Concordant approaches to Resonant Leptogenesis

A scalar prototype

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The scalar prototype

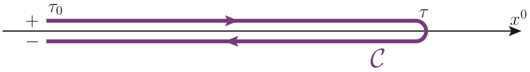
$$\mathcal{L} = \frac{1}{2}\partial^\mu\psi_i\partial_\mu\psi_i - \frac{1}{2}\psi_i M^2_{ij}\psi_j + \partial^\mu\bar{b}\partial_\mu b - m^2\,\bar{b}b \\ - \frac{\lambda}{2!2!}(\bar{b}b)^2 - \frac{h_i}{2!}\psi_i bb - \frac{h_i^*}{2!}\psi_i\bar{b}\bar{b} \tag{1}$$

Approximations:

1. Spatial isotropy: $G(x,y) = G(x^0, y^0, \vec{x} - \vec{y})$
2. Thermal self-energies: $\Pi_{ij}(x^0, y^0) = \Pi_{ij}(x^0 - y^0)$
3. Non-relativistic limit: $\vec{k} = \vec{0}$
4. Weak washout

Toolbox

1. The Closed Time Path (CTP) contour



2. The 2PI Effective Action (2PIEA)

$$\Gamma^{2PI}[\Delta, S] = \frac{i}{2}\mathrm{Tr}\ln\Delta^{-1} + \frac{i}{2}\mathrm{Tr}\ln\Delta_0^{-1}\Delta \tag{2}$$

$$+ \frac{i}{2}\mathrm{Tr}\ln S^{-1} + \frac{i}{2}\mathrm{Tr}\ln S_0^{-1}S + \Gamma_2[\Delta, S] \tag{3}$$

EoM for $\Delta = \langle\psi\psi\rangle$:

$$\frac{\delta\Gamma[\Delta, S]}{\delta\Delta} = 0 \tag{4}$$

EoM for $S = \langle b\bar{b}\rangle$:

$$\frac{\delta\Gamma[\Delta, S]}{\delta S} = 0 \tag{5}$$

The Schwinger-Dyson equations

1. The Kadanoff-Baym equations

$$\left[\Box + M^2\right]\Delta^{\lessgtr} + \Pi^h * \Delta^{\lessgtr} + \Pi^{>} * \Delta^h = \frac{1}{2}\left\{\Pi^{>} * \Delta^{<} - \Pi^{<} * \Delta^{>}\right\} \quad (6)$$

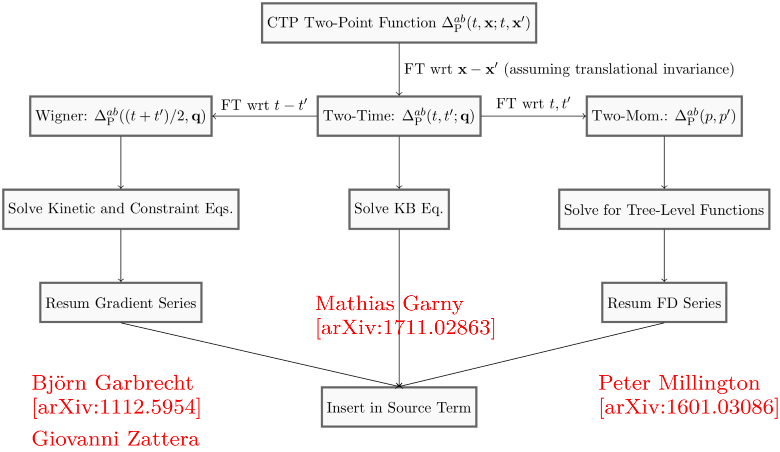
2. The CP-asymmetry

$$\eta(t) \equiv \int \mathrm{d}^3\mathbf{x}\langle j_0(t,\mathbf{x})\rangle \quad (7)$$

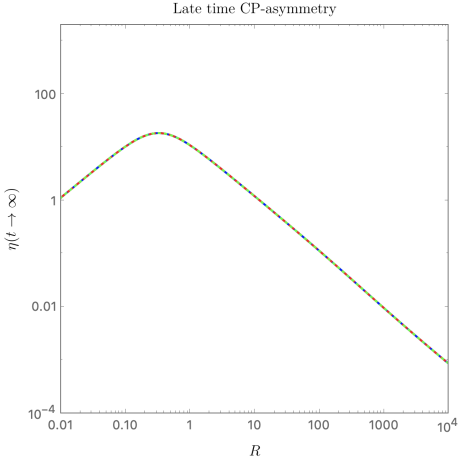
$$= -2\,\mathrm{Im}\,(h^\dagger h)_{12}\int_{-\infty}^{t}\mathrm{d}x^0\int_{-\infty}^{t}\mathrm{d}y^0\int_{\mathbf{q}}$$

$$\times i\Big[\Delta_{12}^{<}(x^0,y^0,\mathbf{q})\Pi^{>}(y^0,x^0,\mathbf{q}) - \Delta_{12}^{>}(x^0,y^0,\mathbf{q})\Pi^{<}(y^0,x^0,\mathbf{q})\Big] \quad (8)$$

The state-of-the-art options



The main result, in brief



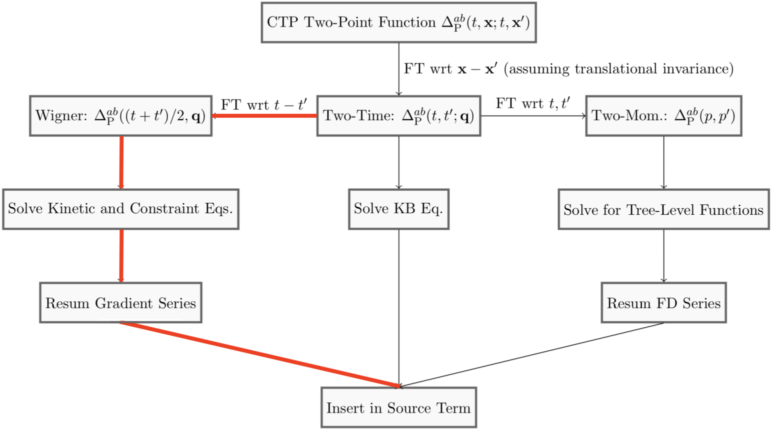
$$R = \frac{\mathrm{Tr}(h^\dagger h)}{\Delta M^2}$$

Wigner space

2-Times

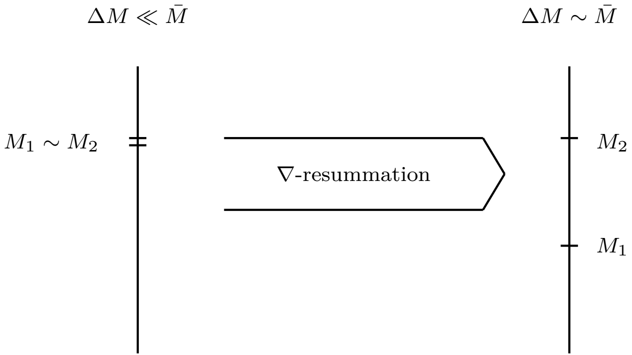
2-P int. pict.

The state-of-the-art options



The Wigner space solution

The mass spectrum



The Wigner space solution

The KB equations in Wigner-Space

$$\begin{cases} t = \frac{x^0+y^0}{2} \\ s = x^0 - y^0 \end{cases} \qquad \delta\Delta(t,k_0) = \int \mathrm{d}s\; e^{ik_0 s}\,\delta\Delta\Big(X - \frac{s}{2}, X + \frac{s}{2}\Big)$$

$$\left[k_0^2 - \frac{1}{4}\partial_t^2 + ik_0\partial_t - M^2\right]\delta\Delta(k_0,t) = -\frac{i}{2}e^{-i\diamondsuit}\{\Pi^{\mathcal{A}}(k_0)\}\{\delta\Delta(k_0,t)\} \tag{9}$$

$$\delta\Delta(k_0,t)\left[k_0^2 - \frac{1}{4}\overleftarrow{\partial_t^2} + ik_0\overleftarrow{\partial_t} - M^2\right] = +\frac{i}{2}e^{-i\diamondsuit}\{\delta\Delta(k_0,t)\}\{\Pi^{\mathcal{A}}(k_0)\} \tag{10}$$

Where the diamond operator is defined by

$$\diamondsuit\{A\}\{B\} = \frac{1}{2}(\partial_t A)(\partial_{k_0} B) - \frac{1}{2}(\partial_{k_0} A)(\partial_t B) \tag{11}$$

The Wigner space solution

The 0^{th}-order complete solution

$$\delta\Delta(k_0,t) = \sum_{i=1}^{N} c_i \, \delta\Lambda_i(k_0,t;k_0^{(i)},\lambda_i) \tag{12}$$

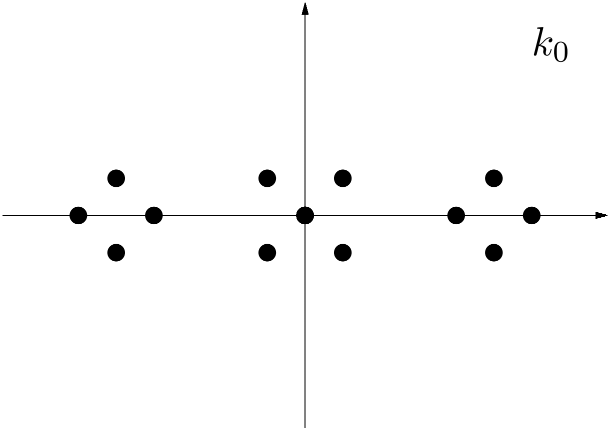
$$\delta\Lambda_i(k_0,t;k_0^{(i)},\lambda_i) = \delta(k_0 - k_0^{(i)}) \; e^{-\lambda_i t} \; \vec{u}(k_0^{(i)},\lambda_i) \tag{13}$$

$$\vec{u}(k_0^{(i)},\lambda_i) = \begin{pmatrix} u_{11}^{(i)} \\ u_{12}^{(i)} \\ u_{21}^{(i)} \\ u_{22}^{(i)} \end{pmatrix} \tag{14}$$

$\{c_i\}$ are 16 $\mathbb{C}$-parameters $\xrightarrow{\text{(H) + (N)}}$ 10 $\mathbb{R}$-parameters

The Wigner space solution

The available shells



The Wigner space solution

The resummation of the gradients

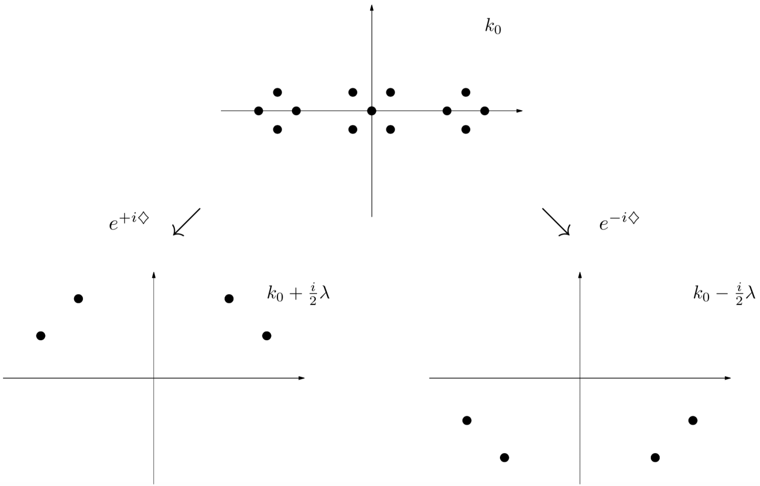
Let us go back to the collisional term of the KB:

$$e^{-i\Diamond}\{\Pi^{\mathcal{A}}(k_0)\}\{\delta\Delta(k_0,t)\} = \Pi^{\mathcal{A}}(k_0)\, e^{\frac{i}{2}\overleftarrow{\partial}_{k_0}\partial_t}\, \delta\Delta(k_0,t) \qquad (15)$$

For solutions as $\delta\Delta(k_0,t) = e^{-\lambda t}\delta\,\bar{\Delta}(k_0)$

$$\begin{aligned} e^{\pm i\Diamond}\{\Pi^{\mathcal{A}}(k_0)\}\{\delta\Delta(k_0,t)\} &= \sum_{n=0}^{\infty}\frac{1}{n!}\left(\pm\frac{i}{2}\right)^n \left(\partial_{k_0}^n\Pi^{\mathcal{A}}\right)\left(\partial_t^n\delta\Delta\right) \\ &= \Pi^{\mathcal{A}}\Big(k_0 \pm \frac{i}{2}\lambda\Big)\,\delta\Delta(k_0,t) \qquad (16) \end{aligned}$$

The Wigner space solution



The Wigner space solution

An example

Let us consider the 12-component of the KB equations:

$$\left[k_0^2 - \frac{1}{4}\partial_t^2 + ik_0\partial_t - M_1^2\right]\delta\Delta_{12}(k_0,t) = -\frac{i}{2}\Pi_{11}^{\mathcal{A}}\Big(k_0 - \frac{M_2 - M_1}{2}\Big)\,\delta\Delta_{12}(k_0,t) \tag{17}$$

But when we evaluate this equation in a distributional sense, k_0 is assigned to the value $\bar{M}$, so that the self energy is now evaluated at the M_1 shell:

$$\Pi_{11}^{\mathcal{A}}\Big(\frac{M_1 + M_2}{2} - \frac{M_2 - M_1}{2}\Big) = \Pi_{11}^{\mathcal{A}}(M_1) \tag{18}$$

Concluding remarks

1. A key element to get the correct CP-asymmetry is the migration of the poles over the complex plane. In Wigner space, this amounts to summing the gradients correctly.
2. This new formulation does not only make the resummation simple, but it also provides the full set of available shells allowing to build the most general possible solution for $\delta\Delta(t, k_0)$.
3. All the first principle approaches based non-equilibrium quantum field theory are equivalent and ready for use.