani, Living Reviews in Relativity 22:1 *(2019)*

$$r = r_+(1+\epsilon)$$

	Constraints		Source
	$\epsilon(\lesssim)$	$\frac{\nu}{\nu_\infty}(\gtrsim)$	
1.	$\mathcal{O}(1)$	1.4	Sgr A* & M87
2.	$\mathcal{O}(0.01)$	10	GW140915
3.	$10^{-4.4}$	158	All with $M > 10^{7.5} M_\odot$
4.	10^{-14}	10^7	Sgr A*
5.	10^{-40}	10^{20}	All with $M < 100 M_\odot$

	Effect and caveats
1.	Uses detected structure in "shadow" of SgrA and M87.
	Spin effects are poorly understood; systematic uncertainties not quantified.
2.	Uses same ringdown as BH and lack of echoes.
	?
3.	Lack of optical/UV transients from tidal disruption events.
	Assumes: all objects are horizonless, have a hard surface, spherical symmetry, and isotropy.
4.	Uses absence of relative low luminosity from Sgr A*, compared to disk.
	Spin effects and interaction of radiation with matter poorly understood; assumes spherical symmetry.
5.	Uses absence of GW stochastic background (from ergoregion instability).
	Assumes: hard surface (perfect reflection); exterior Kerr; all objects are horizonless.

Challenges

i. Are there alternatives to black holes?

ii. Do they form dynamically under reasonable conditions?

iii. Are they stable?

iv. How do they look like? Is GW or EM signal similar to BHs?

v. Are the light-ring modes in pseudospectrum?

Galaxy tomography?

Cardoso + arXiv 2109.00005

Black holes in galaxies: an Einstein Cluster prescription

Assume averaged stress-tensor

$$\langle T^{\mu\nu}\rangle = \frac{n}{m_p}\langle P^\mu P^\nu\rangle \Leftrightarrow T^\mu_\nu = \mathrm{diag}(-\rho, 0, P_t, P_t)$$

Impose spherical symmetry

$$ds^2 = -f dt^2 + \frac{dr^2}{1-2m(r)/r} + r^2 d\Omega^2$$

Assign mass function

Hernquist ApJ356:359 (1990)

$$m(r) = M_{\rm BH} + \frac{Mr^2}{(a_0+r)^2}\left(1-\frac{2M_{\rm BH}}{r}\right)^2$$

Solve field equations

$$f = \left(1-\frac{2M_{\rm BH}}{r}\right)e^{\Upsilon}$$

$$\Upsilon = -\pi\sqrt{\frac{M}{\xi}} + 2\sqrt{\frac{M}{\xi}}\arctan\frac{r+a_0-M}{\sqrt{M\xi}}$$

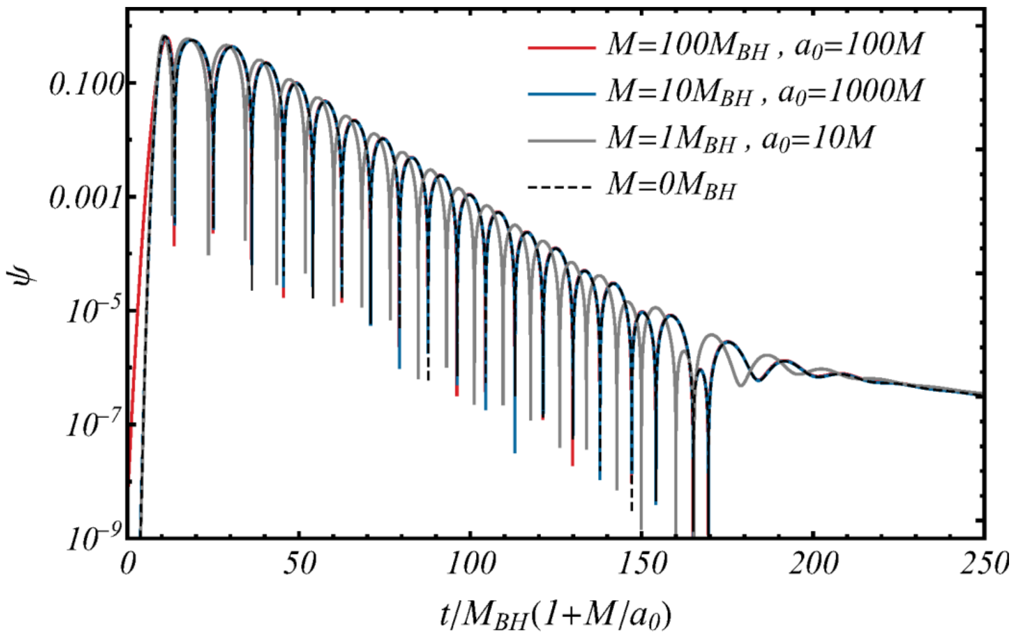
$$\xi = 2a_0 - M + 4M_{\rm BH}$$

Galaxy tomography?

Cardoso + arXiv 2109.00005

$$b_{\rm crit} = 3\sqrt{3}M_{\rm BH}\left(1+\frac{M}{a_0}+\frac{M(5M-18M_{\rm BH})}{6a_0^2}\right)$$

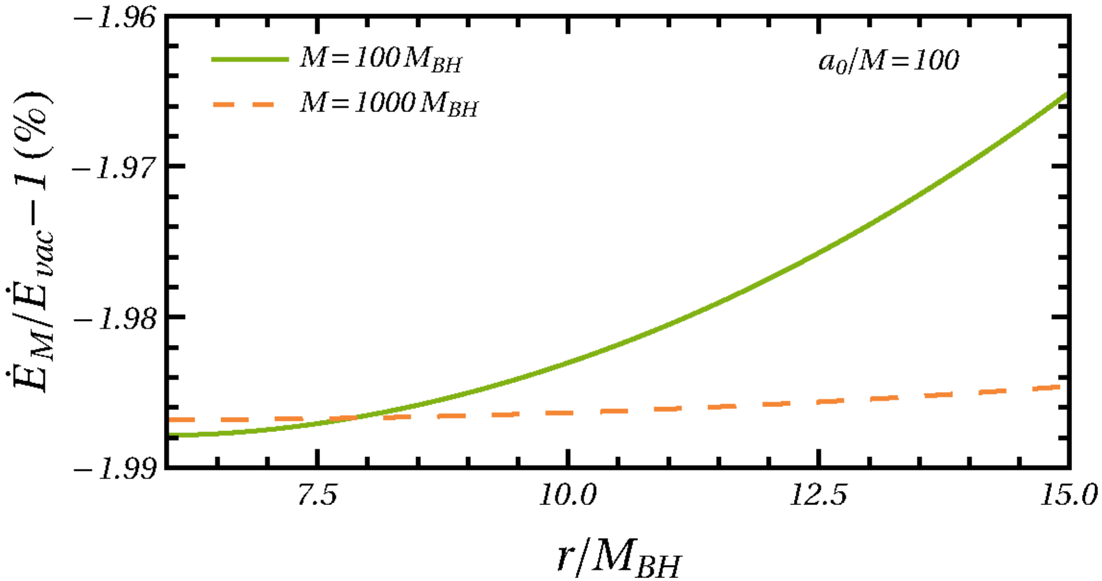
Thus EHT physics affected to levels of 10^-8 only
(tests on nature of compact objects can be done to very good precision)



Ringdown stage: galactic redshift to leading order. No anomalous “echoes”

Galaxy tomography?

Cardoso + arXiv 2109.00005



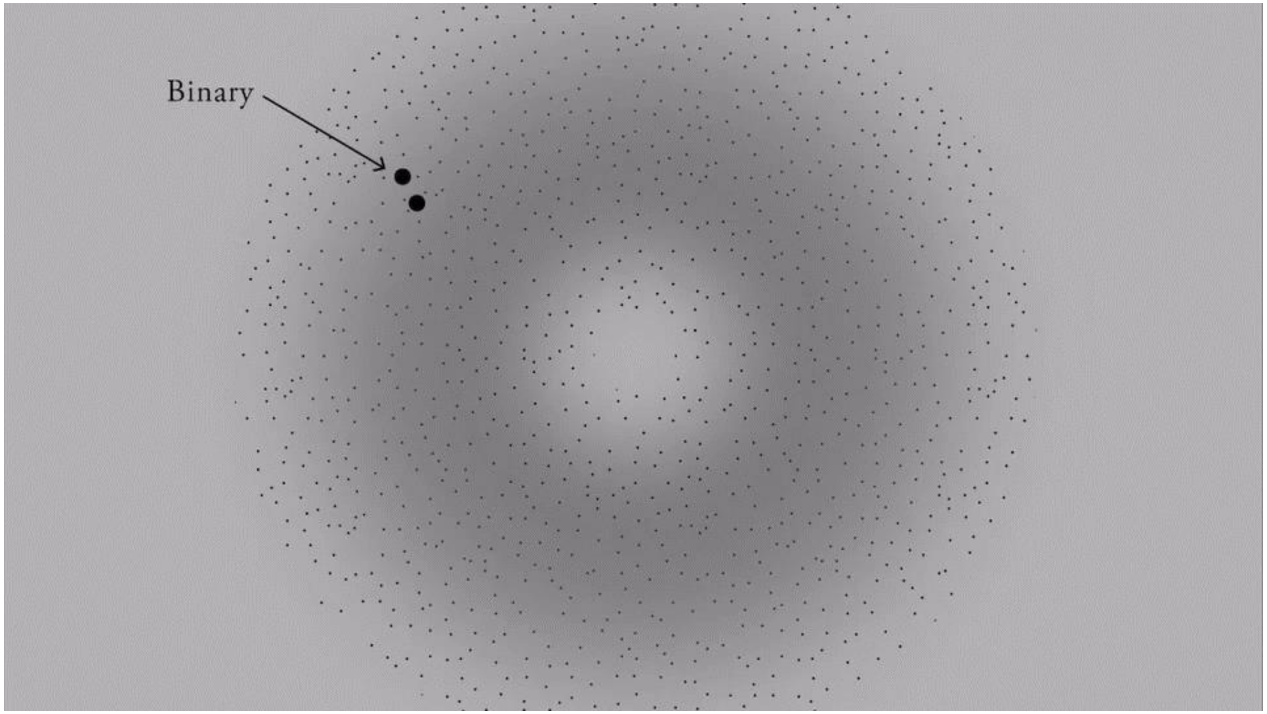
Axial fluxes from particles in circular orbits: for prototype binary with

$$M_{\rm BH} = 10^6\,M_\odot\,, \quad m_2 = 10 M_\odot \Rightarrow \text{cycles :} \quad \delta\mathcal{N} \approx 500$$

If observed one year before merger.

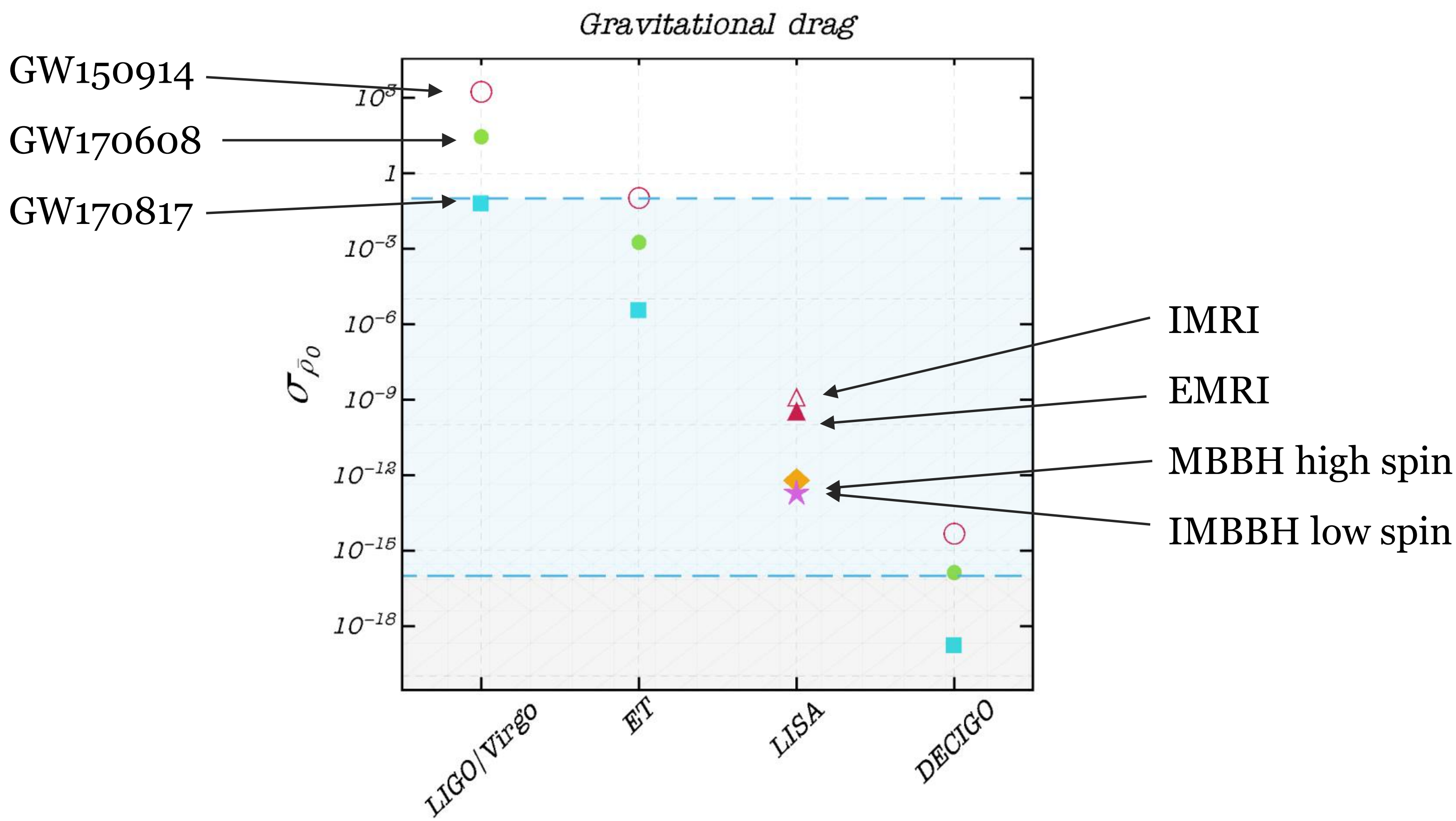
GWs and dark matter I

Inspiral occurs in DM-rich environment and may modify the way inspiral proceeds, given dense-enough media: accretion and gravitational drag play important role.
Eda + PRL110:221101 (2013); Macedo + ApJ774:48 (2013); Cardoso + AA644: A147 (2020) Kavanagh + arXiv 2002.12811; Annulli + PRD102: 063022 (2020)



Animation by Ana Carvalho

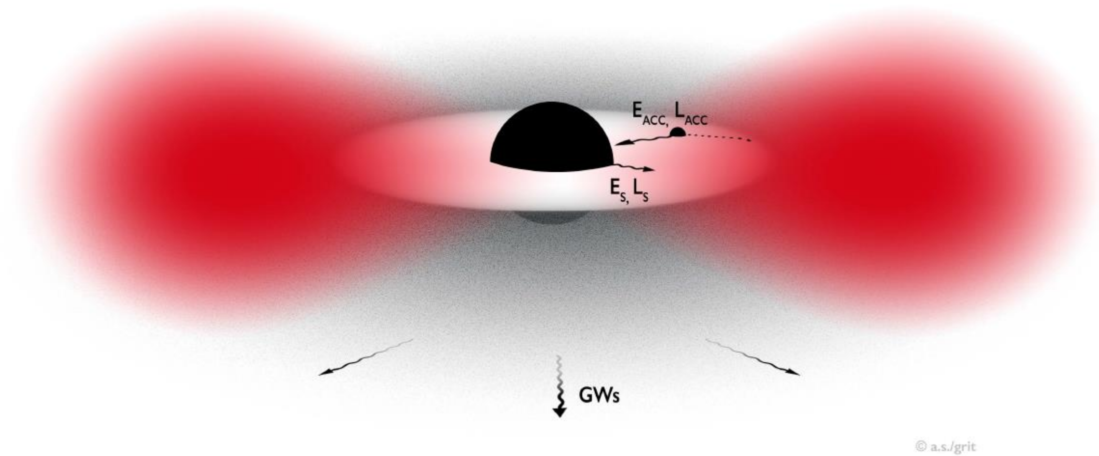
Small Compton wavelength: heavy DM



Effect is -5.5 PN on GW phase

Cardoso & Maselli AA644: A147 (2020) arXiv 1909.05870
Also Eda + PRL 110 (2013) 221101; Macedo+ApJ774 (2013) 48; Annulli+ PRD102;063022 (2020)

Fundamental fields: particle detectors in the sky



$$\nabla_\gamma \nabla^\gamma \Psi = \mu^2 \Psi\,, \quad \nabla_\gamma F^{\gamma\nu} = \mu^2 A^\nu\,, \quad \nabla_\gamma \nabla^\gamma h_{\mu\nu} = \mu^2 h_{\mu\nu}$$

$$\Psi \sim e^{-i\omega t} Y_{lm}$$

$$\omega \sim \mu + i(m\Omega_H - \mu)(M\mu)^{4l+5+S}$$

$$S = -s, -s+1 ..., s-1, s$$

$$\tau \sim 100 \left(\frac{10^6 M_\odot}{M}\right)^8 \left(\frac{10^{-16}\mathrm{eV}}{\mu}\right)^9 \text{ seconds}$$

Wonderful sources of GWs

Brito, Cardoso, Pani, Lecture Notes Physics 971 (2020)

Wonderful sources for different GW-detectors

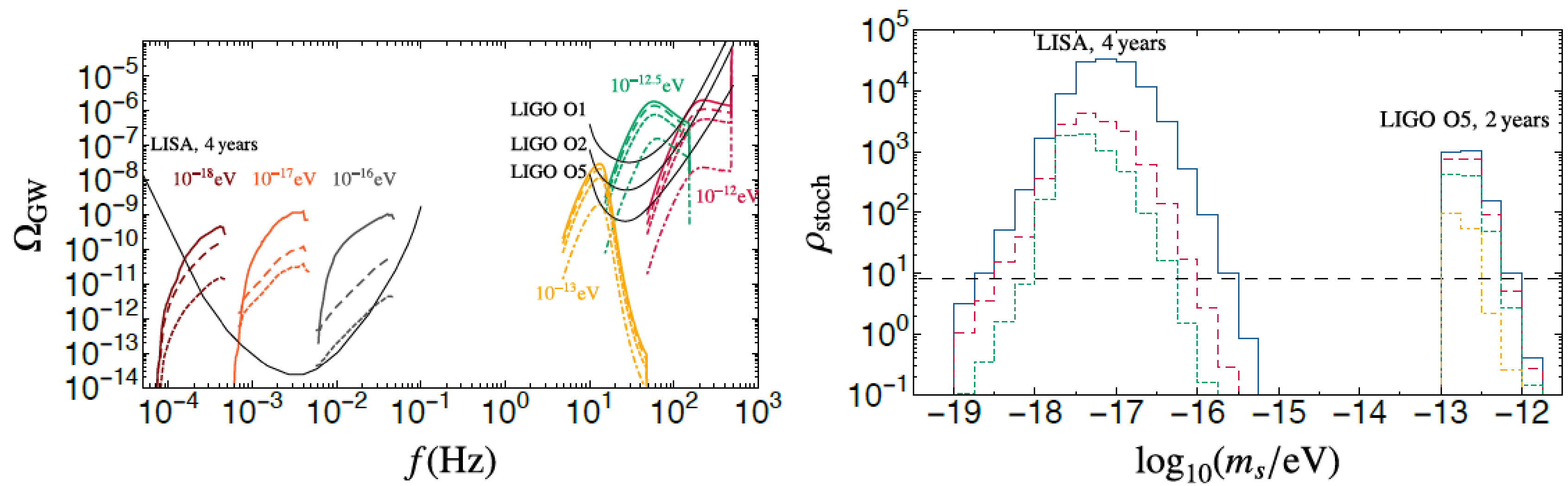
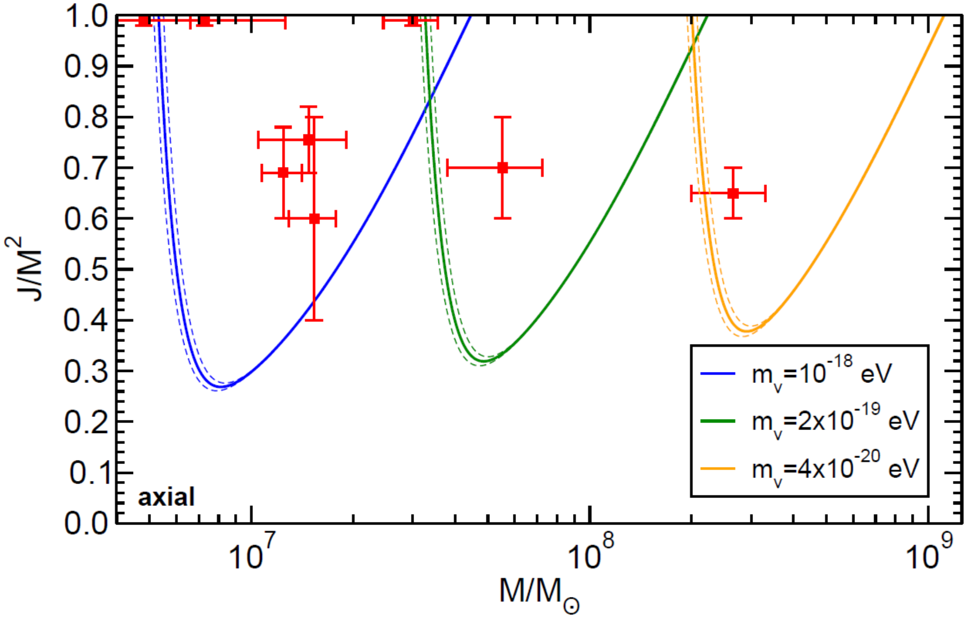


FIG. 2. Left panel: stochastic background in the LIGO and LISA bands. For LISA, the three different signals correspond to the "optimistic" (top), "less optimistic" (middle) and "pessimistic" (bottom) astrophysical models. For LIGO, the different spectra for each scalar field mass correspond to a uniform spin distribution with (from top to bottom) $\chi_i \in [0.8, 1], [0.5, 1], [0, 1]$ and $[0, 0.5]$. The black lines are the power-law integrated curves of Ref. [61], computed using noise PSDs for LISA [9], LIGO's first two observing runs (O1 and O2), and LIGO at design sensitivity (O5) [62]. By definition, $\rho_{\rm stoch} \geq 1$ when a power-law spectrum intersects one of the power-law integrated curves. Right panel: $\rho_{\rm stoch}$ for the backgrounds shown in the left panel. We assumed $T_{\rm obs} = 2$ yr for LIGO and $T_{\rm obs} = 4$ yr for LISA.

Brito + PRL119: 131101 (2017); Tsukada + arXiv: 2011.06995; Yuan + arXiv:2106.00021

Bounding the boson mass with EM observations

Pani + PRL109, 131102 (2012)



Bound on photon mass is model-dependent: details of accretion disks or intergalactic matter are important… but gravitons interact very weakly!

$$m_g < 5 \times 10^{-23}\,\mathrm{eV}$$

Brito + PRD88:023514 (2013); Review of Particle Physics 2014

Constraints on fundamental fields via superradiance

	excluded region (in eV)	source
*	$5.2 \times 10^{-13} < m_S < 6.5 \times 10^{-12}$	Direct bounds from absence of spin down in Cyg X-1.
*	$1.1 \times 10^{-13} < m_V < 8.2 \times 10^{-12}$	
*	$2.9 \times 10^{-13} < m_T < 9.8 \times 10^{-12}$	
	$6 \times 10^{-13} < m_S < 2 \times 10^{-11}$	Indirect bounds from BH mass-spin measurements.
	$7 \times 10^{-20} < m_S < 1 \times 10^{-16}$	
*	$2 \times 10^{-14} < m_V < 1 \times 10^{-11}$	
*	$1 \times 10^{-20} < m_V < 9 \times 10^{-17}$	
*	$6 \times 10^{-14} < m_T < 1 \times 10^{-11}$	
*	$3 \times 10^{-20} < m_T < 9 \times 10^{-17}$	
	$1.2 \times 10^{-13} < m_S < 1.8 \times 10^{-13}$	Null results from blind all-sky searches for continuous GW signals.
	$2.0 \times 10^{-13} < m_S < 2.5 \times 10^{-12}$	
	m_V: NA	
	m_T: NA	
	$5.8 \times 10^{-13} < m_S < 8.6 \times 10^{-13}$	Null results from searches for continuous GW signals from Cygnus X-1.
	m_V: NA	
	m_T: NA	
	$2.0 \times 10^{-13} < m_S < 3.8 \times 10^{-13}$	Negative searches for a GW background.
	m_V: NA	
	m_T: NA	
	$5 \times 10^{-13} < m_S < 3 \times 10^{-12}$	Bounds from pulsar timing.
	$m_V \sim 10^{-12}$	
	m_T: NA	
	$2.9 \times 10^{-21} < m_S < 4.6 \times 10^{-21}$	Bounds from mass and spin measurement of M87 with EHT.
	$8.5 \times 10^{-22} < m_V < 4.6 \times 10^{-21}$	
*	$1.0 \times 10^{-21} < m_T < 8.2 \times 10^{-21}$	

Brito, Cardoso & Pani, Superradiance, Lecture Notes Physics 971 (2020)

Conclusions: exciting times!

Gravitational wave astronomy *will* become a precision discipline, mapping compact objects throughout the entire visible universe.

Strong field gravity is a fascinating topic. From precise maps of Universe to tests of Cosmic Censorship or constraints on dark matter, possibilities are endless & exciting.

Black holes remain the most outstanding object in the universe. BH spectroscopy will allow to test GR and provide strong evidence for the presence of horizons… improved sensitivity pushes putative surface closer to horizon, like probing short-distance structure with accelerators. BHs can play the role of perfect laboratories for particle physics, or high energy physics.

"But a confirmation of the metric of the Kerr spacetime (or some aspect of it) cannot even be contemplated in the foreseeable future."

S. Chandrasekhar, The Karl Schwarzschild Lecture, Astronomischen Gesellschaft, Hamburg, 18 Sept. 1986

Thank you

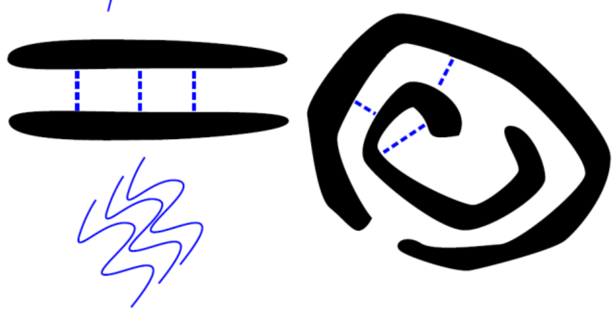
Stability of objects with photospheres

Static objects: *No uniform decay estimate with faster than logarithmic decay can hold for axial perturbations of ultracompact objects.*

Keir CQG33: 135009 (2016); Cardoso + PRD90:044069 (2014)

$$\mathcal{E}^{(N)}_{\text{local}}(t) \lesssim \frac{1}{(\log(2+t))^2}\mathcal{E}^{(N)}_{(2)}(0)$$

$$\Box\phi = 0$$



Burq, Acta Mathematica 180: 1 (1998)

Inspiralling compact objects

$$\text{Binding Energy}: E_b = -\frac{GM\mu}{2L} + \text{other interactions}$$

$$\text{Quadrupole emission}: \dot{E} = -\frac{32}{5}\frac{G\mu^2 L^4 \Omega^6}{c^5} + \text{other emission channels}$$

$$h(f, \text{pars}) = A(f, \text{pars})e^{i\Psi(f,\text{pars})}$$

$$\Psi = \frac{3}{128}\left(G\mathcal{M}\pi f/c^3\right)^{-5/3}\left(... + \alpha_{-4PN}x^{-4} + ... + \alpha_{-1PN}x^{-1} + 1 + \alpha_{1PN}x + ...\right)$$

New physics or extra matter

$$x = (\pi M f)^{2/3}\,, \quad M = m_1 + m_2\,, \quad \nu = m_1 m_2/M^2\,, \quad \mathcal{M} = \nu^{3/5}M$$

Variation of G

Dipole moment (electric charge)

Graviton mass

M. Maggiore, Gravitational waves, Volume I

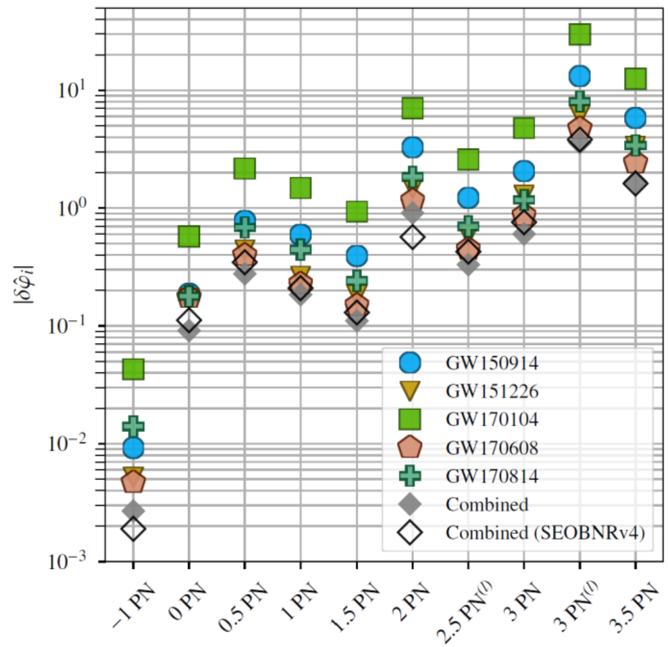
N. Yunes, K. Yagi & F. Pretorius, PRD94:084002 (2016)

Parametrized tests

$$h(f, \text{pars}) = A(f, \text{pars}) e^{i\Psi(f, \text{pars})}$$

$$\Psi = \frac{3}{128} \left(G\mathcal{M}\pi f / c^3 \right)^{-5/3} \left(... + \alpha_{-4PN} x^{-4} + ... + \alpha_{-1PN} x^{-1} + 1 + \alpha_{1PN} x + ... \right)$$

$$x = (\pi M f)^{2/3}\,, \quad M = m_1 + m_2\,, \quad \nu = m_1 m_2 / M^2\,, \quad \mathcal{M} = \nu^{3/5} M$$



LVC arXiv:1903.04467

Parametrized tests

$$h(f, \text{pars}) = A(f, \text{pars}) e^{i\Psi(f,\text{pars})}$$

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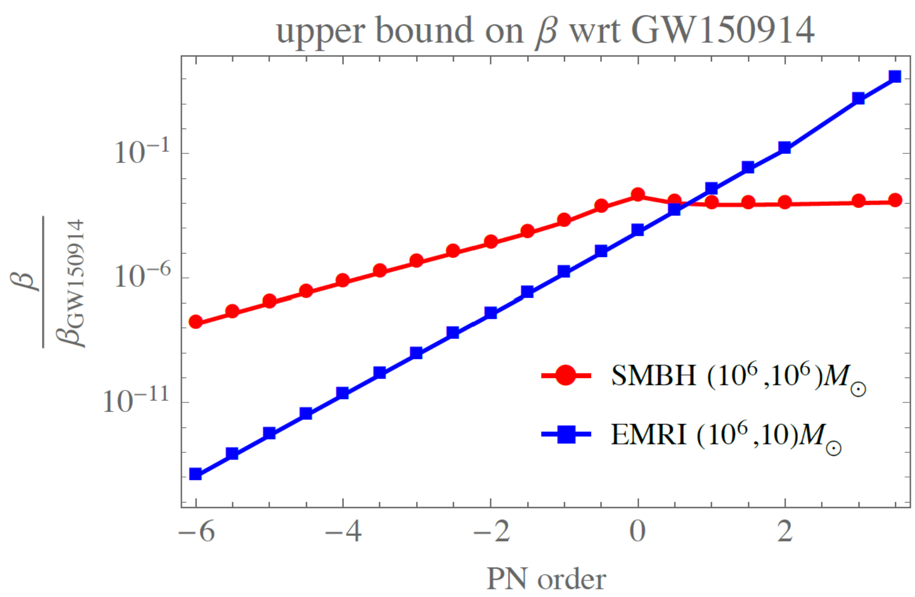
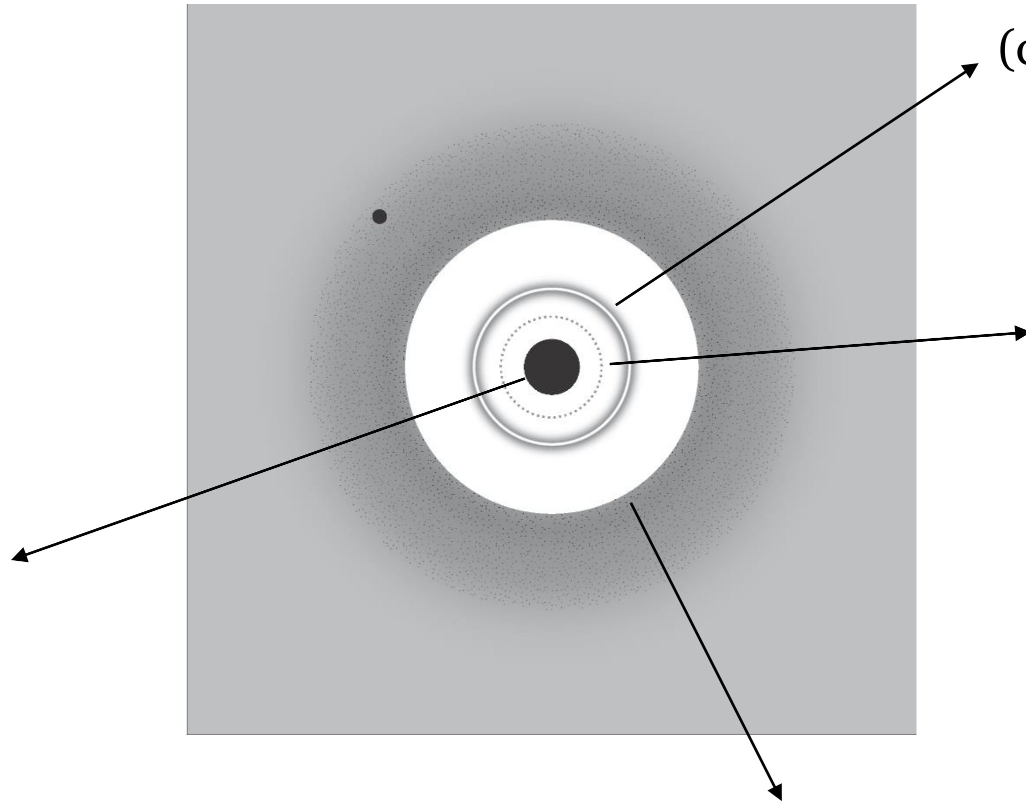
$$x = (\pi M f)^{2/3}, \quad M = m_1 + m_2, \quad \nu = m_1 m_2/M^2, \quad \mathcal{M} = \nu^{3/5} M$$



LISA study on FoMs for binaries 3Gpc away (in preparation, from Kent Yagi)

Black holes are black



Cardoso & Pani, Nature Astronomy 1: 586 (2017); Living Reviews in Relativity 22: 1 (2019)

Image: Ana Carvalho

Energy source?

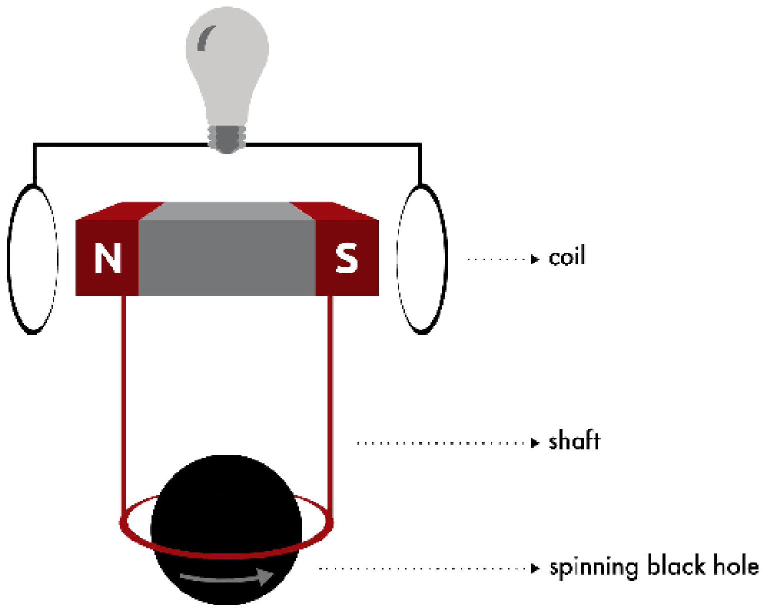


Image: Ana Carvalho

Brito, Cardoso & Pani, *Superradiance* (Springer-Verlag, 2020)

I only wish to make a plea for "black holes" to be taken seriously and their consequences to be explored in full detail. For who is to say, without careful study, that they cannot play some important part in the shaping of observed phenomena?

Penrose, Gravitational Collapse: the role of General Relativity (1969)