Bootstrapping Monodromy Defects in the Wess-Zumino Model

Aleix Gimenez-Grau

DESY Hamburg

Based on:

- AGG, Liendo: 2108.05107.

See also:

- AGG, Liendo, van Vliet: 2012.00018 (Philine's talk).
- Barrat, AGG, Liendo: 2108.13432 (Julien's talk).

Wess-Zumino model

The Wess-Zumino model has one complex scalar and a Weyl fermion:

$$L_{\mathrm{WZ}} = (\partial_\mu \bar{\phi})(\partial_\mu \phi) + \psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi + \frac{g}{2}(\psi\psi\phi + \psi^\dagger\psi^\dagger\bar{\phi}) + \frac{g^2}{4}(\phi\bar{\phi})^2 \,.$$

Interesting properties:

- Supersymmetric and preserves four supercharges.
- Continuum limit of a lattice model.
- A nice example of emergent supersymmetry.
- Might be realized experimentally at the boundary of a topological insulator.

Wess-Zumino model

The Wess-Zumino model has one complex scalar and a Weyl fermion:

$$L_{\mathrm{WZ}} = (\partial_\mu \bar{\phi})(\partial_\mu \phi) + \psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi + \frac{g}{2}(\psi\psi\phi + \psi^\dagger\psi^\dagger\bar{\phi}) + \frac{g^2}{4}(\phi\bar{\phi})^2 \,.$$

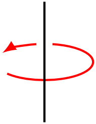
Notice:

- Free in $d = 4$.
- Non-trivial fixed point in $d = 3$.
- Perturbation theory in $d = 4 - \varepsilon$.

We will **not use** the Lagrangian description!

Monodromy defect

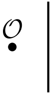
Since the theory has a $U(1)$ symmetry, we can impose a monodromy:
[Söderberg '17; Giombi, Helfenberger, Ji, Khanchandani '21]



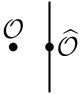
$$\phi(r, \theta + 2\pi, \vec{y}) = e^{2\pi i v} \phi(r, \theta, \vec{y})\,.$$

It breaks conformal symmetry: $SO(d+1,1) \to SO(d-1,1) \oplus SO(2)$.
The monodromy defect preserves two supercharges in the WZ model.

There are two new observables that define any defect



$$= \langle\!\langle\, \mathcal{O} \,\rangle\!\rangle \propto a_{\mathcal{O}}\,,$$

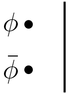


$$= \langle\!\langle\, \mathcal{O}\widehat{\mathcal{O}} \,\rangle\!\rangle \propto b_{\mathcal{O}\widehat{\mathcal{O}}}\,.$$

The defect data $\{a_{\mathcal{O}}, b_{\mathcal{O}\widehat{\mathcal{O}}}\}$ is the analog of $\{\lambda_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}_3}\}$ in regular CFT.

Crossing equation

The simplest correlator not fixed by conformal symmetry is:



$$= \langle\!\langle\, \phi(x_1)\bar{\phi}(x_2)\,\rangle\!\rangle \sim \mathcal{G}(z,\bar{z})\,.$$

There are two possible OPEs

$$\phi \times \bar{\phi} \sim \sum \lambda_{\phi\bar{\phi}\mathcal{O}}\mathcal{O}\,, \qquad \phi \sim \sum b_{\phi\widehat{\mathcal{O}}}\widehat{\mathcal{O}}\,.$$

Their equality is the crossing equation: [Billo, Goncalves, Lauria, Meineri '16]

$$\mathcal{G}(z,\bar{z}) = \sum_{\mathcal{O}} \lambda_{\phi\bar{\phi}\mathcal{O}} a_{\mathcal{O}} F_{\Delta,\ell}(z,\bar{z}) = \sum_{\widehat{\mathcal{O}}} |b_{\phi\widehat{\mathcal{O}}}|^2 \hat{F}_{\hat{\Delta},s}(z,\bar{z})\,.$$

To bootstrap the correlator, information flows from bulk to defect.

Inversion formula

Thanks to the inversion formula, the discontinuity $\operatorname{Disc}\mathcal{G}(z,\bar{z})$ is sufficient to reconstruct the full correlator: [Lemos, Liendo, Meineri, Sarkar '17]

$$\int d^2z\, I_{\hat{\Delta},s}(z,\bar{z})\,\operatorname{Disc}\mathcal{G}(z,\bar{z}) = -\sum_{\widehat{\mathcal{O}}} \frac{b^2_{\phi\widehat{\mathcal{O}}}}{\hat{\Delta}-\hat{\Delta}_{\widehat{\mathcal{O}}}}\,.$$

If the bulk OPE is $\phi\times\bar{\phi}\sim\sum_\ell \mathcal{O}_\ell$, a simple calculation shows

$$\operatorname{Disc}\mathcal{G}(z,\bar{z})|_{O(\varepsilon)} \sim \sum_{\ell=0}^{\infty} (\lambda_{\phi\bar{\phi}\mathcal{O}_\ell} a_{\mathcal{O}_\ell})_{\text{free}}\gamma_\ell \tilde{F}_{\Delta_{\mathcal{O}_\ell},\ell}(z,\bar{z})\,.$$

At this order, the discontinuity depends only on free theory coefficients and bulk anomalous dimensions (independent of the defect!).

We obtained γ_ℓ using standard bootstrap methods:

$$\Delta_\ell = 2\Delta_\phi+\ell+\gamma_\ell = 2\Delta_\phi+\ell+\frac{2}{3}\frac{(-1)^\ell}{\ell+1}\varepsilon+O(\varepsilon^2)\,.$$

Wess-Zumino monodromy defect

Summary:

- Extract the defect data by computing the inversion formula integral.

The leading defect family is of the form $\widehat{\mathcal{O}}_s \sim (\partial_\perp)^s \phi$:

$$\hat{\Delta}_s = \frac{d-1}{3} + |s| + \varepsilon \hat{\gamma}_s^{(1)} + O(\varepsilon^2)\,, \quad \hat{\gamma}_s^{(1)} = \begin{cases} 0 & \text{for } s > 0\,, \\ \frac{2(v-1)}{3|s|} & \text{for } s < 0\,. \end{cases}$$

$$b_{s>0}^2 = 1 + \frac{-(2|s|+1-v)H_{|s|} + (|s|+1-v)H_{|s|+1-v} - (1-v)H_{1-v}}{3|s|}\varepsilon + O(\varepsilon^2)\,,$$

$$b_{s<0}^2 = 1 + \frac{-(2|s|+v-1)H_{|s|} + (|s|+v-1)H_{|s|+v-1} - (v-1)H_{v-1}}{3|s|}\varepsilon + O(\varepsilon^2)\,.$$

Wess-Zumino monodromy defect

Summary:

- Extract the defect data by computing the inversion formula integral.
- Resum the defect channel to obtain the full correlator.

The full result is not particularly simple:

$$
\begin{aligned}
\mathcal{G}_{\mathrm{WZ}}(x,\bar{x}) = -\frac{\varepsilon}{3}\frac{\sqrt{x\bar{x}}}{(1-x\bar{x})}\Bigg[\\
&+ x^{v}(1-v)\big(j_{2v-1,v}(x) - j_{v,v}(x) - H_{v-1}\Phi_v(x) + \Phi_v(x)\log(x\bar{x})\big) \\
&+ \bar{x}^{1-v}(1-v)\big(j_{1-v,1-v}(\bar{x}) - j_{2-2v,1-v}(\bar{x}) + H_{1-v}\Phi_{1-v}(\bar{x})\big) \\
&+ x^{v}\frac{H_{v-1} - H_{2v-2} + \Phi_v(x) - \Phi_{2v-1}(x)}{1-x} \\
&+ \bar{x}^{1-v}\frac{H_{-v} - H_{1-2v} + \Phi_{1-v}(\bar{x}) - \Phi_{2-2v}(\bar{x})}{1-\bar{x}} \\
&- x^{1-v}\bar{x}^{2-2v}\left((v-1)J_{2-2v,1-v}(\bar{x},x) + \frac{\Phi_{2-2v}(\bar{x}) - x\Phi_{2-2v}(x\bar{x})}{1-x}\right) \\
&+ x^{2v-1}\bar{x}^{v-1}\left((v-1)J_{2v-1,v-1}(x,\bar{x}) - \frac{\Phi_{2v-1}(x) - \bar{x}\Phi_{2v-1}(x\bar{x})}{1-\bar{x}}\right) \\
&- \bar{x}x^{v+1}\left((v-1)J_{v+1,1}(x,\bar{x}) - \frac{\Phi_{v+1}(x) - \bar{x}\Phi_{v+1}(x\bar{x})}{1-\bar{x}}\right) \\
&+ x\bar{x}^{2-v}\left((v-1)J_{2-v,1}(\bar{x},x) + \frac{\Phi_{2-v}(\bar{x}) - x\Phi_{2-v}(x\bar{x})}{1-x}\right)\Bigg].
\end{aligned}
$$

Wess-Zumino monodromy defect

Summary:

- Extract the defect data by computing the inversion formula integral.
- Resum the defect channel to obtain the full correlator.
- Extract the bulk OPE coefficients.

For the leading bulk operator $\mathcal{O} = (\phi\bar{\phi})$ we find:

$$\lambda_{\phi\bar{\phi}(\phi\bar{\phi})} a_{(\phi\bar{\phi})} = \frac{v(v-1)}{2} - \varepsilon \frac{2v^3 - 4v^2 + 2v - 1}{6v} + \frac{\varepsilon}{3}(v-1)(2v-1)\left(H_{2v} + H_{-2v} - H_v - H_{-v}\right) + O(\varepsilon^2)\,.$$

We can similarly extract other operators.

Conclusions

Our techniques are very general and do not rely on supersymmetry. With similar ideas we bootrapped two-point functions with:

- Monodromy defect in the $O(N)$ Wilson-Fisher fixed point.
- A supersymmetric boundary in the WZ model (see Philine's talk).
- The Maldacena-Wilson line in $\mathcal{N} = 4$ SYM (see Julien's talk).

In the future, one can look at other interesting setups:

- Monodromy defects in the $O(N)$ model at large N.
- Surface defects in $\mathcal{N} = 4$ SYM at strong coupling.
- Half-BPS Wilson lines in ABJM theory.
- And hopefully many more!