Fractional Dark Energy

Ricardo Landim

Technical University of Munich

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Thermodynamics of a dark fluid

Using the second law of thermodynamics: [Lima, Alcaniz, PLB 600, 191 (2004)]

$$\rho \propto T^{\frac{1+w}{w}} \propto V^{-(1+w)}$$

$$\rho = C_0 \int_0^\infty \frac{\varepsilon^{\frac{1}{w}}}{e^{\beta\varepsilon} + 1} d\varepsilon$$

Constant *w* and valid only for fermions (bosons give a negative ρ)

Fractional dark energy

Density of states: $D(\varepsilon) \propto \varepsilon^{\frac{1}{w}-1}$

Non-canonical kinetic term:

$$\varepsilon \approx m + \frac{p^2}{2m} + \frac{C}{p^{-3w}}$$

$$Cp^{3w} \gg m$$

$$\varepsilon \approx \frac{C}{p^{-3w}}$$

$$N_\varepsilon = -\frac{C^{-\frac{1}{w}} gV}{6\pi^2 w} \int_{\varepsilon_{\min}}^{\varepsilon_{\max}} \frac{\varepsilon^{\frac{1}{w}-1}}{e^{\beta\varepsilon}+1} d\varepsilon$$

ε_{min} ~ m

ε_{max} to avoid divergence when $p \to 0$

Fractional dark energy

$$n = -\frac{C^{-\frac{1}{w}} g}{6\pi^2 w} \int_{\varepsilon_{\min}}^{\varepsilon_{\max}} \frac{\varepsilon^{\frac{1}{w}-1}}{e^{\beta\varepsilon}+1} d\varepsilon$$

$$= -\frac{C^{-\frac{1}{w}} g}{6\pi^2 w} \beta^{-\frac{1}{w}} \mathscr{F}^{u_{\max}}_{u_{\min}, \frac{1}{w}-1}$$

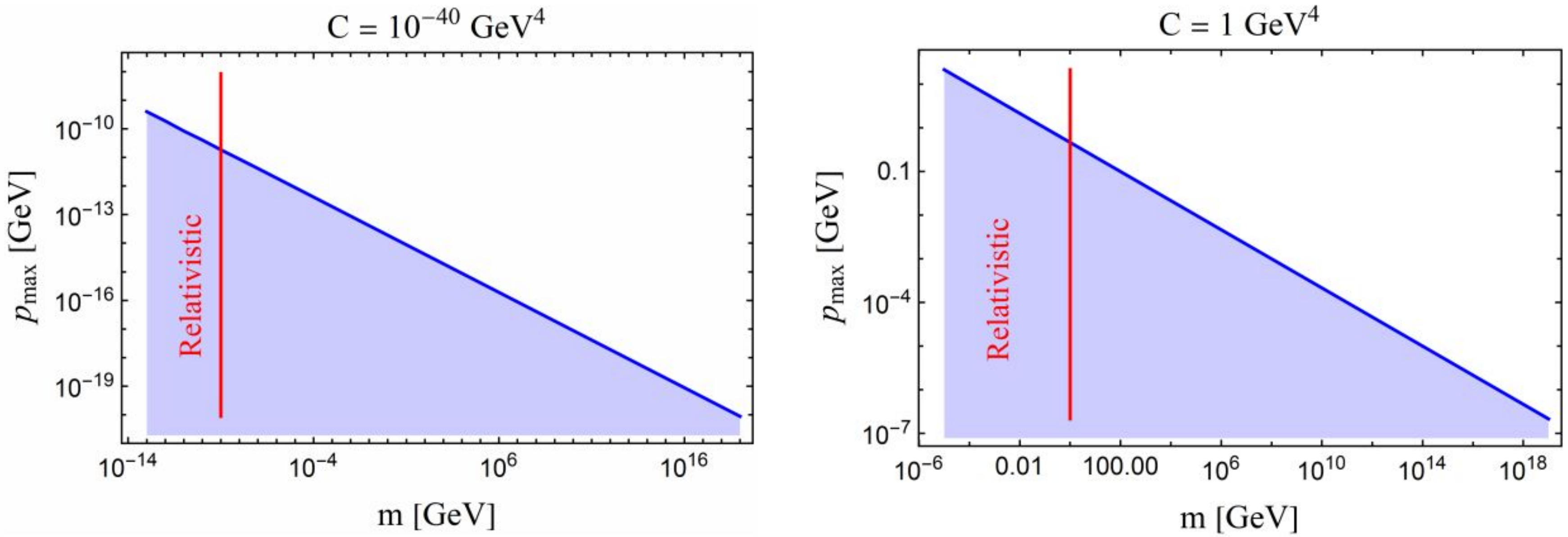
$$\mathscr{F}^{u_{\max}}_{u_{\min}, a} \equiv \int_{u_{\min}}^{u_{\max}} \frac{u^{\frac{1}{a}}}{e^{u}+1} du$$

$$\rho = -\frac{C^{-\frac{1}{w}} g}{6\pi^2 w} \int_{\varepsilon_{\min}}^{\varepsilon_{\max}} \frac{\varepsilon^{\frac{1}{w}}}{e^{\beta\varepsilon}+1} d\varepsilon$$

$$= -\frac{C^{-\frac{1}{w}} g}{6\pi^2 w} \beta^{-\frac{1+w}{w}} \mathscr{F}^{u_{\max}}_{u_{\min}, \frac{1}{w}}$$

$$\rho = \beta^{-1} \frac{\mathscr{F}^{u_{\max}}_{u_{\min}, \frac{1}{w}}}{\mathscr{F}^{u_{\max}}_{u_{\min}, \frac{1}{w}}} n$$

Fractional dark energy



$u_{\min} = 10$ (left) and $u_{\min} = 100$ (right)

Fractional Quantum Mechanics

- Developed by N. Laskin in 2000
- Generalization of QM using fractional calculus

- Applied to several QM problems

Riemann-Liouville derivative:

$$_aD_x^\alpha f(x) = \frac{1}{\Gamma(n+1-\alpha)} \frac{d^{n+1}}{dx^{n+1}} \int_a^x (x-y)^{n-\alpha} f(y)dy$$

$$n \leq \alpha < n+1$$

$$_aD_x^{-\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_a^x (x-y)^{\alpha-1} f(y)dy\,, \quad \alpha > 0$$

$$_aD_b^\alpha (_aD_b^{-\alpha} f(x)) = f(x)$$

$$_aD_b^{\pm\alpha} (_aD_b^{\pm\beta} f(x)) = {_aD_b^{\alpha\pm\beta}} f(x)$$

Fractional Quantum Mechanics

- Riesz fractional derivative

$$\left(-i\hbar\frac{\partial}{\partial x}\right)^{-\alpha} \equiv \frac{1}{2}\left({}_{-\infty}D_x^{-\alpha} +_x D_{\infty}^{-\alpha}\right)$$

- fractional Laplacian operator

$$(-\hbar^2\Delta)^{\alpha/2}\psi(\mathbf{r},t) = \frac{1}{(2\pi\hbar)^3}\int d^3p e^{i\mathbf{p}\cdot\mathbf{r}/\hbar}|\mathbf{p}|^\alpha\varphi(\mathbf{p},t)$$

- fractional Schrödinger equation for FDE

$$C = \lambda^6/M_{Pl}^2,$$

$$i\hbar\frac{\partial\psi(\mathbf{r},t)}{\partial t} = C(-\hbar^2\Delta)^{3w/2}\psi(\mathbf{r},t) \longrightarrow \varepsilon \approx \frac{C}{p^{-3w}}$$

λ~ 0.5- 10^{19} GeV

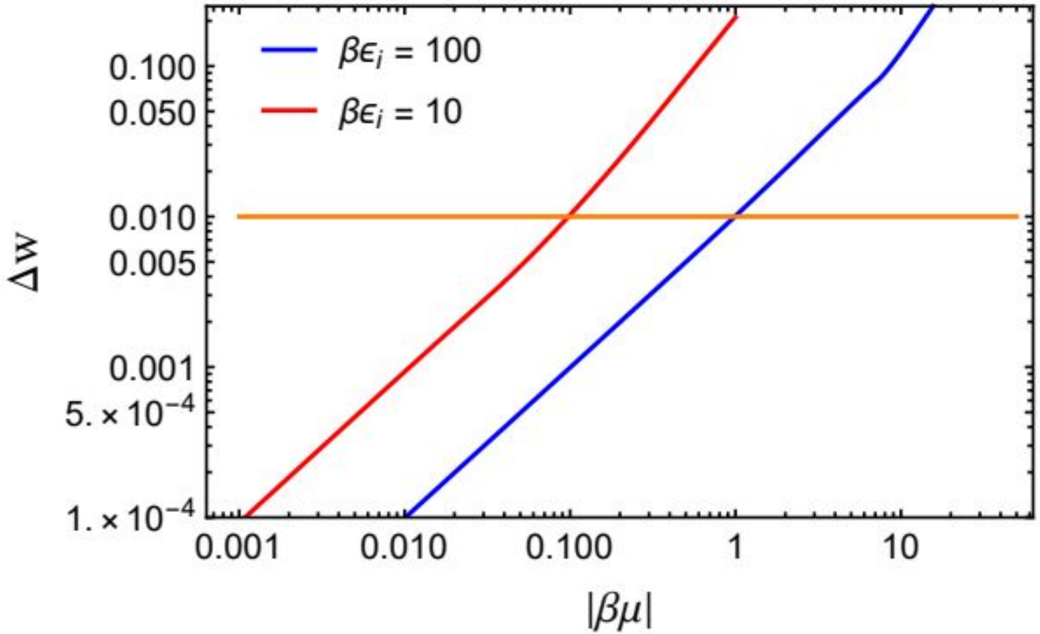
Chemical Potential

$$Ts = \rho + P - \mu n = (1+w)\rho - \mu n$$

$$w \geq -1 + \frac{\mu n}{\rho}$$

$$\Delta w \equiv \frac{\mu_0 n_0}{\rho_0} = \beta\mu \frac{\mathscr{F}^{u_{\max},\,\beta\mu}_{u_{\min},\,\frac{1}{w}-1}}{\mathscr{F}^{u_{\max},\,\beta\mu}_{u_{\min},\,\frac{1}{w}}}$$

$$\mathscr{F}^{u_{\max},\,\beta\mu}_{u_{\min},a} \equiv \int_{u_{\min}}^{u_{\max}} \frac{(u+\beta\mu)^a}{e^u \pm 1} du$$



Negative Absolute Temperature

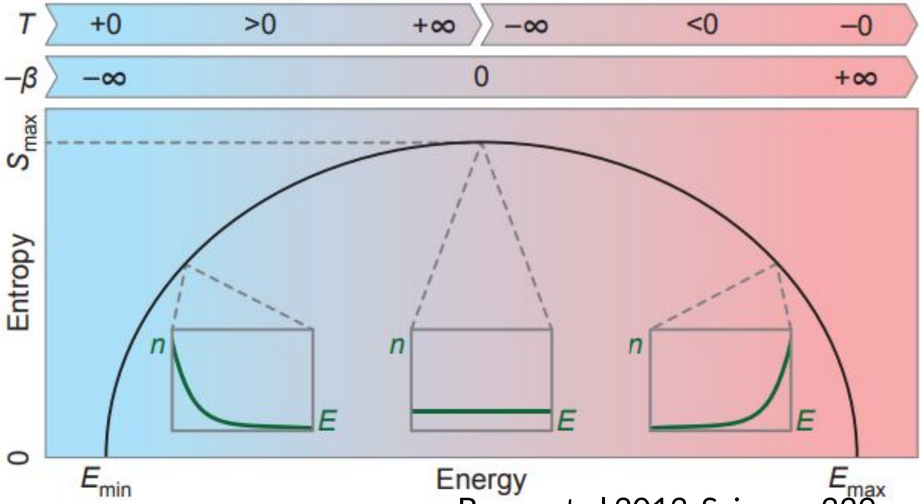
Late 1940’s and 1950’s Pound, Purcell, Onsager and Ramsey studied experimentally and theoretically NAT
[Il Nuovo Cimento 6, 1949, Physical Review, 81, 1951, 103, 1956.]

- Crystals, lasers, motional degrees of freedom, etc. → Negative pressures!

- Lord Kelvin introduced the concept of absolute temperature where absolute zero is the point where particles don't move

$$\frac{1}{T} = \left(\frac{\partial S}{\partial U}\right)_{V,N} \qquad P_i \propto e^{-E_i/k_B T}$$

+0 K, … , +300 K, … , +∞ K, −∞ K, … , −300 K, … , −0 K.



Braun et al 2013, Science, 339

NAT in Cosmology

J. Vieira, C. Byrnes, and A. Lewis. JCAP 2016

$$n(T,\mu) = \int_m^\Lambda D(\epsilon)\mathcal{N}(T,\epsilon,\mu)d\epsilon$$

$$\rho(T,\mu) = \int_m^\Lambda \epsilon D(\epsilon)\mathcal{N}(T,\epsilon,\mu)d\epsilon$$

$$P(T,\mu) = \beta^{-1}\int_m^\Lambda \epsilon D(\epsilon)\ln(1+e^{-\beta(\epsilon-\mu)})d\epsilon$$

Using properties of FD distribution:

$$\mathcal{N}(T,\epsilon,\mu) = \frac{1}{e^{\beta(\epsilon-\mu)}+1} = 1 - \frac{1}{e^{-\beta(\epsilon-\mu)}+1}$$

$$= 1 - \mathcal{N}(-T,\epsilon,\mu)$$

$$\ln\left[1+e^{-\beta(\epsilon-\mu)}\right] = -\beta(\epsilon-\mu) + \ln\left[1+e^{\beta(\epsilon-\mu)}\right]$$

NAT and FDE

$$n(T,\mu) = n_{\max} - n(-T,\mu)$$
$$\rho(T,\mu) = \rho_{\max} - \rho(-T,\mu)$$
$$P(T,\mu) = -\rho_{\max} + \mu n_{\max} - P(-T,\mu)$$

T = -0 K

$$n_{\max} = \int_m^\Lambda D(\epsilon)d\epsilon = \frac{C^{-\frac{1}{w}}g}{6\pi^2}\left(m_0^{\frac{1}{w}} - \Lambda^{\frac{1}{w}}\right)$$
$$\rho_{\max} = \int_m^\Lambda \epsilon D(\epsilon)d\epsilon \overset{w=-1}{=} \frac{Cg}{6\pi^2}\left[\ln(\Lambda) - \ln(m_0)\right]$$

$$n_h = -\frac{C^{-\frac{1}{w}}g}{6\pi^2 w}|\beta|^{-\frac{1}{w}}\mathscr{F}^{u_{\max},\,\beta\mu}_{u_{\min},\,\frac{1}{w}-1}$$
$$\rho_h = -\frac{C^{-\frac{1}{w}}g}{6\pi^2 w}|\beta|^{-\frac{1+w}{w}}\mathscr{F}^{u_{\max},\,\beta\mu}_{u_{\min},\,\frac{1}{w}}$$

} Similar to $+T$

NAT and FDE

$$\dot{n}_p + 3Hn_p = 0$$
$$\dot{n}_h + 3H(n_h - n_{\max}) = 0$$
$$\dot{\rho}_p + 3H(1+w)\rho_p = 0$$
$$\dot{\rho}_h + 3H(1+w)\rho_h = 0$$

$$\rho_h = |\beta|^{-1} \frac{\mathscr{F}^{u_{\max},\, \beta\mu}_{u_{\min},\, \frac{1}{w}}}{\mathscr{F}^{u_{\max},\, \beta\mu}_{u_{\min},\, \frac{1}{w}-1}} n_h$$

Taking the time derivative:

$$\dot{\rho}_p = -3H(1+w)\rho_p$$
$$\dot{\rho}_h = -3H(1+w)\rho_h + 3H(1+w)\rho_h \frac{n_{\max}}{n_h}$$

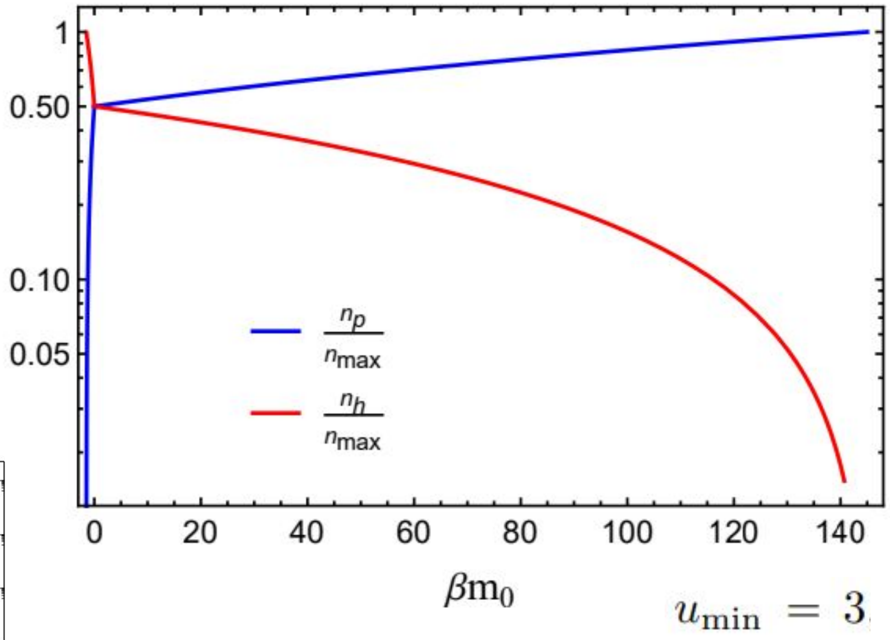
$\downarrow$

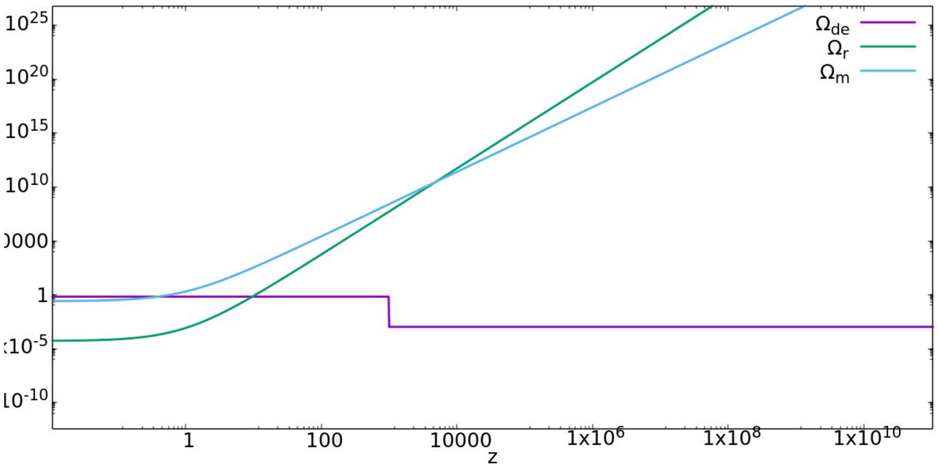
$$w \approx -1$$

FDE and NAT

$$n_p = \frac{Cg}{6\pi^2}\beta\mathscr{F}^{u_{\max},+\beta}_{u_{\min},-2} + \frac{n_{\max}}{2}$$

$$n_h = \frac{Cg}{6\pi^2}|\beta|\mathscr{F}^{u_{\max},-\beta}_{u_{\min},-2} + \frac{n_{\max}}{2}$$





Conclusions

- FDE is a fluid with non-canonical kinetic term and it is described by FQM
- Gives a cosmological constant
- Connection between FDE and NAT

Thank you!!!