

Light-cone distribution amplitudes in QCD \times QED

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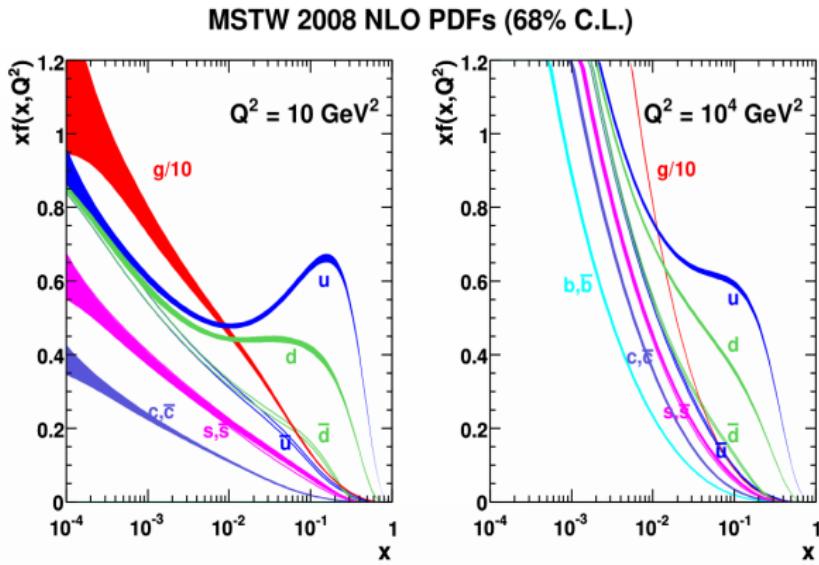
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Idea of factorization

Example to set up the stage: Parton distribution functions (PDFs).

$$\sigma_{pp \rightarrow X} \sim \int_0^1 dx dy f_{q_1/p}(x; \mu) f_{q_2/p}(y; \mu) \sigma_{\text{part.}}(xp_1, yp_2) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{E_{\text{cm}}}\right)$$



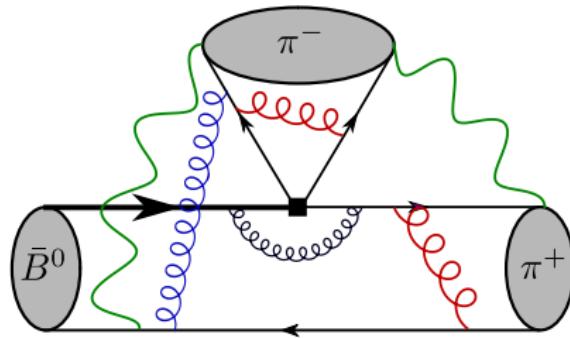
Now, we consider decays $\bar{B} \rightarrow M_1 M_2$ into light meson final states.

Same idea on the amplitude level:

$$\langle M_1 M_2 | Q_i | \bar{B} \rangle = \text{leading power factorization} + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right).$$

Why are these decays interesting?

- Dominated by CKM suppressed tree and penguin amplitudes.
→ Probe the SM loop corrections, sensitive to heavy particles.
- High precision from LHCb/Belle II: theory currently at $\mathcal{O}(\alpha_s^2)$.
→ QED corrections can become relevant.

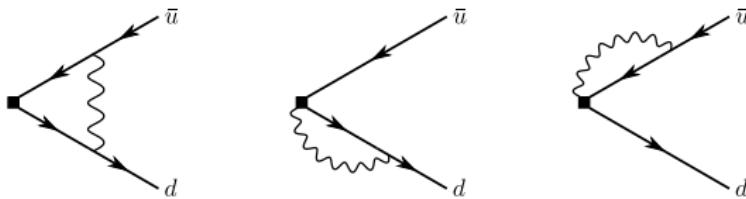


QCD×QED Factorization Formula

[Beneke et. al. 1999+2020]

$$\begin{aligned} \langle M_1 M_2 | Q_i | \bar{B} \rangle &= \mathcal{F}_{Q_2}^{B \rightarrow M_1}(q^2 = 0) \int_0^1 du \mathbf{T}_{i,Q_2}^I(u) f_{M_2} \Phi_{M_2}(u) \\ &+ \int_{-\infty}^{\infty} d\omega \int_0^1 du dv \mathbf{T}_{i,\otimes}^{II}(u, v, \omega) f_{M_1} \Phi_{M_1}(v) f_{M_2} \Phi_{M_2}(u) f_B \Phi_{B,\otimes}(\omega) \end{aligned}$$

$$\otimes = (Q_1, Q_2)$$



QCD

$$\langle M^-(p) | \bar{d}(tn_+) [tn_+, 0] \frac{\not{p}_+}{2} (1 - \gamma_5) u(0) | 0 \rangle = \frac{in+p}{2} \int_0^1 du e^{iu(n+p)t} f_M \phi_M(u; \mu)$$

QCD×QED

$$\langle M | R_c^{(Q_M)} (\bar{d}W^{(d)}) (tn_+) \frac{\not{p}_+}{2} (1 - \gamma_5) (W^{\dagger(u)} u)(0) | 0 \rangle = \frac{in+p}{2} \int_0^1 du e^{iu(n+p)t} f_M \Phi_M(u; \mu)$$

UV anomalous dimension is IR divergent and hence **soft rearrangement** is required:

$$\left| \langle 0 | \left[S_{n+}^{\dagger(Q_M)} S_{n-}^{(Q_M)} \right] (0) | 0 \rangle \right| \equiv R_{\bar{c}}^{(Q_M)} R_c^{(Q_M)} .$$

Renormalization group evolution

Renormalization group equation (RGE)

$$\frac{d}{d \ln \mu} \Phi_M(u; \mu) = - \int_0^1 dv \Gamma(u, v; \mu) \Phi_M(v; \mu) .$$

Integration kernel in QCD×QED [ERBL 1980 + „local“ terms]

$$\begin{aligned} \Gamma(u, v) = & - \frac{\alpha_s C_F + \alpha_{\text{em}} Q_u Q_d}{\pi} \left[\left(1 + \frac{1}{v-u}\right) \frac{u}{v} \theta(v-u) + \left(1 + \frac{1}{u-v}\right) \frac{\bar{u}}{\bar{v}} \theta(u-v) \right]^{(u)}_+ \\ & - \frac{\alpha_{\text{em}}}{\pi} \delta(u-v) Q_M \left(Q_M \left(\ln \frac{\mu}{2E} + \frac{3}{4} \right) - Q_d \ln \textcolor{red}{u} + Q_u \ln \textcolor{red}{\bar{u}} \right) . \end{aligned}$$

Plus-distribution

$$\int_0^1 du \left[\dots \right]^{(u)}_+ f(u) \equiv \int_0^1 du \left[\dots \right] (f(u) - f(v)) .$$

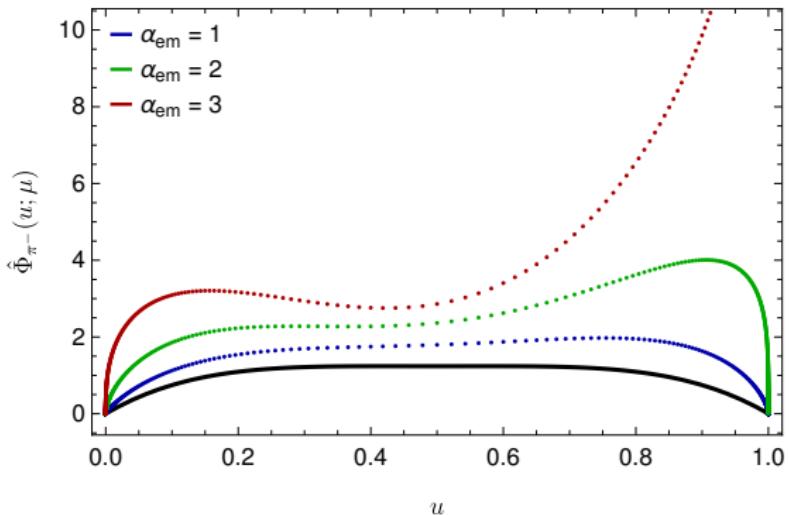
Solution to the RGE

Gegenbauer polynomials are eigenfunctions of the ERBL kernel.

$$\Phi_M(u; \mu) = 6u\bar{u} Z_\ell(\mu) \sum_{n=0}^{\infty} a_n^M(\mu) C_n^{(3/2)}(2u - 1)$$

Lattice data:
[RQCD 2020]

$$\begin{aligned} a_0^\pi(2\text{GeV}) &= 1 \\ a_1^\pi(2\text{GeV}) &= 0 \\ a_2^\pi(2\text{GeV}) &= 0.140^{+23}_{-24} \end{aligned}$$



Inverse moments

Recall that LCDAs appear as convolutions in factorization theorems.
Therefore inverse moments of the LCDAs are relevant:

$$\langle \bar{u}^{-1} \rangle_M(\mu) = \int_0^1 \frac{du}{1-u} \Phi_M(u; \mu) = 3Z_\ell(\mu) \sum_{n=0}^{\infty} a_n^M(\mu).$$

Corrections to Gegenbauer and inverse moments:

$$a_0^{\pi^-}(5.3\text{GeV}) = 1 + 0.0035|_{\text{QED}},$$

$$a_1^{\pi^-}(5.3\text{GeV}) = 0.0006|_{\text{QED}},$$

$$a_2^{\pi^-}(5.3\text{GeV}) = 0.0951|_{\text{LL}} - 0.0084|_{\text{NLL}} + 0.0001|_{\text{NNLL}} + 0.0010|_{\text{QED}},$$

$$\langle \bar{u}^{-1} \rangle_{\pi^-}(5.3\text{ GeV}) = 0.9997|_{\text{point charge}}^{\text{QED}} (3.285^{+0.05}_{-0.05}|_{\text{LL}} - 0.020|_{\text{NLL}} + 0.017|_{\text{partonic}}^{\text{QED}}),$$

$$\langle \bar{u}^{-1} \rangle_{\pi^-}(80.4\text{ GeV}) = 0.985|_{\text{point charge}}^{\text{QED}} (3.197^{+0.03}_{-0.03}|_{\text{LL}} - 0.022|_{\text{NLL}} + 0.042|_{\text{partonic}}^{\text{QED}}).$$

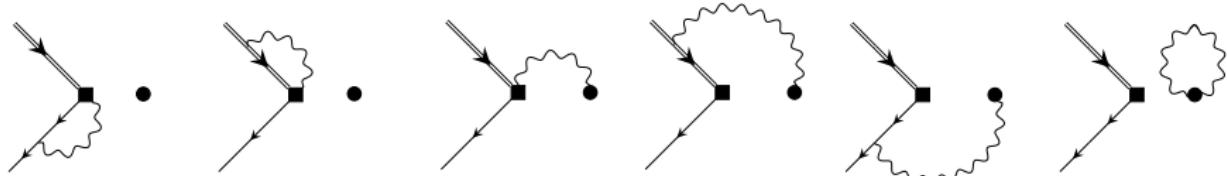
Summary

Light meson LCDAs in QCD \times QED ...

- are IR divergent and need to be regulated properly (**soft rearrangement**+IR subtraction scheme),
- unfold new features, e.g. modified endpoint behaviour and breaking of isospin symmetry,
- and their moments show that QED effects can become sizable and need to be taken into account for future precision calculations.

Thank you for your attention!

Soft function



Definition

$$\begin{aligned} im_B \int_{-\infty}^{\infty} d\omega e^{-i\omega t} \mathcal{F}_{B,\otimes} \Phi_{B,\otimes}(\omega, \mu) \\ = \frac{1}{R_c^{(Q_{M_1})} R_{\bar{c}}^{(Q_{M_2})}} \langle 0 | \bar{q}_s^{(q)}(tn_-)[tn_-, 0]^{(q)} \not{p}_- \gamma_5 h_v(0) S_{n+}^{\dagger(Q_{M_2})} S_{n-}^{\dagger(Q_{M_1})} | \bar{B} \rangle \end{aligned}$$

