Multipoint Conformal Blocks from Gaudin models

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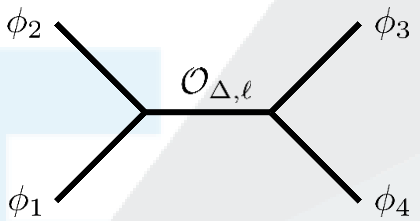
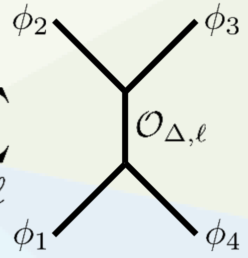
Reminder: Four-point conformal bootstrap

Conformal bootstrap: consistency of different OPE expansions to constrain CFT data

$$\langle \phi_1 \phi_2 \phi_3 \phi_4 \rangle = \sum_{\Delta,\ell}$$



$$= \sum_{\Delta,\ell}$$



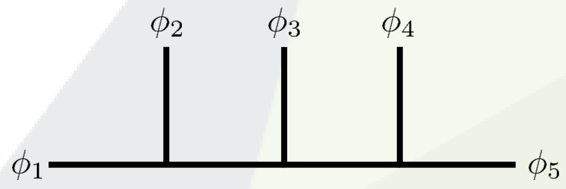
$$\sum_{\Delta,\ell} \lambda_{12\mathcal{O}} \lambda_{34\mathcal{O}}\, g_{\Delta,\ell}^{\Delta_{12},\Delta_{34}}(z,\bar{z}) = \frac{(z\bar{z})^{\frac{\Delta_3+\Delta_4}{2}}}{[(1-z)(1-\bar{z})]^{\frac{\Delta_2+\Delta_3}{2}}} \sum_{\Delta,\ell} \lambda_{23\mathcal{O}} \lambda_{41\mathcal{O}}\, g_{\Delta,\ell}^{\Delta_{23},\Delta_{41}}(1-\bar{z}, 1-z)$$ (crossing equation)

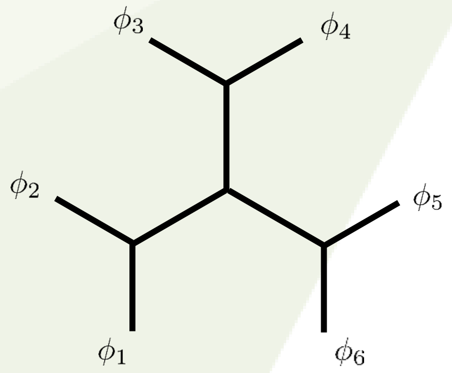
Conformal blocks $g_{\Delta,\ell}^{\Delta_{12},\Delta_{34}}(z,\bar{z})$ are eigenfunctions of quadratic and quartic Casimirs [Dolan,Osborn]

$$\frac{1}{2}\kappa^{ab} T_a^{(12)} T_b^{(12)} \mathcal{O}_{\Delta,\ell} = c_{\Delta,\ell}^{(2)} \mathcal{O}_{\Delta,\ell} \implies \mathcal{D}_2^{(12)}\, g_{\Delta,\ell}^{\Delta_{12},\Delta_{34}}(z,\bar{z}) = c_{\Delta,\ell}^{(2)}\, g_{\Delta,\ell}^{\Delta_{12},\Delta_{34}}$$

$$\kappa^{abcd} T_a^{(12)} T_b^{(12)} T_c^{(12)} T_d^{(12)} \mathcal{O}_{\Delta,\ell} = c_{\Delta,\ell}^{(4)} \mathcal{O}_{\Delta,\ell} \implies \mathcal{D}_4^{(12)}\, g_{\Delta,\ell}^{\Delta_{12},\Delta_{34}}(z,\bar{z}) = c_{\Delta,\ell}^{(4)}\, g_{\Delta,\ell}^{\Delta_{12},\Delta_{34}}$$

Goal: Multipoint Conformal Blocks





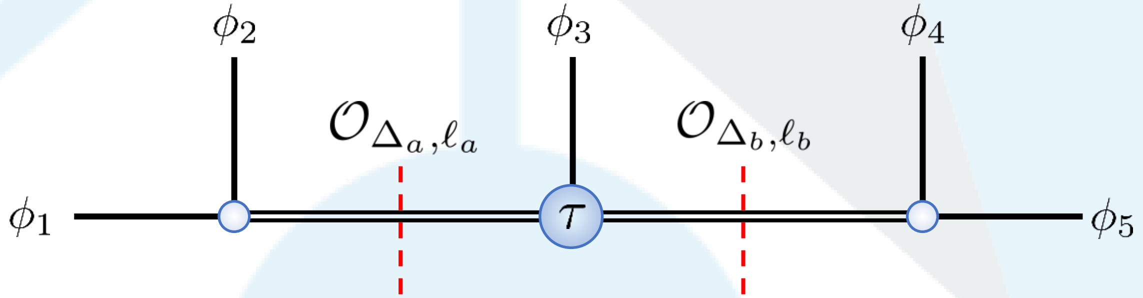
Can we extend this to a higher number of points?

- One multi-point function $\longrightarrow$ infinitely many four-point functions!
- Alternative path to spinning four-point functions
- Interplay between analytic bootstrap & scattering amplitudes [Bercini,Gonçalves,Vieira]

Multipoint blocks are a crucial element

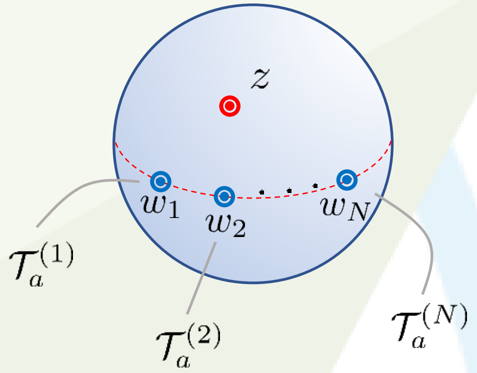
$$\#(\text{cross ratios}) \overset{*}{=} Nd - \dim\left[SO(d+1,1)\right] \quad \Longrightarrow$$ Need more commuting operators than just Casimirs!

$$N = 5 \quad \Rightarrow \quad \#(\text{c.r.}) = 5$$



$^*N$ large enough

Strategy: $\mathfrak{so}(d+1,1)$ Gaudin Models



Lax Matrix:

$$\mathcal{L}_a(z) = \sum_{i=1}^{N} \frac{\mathcal{T}_a^{(i)}}{z - w_i} \quad \Longrightarrow$$

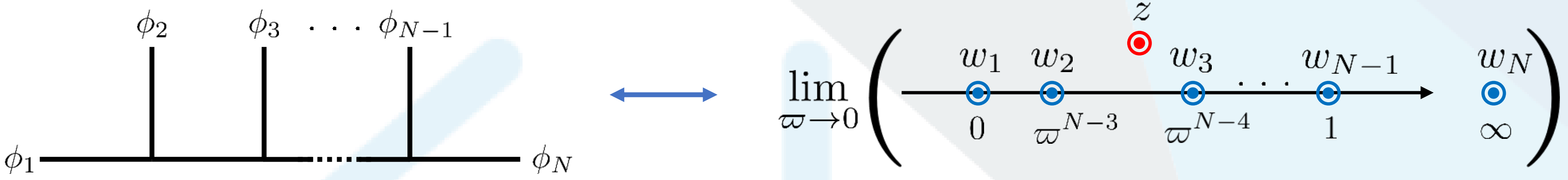
Gaudin Hamiltonians:

$$\mathcal{H}_p(z) = \kappa_p^{a_1 \cdots a_p} \mathcal{L}_{a_1}(z) \cdots \mathcal{L}_{a_p}(z) + \dots$$

Commute among themselves and with diagonal action of the conformal group

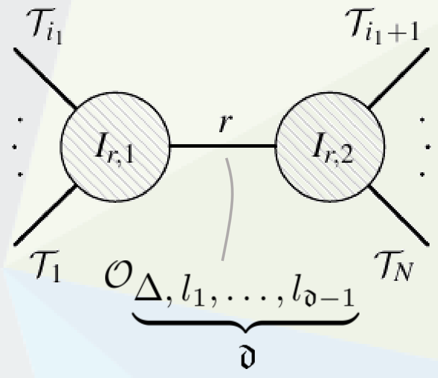
$$[\mathcal{H}_p(z), \mathcal{H}_q(z')] = 0 \qquad \left[\sum_{i=1}^{N} \mathcal{T}_a^{(i)}, \mathcal{H}_p(z)\right] = 0$$

Application to conformal blocks is obtained by taking limits on the w_i that reproduce the OPE channel.



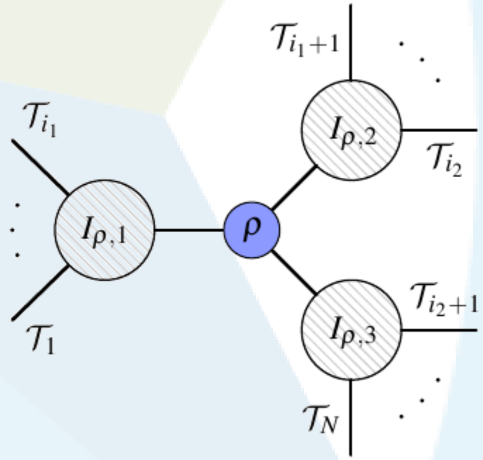
The individual Casimirs and “vertex operators” are extracted manipulating the z dependence

Result: operators for any number of points



Casimir operators at every internal leg r

$$\mathcal{D}^p_{r,1} = \mathcal{D}^p_{r,2} = \kappa_p^{a_1 \dots a_p} \left[\mathcal{T}_{a_1}^{(I_{r,1})} \dots \mathcal{T}_{a_p}^{(I_{r,1})} \right]_{|\mathcal{G}}$$



"Vertex operators" for non-trivial vertices ρ

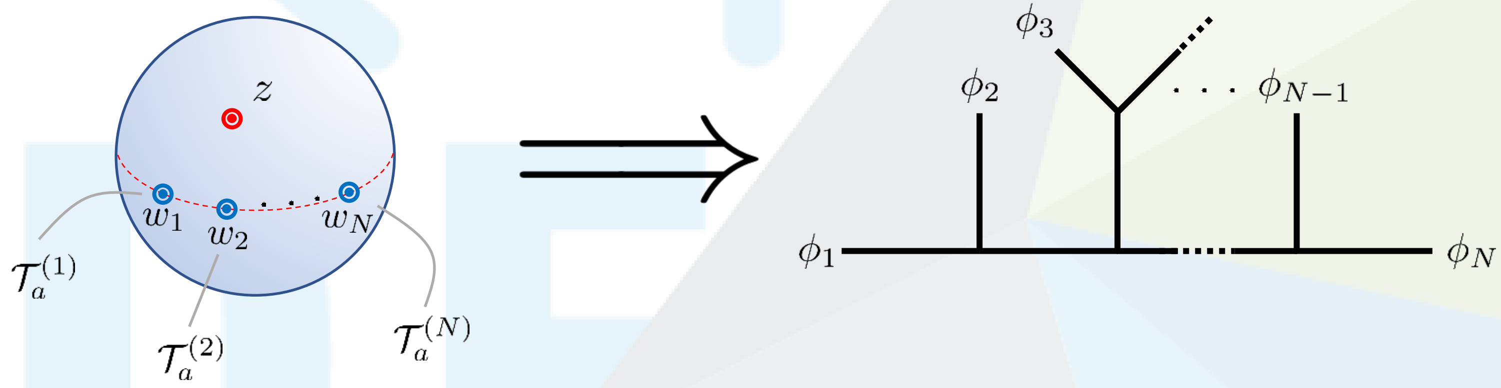
$$\mathcal{D}^{p,\nu}_{\rho,12} = \kappa_p^{a_1 \dots a_\nu a_{\nu+1} \dots a_p} \left[\mathcal{T}_{a_1}^{(I_{\rho,1})} \dots \mathcal{T}_{a_\nu}^{(I_{\rho,1})} \mathcal{T}_{a_{\nu+1}}^{(I_{\rho,2})} \dots \mathcal{T}_{a_p}^{(I_{\rho,2})} \right]_{|\mathcal{G}}$$

Number of independent vertex operators matches number of tensor structures

$$n^{(p)}_{vdo,\rho} = \max\left[\left((p-2) - \sum_{i=1}^{3} \Theta_0(p - 2\mathfrak{d}_i)(p - 2\mathfrak{d}_i - 1) \right), 0 \right] \qquad p \in 2\mathbb{N}\,, \quad p \geq 4$$

$\Longrightarrow$ distinguished basis of tensor structures: eigenfunctions of reduced vertex operators (OPE limits)

Conclusions & Outlook



- Multipoint conformal blocks $=$ eigenfunctions of special limits of Gaudin models
- Gaudin eigenfunctions $\longrightarrow$ interpolation between OPE channels [Eberhardt, Komatsu, Mizera] [Roehrig,Skinner]
- Fourth-order reduced vertex system $\longrightarrow$ Elliptic Calogero-Moser models [2108.00023]
- Partial light-cone limits?