Portal Chiral Perturbation Theory

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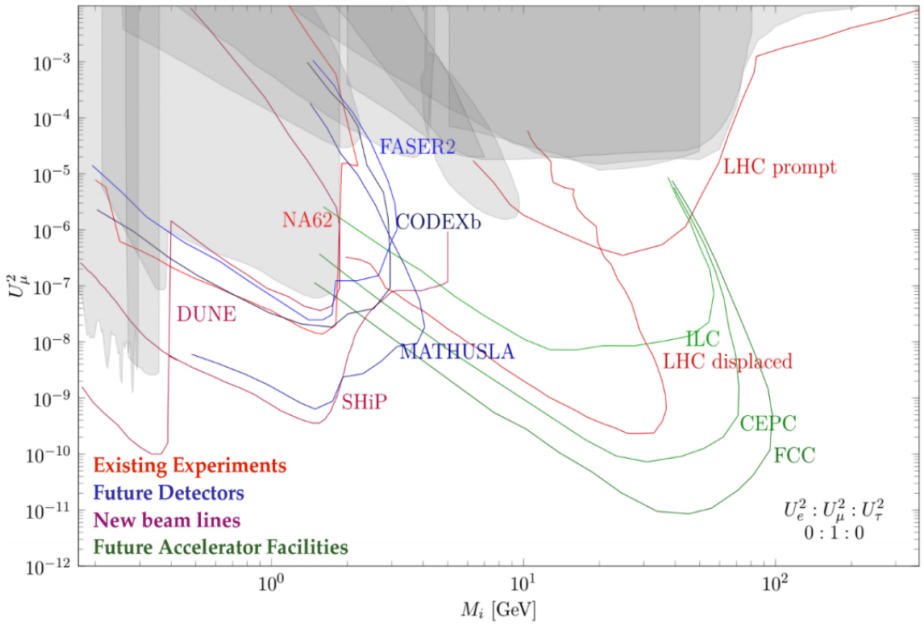
DESY Theory Workshop 2021: Bright Ideas for a Dark Universe

arXiv:2105.06477

Collaborators: Chiara Arina, Jan Hajer

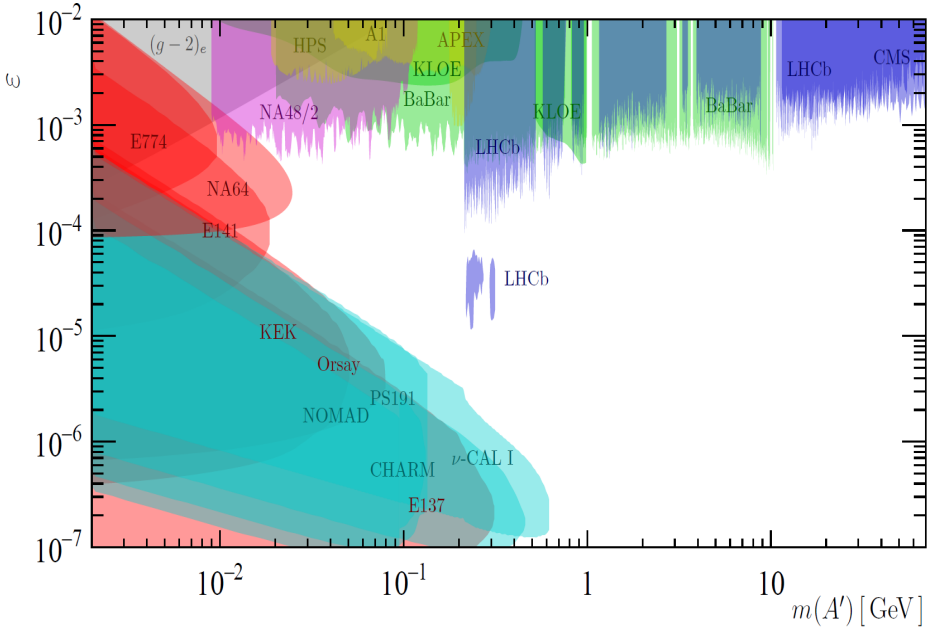
Model-Independent Meson Portal Interactions

Heavy Neutral Leptons



arXiv:2102.12143

Dark Photons



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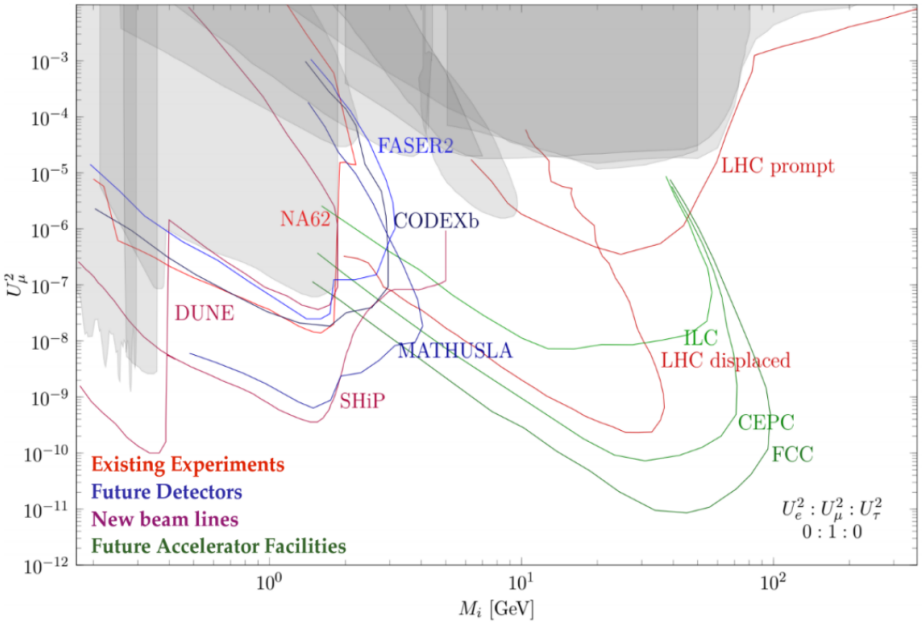

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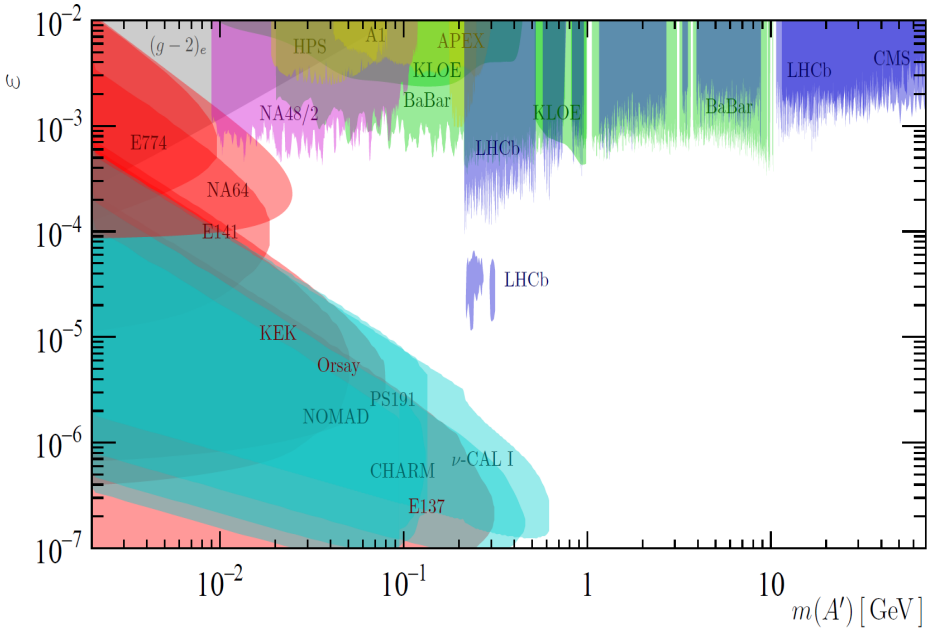

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(E.g. for ALPs, HNLs, dark photons, light Higgs)

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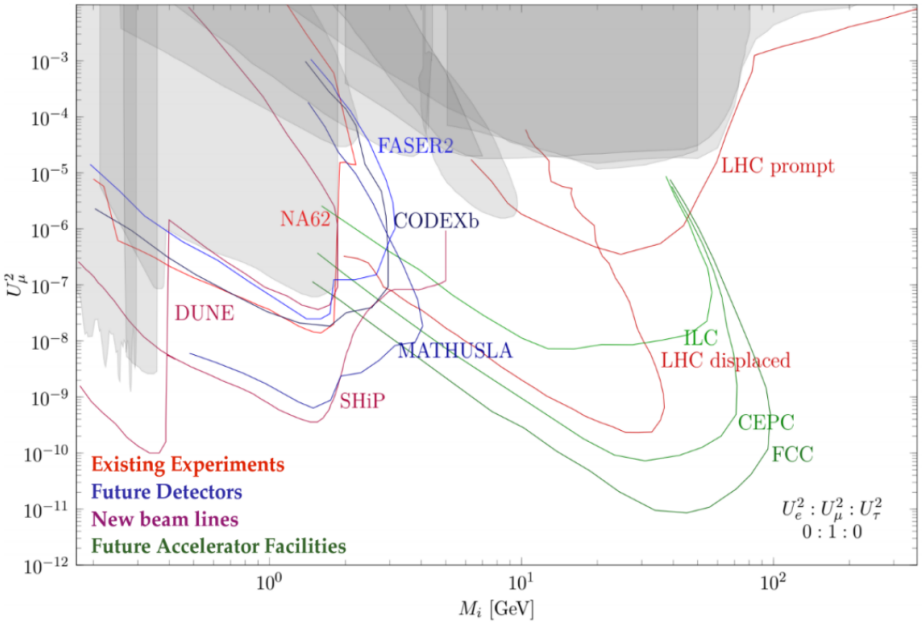

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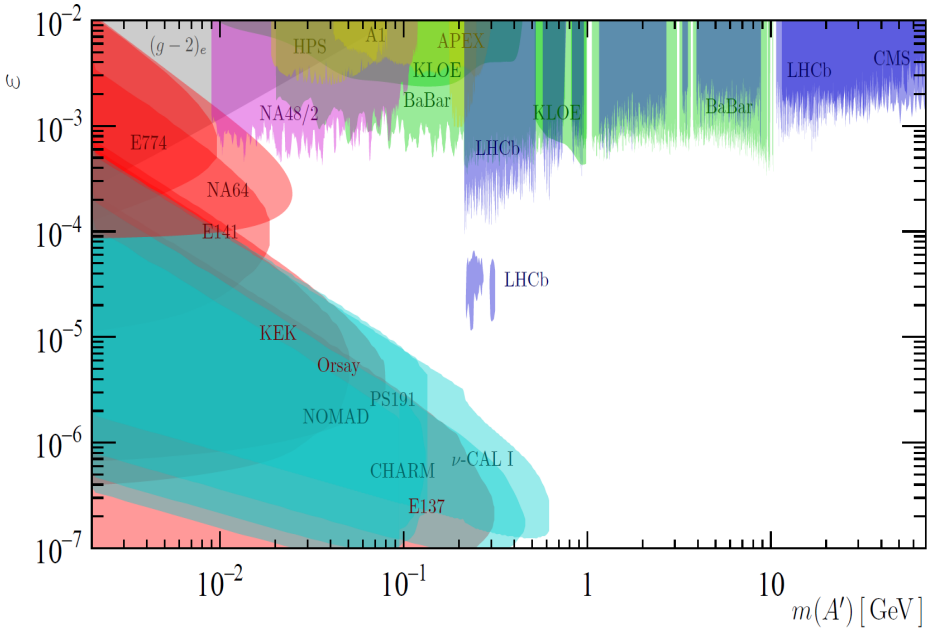

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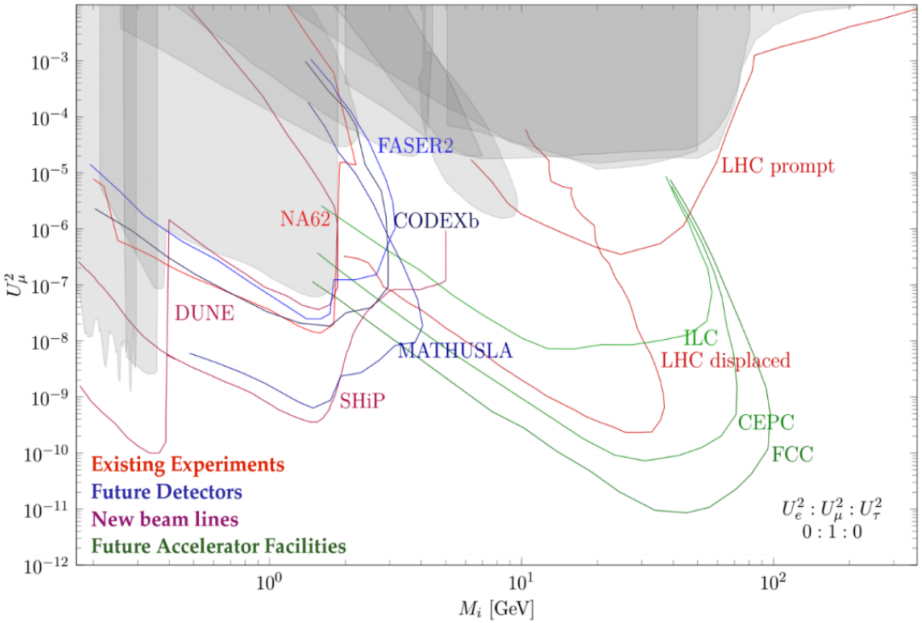

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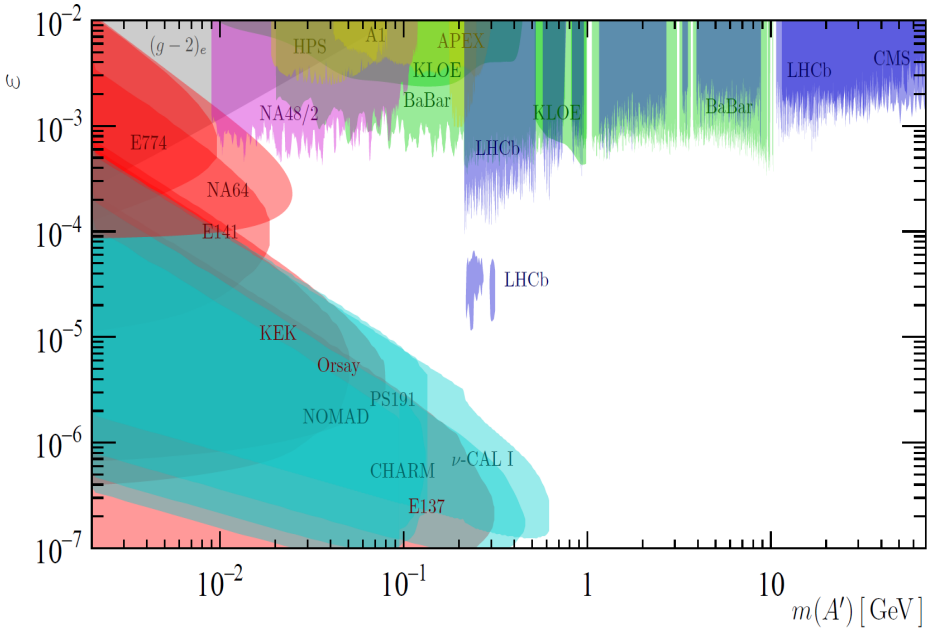

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Better: More model-independent predictions

$\Rightarrow$ Use **Portal EFT Framework** to couple mesons to generic messengers
(see also talk by Jan Hajer)

Portal Effective Theory Setup

Physics Focus:

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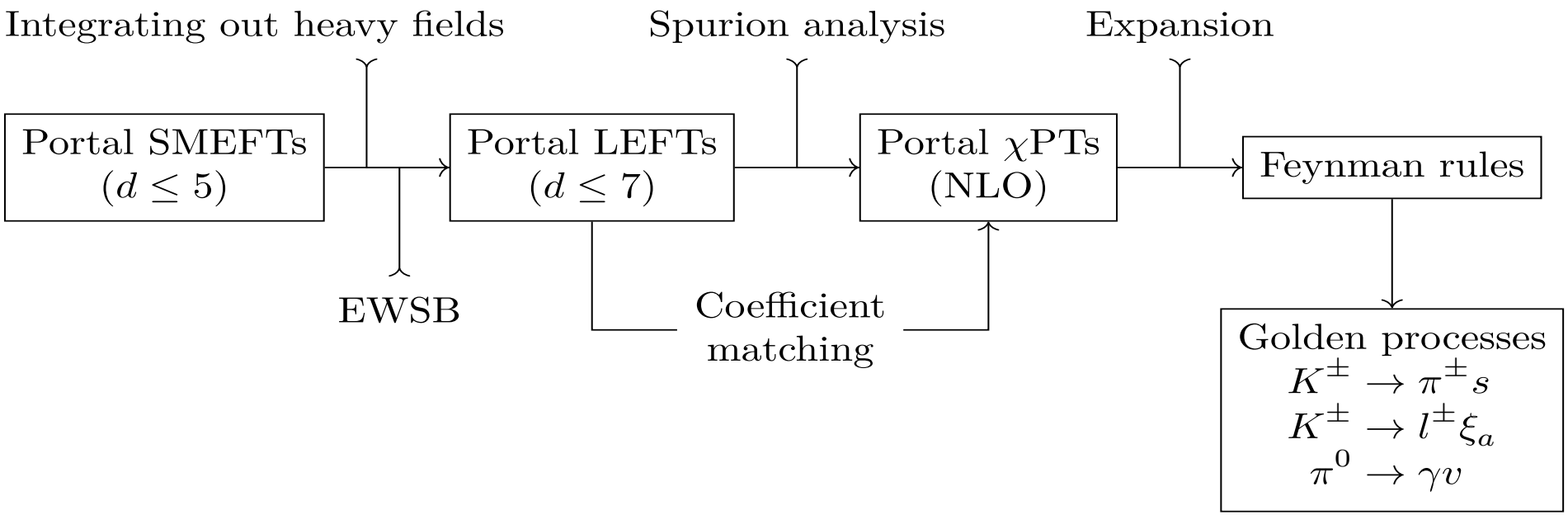

Constructing Portal Chiral Perturbation Theory

QCD $\supset$ SM: 4 currents capture $\bar{q}q$ masses, photons, θ angle

$\Rightarrow$ Use **spurion analysis** to fix meson couplings

$$\mathcal{L}_{\text{SM}} \supset -q^\dagger \bar{\sigma}_\mu \boldsymbol{l}^\mu q - \bar{q}\sigma_\mu \boldsymbol{r}^\mu \bar{q}^\dagger - \theta\, G\widetilde{G} - \left[\bar{q}\boldsymbol{m}q + \text{h.c.}\right]$$

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Portal LEFT: 10 currents capture leading QCD portal operators
(both flavour conserving + violating interactions)

$$\delta\mathcal{L}_{\text{portals}} \supset -q^\dagger\bar{\sigma}_\mu \boldsymbol{L}^\mu q - \bar{q}\sigma_\mu \boldsymbol{R}^\mu \bar{q}^\dagger - \Omega\, GG - \Theta\, G\widetilde{G}$$
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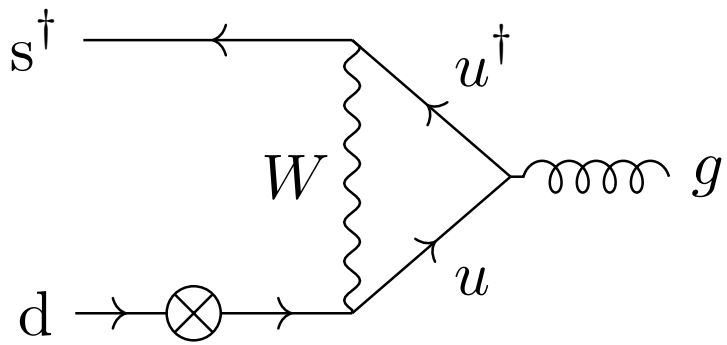
1. We extend spurion appraoch to currents Ω, $\boldsymbol{\Gamma}$, H_x
   $\Rightarrow$ approach now used for all 10 currents
2. We estimate resulting new ChPT coefficients
   (Using trace anomaly, large n_c, matching to lattice *etc.*)

Example: Coupling to chromomagnetic current Γ



$$\xrightarrow{p^2 \ll M_w^2} \quad s_i \; \overline{q} \, \boldsymbol{c}_i^{\gamma} \overline{\sigma}^{\mu\nu} G_{\mu\nu} q$$

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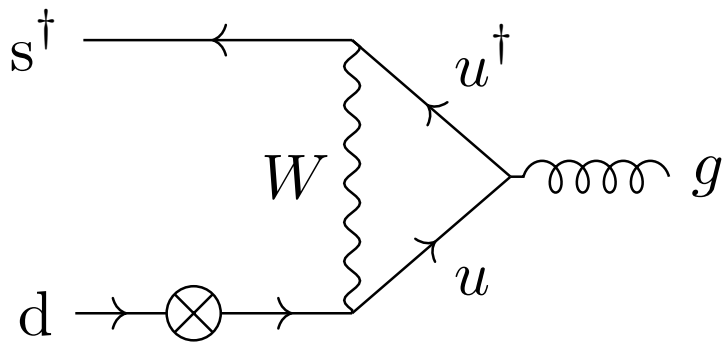

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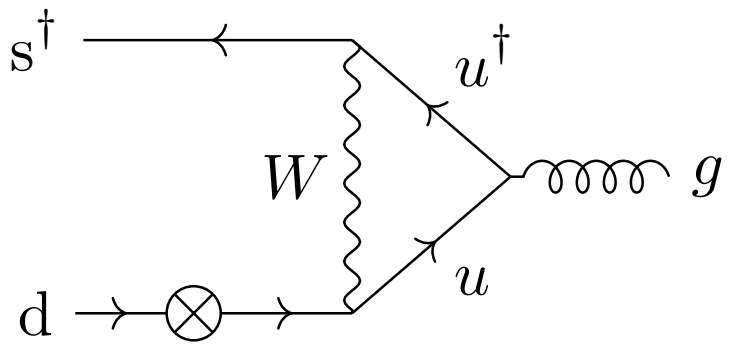

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$$\underbrace{\bar{q}\boldsymbol{m}q \xrightarrow{\text{ChPT}} \langle \bar{u}u \rangle \,\mathrm{tr}(g\boldsymbol{m})}_{\text{textbook}}$$

g = coset matrix (encodes mesons), f = pion decay const., $\langle \bar{u}u \rangle$ quark condensate

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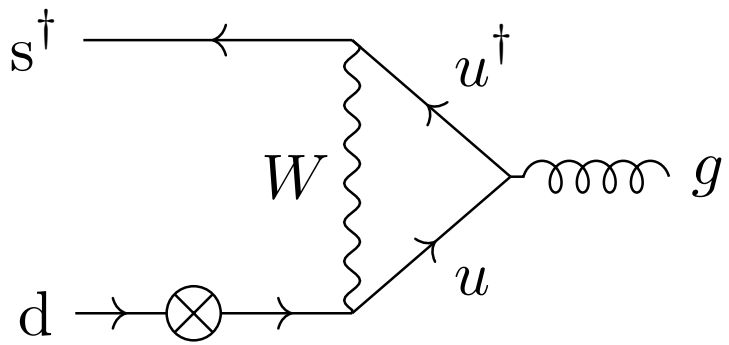

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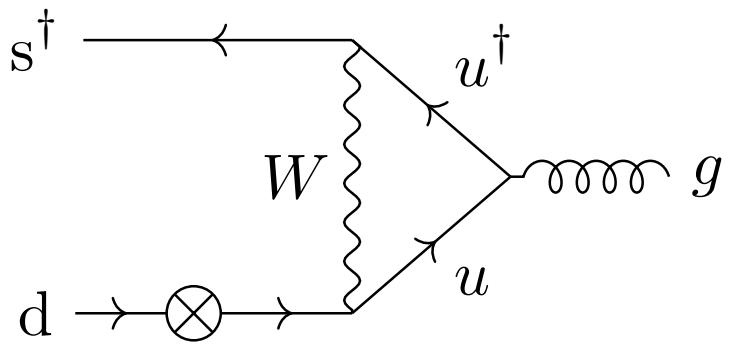

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2. $\langle\bar{u}\bar{\sigma}^{\mu\nu} G_{\mu\nu} u\rangle$ condensate fixes κ:

$$\langle\bar{u}\bar{\sigma}^{\mu\nu} G_{\mu\nu} u\rangle = \kappa(4\pi f)^2 \langle\bar{u}u\rangle + \text{NLO} \quad \Rightarrow \quad \kappa = 1.21 + \text{NLO}$$

We computed $K^+ \to \pi^+ s$ master-amplitude

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$$\begin{aligned} A_{\text{direct}} = -\frac{1}{4v} \Big[& 2bv\ c_{K\pi i} + \epsilon_{\text{EW}} \left(h_{8i} + 2h_{27i}\right) \left(m_K^2 + m_\pi^2 - m_s^2\right) \\ & - \epsilon_{\text{EW}} c_{\Omega i} \left(h'_b m_K^2 - \left(h_8 + 2h_{27}\right) \left(m_K^2 + m_\pi^2 - m_s^2\right)\right) \Big] \end{aligned}$$

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- Amplitude constrains quark, gluon portal interactions

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   - Include mixing + full flavour dependence
   - Reproduces prior model-dependent results (ALPs, light Higgs, *etc.*)
   - Matching to QCD $\Rightarrow$ constrains quark, gluon portal interactions

Thank you for your attention!

Portal SMEFT Operators

	d	Higgs	Yukawa + h.c.	Fermions	Gauge bosons
s_i	3	$s_i\lvert H\rvert^2$			
	4	$s_i s_j\lvert H\rvert^2$			
	5	$s_i s_j s_k\lvert H\rvert^2$	$s_i q_a \overline{u}_b \widetilde{H}^\dagger$		$s_i G^a_{\mu\nu} G_a^{\mu\nu}$
		$s_i D^\mu H^\dagger D_\mu H$	$s_i q_a \overline{d}_b H^\dagger$		$s_i W^a_{\mu\nu} W_a^{\mu\nu}$
		$s_i\lvert H\rvert^4$	$s_i \ell_a \overline{e}_b H^\dagger$		$s_i B_{\mu\nu} B^{\mu\nu}$
					$s_i G^a_{\mu\nu} \widetilde{G}_a^{\mu\nu}$
					$s_i W^a_{\mu\nu} \widetilde{W}_a^{\mu\nu}$
					$s_i B_{\mu\nu} \widetilde{B}^{\mu\nu}$
ξ_a + h.c.	4		$\xi_a \ell_b \widetilde{H}^\dagger$		
	5	$\xi_a \xi_b\lvert H\rvert^2$	$\xi_a^\dagger \overline{\sigma}^\mu \ell_b D_\mu \widetilde{H}^\dagger$		$\xi_a \sigma^{\mu\nu} \xi_b B_{\mu\nu}$
v^μ	4	$v_\mu v^\mu\lvert H\rvert^2$		$v^\mu q_a^\dagger \overline{\sigma}_\mu q_b$	
		$\partial_\mu v^\mu\lvert H\rvert^2$		$v^\mu \overline{u}_a^\dagger \sigma_\mu \overline{u}_b$	
		$v^\mu H^\dagger \overleftrightarrow{D}_\mu H$		$v^\mu \overline{d}_a^\dagger \sigma_\mu \overline{d}_b$	
				$v^\mu \ell_a^\dagger \overline{\sigma}_\mu \ell_b$	
				$v^\mu \overline{e}_a^\dagger \sigma_\mu \overline{e}_b$	

Leading QCD Portal Operators

	d	Scalar	Vector	Gauge
s_i	4	$s_i\,\overline{\psi}\psi$		
	5	$s_i s_j\,\overline{\psi}\psi$		$s_i\,F_{\mu\nu}F^{\mu\nu}$ $s_i\,F_{\mu\nu}\widetilde{F}^{\mu\nu}$ $s_i\,G_{\mu\nu}G^{\mu\nu}$ $s_i\,G_{\mu\nu}\widetilde{G}^{\mu\nu}$
ξ_a + h.c.	3	$\xi_a\,\nu$		
	5			$\xi_a\overline{\sigma}_{\mu\nu}\nu\,F^{\mu\nu}$ $\xi_a\overline{\sigma}_{\mu\nu}\xi_b\,F^{\mu\nu}$
v_μ	4		$v_\mu\,\psi^\dagger\overline{\sigma}^\mu\psi$	

	d	Two quarks	Quark dipole	Four fermions
s_i	6	$s_i s_j s_k\,\overline{d}d$ $\partial^2 s_i\,\overline{d}d$ $s_i\partial_\mu s_j\,d^\dagger\overline{\sigma}^\mu d$	$s_i\,F^{\mu\nu}\overline{d}\sigma_{\mu\nu}d$ $s_i\,G^{\mu\nu}\overline{d}\sigma_{\mu\nu}d$	
	7	$s_i s_j s_k s_l\,\overline{d}d$		$s_i\,d^\dagger\overline{q}^\dagger\overline{q}d$ $s_i\,q^\dagger\overline{\sigma}^\mu q q^\dagger\overline{\sigma}_\mu q$ $s_i\,d^\dagger\overline{\sigma}^\mu d\overline{q}\sigma_\mu\overline{q}^\dagger$ $s_i\,e^\dagger\overline{\sigma}_\mu\nu u^\dagger\overline{\sigma}^\mu d$ $s_i\,\nu^\dagger\overline{\sigma}_\mu\nu d^\dagger\overline{\sigma}^\mu d$
ξ_a h.c.	6	$\xi_a^\dagger\overline{\sigma}_\mu\,e d^\dagger\overline{\sigma}^\mu u$ $\xi_a^\dagger\overline{\sigma}_\mu\,\nu d^\dagger\overline{\sigma}^\mu d$		

$K^+ \to \pi^+ s_i$ Interactions

Scalar messenger contributions to currents:

$$\Omega \supset \frac{1}{4v}\beta_0 c_{\Omega i} s_i \ , \qquad \boldsymbol{M} \supset \left(\boldsymbol{c}_i^{S_m} + \boldsymbol{c}_{\partial^2 i}^{S_m} \frac{1}{v^2}\partial^2 \right) s_i \ , \qquad H_x \supset \frac{h_{xi}}{v} s_i \ ,$$

$$\Theta \supset \frac{1}{v} c_i^{S_\theta} s_i \ , \qquad \boldsymbol{\Gamma} \supset \left(\boldsymbol{\lambda}_d^s c_{i\bar{s}d}^{\gamma} + \boldsymbol{\lambda}_s^d c_{i\bar{d}s}^{\gamma} \right) s_i$$

$K^+ \to \pi^+ s_i$ Parameters

$$\mathcal{L}_{\text{portal}} \supset -\frac{b}{2} K^+ \pi^- c_{K\pi i} s_i = -\frac{b}{2} K^+ \pi^- \left(\overline{c}_{K\pi i} + \text{Re}\, \boldsymbol{c}^{S_m\, d}_{\partial^2\, i\, s} \frac{\partial^2}{v^2} \right) s_i \ ,$$

$$\overline{c}_{K\pi i} = \text{Re}\, \boldsymbol{c}^{S_m\, d}_{i\quad s} + \frac{\epsilon_{\text{EW}}}{2} \left((m_K^2 - m_\pi^2)\, \text{Re}\, \boldsymbol{c}^{S_m\, u}_{i\quad u} + m_K^2\, \text{Re}\, \boldsymbol{c}^{S_m\, d}_{i\quad d} - m_\pi^2\, \text{Re}\, \boldsymbol{c}^{S_m\, s}_{i\quad s} \right) \theta_{K^\pm \pi^\mp}$$
$$- \frac{\epsilon_{\text{EW}}}{2} \left(2 v h_b \left(\frac{m_{ud}}{m_s} \left(\boldsymbol{c}^{S_m\, d}_{i\quad d} - \boldsymbol{c}^{S_m\, s\,\dagger}_{i\quad s} \right) + \boldsymbol{c}^{S_m\, d\,\dagger}_{i\quad d} \right) + \frac{m_{ud} + m_s}{v} h_{bi} - \kappa_\gamma \left(c^{\gamma}_{i\bar{d}s} + c^{\gamma\,\dagger}_{i\bar{s}d} \right) \right)$$

$$\theta_{\pi s_i} = f \frac{b c_{s_i \pi}}{m_s^2 - m_\pi^2} \qquad \theta_{\eta s_i} = f \frac{b c_{s_i \eta} + c_i^{S_\theta} s_\eta \frac{m_0^2}{v}}{m_s^2 - m_\eta^2} \qquad \theta_{\eta' s_i} = f \frac{b c_{s_i \eta'} - c_i^{S_\theta} c_\eta \frac{m_0^2}{v}}{m_s^2 - m_{\eta'}^2}$$

- $\theta_{K^\pm \pi^\mp}$ = kaon to pion mixing angle, s_η, c_η (co-)sine of η-η' mixing angles
- β_0 = leading coeff. of QCD β-function
- m_{ud} = light quark mass, m_0^2 = singlet η mass parameter

$K^+ \to \nu_a \xi_b$ Width

Interaction:

$$\mathcal{L}_{\text{portal}} \supset -v c^{\nu}_{ba}\ \nu_b \xi_a - \frac{f}{v^2} c^{L\dagger}_{\bar{u}s,ba}\ \xi^\dagger_a \overline{\sigma}_\mu e_b \partial^\mu K^+ + \text{h.c.}$$

Width:

$$\Gamma = \left|\theta'_{ba}\right|^2 \Gamma(K^+ \to \ell^+_b \nu_b) \frac{\rho(x_\ell, x_\xi)}{\rho(x_\ell, 0)}, \qquad \theta'_{ba} = \left(\frac{c^{\nu}_{ba} v}{m_\xi} + \frac{c^{L}_{\bar{u}s,ba}}{V_{us}}\right)$$

Phasespace factor:

$$\rho(x_\ell, x_\xi) = \left(x_\ell + x_\xi - (x_\ell - x_\xi)^2\right) \sqrt{\left(\frac{1 - x_\ell - x_\xi}{2}\right)^2 - x_\ell x_\xi}\ , \qquad x_i = \frac{m_i^2}{m_K^2}$$

$\pi^0 \to \gamma v_i$ Width

Interaction:

$$\mathcal{L}_{\text{portal}} \supset \frac{2}{(4\pi)^2 f}\left(2\partial^\mu \boldsymbol{V}^\nu_{vu} + \partial^\mu \boldsymbol{V}^\nu_{vd}\right)\frac{\pi^0}{\sqrt{2}} e\widetilde{F}_{\mu\nu}, \qquad \boldsymbol{V}^\mu_v = \left(\boldsymbol{c}^L_v + \boldsymbol{c}^R_v\right) v^\mu$$

Width:

$$\Gamma(\pi^0 \to \gamma v_i) = 2\epsilon^2_{\text{eff}}\Gamma_{\pi^0\to\gamma\gamma}\left(1 - \frac{m_v^2}{m_\pi^2}\right)^3, \quad \epsilon_{\text{eff}} = \frac{2(\boldsymbol{c}^R_v + \boldsymbol{c}^L_v)_u + (\boldsymbol{c}^R_v + \boldsymbol{c}^L_v)_d}{2e\,(2q_u + q_d)}$$