Supersymmetric Alignment Models for $(g-2)_\mu$

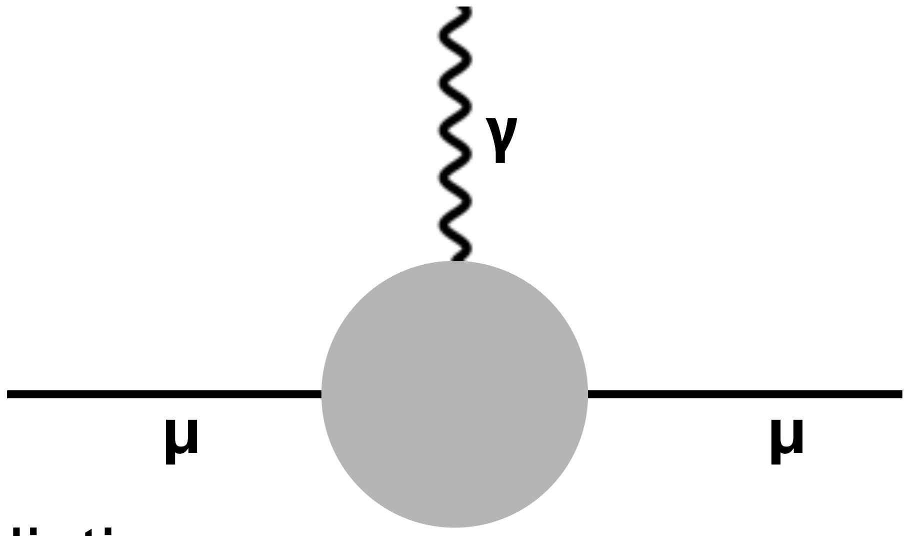
Motoo Suzuki

T. D. Lee Institute & Shanghai Jiao Tong U.

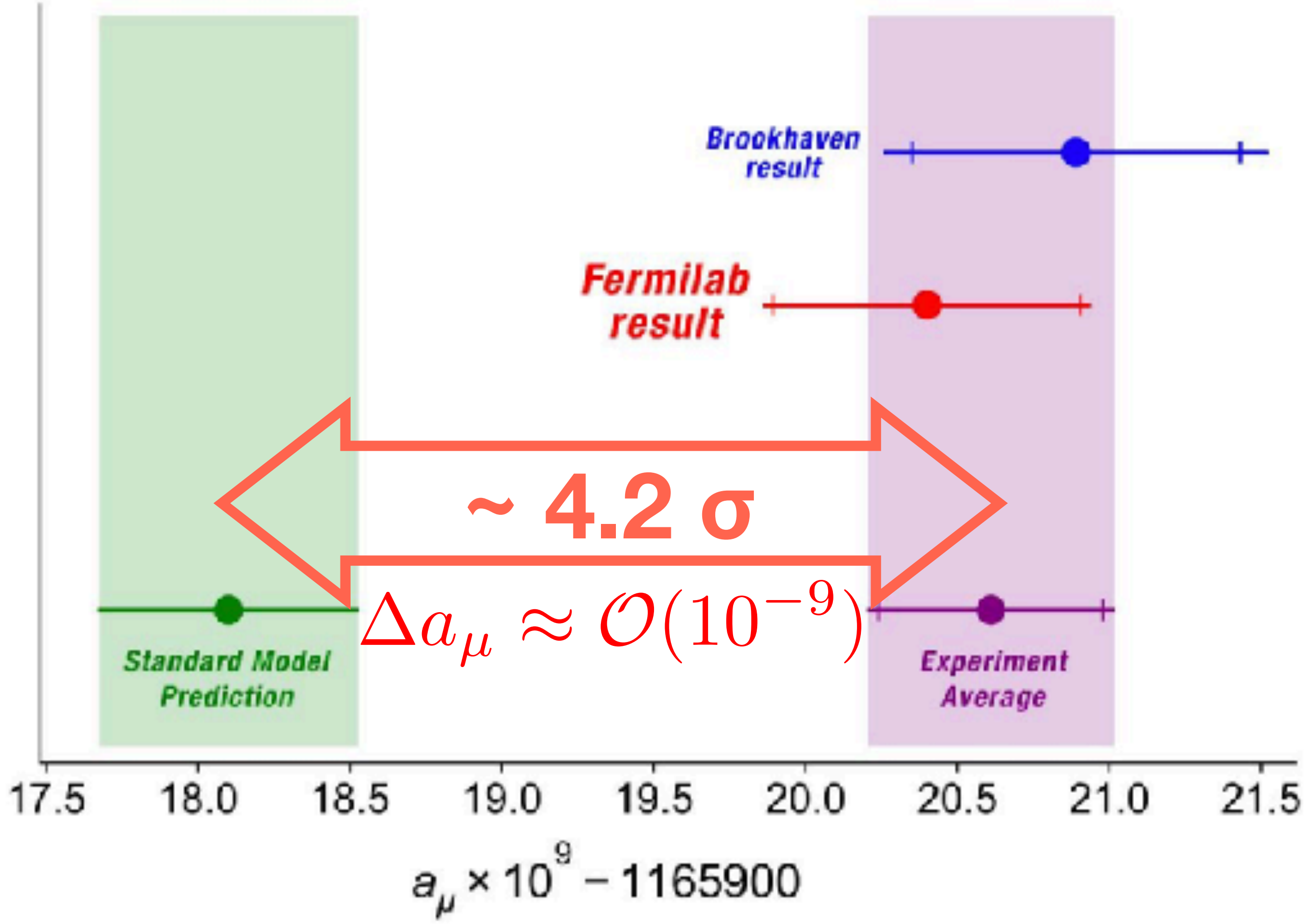
Based on Y. Nakai (TDLI&SJTU), M. Reece (Harvard U.), arXiv: 2107.10268 .

Muon g-2 anomaly

- g-factor : spin-magnetic field interaction
  - g=2 : tree level
  - a=(g-2)/2 : radiative correction



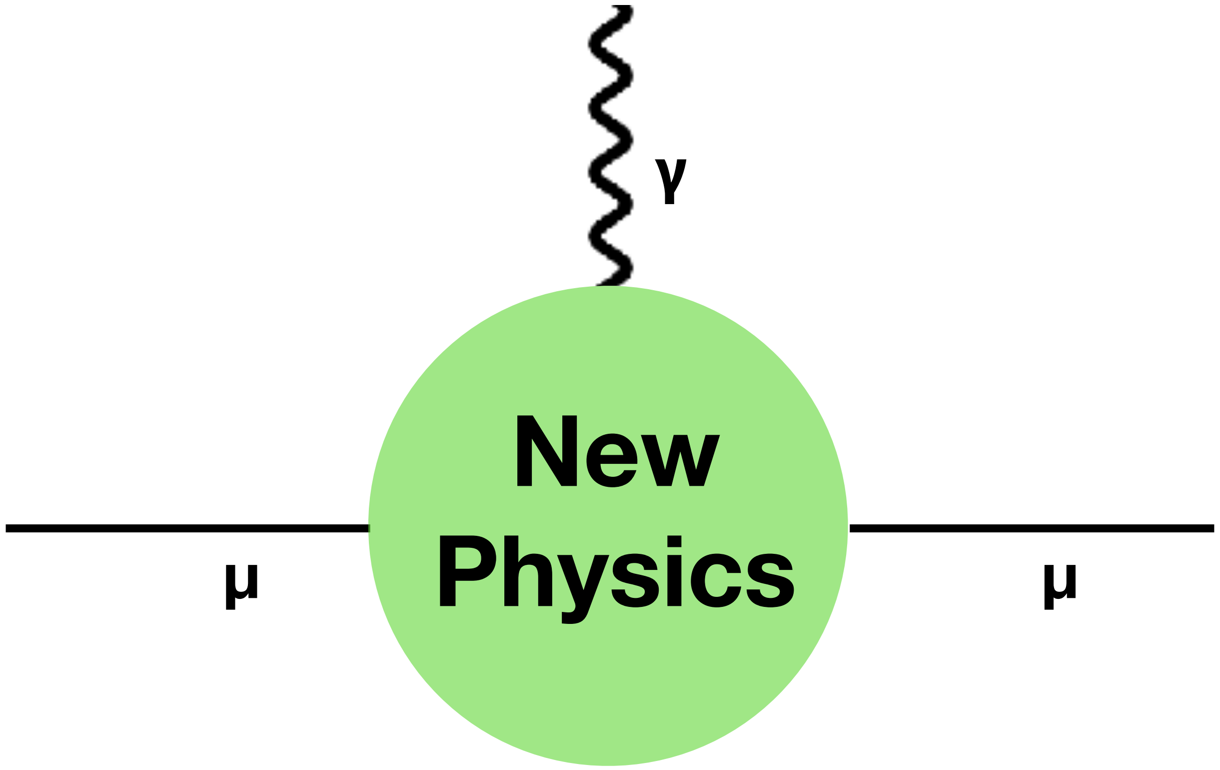
- Muon g-2: experimental results & SM prediction



From Fermilab webpage

New physics contribution

- New physics contribution to muon g-2



$$\Delta a_{\mu,\mathrm{NP}} \approx \frac{1}{16\pi^2}\mathcal{O}(m_\mu^2/m_{\mathrm{NP}}^2)$$

$$\Delta a_{\mu,\mathrm{NP}} \approx \mathcal{O}(10^{-9})$$

$$m_{\mathrm{NP}} \lesssim 1\,\mathrm{TeV}$$

SUSY contributions

- We focus on SUSY models

- electroweakinos-smuon loops

e.g.

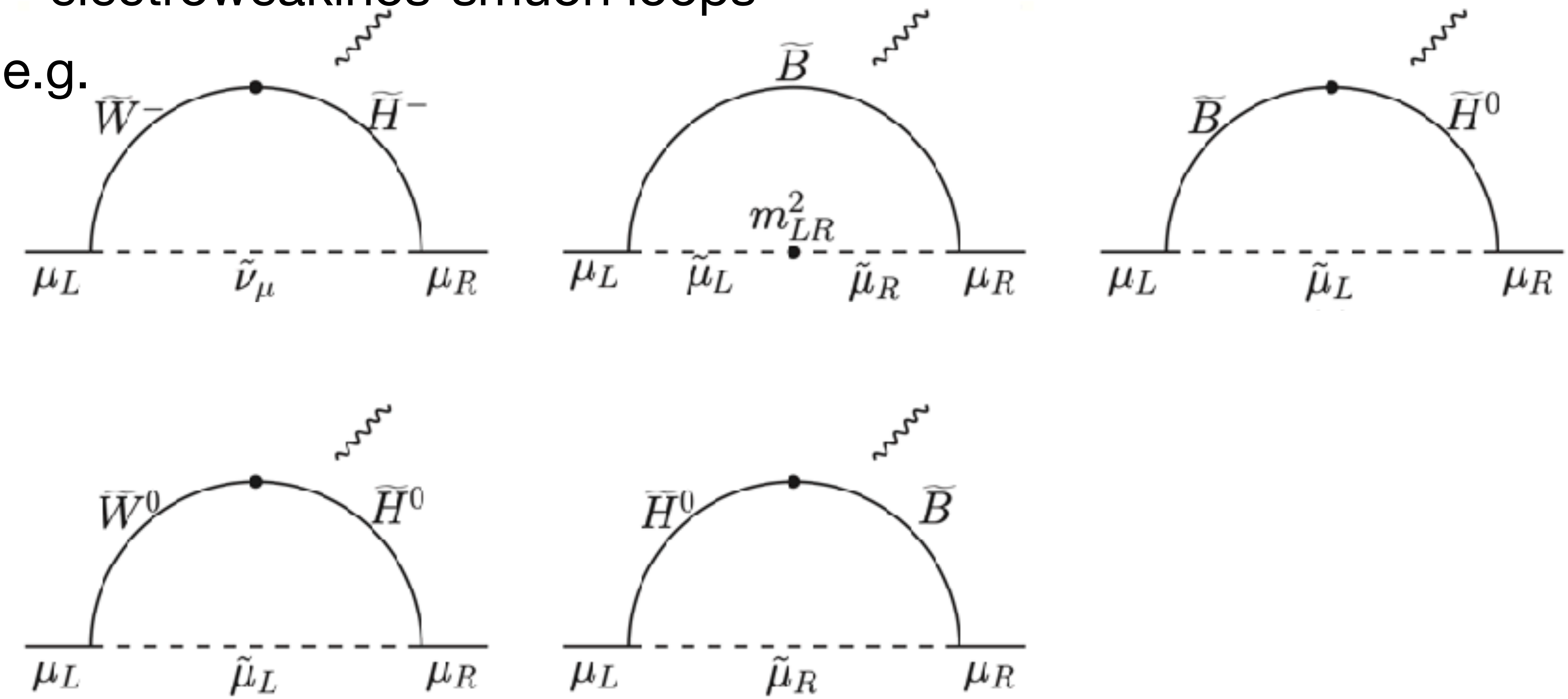


Figure from Gi. Cho, K. Hagiwara, Y. Matsumoto, D. Nomura (2011)

FCNC

e.g. Gravity mediation

$$\delta \sim 0.3$$

$$m^2_{\text{slepton}} \sim \tilde{m}^2 \begin{pmatrix} 1 & \delta & \delta \\ \delta & 1 & \delta \\ \delta & \delta & 1 \end{pmatrix}$$

$\mu \to e + \gamma$ ➡ $\tilde{m} \gg 1\,\text{TeV}$

Alignment

Alignment : a mechanism to suppress FCNCs

- In the basis that Yukawa diagonal

$$Y_e \sim \begin{pmatrix} Y_e & 0 & 0 \\ 0 & Y_\mu & 0 \\ 0 & 0 & Y_\tau \end{pmatrix}$$

$$\Downarrow$$

$$m^2_{\text{slepton}} \sim \tilde{m}^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ and } A_e \propto Y_e$$

FCNC constraints can be relaxed by approximate alignment

Alignment

Horizontal (flavor) symmetries

— Flavor structure of SM fermions + alignment

- Approximate alignment of quark-squark by horizontal symmetries

  Nir and Seiberg (1993)

- Approximate alignment of lepton-slepton by horizontal symmetries

  Ben-Hamo and Nir (1994)

  Grossman and Nir (1995)

SM flavor structure

- CKM matrix is presented in the Wolfenstein's parameterization — the order of magnitude of mixing angle is given by

$$|V_{12}^{\rm CKM}| \sim \lambda \ , \quad |V_{23}^{\rm CKM}| \sim \lambda^2 \ , \quad |V_{13}^{\rm CKM}| \sim \lambda^3$$

$$\lambda \sim 0.2$$

- Quark mass ratio can be also expressed in powers of λ

$$m_c/m_t \sim \lambda^3 \ , \quad m_u/m_t \sim \lambda^6 - \lambda^7,$$

$$m_b/m_t \sim \lambda^2 \ , \quad m_s/m_b \sim \lambda^2 \ , \quad m_d/m_b \sim \lambda^4$$

- Lepton mass ratio and mixing angles can be also expressed by λ

Horizontal symmetry

- Hierarchy and smallness of quark and lepton sector parameters

’tHooft Naturalness

Small numbers are natural only if an exact symmetry is acquired when they are set to zero

- Let us consider **U(1)H horizontal symmetry** that acts on quarks and leptons

$H(X)$: U(1)H charge of X

- e.g. up-type Yukawa terms

$$W \sim H_u Q_i \bar{U}_j$$

$$H(H_u) + H(Q_i) + H(\bar{U}_j) \neq 0$$

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Horizontal symmetry

- Introducing flavon "S" which has a non-zero U(1)H charge and VEV

$$H(S) \neq 0$$

$$\langle S \rangle = \lambda \, \Lambda_{\rm UV} \ \ (\lambda \sim 0.2)$$

- Fermion mass ratio and mixing can be explained by couplings with flavon

$$c.f. \ \ W \sim S^{a_{ij}} H_u Q_i \bar{U}_j \ \ (\Lambda_{\rm UV} = 1 \text{ unit})$$

$$Y_u = c^u_{ij} \langle S \rangle^{a_{ij}}$$

A simple model

- Introducing a flavon S1

$$H(S_1) = -1$$

- Assuming non-negative U(1)H charges for matter,
the mass ratios and mixing matrices are estimated by simple formula

$$|V_{ij}^{\mathrm{PMNS}}| \sim \lambda^{|H(L_i)-H(L_j)|}$$

- We also obtain soft mass squared matrices

$$M^2_{\tilde{L}\,ij} = \tilde{m}^2_{\ell}\,\lambda^{|H(L_i)-H(L_j)|}$$

— relation between mixing matrices and soft mass squared

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FCNC in lepton sector

- Mixing matrices - soft mass matrices relation

$$|V_{12}^{\mathrm{PMNS}}| \sim \lambda \quad \longrightarrow \quad M^2_{\tilde{L}\,12} \sim \tilde{m}_\ell^2 \lambda$$

$$\lambda \sim 0.2 \quad \longrightarrow \quad \tilde{m}_\ell \gg 1\,\mathrm{TeV}$$

$$\mu \to e + \gamma$$

Our work

Y. Nakai, M. Reece, M.S. (2021)

We study horizontal symmetry models to explain muon g-2

- one horizontal symmetry and several flavons
- two horizontal symmetries and several flavons

Two horizontal symmetries

$$U(1)_{H_1} \times U(1)_{H_2}$$

- Two flavons

$$S_1 \quad (-1,0) \qquad S_2 \quad (0,-1)$$

- Flavon VEVs

$$\langle S_1 \rangle \sim \lambda, \ \langle S_2 \rangle \sim \lambda^2$$

- An example charge assignment

Q_1	Q_2	Q_3	$\bar{u}_1$	$\bar{u}_2$	$\bar{u}_3$
$(3,0)$	$(0,1)$	$(0,0)$	$(-2,3)$	$(1,0)$	$(0,0)$

$\bar{d}_1$	$\bar{d}_2$	$\bar{d}_3$
$(-3,2)$	$(2,-1)$	$(0,0)$

L_1	L_2	L_3	$\bar{e}_1$	$\bar{e}_2$	$\bar{e}_3$
$(5,0)$	$(0,2)$	$(0,2)$	$(-4,2)$	$(2,-2)$	$(0,-2)$

Two horizontal symmetries

- Soft mass matrices are suppressed well

$$M^2_{\widetilde{L}} \sim \widetilde{m}^2 \begin{pmatrix} 1 & \lambda^9 & \lambda^9 \\ \lambda^9 & 1 & 1 \\ \lambda^9 & 1 & 1 \end{pmatrix}, \quad M^2_{\widetilde{\bar{e}}} \sim \widetilde{m}^2 \begin{pmatrix} 1 & \lambda^{14} & \lambda^{12} \\ \lambda^{14} & 1 & \lambda^2 \\ \lambda^{12} & \lambda^2 & 1 \end{pmatrix}$$

- $\mu \to e + \gamma$

$$\tilde{m}_\ell < 1\,\mathrm{TeV}$$

- However, eEDM constraint is also stringent…

$$\tilde{m}_\ell \gg 1\,\mathrm{TeV}$$

Spontaneous CP violation

- Suppression of eEDM by SCPV

- Consider an additional flavon

$$\begin{matrix} S_N \\ (-N, 0) \end{matrix}$$

- Superpotential is

$$W_S = Z(aS_N^2 + bS_N S_1^N + cS_1^{2N}) \, ,$$

- S_N can obtain a complex phase

$$\frac{\langle S_N \rangle}{\langle S_1 \rangle^N} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad b^2 - 4ac < 0$$

$$|\langle S_N \rangle| \sim \lambda^N$$

eEDM

- Flavon with complex phase

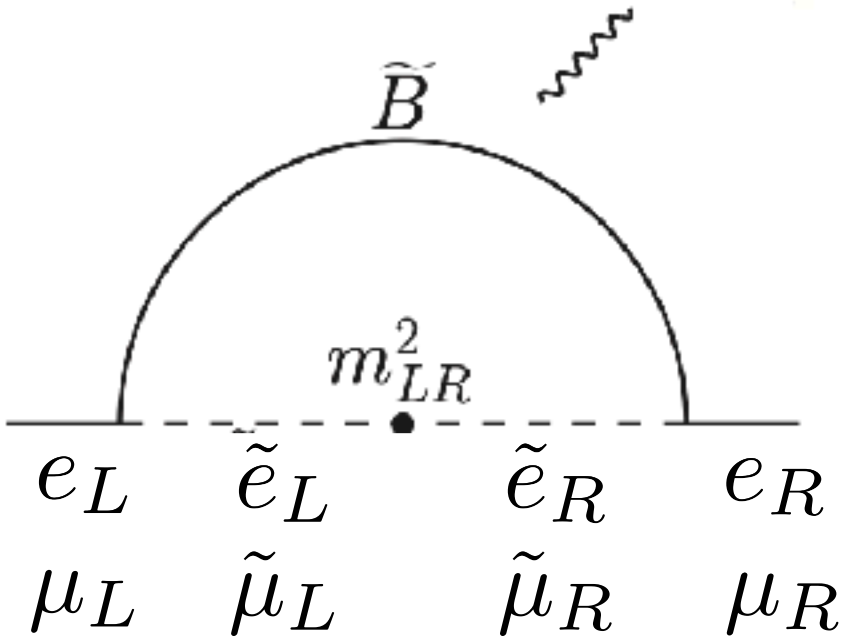
$$S_4$$
$$(-4, 0)$$

- Dominant flavor diagonal contribution
- Electron electric dipole moment

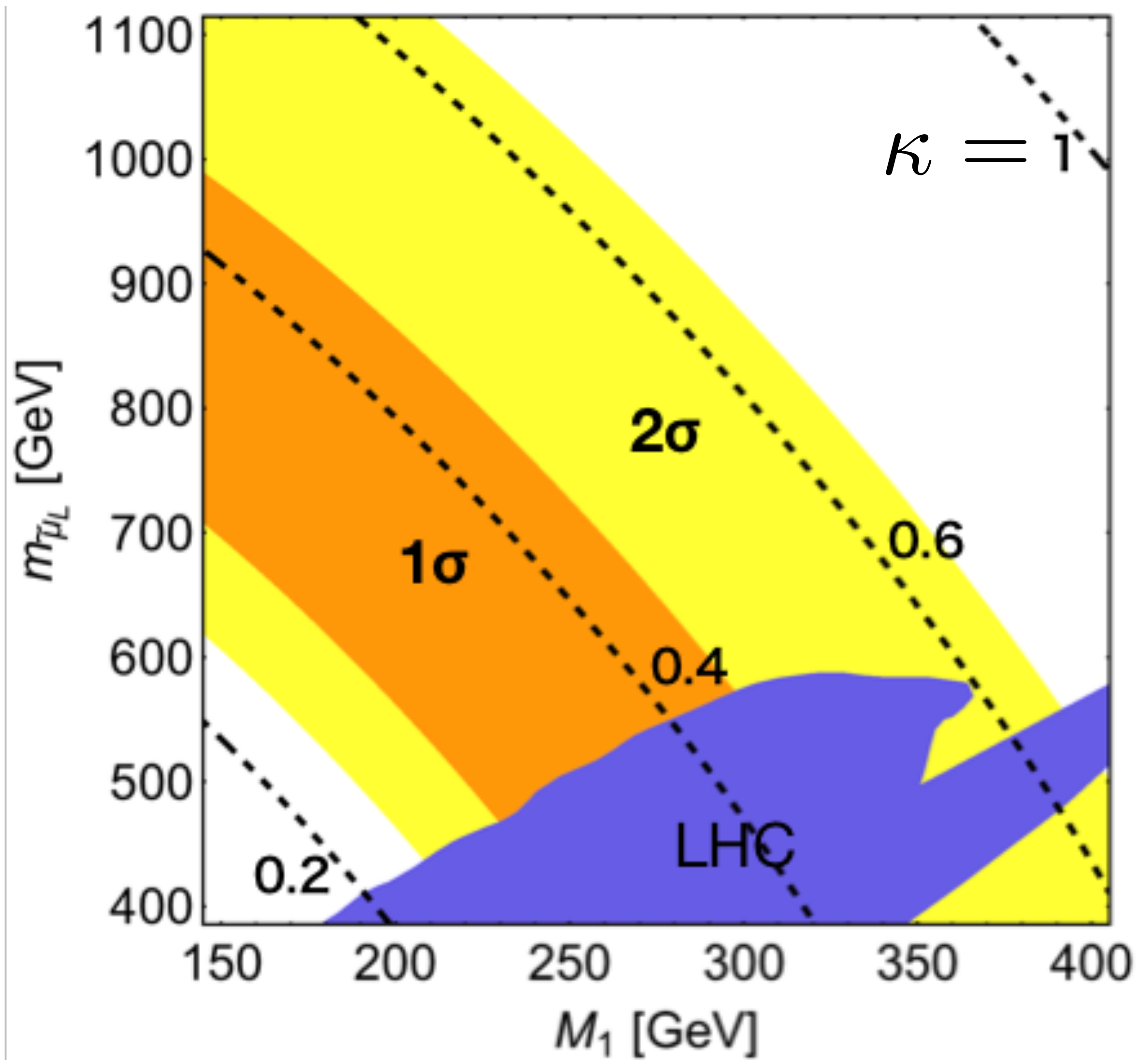
$$|d_e^{\rm SUSY}| \simeq 10^{-24} \left(\frac{a_\mu}{2 \times 10^{-9}} \right) |\arg(\mu)| \, e \, {\rm cm}$$

- CP phase in mu-term

$$\mu = |\mu|(1 + i\kappa \lambda^8)$$



Result



future expected reach

$$|d_e| = 10^{-30}\, e\,\mathrm{cm}$$

$$\arg(\mu) = \kappa\,\lambda^8$$

- decoupling of $\tilde{e}_i$
- parameters of

$$\tan\beta = 50,\ \ \mu = M_2 = 2M_1$$

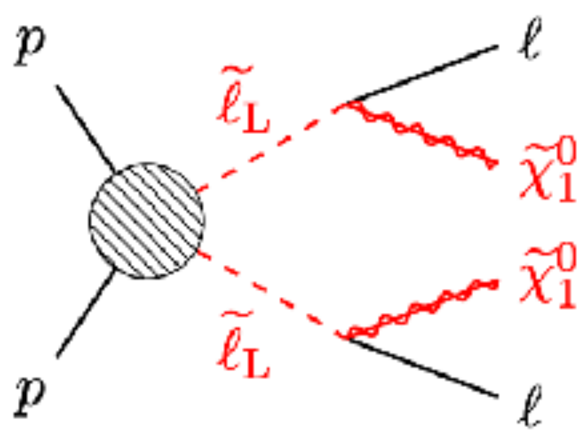
Future eEDM experiment will search for favored region

Conclusion

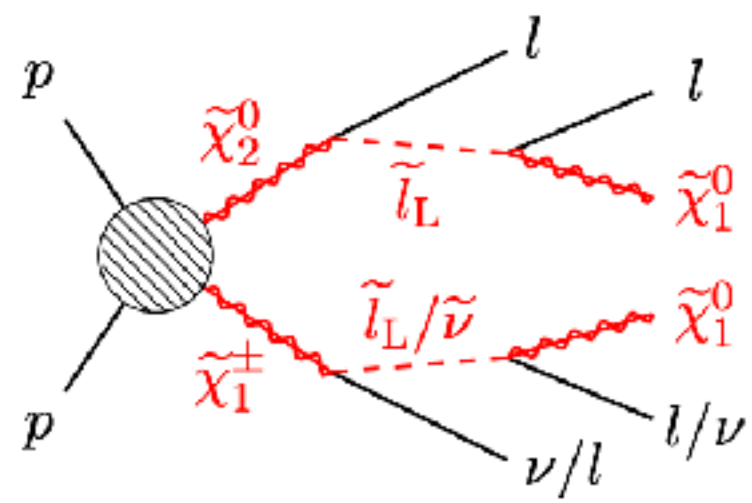
- New physics to explain muon g-2
- SUSY model is a prime candidate
- Horizontal symmetry provides fermion flavor structure and suppressed FCNCs = alignment
- A simple model predicts large FCNCs
- We considered SUSY horizontal symmetry models by one or two horizontal symmetries with multi-flavons
- We have developed models where muon g-2 can be explained by avoiding FCNC and CP observable constraints
- Future eEDM experiment will search for the favored parameter space
- Future study : Dark matter?

Back up

Collider constraint



(a) SLSL



(e) NC/3L

From M. Endo, K. Hamaguchi, S. Iwamoto, T. Kitahara (2021)

Flavor & CP observables

Observable	Experimental bound	Model with S_3	Model with S_4
$\|\epsilon_K\|$	$2.228(11)\times10^{-3}$ [56]	$\sim10^{-3}$	$\sim10^{-7}$
$\|\Delta M_D\|$	$0.63^{+0.27}_{-0.29}\times10^{-14}$ GeV [56]	$\sim5\times10^{-17}$ GeV	$\sim5\times10^{-17}$ GeV
nEDM	$\leqslant10^{-26}$ e cm [62]	$\sim10^{-28}$ e cm	$\sim10^{-28}$ e cm
Br$(\mu\to e+\gamma)$	$\leqslant4.2\times10^{-13}$ [64]	$\sim10^{-16}$	$\sim10^{-16}$
eEDM	$\leqslant1.1\times10^{-29}$ e cm [23]	$\sim5\times10^{-29}$ e cm	$\sim10^{-30}$ e cm

TABLE I. CP and flavor observables and their current experimental bounds. The estimation of the SUSY contribution to each observable in the models with $U(1)_{H_1}\times U(1)_{H_2}$ is also shown. We take $\widetilde{m}_q=M_3=5\,\text{TeV}$, $\widetilde{m}_\ell=M_{1,2}=\mu=500\,\text{GeV}$ and $\tan\beta=50$. The typical mass scales of the trilinear soft SUSY breaking terms are taken as $\widetilde{m}_q$, $\widetilde{m}_\ell$ for squarks and sleptons, respectively.

Q_1	Q_2	Q_3	$\bar{u}_1$	$\bar{u}_2$	$\bar{u}_3$
$(3,0)$	$(0,1)$	$(0,0)$	$(-2,3)$	$(1,0)$	$(0,0)$

$\bar{d}_1$	$\bar{d}_2$	$\bar{d}_3$
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$(5,0)$	$(0,2)$	$(0,2)$	$(-4,2)$	$(2,-2)$	$(0,-2)$

Flavor structure

$$Y_u \sim \begin{pmatrix} \lambda^7 & \lambda^4 & \lambda^3 \\ 0 & \lambda^3 & \lambda^2 \\ 0 & \lambda & 1 \end{pmatrix}, \quad Y_d \sim \begin{pmatrix} \lambda^4 & 0 & \lambda^3 \\ 0 & \lambda^2 & \lambda^2 \\ 0 & 0 & 1 \end{pmatrix},$$

$$Y_e \sim \begin{pmatrix} \lambda^5 & 0 & 0 \\ 0 & \lambda^2 & 1 \\ 0 & \lambda^2 & 1 \end{pmatrix}, \quad Y_\nu \sim \begin{pmatrix} \lambda^{10} & \lambda^9 & \lambda^9 \\ \lambda^9 & \lambda^8 & \lambda^8 \\ \lambda^9 & \lambda^8 & \lambda^8 \end{pmatrix}.$$

$$M^2_{\widetilde{Q}} \sim \widetilde{m}^2 \begin{pmatrix} 1 & \lambda^5 & \lambda^3 \\ \lambda^5 & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \; M^2_{\widetilde{\bar{u}}} \sim \widetilde{m}^2 \begin{pmatrix} 1 & \lambda^9 & \lambda^8 \\ \lambda^9 & 1 & \lambda \\ \lambda^8 & \lambda & 1 \end{pmatrix},$$

$$M^2_{\widetilde{\bar{d}}} \sim \widetilde{m}^2 \begin{pmatrix} 1 & \lambda^{11} & \lambda^7 \\ \lambda^{11} & 1 & \lambda^4 \\ \lambda^7 & \lambda^4 & 1 \end{pmatrix},$$

$$M^2_{\widetilde{L}} \sim \widetilde{m}^2 \begin{pmatrix} 1 & \lambda^9 & \lambda^9 \\ \lambda^9 & 1 & 1 \\ \lambda^9 & 1 & 1 \end{pmatrix}, \; M^2_{\widetilde{\bar{e}}} \sim \widetilde{m}^2 \begin{pmatrix} 1 & \lambda^{14} & \lambda^{12} \\ \lambda^{14} & 1 & \lambda^2 \\ \lambda^{12} & \lambda^2 & 1 \end{pmatrix}.$$

Another charge assignment

- Charge assignment

$$\begin{array}{cccccc} L_1 & L_2 & L_3 & \bar{e}_1 & \bar{e}_2 & \bar{e}_3 \\ (7,0) & (2,2) & (0,3) & (-6,2) & (-2,-1) & (0,-3) \end{array}$$

- Yukawa matrices

$$Y_e \sim \begin{pmatrix} \lambda^5 & 0 & 0 \\ 0 & \lambda^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad Y_\nu \sim \begin{pmatrix} \lambda^{14} & \lambda^{13} & \lambda^{13} \\ \lambda^{13} & \lambda^{12} & \lambda^{12} \\ \lambda^{13} & \lambda^{12} & \lambda^{12} \end{pmatrix}$$

- Soft mass squared mass matrices

$$M^2_{\widetilde{L}} \sim \widetilde{m}^2 \begin{pmatrix} 1 & \lambda^9 & \lambda^{13} \\ \lambda^9 & 1 & \lambda^4 \\ \lambda^{13} & \lambda^4 & 1 \end{pmatrix}, \; M^2_{\widetilde{\bar{e}}} \sim \widetilde{m}^2 \begin{pmatrix} 1 & \lambda^{10} & \lambda^{16} \\ \lambda^{10} & 1 & \lambda^6 \\ \lambda^{16} & \lambda^6 & 1 \end{pmatrix}$$