Moduli Stabilization near the Boundary

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Based on:

2105.02232 and 2108.11962, with **Brice Bastian** and **Thomas Grimm**

work in progress, with **Erik Plauschinn** and **Thomas Grimm**

DESY Theory Workshop:
Bright ideas for a dark universe

Setting the stage

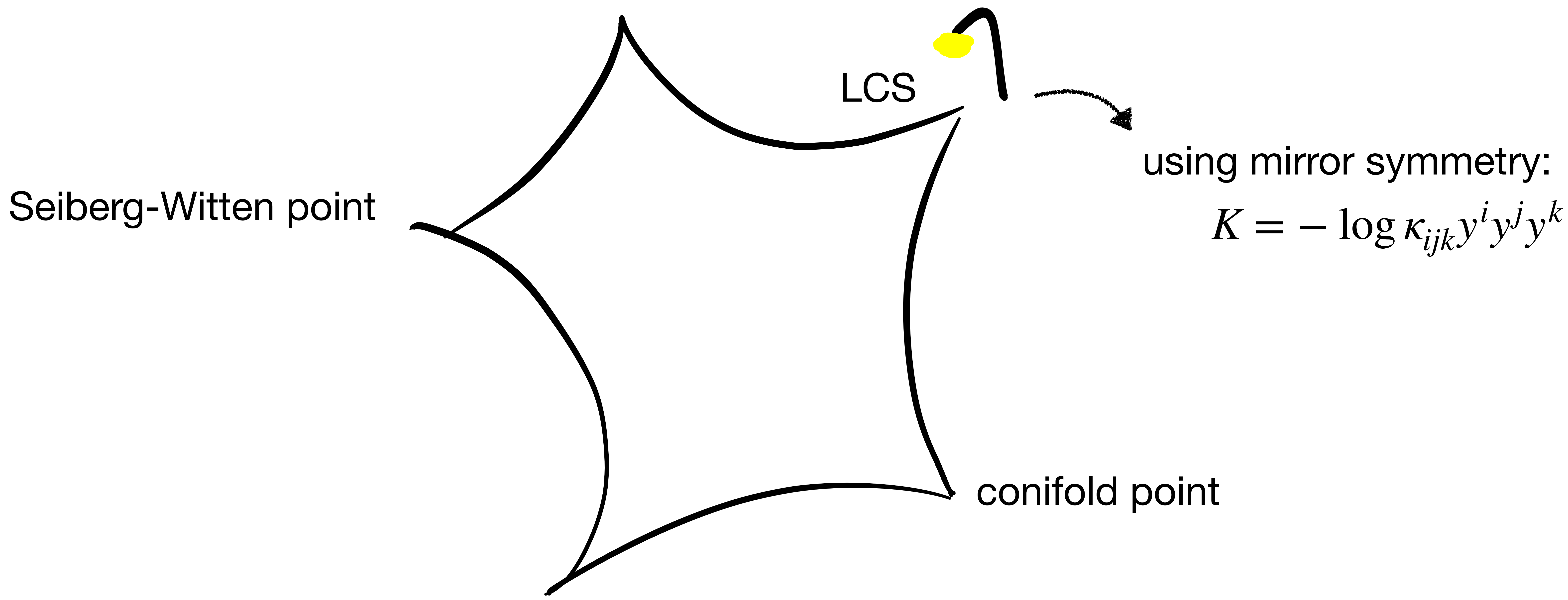
Arena: Calabi-Yau compactifications of Type IIB string theory

$\Longrightarrow$ Kähler and complex structure deformations produce **massless** fields in 4d

Three-form flux $G_3 = F_3 - \tau H_3$ induces potential for complex structure moduli (and axio-dilaton)

[Gukov, Vafa, Witten, '99]

Our goal: study scalar potential near boundaries **away** from large complex structure



Moduli stabilization:

(1) with a small vacuum superpotential [Bastian, Grimm, DH, '21]

(2) using boundary sl(2)-structures [Grimm, Plauschinn, DH, to appear]

Moduli stabilization (1): small superpotentials

[Bastian, Grimm, DH, ’21]

Finding vacua with small superpotentials

- Important step in realizing the KKLT scenario

Recently new methods for constructing such vacua:

- Strategy near large complex structure [Demirtas, Kim, McAllister, Moritz, '19]
- Extended to coni-LCS boundaries [Demirtas, Kim, McAllister, Moritz, '20], [Alvarez-Garcia, Blumenhagen, Brinkmann, Schlechter '20]

For related work see: [Honma, Otsuka, '21], [Demirtas, Kim, McAllister, Moritz, Rios-Tascon '21], [Broeckel, Cicoli, Maharana, Singh, Sinha, '21]

Constructions rely heavily on coefficients κ_{ijk} at LCS

$\Longrightarrow$ what about other boundaries in moduli space?

Periods near the boundary

- Nilpotent orbit expansion of periods [Schmid, '70]

$$\Omega = \mathbf{\Pi}^I \gamma_I\,, \qquad \mathbf{\Pi}(t^i) = \mathbf{\Pi}_{\text{pol}} + \mathbf{\Pi}_{\text{inst}} = e^{t^i N_i}\Big(\mathbf{a}_0 + \sum_{r_i} e^{2\pi i r_i t^i}\mathbf{a}_{r_1\cdots r_n}\Big)$$

coordinates $t = x + iy$:

- $x \to x+1$ circles boundary (axion)
- boundary at $y = \infty$ (saxion/dilaton)

- Periods encode 4d N=1 supergravity data

Kahler potential $\quad K = -\log i\langle \bar{\mathbf{\Pi}}, \mathbf{\Pi}\rangle = -\log\big[K_{\text{pol}} + K_{\text{inst}}\big]$

Flux superpotential $\quad W = \langle G_3, \mathbf{\Pi}\rangle = W_{\text{pol}} + W_{\text{inst}}$

- Exponential terms $\mathbf{a}_{r_1\cdots r_n}$ must be present near boundaries other than LCS

(example: $N\mathbf{a}_0 = 0$ for the conifold point)

fits nicely with [Palti, Vafa, Weigand '20], [Cecotti (1), '20]

- Construct **general models** for asymptotic one- and two-moduli periods by using boundary classification

[Bastian, Grimm, DH (1), '21]

An example

Consider a two-moduli boundary (Seiberg-Witten point for $\mathbb{P}_4^{1,1,2,2,6}[12]$ with $n_2 = \frac{1}{4}$) [Kachru, Klemm, Lerche, Mayr, Vafa, '95] [Curio, Klemm, Lüst, Theisen, '00]

$$K = -\log(y_1 + n_2 y_2)$$

$\Longrightarrow$ Kahler metric is degenerate $K_{ij} = -\partial_i \partial_j \log K_{\text{pol}} = \frac{1}{(y_1 + n_2 y_2)^2} \begin{pmatrix} 1 & n_2 \\ n_2 & (n_2)^2 \end{pmatrix}$

$\Longrightarrow$ exponential terms must be included at **leading order:**

$$K_{\text{inst}} = -2a^2 e^{-4\pi y_2}\left(n_1 y_1 + y_2 + \frac{1 - n_1 n_2}{2\pi}\right)$$

Important lessons for scalar potential: $V = e^K K^{I\bar{J}} D_I W \overline{D_J W}$

exponential factors cancel $\Longrightarrow$ W_{inst} contributes to V_{pol}

Flux vacua with small superpotentials

[Bastian, Grimm, DH (2) '21]

Extremization conditions

- Starting point: impose $D_I W_{\text{pol}} = 0$ and $W_{\text{pol}} = 0$
- For metric-essential instantons: $V^I D_I W_{\text{inst}} = 0$ (V^I: eigenvector of K^{pol}_{IJ} w/ vanishing eigenvalue)

  - Vacuum superpotential $W_0 = \langle W_{\text{inst}} \rangle$
  - For boundaries with linear K_{pol} : stabilizes all complex structure + axio-dilaton moduli
  - Moduli stabilized by V_{pol} : masses of polynomial order

$\Longrightarrow$ no racetrack potential for a perturbatively flat direction

$\Longrightarrow$ separation of mass scale compared to Kähler moduli in $W_{\text{np}} = W_0 + Ae^{-\rho}$

Moduli stabilization (2): using sl(2)-structures

[Grimm, Plauschinn, DH, to appear]

Asymptotic behavior of the scalar potential

(think of as expansion in $\frac{y^{i+1}}{y^i}, \frac{1}{y^n}$)

Strict asymptotic regime $y^1 \gg \ldots \gg y^n \gg 1 \quad \Longrightarrow$ sl(2)^n-structure emerges [Cattani, Kaplan, Schmid '86]

Decompose space of three-forms as $H^3(Y_3, \mathbb{R}) = \oplus_{\ell_1 \cdots \ell_n} V_{\ell_1 \cdots \ell_n}$ ← eigenspaces of sl(2)-weight operators

Scalar potential asymptotes towards:

$$V_{\text{pol}} = \int_{Y_3} \bar{G}_3 \wedge \star G_3 \simeq \langle \bar{G}_3, C_{\text{sl}(2)}\, G_3 \rangle = \sum_{\ell} (y_1)^{\ell_1} \cdots (y_n)^{\ell_n} \langle \bar{G}_3^{\ell}, C_\infty\, G_3^{\ell} \rangle$$

$\Longrightarrow$ can be constructed **explicitly** near any boundary

An example

Two-parameter Calabi-Yau threefold in $\mathbb{P}_4^{1,1,2,2,2,2}[8]$

$\Longrightarrow$ large complex structure regime $\kappa_{122} = 4\,,\ \ \kappa_{222} = 8$

$$C_{\text{pol}} = \begin{pmatrix} 0 & 0 & 0 & -\frac{3}{6y_1y_2^2+4y_2^3} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{3y_1^2+4y_1y_2+2y_2^2}{6y_1y_2^2+4y_2^3} & \frac{1}{6y_1+4y_2} \\ 0 & 0 & 0 & 0 & \frac{1}{6y_1+4y_2} & -\frac{3}{12y_1+8y_2} \\ \frac{2}{3}y_2^2(3y_1+2y_2) & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{6y_2^2}{3y_1+2y_2} & \frac{4y_2^2}{3y_1+2y_2} & 0 & 0 & 0 \\ 0 & \frac{4y_2^2}{3y_1+2y_2} & \frac{4\left(3y_1^2+4y_1y_2+2y_2^2\right)}{3y_1+2y_2} & 0 & 0 & 0 \end{pmatrix}$$

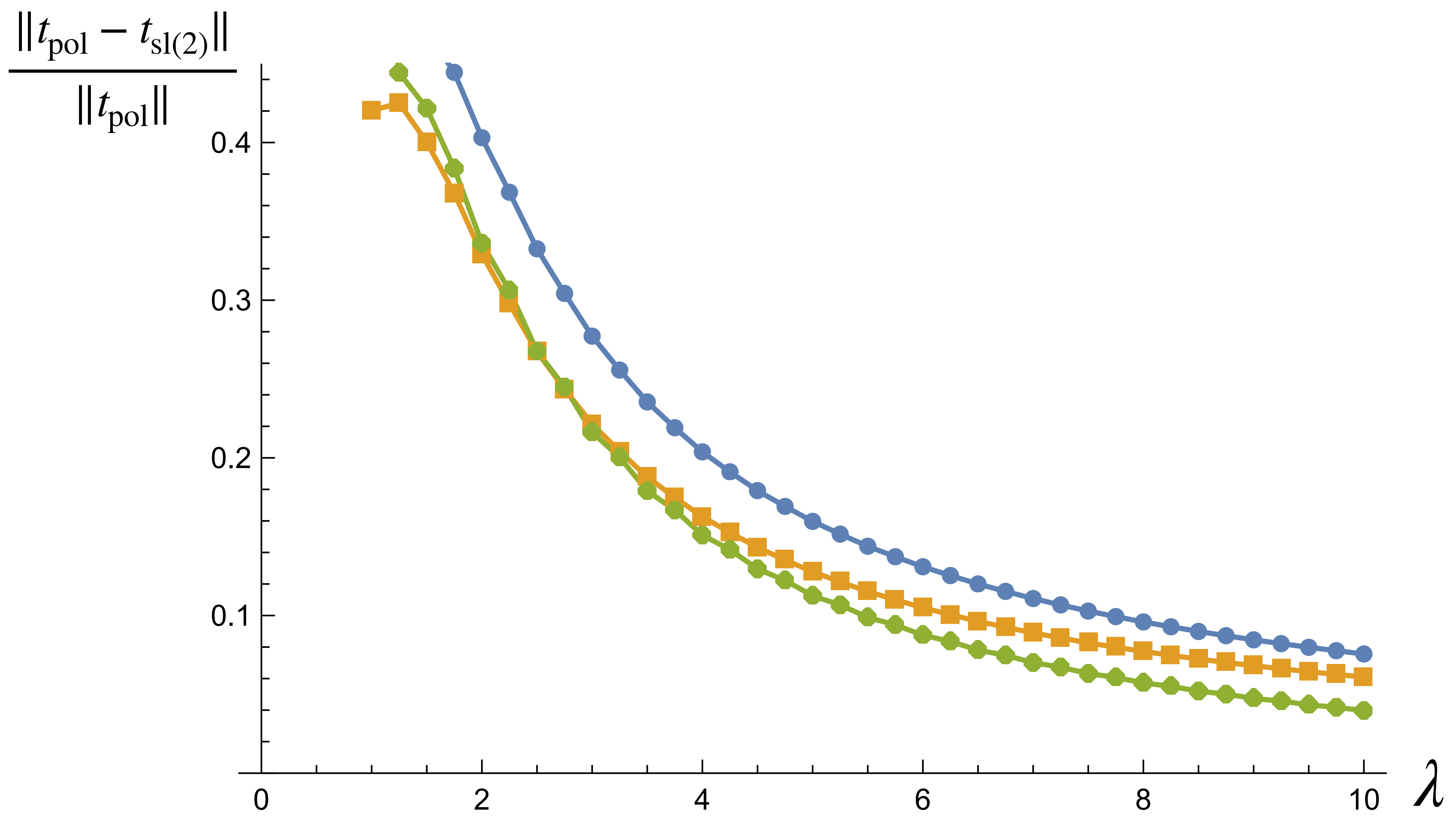
$$y_1, y_2 \gg 1$$

$$C_{\text{sl}(2)} = \begin{pmatrix} 0 & 0 & 0 & -\frac{1}{2y_1y_2^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{y_1}{2y_2^2}-\frac{1}{9y_1} & \frac{1}{6y_1} \\ 0 & 0 & 0 & 0 & \frac{1}{6y_1} & -\frac{1}{4y_1} \\ 2y_1y_2^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{2y_2^2}{y_1} & \frac{4y_2^2}{3y_1} & 0 & 0 & 0 \\ 0 & \frac{4y_2^2}{3y_1} & \frac{8y_2^2}{9y_1}+4y_1 & 0 & 0 & 0 \end{pmatrix}$$

$$y_1 \gg y_2 \gg 1$$

Comparing C_{pol} and $C_{\text{sl(2)}}$ vacua

Families of vacua with: $y_1 = \lambda^2\,,\ \ y_2 = \lambda$



$\Longrightarrow$ sl(2)-approximation agrees reasonably quickly!

Conclusions

- Modeling asymptotic periods provides us with new exciting setups

$\Longrightarrow$ exponential corrections open up new windows for pheno applications

- Vacua with small flux superpotentials
- Axion monodromy inflation?
- …

- Asymptotic sl(2)-structures give a systematic approach to moduli stabilization

$\Longrightarrow$ control over scalar potential in complicated moduli spaces

Thanks for your attention!