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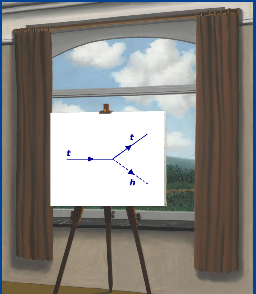
Top-quark fragmentation into a Higgs boson with next-to-leading order accuracy

Colomba Brancaccio
Terry Generet, Michał Czakon, Michael Krämer, Alexander Mück

DESY Theory Workshop, September 22nd, 2021



Goals & Motivations

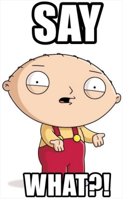
CURRENT THEORETICAL DESCRIPTION

WHAT?

Compute NNLO QCD corrections to $pp \to t\bar{t}h$ by using Fragmentation Functions (FFs).

WHY $t\bar{t}h$?

Study the top-Higgs Yukawa coupling.



$t\bar{t}h$ production

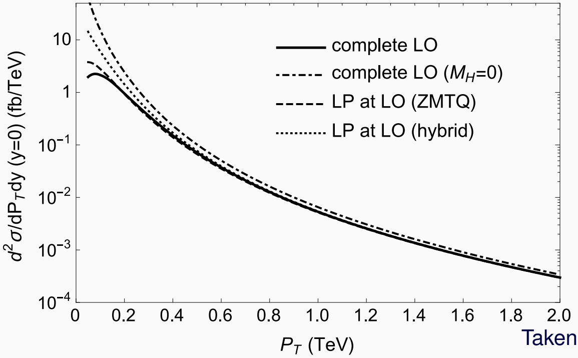
- ✓ NLO QCD corrections[1],
- ✓ NLO QCD corrections of $t\bar{t}h + jets$[2]
- ✓ NLO EW corrections[3],
- × NNLO QCD corrections.

$t \to th$ fragmentation

- ✓ LO top-Higgs FF[4],
- ✓ NLO top-Higgs FF in the limit $m_h^2 \ll m_t^2$ and based on soft-gluon approximation [5],
- ✓ NLO top-Higgs FF[6].

[1] Beenakker *et al.*, '01,
[2] Cullen *et al*, '13,
[3] Zhang *et al*, '14,
[4] Braaten, Zhang, '16,
[5] Dawson, Reina, '98,
[6] CB, Czakon, Generet, Krämer, '21.

CURRENT THEORETICAL DESCRIPTION



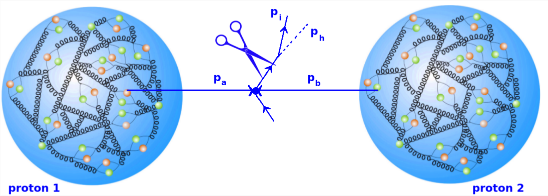
Taken from [4].

- **Good approximation at large p_T** $\rightarrow$ errors decreasing to below 5% for $p_T > 600$ GeV.
- Enables to **resum logarithms** at high $p_T \rightarrow$ necessary for future colliders.

[4] Braaten, Zhang, '16.

The fragmentation function

THE FINAL STATE FACTORISATION



In the collinear limit:

$$\hat{s} \gg q^2 \sim m_t^2$$

hard scattering and collinear emission factorise.

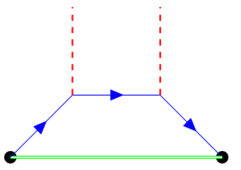
$$d\hat{\sigma}_{ab\to h+X}(p_a, p_b, p_h) = \sum_i \int_0^1 dz\, d\tilde{\sigma}_{ab\to i+X}(p_a, p_b, p_i; \mu) D_{i\to h}(z; \mu),$$

with $z = \dfrac{n \cdot p_h}{n \cdot p_i}$ and $n^\mu = \dfrac{1}{\sqrt{2}}(1, 0, 0, 1)$ is the light-cone vector in the Higgs direction.

$\boxed{\rightarrow}$ Analogous to the initial state factorisation (PDFs).

$\boxed{!}$ $D_{i\to h}(z; \mu)$ can be perturbatively computed.

Definition of the fragmentation function



[LO example of the following general formula]

$$D_{q\to h}(z) = \frac{z^{d-3}}{4\pi} \int dx^- e^{-ip_h^+ x^-/z} \frac{1}{2N_c} Tr_{colour} Tr_{Dirac} \Big[\not{n} \langle 0 | \psi_q(0)$$

$$\bar{\mathrm{P}} \exp\left(ig \int_0^\infty dy^- n \cdot A_a(y^- n) T_a^T \right) a_h^\dagger(p_h) a_h(p_h)$$

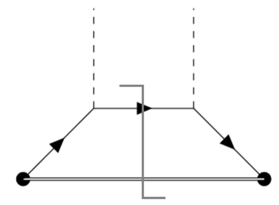
$$\mathrm{P} \exp\left(-ig \int_{x^-}^\infty dy^- n \cdot A_b(y^- n) T_b^T \right) \bar{\psi}_q(x^- n) |0\rangle \Big]$$

Wilson Lines

This definition is Gauge Invariant!

[7] Collins, Soper, '82.

EXAMPLE: THE LO FRAGMENTATION FUNCTION

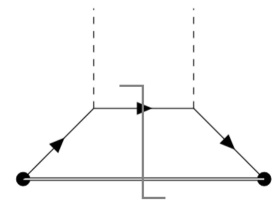


Applying the formula of the previous slide at LO, the fragmentation $D_{t\to h}$ reads:

$$D_{t\to h} = \frac{z^{d-3}}{4\pi} \int \frac{d^d p_t}{(2\pi)^d} (2\pi)\delta^+(p_t^2 - m_t^2)(2\pi)\delta^+(p_h^+/z - (p_t + p_h)^+) \, \frac{y_t^2 \tilde{\mu}^{2\epsilon}}{2N_c}$$

$$\times \sum_{spins,colours} Tr\left[\not{n} \frac{\not{p}_t + \not{p}_h + m_t}{(p_t+p_h)^2 - m_t^2} (\not{p}_t + m_t) \frac{\not{p}_t + \not{p}_h + m_t}{(p_t+p_h)^2 - m_t^2} \right].$$

Example: The LO fragmentation function



Using reverse unitarity, the phase-space becomes a loop integral[8]:

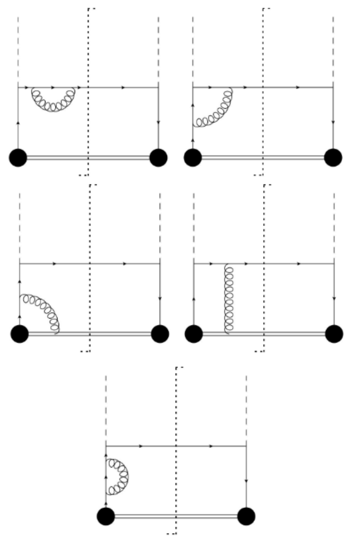
$$\delta(x) \to \frac{1}{2\pi i}\left(\frac{1}{x - i\varepsilon} - \frac{1}{x + i\varepsilon}\right)$$

$$D_{t\to h} = \frac{z^{d-3}}{4\pi}\int \frac{d^d p_t}{(2\pi)^d}(2\pi)\,\boxed{\delta^+(p_t^2 - m_t^2)}\,(2\pi)\,\boxed{\delta^+(p_h^+/z - (p_t + p_h)^+)}\,\frac{y_t^2\tilde{\mu}^{2\epsilon}}{2N_c}$$
$$\times \sum_{spins,colours} Tr\left[\not{n}\frac{\not{p}_t + \not{p}_h + m_t}{(p_t+p_h)^2 - m_t^2}(\not{p}_t + m_t)\frac{\not{p}_t + \not{p}_h + m_t}{(p_t+p_h)^2 - m_t^2}\right].$$

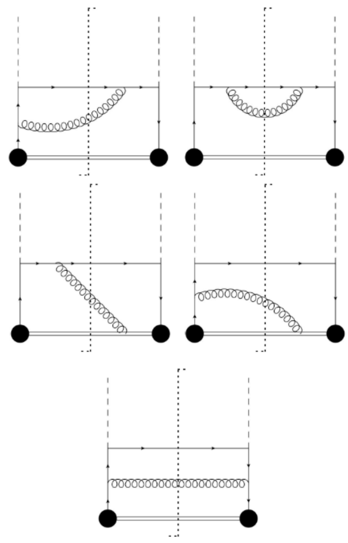
[8] Anastasiou, Melnikov, '02.

THE NLO FRAGMENTATION FUNCTION CONTRIBUTIONS

Virtual corrections



Real corrections



Loop computation

Differential equations method

- **Reduction to MIs** can be performed with the usual techniques[9,10]:
  - ✓ 8 master found for virtual corrections,
  - ✓ 8 master found for real corrections.
- MI derivatives with respect to each kinematic invariant x_i can be computed by introducing the differential operators:

$$O_{jk} = p_j^{\mu} \sum_{i=1}^{n} \frac{\partial x_i}{\partial p_k^{\mu}} \frac{\partial f(\vec{x}, \epsilon)}{\partial x_i} = \sum_{i=1}^{n} a_{i,jk}(x_i) \frac{\partial f(\vec{x}, \epsilon)}{\partial x_i}.$$

- A **system of first order linear differential equations** for the MIs can be derived:

$$\partial_{x_i} \vec{f}(\vec{x}, \epsilon) = A_{x_i}(\vec{x}, \epsilon)\, \vec{f}(\vec{x}, \epsilon).$$

[9] *LiteRed*. Lee, '13, [10] *FIRE*. Smirnov, '15.

Canonical basis approach

- It is possible to choose a basis of MIs, the **Canonical basis**[11], such that:

$$\mathrm{d}\vec{f}(\vec{x},\epsilon) = \epsilon\, \mathrm{d}A(\vec{x})\, \vec{f}(\vec{x},\epsilon),$$

with

$$\mathrm{d}A(\vec{x})_{ij} = \sum_k c_{ijk}\, \mathrm{dlog}(\alpha_k(\vec{x})).$$

- The **solution of the differential equations system** is:

$$\vec{f}(\vec{x},\epsilon) = \mathrm{P}\exp\left[\epsilon \int_\gamma \mathrm{d}\tilde{A}(\vec{x}')\right] \vec{f}(\vec{x}_0,\epsilon).$$

Canonical MIs can be expanded in Taylor series around $\epsilon = 0$:

$$\vec{f}(\vec{x},\epsilon) = \sum_{k=0}^{\infty} \epsilon^k\, \vec{f}^{(k)}(\vec{x}) \rightarrow \vec{f}^{(k)}(\vec{x}) = \int_\gamma \mathrm{d}\tilde{A}(\vec{x}')\, \vec{f}^{(k-1)}(\vec{x}') + \vec{f}(\vec{x}_0,\epsilon)$$

[11] Henn, '13.

CANONICAL MATRIX FOR VIRTUAL CORRECTIONS

- For a dlog shape of the canonical matrix, the change of variable $m_t^2 \to \frac{m_h^2}{4}(-\tau^2+1)$ is performed.
- The canonical matrix reads:

$$
\begin{aligned}
\mathrm{d}A_\tau = & M_1 \,\mathrm{dlog}\,(\tau) + M_2 \,\mathrm{dlog}\,(1-\tau) + M_3 \,\mathrm{dlog}\,(1+\tau) \\
& + M_4 \,\mathrm{dlog}\,(2 - z\,(1-\tau)) + M_5 \,\mathrm{dlog}\,(-2 + z\,(1+\tau)) \\
& + M_6 \,\mathrm{dlog}\,\left(-4 + z\left(3+\tau^2\right)\right),
\end{aligned}
$$

where the M_i are 8×8 matrices with purely rational entries.

- Solution given in terms of GPLs.
- Integration constants are m_h and z dependent!

Introduction to symbols

Master Integrals Results

$$
\begin{aligned}
I_{0,0,1,1,1,1,1} = & -\frac{1}{4\epsilon^2}\ln(z) + \frac{1}{\epsilon}\Big\{ -\mathrm{Re}\Big[\mathrm{Li}_2\Big(\frac{z}{x^+}\Big)\Big] - \frac{1}{8}\arg^2\Big(\frac{x^+}{x^-}\Big) - \frac{1}{8}\ln^2(1-r) - \frac{1}{8}\ln^2(r) \\
& - \frac{1}{8}\ln^2(1-z) - \frac{1}{8}\ln^2(z) + \frac{1}{2}\ln(z)\ln\Big(\frac{m_t^2}{\mu^2}\Big) + \frac{1}{4}\ln(1-r)\ln(1-z) \\
& - \frac{1}{4}\ln(r)\ln(1-z) + \frac{1}{4}\ln(1-r)\ln(r) + \frac{\pi^2}{12}\Big\} + \Big\{ -\mathrm{Re}\Big[\mathrm{Li}_3\Big(\frac{z-1}{z-x^+}\Big)\Big] \\
& - 4\,\mathrm{Re}\Big[\mathrm{Li}_3\Big(\frac{z}{z-x^+}\Big)\Big] - \mathrm{Re}\Big[\mathrm{Li}_3\Big(\frac{z(1-x^+)}{z-x^+}\Big)\Big] - \mathrm{Re}\Big[\mathrm{Li}_3\Big(\frac{z}{x^+}\Big)\Big] \\
& - 3\,\mathrm{Re}[\mathrm{Li}_3(x^+)] + 3\,\mathrm{Re}\Big[\mathrm{Li}_3\Big(1-\frac{x^+}{x^-}\Big)\Big] + \frac{1}{2}\mathrm{Re}\Big[\mathrm{Li}_3\Big(\frac{z-x^+}{z-x^-}\Big)\Big] \\
& - \mathrm{Re}\Big[\mathrm{Li}_3\Big(1-\frac{z}{1-w^+}\Big)\Big] + \frac{1}{2}\mathrm{Re}\Big[\mathrm{Li}_3\Big(\frac{r+z(w^+-1)}{zw^+}\Big)\Big] \\
& + \frac{1}{2}\mathrm{Re}\Big[\mathrm{Li}_3\Big(\frac{zw^+}{r+z(w^+-1)}\Big)\Big] - \frac{1}{2}\mathrm{Re}\Big[\mathrm{Li}_3\Big(\frac{rw^+}{w^+z-z+r}\Big)\Big] \\
& - \frac{1}{2}\mathrm{Re}\Big[\mathrm{Li}_3\Big(\frac{w^+z-z+r}{rw^+}\Big)\Big] - \frac{1}{2}\mathrm{Re}\Big[\mathrm{Li}_3\Big(\frac{w^+}{w^-}\Big)\Big] + 2\arg\Big(\frac{x^+}{x^-}\Big)\mathrm{Im}\Big[\mathrm{Li}_2\Big(\frac{z}{x^+}\Big)\Big] \\
& - 2\,\mathrm{Im}\Big[\ln\Big(1-\frac{z-x^+}{z-x^-}\Big)\Big]\mathrm{Im}\Big[\mathrm{Li}_2\Big(\frac{z}{x^+}\Big)\Big] + \arg\Big(\frac{x^+}{x^-}\Big)\mathrm{Im}\Big[\mathrm{Li}_2\Big(1-\frac{x^+}{x^-}\Big)\Big] \\
& - \mathrm{Im}\Big[\ln\Big(1-\frac{z-x^+}{z-x^-}\Big)\Big]\mathrm{Im}\Big[\mathrm{Li}_2\Big(1-\frac{x^+}{x^-}\Big)\Big] + \frac{1}{2}\ln(z)\mathrm{Li}_2\Big(1-\frac{1-r}{1-z}\Big) \\
& - \frac{1}{2}\ln(r)\mathrm{Li}_2\Big(1-\frac{1-r}{1-z}\Big) - \frac{1}{2}\ln(z)\mathrm{Li}_2(r) + \frac{1}{2}\ln(r)\mathrm{Li}_2(r) - \frac{1}{2}\ln(z)\mathrm{Li}_2\Big(1-\frac{r}{z}\Big)
\end{aligned}
$$

SYMBOLS

Symbols[12] were used as a systematic way of simplifying the polylogarithms appearing in the MIs analytic expressions.

Definition

- $symbol(Li_n(x)) = -ln(1-x) \otimes \underbrace{ln(x) \otimes ... \otimes ln(x)}_{\text{n-1 times}}$,
- $symbol(ln(x)ln(y)) = ln(x) \otimes ln(y) + ln(y) \otimes ln(x)$,
- $... \otimes ln(x \cdot y) \otimes ... = (... \otimes ln(x) \otimes ...) + (... \otimes ln(y) \otimes ...)$,
- $symbol(\pi^n) = 0$ with $n \geq 2$,
- Symbols are unique up to $\sim \pi^n$ with $n \geq 2$.

[12] Duhr, Gangl, Rhodes,'12.

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- $\ldots \otimes ln(x \cdot y) \otimes \ldots = (\ldots \otimes ln(x) \otimes \ldots) + (\ldots \otimes ln(y) \otimes \ldots)$,
- $symbol(\pi^n) = 0$ with $n \geq 2$,
- Symbols are unique up to $\sim \pi^n$ with $n \geq 2$.

Example

$$symbol(\,-Li_2(x) - ln(1-x)ln(x) + \frac{\pi^2}{6}\,) =$$
$$ln(1-x) \otimes ln(x) - (ln(1-x) \otimes ln(x) + ln(x) \otimes ln(1-x)) =$$
$$symbol(Li_2(1-x))$$

$$\downarrow$$

$$Li_2(1-x) = -Li_2(x) - ln(1-x)ln(x) + A\pi^2$$

$A = \frac{1}{6} \rightarrow$ Euler's reflection formula

[12] Duhr, Gangl, Rhodes,'12.

Collinear renormalisation

Collinear renormalisation

Similarly to the heavy quark case, the bare $t \to h$ fragmentation function reads: $D_{h\to h} = \delta(1-z) + \mathcal{O}(y_t^2)$

$$D^B_{t\to h}(z) = (Z_{th} \otimes D_{h\to h})(z) + (Z_{tt} \otimes D_{t\to h})(z) + \mathcal{O}(y_t^2\alpha_s^2, y_t^4),$$

where the renormalisation constants in terms of splitting functions are:

$$Z_{th}(z) = \frac{y_t^2}{16\pi^2}\frac{1}{\epsilon}P^{(0)T}_{th}(z) + \frac{y_t^2}{16\pi^2}\frac{\alpha_s}{2\pi}\left(\underbrace{\frac{1}{2\epsilon}P^{(1)T}_{th}(z)}_{?} + \frac{1}{2\epsilon^2}(P^{(0)T}_{qq} \otimes P^{(0)T}_{th})(z) - \frac{\beta^{(0)}_{th}}{4\epsilon^2}P^{(0)T}_{th}(z)\right) + \mathcal{O}(y_t^2\alpha_s^2, y_t^4),$$

$$Z_{tt}(z) = \delta(1-z) + \frac{\alpha_s}{2\pi}\frac{1}{\epsilon}P^{(0)T}_{qq} + \mathcal{O}(\alpha_s^2, y_t^2).$$

$P^{(0)T}_{qq}$ and $P^{(0)T}_{th}$ known $\to$ $P^{(1)T}_{th}$ derived as a by-product of our computation.

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where the renormalisation constants in terms of splitting functions are:

$$Z_{th}(z) = \frac{y_t^2}{16\pi^2}\frac{1}{\epsilon}P_{th}^{(0)T}(z) + \frac{y_t^2}{16\pi^2}\frac{\alpha_s}{2\pi}\left(\frac{1}{2\epsilon}P_{th}^{(1)T}(z) + \frac{1}{2\epsilon^2}(P_{qq}^{(0)T}\otimes P_{th}^{(0)T})(z) - \frac{\beta_{th}^{(0)}}{4\epsilon^2}P_{th}^{(0)T}(z)\right) + \mathcal{O}(y_t^2\alpha_s^2, y_t^4),$$

$$Z_{tt}(z) = \delta(1-z) + \frac{\alpha_s}{2\pi}\frac{1}{\epsilon}P_{qq}^{(0)T} + \mathcal{O}(\alpha_s^2, y_t^2).$$

$P_{qq}^{(0)T}$ and $P_{th}^{(0)T}$ known $\to$ $P_{th}^{(1)T}$ derived as a by-product of our computation.

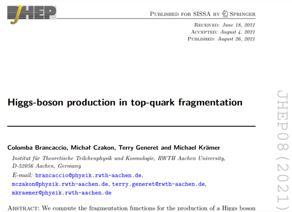
$$P_{th}^{(1)T}(z) = C_F\Big[-8z\, Li_2(z) + z\, ln^2(1-z) - \frac{1}{2}z\, ln^2(z) + 3z\, ln(1-z) - 4z\, ln(z)\, ln(1-z) + (-1 + \frac{1}{2}z)\, ln(z) + (-\frac{13}{2} + 15z)\Big]$$

Conclusions

SUMMARY & OUTLOOK

ACHIEVEMENTS

- ✓ Analytic computation of the MIs,
- ✓ $D_{t\to h}(z)$ fragmentation function at $\mathcal{O}(y_t^2\alpha_s)$,
- ✓ $D_{g\to h}(z)$ fragmentation function at $\mathcal{O}(y_t^2\alpha_s)$,
- ✓ $P_{th}^{T}(z)$ splitting function at $\mathcal{O}(y_t^2\alpha_s)$,
- ✓ $P_{gh}^{T}(z)$ splitting function at $\mathcal{O}(y_t^2\alpha_s)$.



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Higgs-boson production in top-quark fragmentation

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Abstract: We compute the fragmentation functions for the production of a Higgs boson

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OUTLOOK

- × Testing the validity of the fragmentation approximation for the $t\bar{t}h$ production,
- × NNLO computation of the fragmentation functions in $m_h \to 0$.


That's
all folks!