Loops in (A)dS/CFT

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based on 2009.06511 with I. Sachs
and ongoing work with I. Sachs, E. Skvortsov and P. Vanhove

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Idea and Motivation

- Central question: Given a QFT on $EAdS_4/dS_{3,1}$ what is the corresponding CFT?
- Calculate loop corrections to Witten diagrams in $EAdS_4$
- Obtain generating functional of a hopefully useful CFT.
- Use results from EAdS to get information about the dual CFT in de Sitter

We consider a real scalar field with $V(\phi) = m^2\phi^2 + \lambda\phi^4$ in EAdS.

Similarities and differences between AdS/CFT and dS/CFT

AdS [Maldacena '98, Witten '98, Klebanov, Polyakov '02, ...]

- The partition function Z_{EAdS} of a QFT in EAdS is equivalent to the generating functional of a CFT on the boundary
- Correlation functions of the CFT can be calculated by taking functional derivatives of Z_{EAdS}

$$\langle \mathcal{O}(\underline{x}_1)\mathcal{O}(\underline{x}_2)...\mathcal{O}(\underline{x}_n)\rangle = \lim_{z\to 0} \frac{\delta^n}{\delta\phi_0(\underline{x}_1)...\delta\phi_0(\underline{x}_n)} Z_{EAdS}[\phi_0(z,\underline{x})]$$

dS [Strominger '01, Maldacena '02,...]

- The wave function of the Bunch-Davies vacuum $\Psi[\eta, \phi_0(\underline{x})]$ is equivalent to the generating functional of a CFT at future infinity
- $\langle \mathcal{O}(\underline{x}_1)\mathcal{O}(\underline{x}_2)...\mathcal{O}(\underline{x}_n)\rangle = \lim_{\eta\to 0} \frac{\delta^n}{\delta\phi_0(\underline{x}_1)\delta\phi_0(\underline{x}_2)...\delta\phi_0(\underline{x}_n)} \Psi[\eta, \phi_0(\underline{x})]$

Transformation for going from dS to EAdS: $\eta \to iz$ and $\ell_{dS} \to i\ell_{AdS}$

Difference between dS and AdS

Important difference:
In AdS the CFT correlation functions can be extrapolated from the bulk n-point function:

$$\langle \mathscr{O}(\underline{x}_1)\mathscr{O}(\underline{x}_2)...\mathscr{O}(\underline{x}_n)\rangle \propto \lim_{z_i \to 0} \langle \phi(z_1, \underline{x}_1), \phi(z_2, \underline{x}_2)...\phi(z_n, \underline{x}_n)\rangle$$

In dS the n-point function in the Bunch-Davies vacuum at time η is given by:

$$\langle \phi(\eta, \underline{x}_1), \phi(\eta, \underline{x}_2)...\phi(\eta, \underline{x}_n)\rangle \propto \int \mathscr{D}\phi(\underline{x}) \left|\Psi[\eta, \phi(\underline{x})]\right|^2 \phi(\underline{x}_1)\phi(\underline{x}_2)...\phi(\underline{x}_n)$$

where as the conformal correlators are more constructed of matrix elements between the Bunch-Davies vacuum and an out vacuum the enforces the Dirichlet boundary condition.

Propagators in position space

The general propagator of a scalar field in EAdS is given by:

$$\Lambda(K,\Delta) \propto K(\mathbf{X},\mathbf{Y})^{\Delta}\,{}_2F_1\left(\frac{\Delta}{2},\frac{\Delta+1}{2};-\frac{d}{2}+\Delta+1;K(\mathbf{X},\mathbf{Y})^2\right)$$

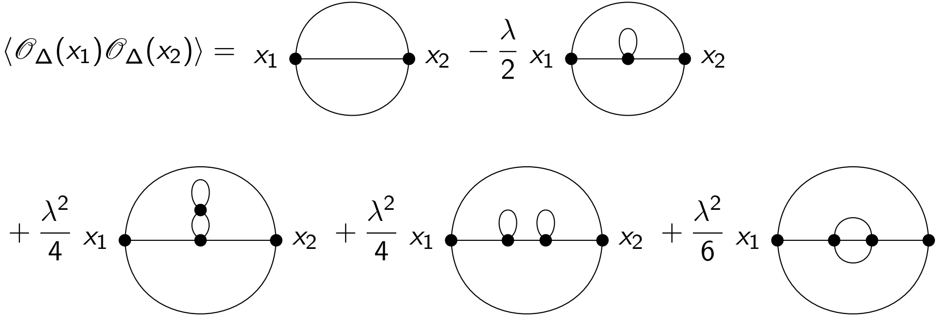
$$\text{with: }\ \Delta = \frac{d}{2} \pm \sqrt{\frac{d^2}{4}+m^2\ell^2}$$

Almost impossible to do any calculations with general Δ
$\Rightarrow$ restrictions to conformal coupling, i.e. $\Delta \in \{1,2\}$.

$$\Rightarrow \Lambda(K,\Delta) \propto \frac{K^{\Delta}}{1-K^2} = \frac{1}{2}\left(\frac{K}{1-K} - (-1)^{\Delta}\frac{K}{1+K}\right)$$

Similar to method of mirror charges in electrostatics.

Quantum corrections (two point function)

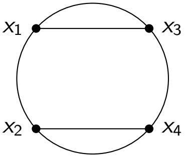


Results:

- all loop corrections are proportional to the massshift
- can be absorbed by fixing the dimension of the boundary operator to be $\Delta = 2$ as renormalization condition

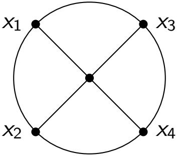
Quantum corrections (four point function)

$$\langle \mathcal{O}_\Delta(x_1)\mathcal{O}_\Delta(x_2)\mathcal{O}_\Delta(x_3)\mathcal{O}_\Delta(x_4)\rangle = \Bigg($$

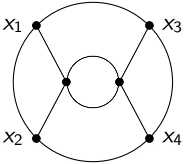


$$+ 2 \text{ perm.}\Bigg)$$

$$-\lambda$$



$$+\frac{\lambda^2}{2}\Bigg($$



$$+ 2 \text{ perm.}\Bigg)$$

After integration this gives:

$$\langle \mathcal{O}(\underline{x}_1)\mathcal{O}(\underline{x}_2)\mathcal{O}(\underline{x}_3)\mathcal{O}(\underline{x}_4)\rangle = \frac{N_\phi^2}{(x_{12}x_{34})^4}\left[1 + u^2 + \left(\frac{u}{v}\right)^2 - \frac{3\lambda_R}{4\pi^2}\left(\frac{u}{v}\right)^2 L_0 \right.$$
$$\left. + \frac{3\lambda_R^2}{4^4\pi^4}\left(\frac{u}{v}\right)^2\left(\sum_{s,t,u}\tilde{L}_1 - 12L_0\right)\right] + \mathcal{O}(\lambda_R^3)$$

Conformal block expansion of free theory

The free four point function has the form of a generalized free theory [Greenberg '61] and can be expanded in $\underline{x}_1 \to \underline{x}_2$ and $\underline{x}_3 \to \underline{x}_4$:

$$\langle \mathcal{O}(\underline{x_1})\mathcal{O}(\underline{x_2})\mathcal{O}(\underline{x_3})\mathcal{O}(\underline{x_4})\rangle = \frac{N_\phi^2}{(x_{12}x_{34})^4}\left(1 + \left(\frac{u}{v}\right)^2 \left(2 + \sum_{n=1}^{\infty}(n+1)\left(1 - \frac{1}{v}\right)\right.\right.$$

This has the form of a conformal block expansion [Dolan, Osborne '01]:

$$\langle \mathcal{O}(\underline{x_1})\mathcal{O}(\underline{x_2})\mathcal{O}(\underline{x_3})\mathcal{O}(\underline{x_4})\rangle = \frac{1}{(x_{12}x_{34})^{2\Delta_\mathcal{O}}} \sum_{\tilde{\mathcal{O}}} \lambda_{\tilde{\mathcal{O}}}^2 G_{\tilde{\mathcal{O}}}(u,v)$$

Comparing the two equations we find that this expansion corresponds to double trace primary operators:

$$:\mathcal{O}^2:_{n,l} = :\mathcal{O}\Box^n\partial^l\mathcal{O}: \quad \text{with: } \Delta(n,l) = 2\Delta_\mathcal{O} + 2n + l$$

Anomalous dimensions from interactions

To account for interactions, we look at general conformal block expansions:

$$\langle \mathcal{O}_\Delta(x_1)\mathcal{O}_\Delta(x_2)\mathcal{O}_\Delta(x_3)\mathcal{O}_\Delta(x_4)\rangle = \frac{1}{(x_{12}x_{34})^{2\Delta}}\left(1+\sum_{\Delta,l}\mathfrak{C}_{\Delta,l}\mathfrak{G}_{\Delta,l}\right)$$

We expect the interactions to generate anomalous dimensions for the double trace operators:

$$\tilde{\Delta}_{n,l} = \Delta_{n,l} + \gamma^{(1)}_{n,l} + \gamma^{(2)}_{n,l} + \dots$$

We can now expand the conformal blocks and the OPE coefficients around $\Delta_{n,l}$ [Heemskerk, Penedones, Polchinski, Sully '09]:

$$\mathfrak{C}_{\Delta(n,l),l} = C_{\Delta(n,l),l} + (\gamma^{(1)}_{n,l}+\gamma^{(2)}_{n,l})C^{(1)}_{n,l} + \frac{1}{2}(\gamma^{(1)}_{n,l})^2 C^{(2)}_{n,l} + \dots$$

$$\mathfrak{G}_{\Delta(n,l),l} = G_{\Delta(n,l),l} + (\gamma^{(1)}_{n,l}+\gamma^{(2)}_{n,l})\underbrace{\left.\frac{\partial G_{\Delta,l}}{\partial\Delta}\right|_{\Delta(n,l)}}_{G'_{\Delta(n,l),l}} + \frac{1}{2}(\gamma^{(1)}_{n,l})^2\underbrace{\left.\frac{\partial^2 G_{\Delta,l}}{\partial\Delta^2}\right|_{\Delta(n,l)}}_{G''_{\Delta(n,l),l}} + \dots$$

Results

Results from $\mathscr{O}(\lambda_R)$ terms:

$$\gamma_{n,l}^{(1)} = \frac{\lambda_R}{16\pi^2}\delta_{l,0}; \quad C_{n,0}^{(1)} = \frac{1}{2}\frac{\partial C_{\Delta(n,0),0}}{\partial n}$$

Results from $\mathscr{O}(\lambda_R^2)$ terms [Bertan, Sachs, Skvortsov '18, T.H., Sachs '20]:

$$\gamma_{n,l}^{(2)} = \frac{\lambda_R^2}{256\pi^4} \times \frac{1}{(l)_{4+2n}} \sum_{m=0}^{n} \mathfrak{a}_{n-m}^{(n)} (l+2+m)_{2(n-m)}$$

With $(x)_y = \frac{\Gamma(x+y)}{\Gamma(x)}$ and $\mathfrak{a}_{n-m}^{(n)}$ a function of n and m.

Summary and outlook

- Different approach to works reverse engineering the conformal bootstrap [Aharony, Alday, Aprile, Bissi, Drumond, Perlmutter,...] and using unitarity cuts [Fitzpatrick, Kaplan '12, Meltzer, Perlmutter, Sivaramakrishnan '20,...]
- All calculations can be made without assuming $(A)dS/CFT$
- Technically very involved and cumbersome
- Only possible for $\Delta \in \{1, 2\}$ so far.
- We are able to calculate analytical expressions for anomalous dimensions

Outlook:

- Go to higher loops (very hard)
- Find results for other values of Δ
- Calculate loop corrections to the cosmological correlator CFT in de Sitter (Schwinger-Keldysh formalism)

Thank you for your attention!