Small Field Polynomial Inflation

Yong Xu

based on 2104.03977 [JCAP09(2021)012] with Manuel Drees

Sep. 22 DESY Theory Workshop 2021

Outline

1. Why polynomial model?
2. Predictions?

Why Small Field Polynomial Model?

- Monomial: $V(\phi) \sim \phi^{n}$, tensor-to-scalar ratio

$$r \propto \left(\frac{V'}{V}\right)^2 \sim \frac{4n}{N_{\mathrm{CMB}}}$$

Why Small Field Polynomial Model?

- Monomial: $V(\phi) \sim \phi^{n}$, tensor-to-scalar ratio

$$r \propto \left(\frac{V'}{V}\right)^2 \sim \frac{4n}{N_{\mathrm{CMB}}}$$

- Planck 18: $r < 0.061$; ruled out $n > 1 \Rightarrow$ less steep potentials favored:
    - fraction power e.g. $V \sim \phi^{2/3}$
    - non-minimal coupling $V \sim \phi^n/(1+\xi\phi^2 R)^2$
    - ...

Why Small Field Polynomial Model?

- Monomial: $V(\phi) \sim \phi^{n}$, tensor-to-scalar ratio

$$r \propto \left(\frac{V'}{V}\right)^2 \sim \frac{4n}{N_{\rm CMB}}$$

- Planck 18: $r < 0.061$; ruled out $n > 1 \Rightarrow$ less steep potentials favored:
    - fraction power e.g. $V \sim \phi^{2/3}$
    - non-minimal coupling $V \sim \phi^n/(1+\xi\phi^2 R)^2$
    - ...
- Alternatively

$$V(\phi) = \sum_{n=0}^{4} \alpha_n \phi^n$$

Why Small Field Polynomial Model?

- Monomial: $V(\phi) \sim \phi^{n}$, tensor-to-scalar ratio

$$r \propto \left(\frac{V'}{V}\right)^2 \sim \frac{4n}{N_{\rm CMB}}$$

- Planck 18: $r < 0.061$; ruled out $n > 1 \Rightarrow$ less steep potentials favored:
  - fraction power e.g. $V \sim \phi^{2/3}$
  - non-minimal coupling $V \sim \phi^n/(1+\xi\phi^2 R)^2$
  - ...
- Alternatively

$$V(\phi) = \sum_{n=0}^{4} \alpha_n \phi^n$$

  - Avoid trans-Planckian $\Rightarrow \phi < M_p \Rightarrow$ Small Field

Why Small Field Polynomial Model?

- Monomial: $V(\phi) \sim \phi^{n}$, tensor-to-scalar ratio

$$r \propto \left(\frac{V'}{V}\right)^2 \sim \frac{4n}{N_{\mathrm{CMB}}}$$

- Planck 18: $r < 0.061$; ruled out $n > 1 \Rightarrow$ less steep potentials favored:
  - fraction power e.g. $V \sim \phi^{2/3}$
  - non-minimal coupling $V \sim \phi^n/(1+\xi\phi^2 R)^2$
  - ...
- Alternatively

$$V(\phi) = \sum_{n=0}^{4} \alpha_n \phi^n$$

  - Avoid trans-Planckian $\Rightarrow \phi < M_p \Rightarrow$ Small Field
  - Reasonable to insist on renormalizability (especially ϕ in UV complete theory)

Why Small Field Polynomial Model?

- Monomial: $V(\phi) \sim \phi^{n}$, tensor-to-scalar ratio

$$r \propto \left(\frac{V'}{V}\right)^2 \sim \frac{4n}{N_{\rm CMB}}$$

- Planck 18: $r < 0.061$; ruled out $n > 1 \Rightarrow$ less steep potentials favored:
    - fraction power e.g. $V \sim \phi^{2/3}$
    - non-minimal coupling $V \sim \phi^n/(1+\xi\phi^2 R)^2$
    - ...
- Alternatively

$$V(\phi) = \sum_{n=0}^{4} \alpha_n \phi^n$$

    - Avoid trans-Planckian $\Rightarrow \phi < M_p \Rightarrow$ Small Field
    - Reasonable to insist on renormalizability (especially ϕ in UV complete theory)
    - $V(\phi)$: most general renormalizable inflaton potential

Polynomial Inflation Analysis

- Potential

$$V(\phi) = \cancelto{\text{negligible}}{C} + d\,\phi^4 + c\,\phi^3 + b\,\phi^2 + e\,\cancelto{\text{shifted away}}{\phi^1}$$

Polynomial Inflation Analysis

- Potential

$$V(\phi) = \overset{\text{negligible}}{\cancel{C}} + d\,\phi^4 + c\,\phi^3 + b\,\phi^2 + e\,\overset{\text{shifted away}}{\cancel{\phi^1}}$$

  - Large ϕ: $V \sim \phi^4 \Rightarrow$ Too steep ☹
  - Small ϕ: $V \sim \phi^2 \Rightarrow$ Too steep ☹

Polynomial Inflation Analysis

- Potential

$$V(\phi) = \cancelto{\text{negligible}}{C} + d\,\phi^4 + c\,\phi^3 + b\,\phi^2 + e\,\cancelto{\text{shifted away}}{\phi^1}$$

  - Large ϕ: $V \sim \phi^4 \Rightarrow$ Too steep ☹
  - Small ϕ: $V \sim \phi^2 \Rightarrow$ Too steep ☹
  - Intermediate ϕ: V flat $\Leftarrow \phi^3$ term ☺

Polynomial Inflation Analysis

- Potential

$$V(\phi) = \cancelto{\text{negligible}}{C} + d\,\phi^4 + c\,\phi^3 + b\,\phi^2 + e\,\cancelto{\text{shifted away}}{\phi^1}$$

  - Large ϕ: $V \sim \phi^4 \Rightarrow$ Too steep ☹
  - Small ϕ: $V \sim \phi^2 \Rightarrow$ Too steep ☹
  - Intermediate ϕ: V flat $\Leftarrow \phi^3$ term ☺
- If $b = \frac{9c^2}{32d} \Rightarrow$ inflection-point $\phi_0 = -\frac{3c}{8d}$

Polynomial Inflation Analysis

- Potential

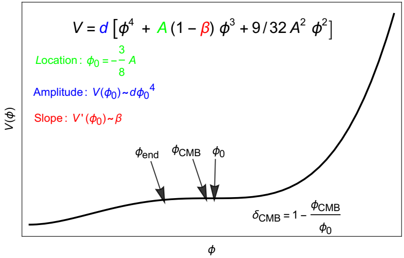
$$V(\phi) = \overset{\text{negligible}}{\cancel{C}} + d\,\phi^4 + c\,\phi^3 + b\,\phi^2 + e\,\overset{\text{shifted away}}{\cancel{\phi^1}}$$

  - Large ϕ: $V \sim \phi^4 \Rightarrow$ Too steep ☹
  - Small ϕ: $V \sim \phi^2 \Rightarrow$ Too steep ☹
  - Intermediate ϕ: V flat $\Leftarrow \phi^3$ term ☺
- If $b = \frac{9c^2}{32d} \Rightarrow$ inflection-point $\phi_0 = -\frac{3c}{8d}$
- Three parameters (d, A, β) reparametrization:

$$V(\phi) = d\left[\phi^4 + \frac{c}{d}(1-\beta)\,\phi^3 + \frac{9}{32}\left(\frac{c}{d}\right)^2\phi^2\right] \equiv d\left[\phi^4 + A\,(1-\beta)\,\phi^3 + \frac{9}{32}A^2\phi^2\right]$$

  - $A \equiv c/d = -8/3\phi_0 \longleftrightarrow$ location ϕ_0
  - $\beta > 0$: $\longleftrightarrow$ flatness $V(\phi_0)$
  - d: $\longleftrightarrow$ amplitude (power spectrum)

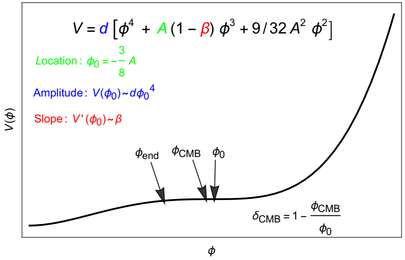
Slow-Roll Predictions



- Need $\phi_{\rm CMB} \Rightarrow$ introduce δ:

$$\phi_{\rm CMB} = \phi_0(1 - \delta_{\rm CMB})$$

Slow-Roll Predictions

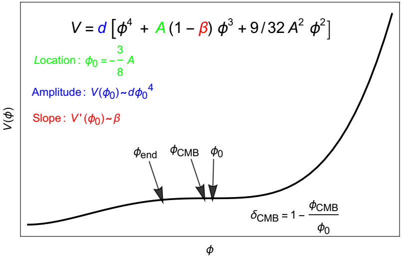


- Need ϕ_{CMB} ⇒ introduce δ:

$$\phi_{\mathrm{CMB}} = \phi_0\left(1 - \delta_{\mathrm{CMB}}\right)$$

- Predictions:
  - $n_s \simeq 1 - 48\delta_{\mathrm{CMB}}/\phi_0^2$

Slow-Roll Predictions

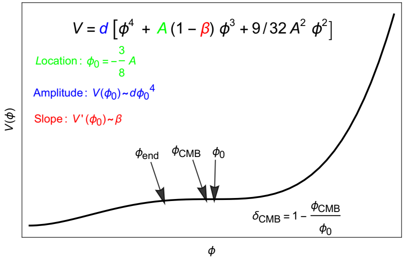


- Need $\phi_{\rm CMB}$ ⇒ introduce δ:

$$\phi_{\rm CMB} = \phi_0(1 - \delta_{\rm CMB})$$

- Predictions:
  - $n_s \simeq 1 - 48\delta_{\rm CMB}/\phi_0^2$
  - $N_{\rm CMB} \propto \left[\frac{\pi}{2} - \arctan\left(\frac{\delta_{\rm CMB}}{\sqrt{2\beta}}\right)\right]$

Slow-Roll Predictions

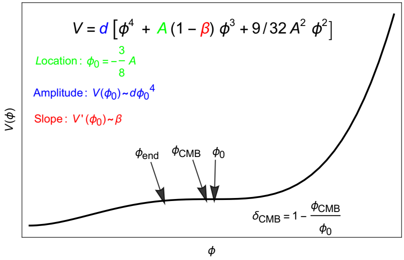


- Need $\phi_{\rm CMB} \Rightarrow$ introduce δ:

$$\phi_{\rm CMB} = \phi_0\left(1 - \delta_{\rm CMB}\right)$$

- Predictions:
  - $n_s \simeq 1 - 48\delta_{\rm CMB}/\phi_0^2$
  - $N_{\rm CMB} \propto \left[\frac{\pi}{2} - \arctan\left(\frac{\delta_{\rm CMB}}{\sqrt{2\beta}}\right)\right]$
  - $r \propto \left(2\beta + \delta^2\right)^2/\phi_0^2$

Slow-Roll Predictions



- Need $\phi_{\rm CMB} \Rightarrow$ introduce δ:

$$\phi_{\rm CMB} = \phi_0(1 - \delta_{\rm CMB})$$

- Predictions:
  - $n_s \simeq 1 - 48\delta_{\rm CMB}/\phi_0^2$
  - $N_{\rm CMB} \propto \left[\frac{\pi}{2} - \arctan\left(\frac{\delta_{\rm CMB}}{\sqrt{2\beta}}\right)\right]$
  - $r \propto \left(2\beta + \delta^2\right)^2 / \phi_0^2$
  - $\alpha \simeq -\frac{576(2\beta+\delta^2)}{\phi_0^4}$
  - $\mathcal{P}_\zeta \simeq \frac{d\,\phi_0^6}{5184\pi^2(\delta^2+2\beta)^2}$
- $n_s = 0.9649,\ N_{\rm CMB} = 65,\ \mathcal{P}_\zeta = 2.1 \cdot 10^{-9}$

$$\begin{aligned}\delta_{\rm CMB} &= 7.31 \times 10^{-4}\phi_0^2 \\ \beta &= 9.73 \times 10^{-7}\phi_0^4 \\ d &= 6.61 \times 10^{-16}\phi_0^2\end{aligned}$$

- r and α?

Predictions for r and α

- r not detectable ☹

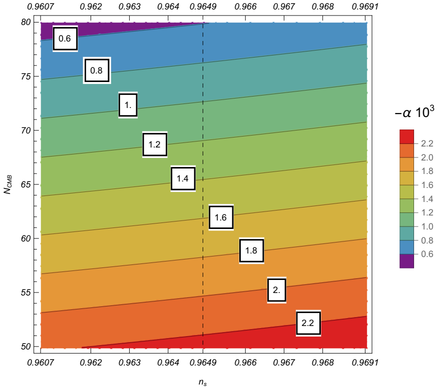
$$\frac{r}{7.09\times10^{-9}\,\phi_0^6} = 1 - 3.9\cdot10^{-2}(65 - N_{\mathrm{CMB}})$$
$$+\,15.0\,(0.9649 - n_s) + 175\,(0.9649 - n_s)^2$$

- α hopefully testable [S4 CMB]

$$\alpha = -1.43\cdot10^{-3} - 5.56\cdot10^{-5}\,(65 - N_{\mathrm{CMB}})$$
$$+\,0.02\,(0.9649 - n_s) - 0.25\,(0.9649 - n_s)^2$$

- Central model parameters

$$\delta_{\mathrm{CMB}} = 7.31\times10^{-4}\,\phi_0^2;\ \beta = 9.73\times10^{-7}\,\phi_0^4;$$
$$d = 6.61\times10^{-16}\,\phi_0^2$$



- Inflaton mass: $m_\phi^2 = \left.\frac{\partial^2 V}{\partial\phi^2}\right|_{\phi=0} \simeq 4d\phi_0^2;$

  Inflationary scale: $H_{\mathrm{inf}} = \sqrt{\frac{V(\phi_0)}{3}} \simeq 8.6\cdot10^{-9}\phi_0^3$
- Question: What's the lower bound for ϕ_0?

Reheating $\Rightarrow$ Lower Bound

- After inflation ends:
  - $V \sim m_\phi^2 \phi^2$
  - ϕ oscillates and transfers energy
- Decays to Bosons (e.g. Higgs) or Fermions (e.g. RHN)

$$\mathcal{L} \supset -g\phi|\phi'|^2 - y\phi\bar{\chi}\chi$$

- Decay rate ($m_\phi \sim \phi_0^2$):

$$\Gamma_\phi \simeq \frac{g^2}{8\pi m_\phi}; \ \frac{y^2}{8\pi} m_\phi$$

- Reheating Tem:

$$T_{\rm re} \simeq 1.41 g_\star^{-1/4} \Gamma_\phi^{1/2}$$

Reheating $\Rightarrow$ Lower Bound

- After inflation ends:
  - $V \sim m_\phi^2 \phi^2$
  - ϕ oscillates and transfers energy
- Decays to Bosons (e.g. Higgs) or Fermions (e.g. RHN)

$$\mathcal{L} \supset -g\phi|\phi'|^2 - y\phi\bar{\chi}\chi$$

- Decay rate ($m_\phi \sim \phi_0^2$):

$$\Gamma_\phi \simeq \frac{g^2}{8\pi m_\phi}; \ \frac{y^2}{8\pi} m_\phi$$

- Reheating Tem:

$$T_{\rm re} \simeq 1.41 g_\star^{-1/4} \Gamma_\phi^{1/2}$$

- BBN requires $T_{\rm re} \gtrsim 4$ MeV $\Rightarrow$ Lower bounds

$$\boxed{y\phi_0 \gtrsim 4.7 \times 10^{-17}; \ \frac{g}{\phi_0} \gtrsim 2.4 \times 10^{-24}}$$

Reheating ⇒ Lower Bound

- After inflation ends:
  - $V \sim m_\phi^2 \phi^2$
  - ϕ oscillates and transfers energy
- Decays to Bosons (e.g. Higgs) or Fermions (e.g. RHN)

$$\mathcal{L} \supset -g\phi|\phi'|^2 - y\phi\bar{\chi}\chi$$

- Decay rate ($m_\phi \sim \phi_0^2$):

$$\Gamma_\phi \simeq \frac{g^2}{8\pi m_\phi};\ \frac{y^2}{8\pi} m_\phi$$

- Reheating Tem:

$$T_{\rm re} \simeq 1.41 g_\star^{-1/4} \Gamma_\phi^{1/2}$$

- BBN requires $T_{\rm re} \gtrsim 4$ MeV ⇒ Lower bounds

$$\boxed{y\phi_0 \gtrsim 4.7 \times 10^{-17};\ \frac{g}{\phi_0} \gtrsim 2.4 \times 10^{-24}}$$

- Remarks:
  - Preheating negligible
  - Though $m_{\phi'}^2 \sim g\phi$ ⇒ tachyonic resonance, still ok here, due to (sizeable) self-coupling $\lambda\phi'^4$ (⇒ back-reaction $m_{\phi'}^2 \sim \lambda\langle\phi'^2\rangle$); Pauli blocking for χ ⇒ Preheating not efficient here

Reheating ⇒ Lower Bound

- After inflation ends:
  - $V \sim m_\phi^2 \phi^2$
  - ϕ oscillates and transfers energy
- Decays to Bosons (e.g. Higgs) or Fermions (e.g. RHN)

$$\mathcal{L} \supset -g\phi|\phi'|^2 - y\phi\bar{\chi}\chi$$

- Decay rate ($m_\phi \sim \phi_0^2$):

$$\Gamma_\phi \simeq \frac{g^2}{8\pi m_\phi}; \ \frac{y^2}{8\pi} m_\phi$$

- Reheating Tem:

$$T_{\rm re} \simeq 1.41 g_\star^{-1/4} \Gamma_\phi^{1/2}$$

- BBN requires $T_{\rm re} \gtrsim 4$ MeV ⇒ Lower bounds

$$\boxed{y\phi_0 \gtrsim 4.7 \times 10^{-17}; \ \frac{g}{\phi_0} \gtrsim 2.4 \times 10^{-24}}$$

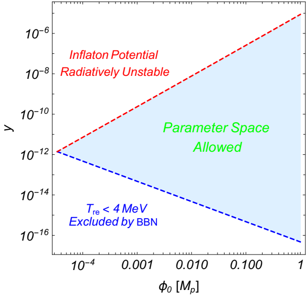
- Remarks:
  - Preheating negligible
  - Though $m_{\phi'}^2 \sim g\phi \Rightarrow$ tachyonic resonance, still ok here, due to (sizeable) self-coupling $\lambda\phi'^4$ ($\Rightarrow$ back-reaction $m_{\phi'}^2 \sim \lambda\langle\phi'^2\rangle$); Pauli blocking for $\chi \Rightarrow$ Preheating not efficient here
- Question: What are the upper bounds for the couplings? ⇒ Radiative stability

Radiative Stability ⇒ Upper Bound

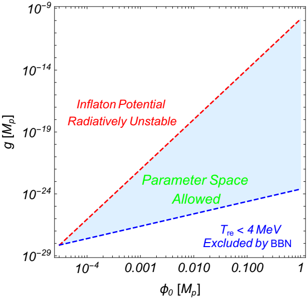
- Require: $\Delta V(\phi_0) \ll V(\phi_0)\,; \Delta V'(\phi_0) \ll V'(\phi_0)\,; \Delta V''(\phi_0) \ll V''(\phi_0),$

$$\Delta V = \frac{1}{64\pi^2} \sum_{\psi=\phi',\chi} (-1)^{2s_\psi} g_\psi \widetilde{m}_\psi(\phi)^4 \left(\ln\left(\frac{\widetilde{m}_\psi(\phi)^2}{Q_0^2} \right) - \frac{3}{2} \right)$$

- Upper bounds for y (coupling $y\phi\bar{\chi}\chi$): $\left|\frac{y^4 - 3y^4 \ln(y^2)}{4\pi^2}\right| < 16 d\beta$



- Upper bounds for g (coupling $g\phi|\phi'|^2$): $\frac{g^2}{8\pi^2}\left|\ln\left(\frac{g}{\phi_0}\right) - 1\right| < 8 d\beta\phi_0^2$



- Radiative Stability + Reheating ⇒ Lower bound $\phi_0 > 3 \cdot 10^{-5} M_p$

Summary

- A simple polynomial model fits data very well:

$$V \equiv d\left[\phi^4 + A\left(1-\beta\right)\phi^3 + \frac{9}{32}A^2\phi^2\right]$$

with $A = -8/3\phi_0$; $\beta = 9.73 \times 10^{-7}\phi_0^4/M_p^4$; $d = 6.61 \times 10^{-16}\phi_0^2/M_p^2$.

Summary

- A simple polynomial model fits data very well:

$$V \equiv d\left[\phi^4 + A\,(1-\beta)\,\phi^3 + \frac{9}{32}A^2\phi^2\right]$$

with $A = -8/3\phi_0\,;\, \beta = 9.73\times 10^{-7}\phi_0^4/M_p^4\,;\, d = 6.61\times 10^{-16}\phi_0^2/M_p^2.$

  - Parameter space: Reheating +Radiative Stability ⇒ $\phi_0 > 3\cdot 10^{-5}M_p$
  - Predictions:
    1. $r \simeq 7.1\cdot 10^{-9}\phi_0^6/M_p^6$ ☹
    2. $\alpha \simeq -1.43\cdot 10^{-3}$ ⇒ testable in future [S4 CMB] ☺

Summary

- A simple polynomial model fits data very well:

$$V \equiv d\left[\phi^4 + A\,(1-\beta)\,\phi^3 + \frac{9}{32}A^2\phi^2\right]$$

with $A = -8/3\phi_0\,;\ \beta = 9.73\times10^{-7}\phi_0^4/M_p^4\,;\ d = 6.61\times10^{-16}\phi_0^2/M_p^2.$

  - Parameter space: Reheating +Radiative Stability $\Rightarrow$ $\phi_0 > 3\cdot10^{-5}M_p$
  - Predictions:
    1. $r \simeq 7.1\cdot10^{-9}\phi_0^6/M_p^6$ ☹
    2. $\alpha \simeq -1.43\cdot10^{-3}$ $\Rightarrow$ testable in future [S4 CMB] ☺
  - Implications:
    1. $m_\phi \simeq 5.1\times10^{-8}\phi_0^2/M_p$ $\Rightarrow$ as light as $\mathcal{O}(100)$ GeV (EW scale);
       For comparison: monomial $m^2\phi^2$ model, $m \sim \mathcal{O}(10^{13})$ GeV
    2. Inflationary scale: $H_{\rm inf} \simeq 8.6\cdot10^{-9}\phi_0^3/M_p^2 \Rightarrow H_{\rm inf}$ as low as 1 MeV!
    3. Reheating Tem: $T_{\rm re} \in \left[4\ \text{MeV}, 10^{11}\ \text{GeV}\right]$

Summary

- A simple polynomial model fits data very well:

$$V \equiv d\left[\phi^4 + A\left(1-\beta\right)\phi^3 + \frac{9}{32}A^2\phi^2\right]$$

with $A = -8/3\phi_0$; $\beta = 9.73 \times 10^{-7}\phi_0^4/M_p^4$; $d = 6.61 \times 10^{-16}\phi_0^2/M_p^2$.

  - Parameter space: Reheating +Radiative Stability $\Rightarrow \phi_0 > 3 \cdot 10^{-5} M_p$
  - Predictions:
    1. $r \simeq 7.1 \cdot 10^{-9}\phi_0^6/M_p^6$ ☹
    2. $\alpha \simeq -1.43 \cdot 10^{-3} \Rightarrow$ testable in future [S4 CMB] ☺
  - Implications:
    1. $m_\phi \simeq 5.1 \times 10^{-8}\phi_0^2/M_p \Rightarrow$ as light as $\mathcal{O}(100)$ GeV (EW scale);
       For comparison: monomial $m^2\phi^2$ model, $m \sim \mathcal{O}(10^{13})$ GeV
    2. Inflationary scale: $H_{\rm inf} \simeq 8.6 \cdot 10^{-9}\phi_0^3/M_p^2 \Rightarrow H_{\rm inf}$ as low as 1 MeV!
    3. Reheating Tem: $T_{\rm re} \in \left[4 \text{ MeV}, 10^{11} \text{ GeV}\right]$
- Renormalizable, easy to extend $\Rightarrow$ DM [2106.03950], Leptogenesis [in preparation]

Summary

- A simple polynomial model fits data very well:

$$V \equiv d\left[\phi^4 + A\,(1-\beta)\,\phi^3 + \frac{9}{32}A^2\phi^2\right]$$

with $A = -8/3\phi_0$; $\beta = 9.73 \times 10^{-7}\phi_0^4/M_p^4$; $d = 6.61 \times 10^{-16}\phi_0^2/M_p^2$.

  - Parameter space: Reheating +Radiative Stability ⇒ $\phi_0 > 3 \cdot 10^{-5} M_p$
  - Predictions:
    1. $r \simeq 7.1 \cdot 10^{-9}\phi_0^6/M_p^6$ ☹
    2. $\alpha \simeq -1.43 \cdot 10^{-3}$ ⇒ testable in future [S4 CMB] ☺
  - Implications:
    1. $m_\phi \simeq 5.1 \times 10^{-8}\phi_0^2/M_p$ ⇒ as light as $\mathcal{O}(100)$ GeV (EW scale);
       For comparison: monomial $m^2\phi^2$ model, $m \sim \mathcal{O}(10^{13})$ GeV
    2. Inflationary scale: $H_{\rm inf} \simeq 8.6 \cdot 10^{-9}\phi_0^3/M_p^2$ ⇒ $H_{\rm inf}$ as low as 1 MeV!
    3. Reheating Tem: $T_{\rm re} \in \left[4 \text{ MeV}, 10^{11} \text{ GeV}\right]$
- Renormalizable, easy to extend ⇒ DM [2106.03950], Leptogenesis [in preparation]

Thank you for your attention!