Turn up the volume:
Listening to phase transitions in hot dark sectors

DESY Theory Workshop 2021

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22 September 2021

Based on 2109.06208, in collaboration
with Fatih Ertas and Felix Kahlhöfer

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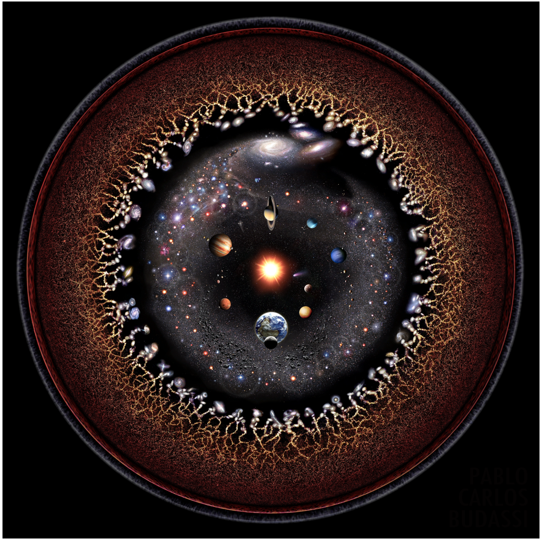
Why consider gravitational waves from hot dark sector phase transitions?



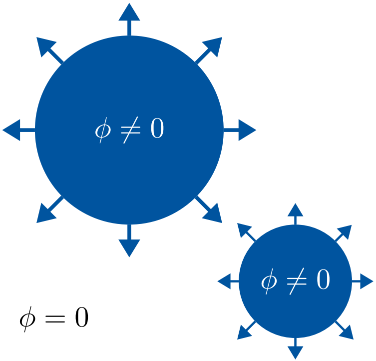
[Pablo Carlos Budassi, 2020]

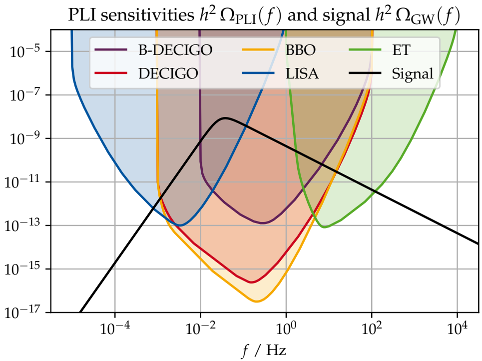
How can we observe what happened beyond the surface of last scattering?

$\rightsquigarrow$ Need messenger that comes straight from the Early Universe: **Gravitational waves**

Why consider gravitational waves from hot dark sector phase transitions?

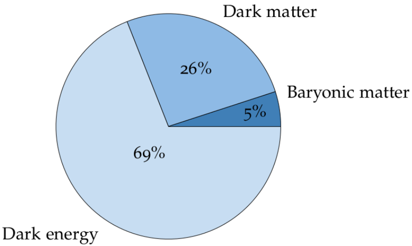
Bubbles of the new phase nucleate and eventually collide...





... giving rise to a stochastic gravitational wave background.

Why consider gravitational waves from hot dark sector phase transitions?



⇝ What kind of **dark sector** could produce observable GW signals?

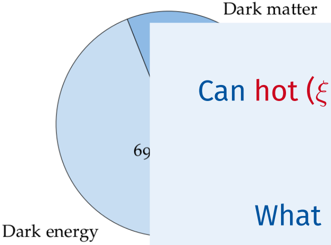
Dark sector: particle bath without thermal contact to SM particles:

$$T_{\mathsf{DS}} = \xi \, T_{\mathsf{SM}}$$

Breitbach et al. [1811.11175] showed that cold ($\xi < 1$) dark sectors produce weak signals, since

$$\alpha = \frac{\text{Latent heat}}{\text{Plasma energy density}} \propto \xi^4$$

Why consider gravitational waves from hot dark sector phase transitions?



Dark sector: particle bath without thermal contact to SM particles:

SM

5] showed that

produce weak

at

$$\alpha = \frac{}{\text{Plasma energy density}} \propto \xi^4$$

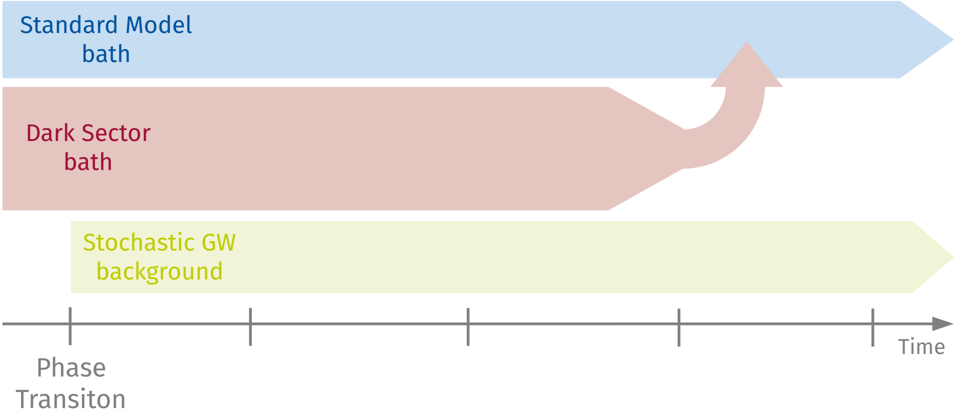
Can hot ($\xi > 1$) dark sector phase transitions emit strong GW signals?

What happens when the dark sector finally decays to SM particles?

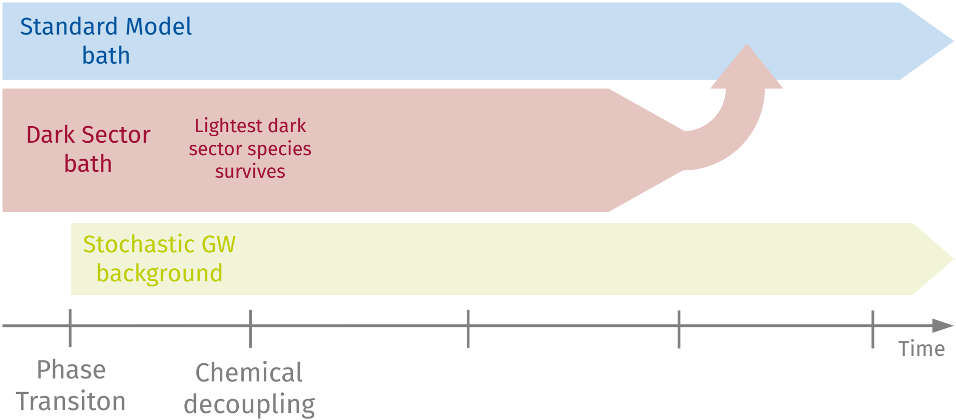
⇝ What kind o

produce observable GW signals?

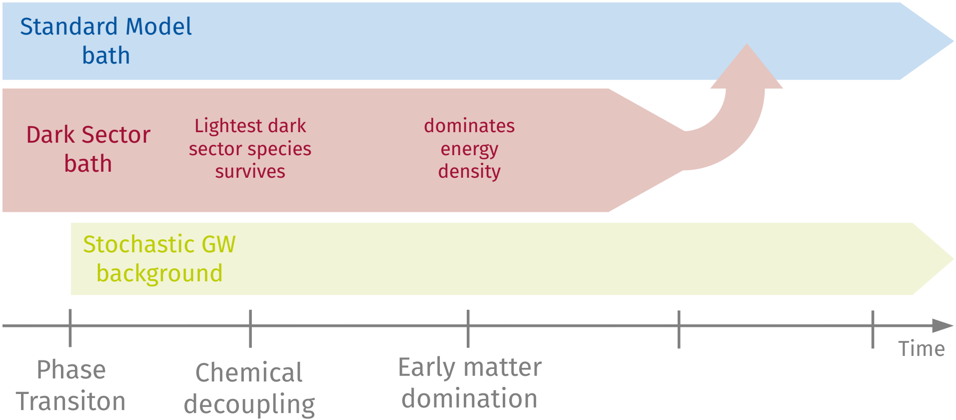
Long-lived dark sector evolution after a phase transition



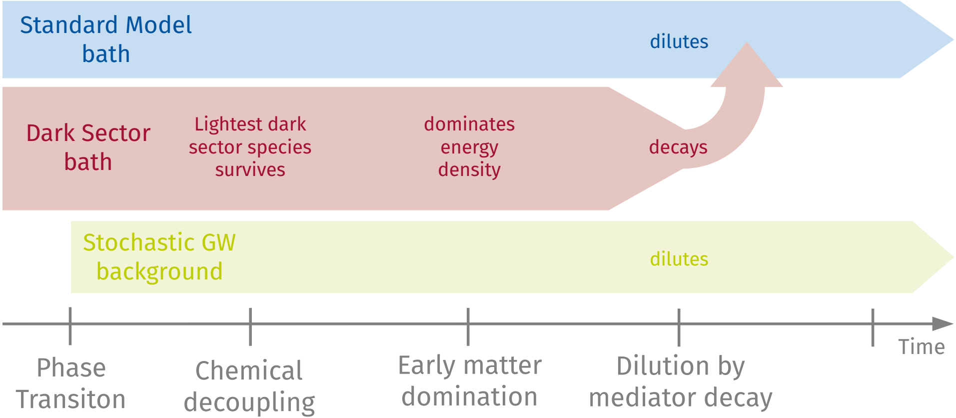
Long-lived dark sector evolution after a phase transition



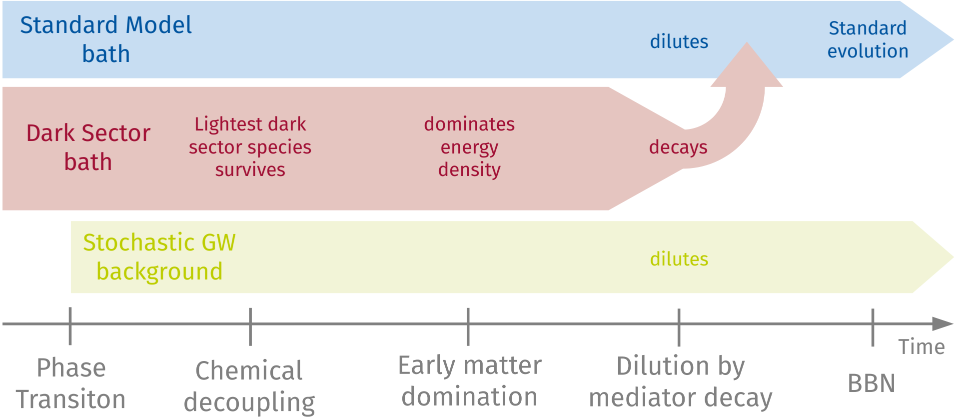
Long-lived dark sector evolution after a phase transition



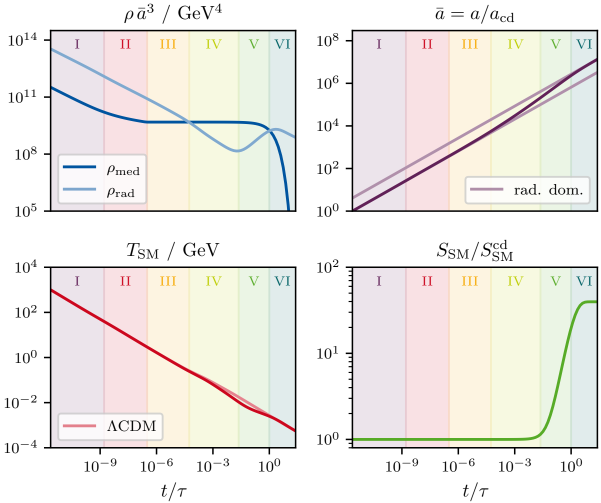
Long-lived dark sector evolution after a phase transition



Long-lived dark sector evolution after a phase transition



The out-of-equilibrium decay of a dark mediator



I Relativistic mediator

II Cannibalistic mediator

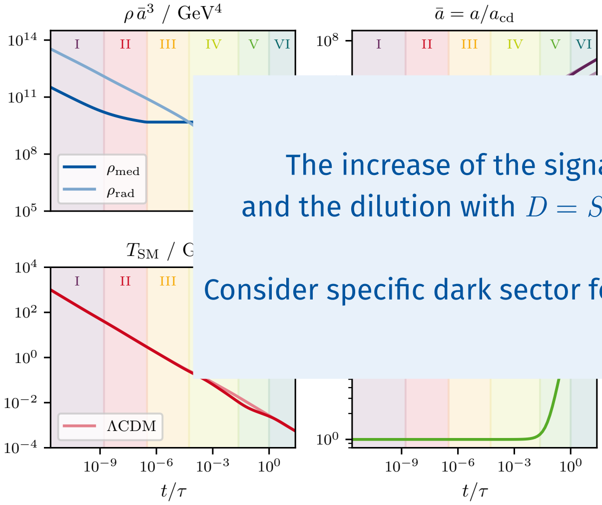
III Non-relativistic mediator

IV Early matter domination

V Entropy injection

VI Mediator decay

The out-of-equilibrium decay of a dark mediator



…diator

…ediator

…mediator

…mination

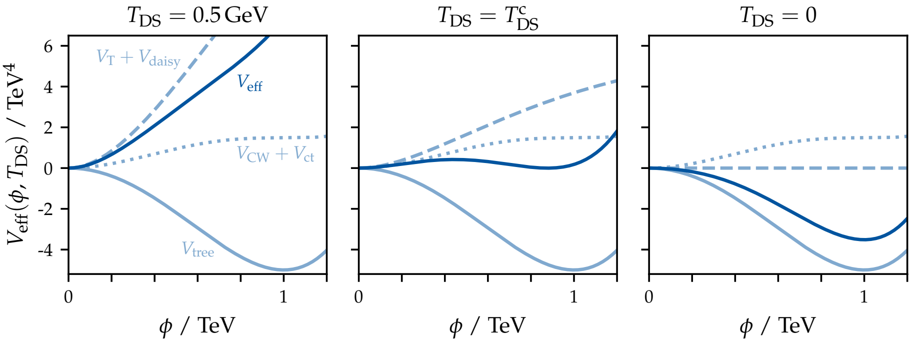
…on

VI Mediator decay

The increase of the signal strength $\alpha \propto \xi^4$ and the dilution with $D = S_{\mathrm{SM}}^{\mathrm{after}} / S_{\mathrm{tot}}^{\mathrm{before}}$ compete.

Consider specific dark sector for quantitative analysis!

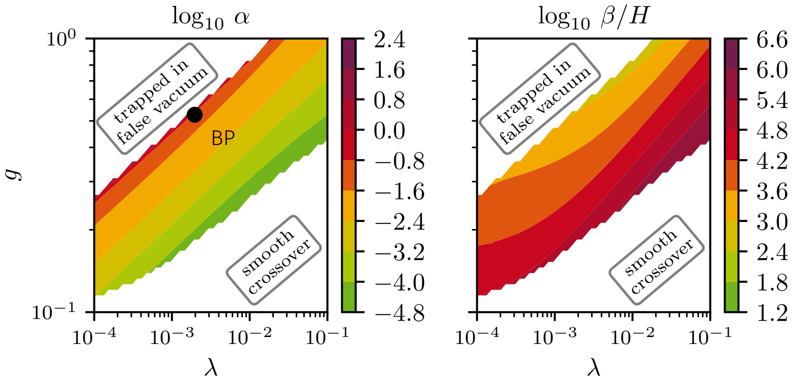
The dark photon model



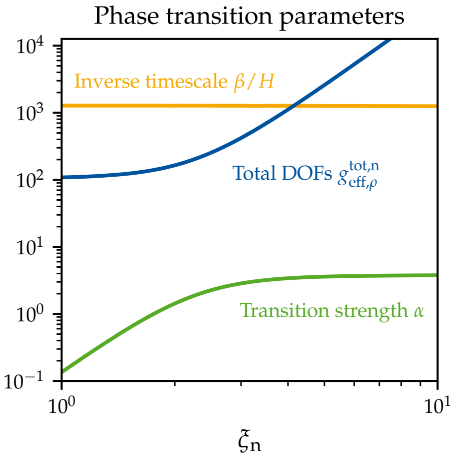
Add a $U(1)_D$ to the SM gauge groups. Its gauge boson, the "dark photon", gets massive when a "dark Higgs" obtains $\phi \neq 0$. Effective potential controlled by the tree-level VEV v, dark Higgs quartic coupling λ and gauge coupling g.

Strength and time scale of the transition

Analyze the phase structure and determine the strength α and inverse time scale β/H. Vary quartic coupling λ and gauge coupling g to identify region of strong and slow transitions. Consider case of dark higgs mediator.

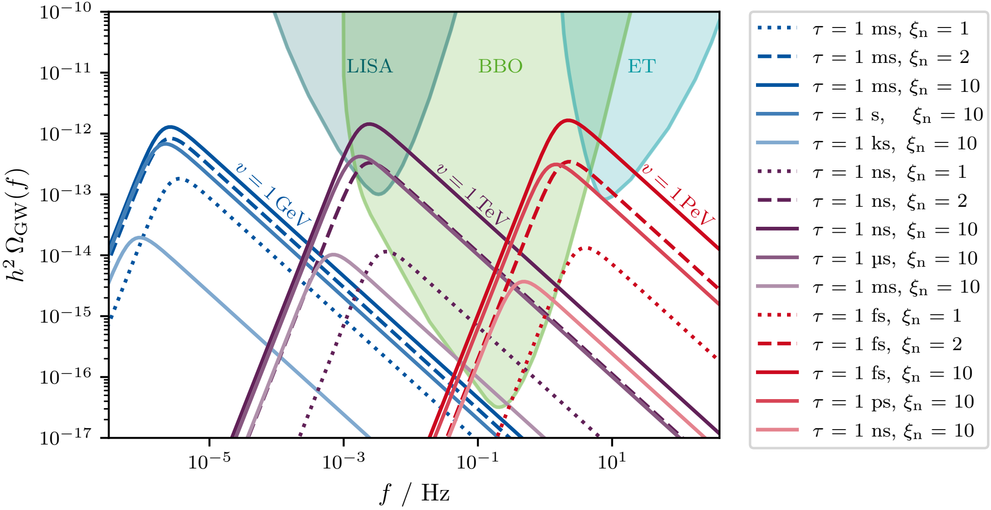


The temperature ratio's impact on α and β/H

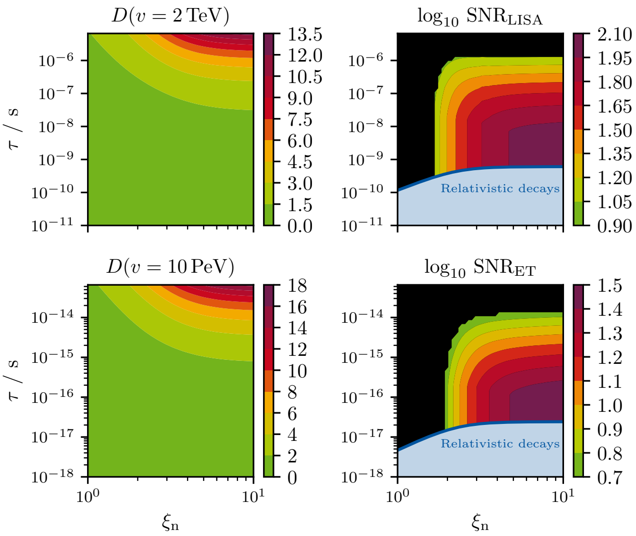


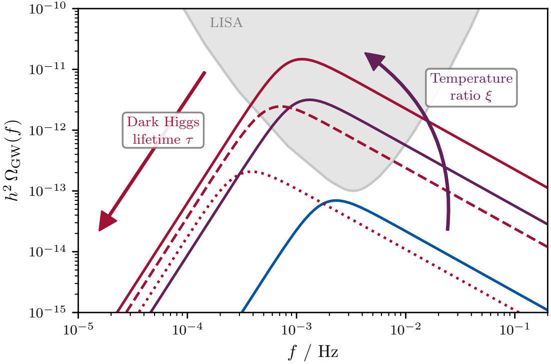
The transition strength α increases $\propto \xi_\mathrm{n}^4$, but only until the Universe is completely dominated by the dark sector! Then, the temperature ratio becomes irrelevant. The inverse timescale is virtually independent of ξ_n.

Influence of VEV v, dark Higgs lifetime τ, and temperature ratio ξ on GW signal



Benchmark point study





- Hot dark sectors are loud
- Long-lived dark sector decays can dilute the signals
- Presented effects are largely model-independent
- Cannibalism in the dark sector is relevant
- LISA and ET can partially test the $U(1)_D$ parameter space

Thank you very much for your attention!

Do you have any questions?

Backup slides

Describing the dark sector in equilibrium

For several dark sector species in thermal equilibrium: can define effective DOFs

$$\rho_{\mathrm{tot}}(T_{\mathrm{SM}}) = \left[g^{\mathrm{SM}}_{\mathrm{eff},\rho}(T_{\mathrm{SM}}) + g^{\mathrm{DS}}_{\mathrm{eff},\rho}(T_{\mathrm{SM}})\,\xi^4(T_{\mathrm{SM}})\right]\,\frac{\pi^2}{30}\,T_{\mathrm{SM}}^4$$

$$s_{\mathrm{tot}}(T_{\mathrm{SM}}) = \left[g^{\mathrm{SM}}_{\mathrm{eff},s}(T_{\mathrm{SM}}) + g^{\mathrm{DS}}_{\mathrm{eff},s}(T_{\mathrm{SM}})\,\xi^3(T_{\mathrm{SM}})\right]\,\frac{2\,\pi^2}{45}\,T_{\mathrm{SM}}^3$$

Describing the dark sector in equilibrium

For several dark sector species in thermal equilibrium: can define effective DOFs

$$\rho_{\text{tot}}(T_{\text{SM}}) = \left[g^{\text{SM}}_{\text{eff},\rho}(T_{\text{SM}}) + g^{\text{DS}}_{\text{eff},\rho}(T_{\text{SM}})\,\xi^4(T_{\text{SM}})\right] \frac{\pi^2}{30}\, T^4_{\text{SM}}$$

$$s_{\text{tot}}(T_{\text{SM}}) = \left[g^{\text{SM}}_{\text{eff},s}(T_{\text{SM}}) + g^{\text{DS}}_{\text{eff},s}(T_{\text{SM}})\,\xi^3(T_{\text{SM}})\right] \frac{2\,\pi^2}{45}\, T^3_{\text{SM}}$$

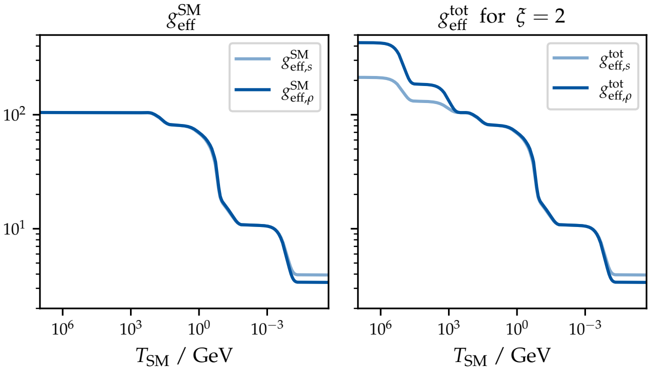
As entropy is conserved separately in the two baths, the temperature ratio follows

$$\xi(T_{\text{SM}}) = \tilde{\xi} \left(\frac{g^{\text{SM}}_{\text{eff},s}}{\tilde{g}^{\text{SM}}_{\text{eff},s}}\right)^{1/3} \left(\frac{\tilde{g}^{\text{DS}}_{\text{eff},s}}{g^{\text{DS}}_{\text{eff},s}}\right)^{1/3}$$

When SM particles annihilate , ξ decreases.

When dark sector DOF decrease , ξ increases.

Describing the dark sector in equilibrium



Example: Thermal evolution of a hot ($\xi = 2$) dark sector consisting of a dark photon ($m_{\text{DP}} = 10^6$ GeV) and a dark Higgs boson ($m_{\text{DH}} = 10^4$ GeV).

The out-of-equilibrium decay of a dark mediator

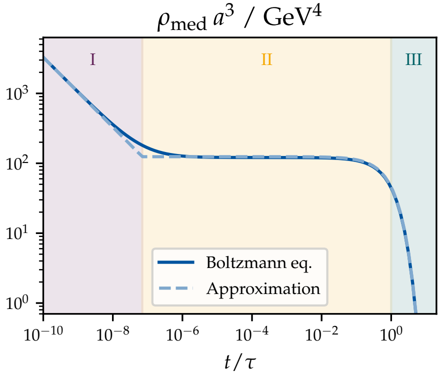
Evolution of the lightest dark sector state ("mediator") after chemical decoupling:

$$\dot{\rho}_{\text{med}} \simeq -3\,\zeta\,H\,\rho_{\text{med}} - \Gamma\,\rho_{\text{med}}$$

with

$$\zeta = 1 + \frac{P_{\text{med}}}{\rho_{\text{med}}} = \begin{cases} 4/3 & \text{rel.} \\ 1 & \text{non-rel.} \end{cases}$$

Three phases: Relativistic, non-relativistic and decaying mediator



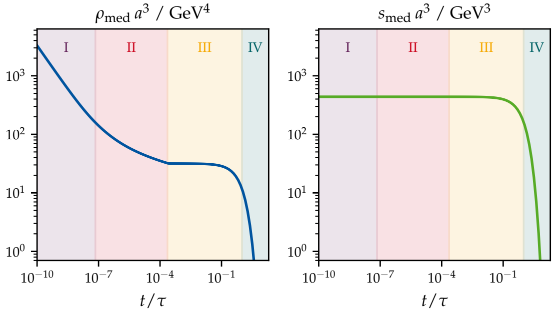
The out-of-equilibrium decay of a dark mediator

Number-changing processes of the mediator lead to a "cannibalistic" phase with $\mu_{\text{med}} = 0$. Therefore, the unique function $\rho_{\text{med}}(s_{\text{med}})$ exists. We found:

$$\zeta = \begin{cases} \dfrac{\mathrm{d}\ln \rho_{\text{med}}}{\mathrm{d}\ln s_{\text{med}}} & 3 \to 2 \text{ efficient} \\ 1 & 3 \to 2 \text{ inefficient} \end{cases}$$

During cannibalism, ζ goes smoothly from $4/3$ to 1.

Four phases: Relativistic, cannibalistic, non-relativistic and decaying mediator



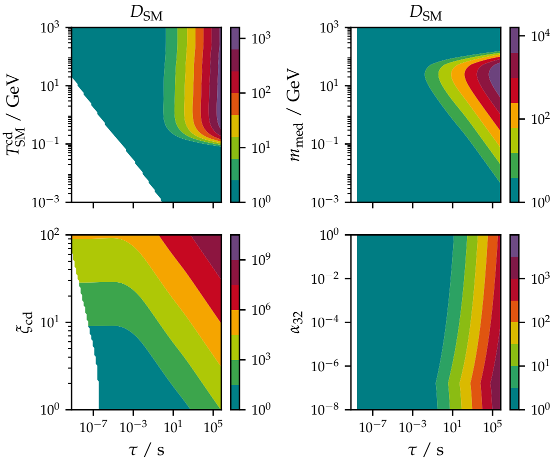
The out-of-equilibrium decay of a dark mediator

Dark sector parameters:

- SM temperature $T_{\text{SM}}^{\text{cd}}$ at chemical decoupling
- Mediator mass m_{med}
- Temperature ratio ξ_{cd} at chemical decoupling
- Effective $3 \to 2$ coupling α_{32}

Define dilution factor:

$$D_{\text{SM}} = \frac{S_{\text{SM}}^{\text{after decay}}}{S_{\text{SM}}^{\text{before decay}}}$$



Parametrization of the GW signal

Assuming strong[1] phase transitions, the GW spectrum can be parameterized by

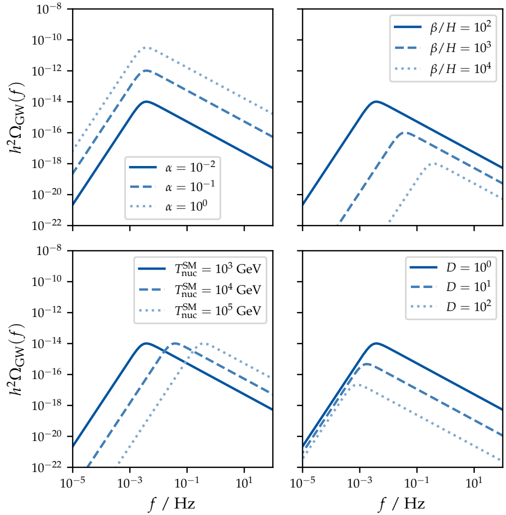
$$h^2\,\Omega_{\mathrm{GW}}(f) \simeq \frac{\mathcal{O}(10^{-6})}{D^{4/3}} \left(\frac{\alpha}{1+\alpha}\right)^2 \left(\frac{\beta}{H}\right)^{-2} \frac{3.8\,(f/f_{\mathrm{p}})^{2.8}}{1+2.8\,(f/f_{\mathrm{p}})^{3.8}}\,, \quad \text{where}$$

$$D \equiv \frac{g_{\mathrm{eff},s}^{\mathrm{SM,n}}}{g_{\mathrm{eff},s}^{\mathrm{tot,n}}}\, D_{\mathrm{SM}} \qquad \text{and} \qquad f_{\mathrm{p}} \simeq \frac{\mathcal{O}(\mu\mathrm{Hz})}{D^{1/3}} \left(\frac{\beta}{H}\right) \left(\frac{T_{\mathrm{SM}}^{\mathrm{n}}}{100\,\mathrm{GeV}}\right)$$

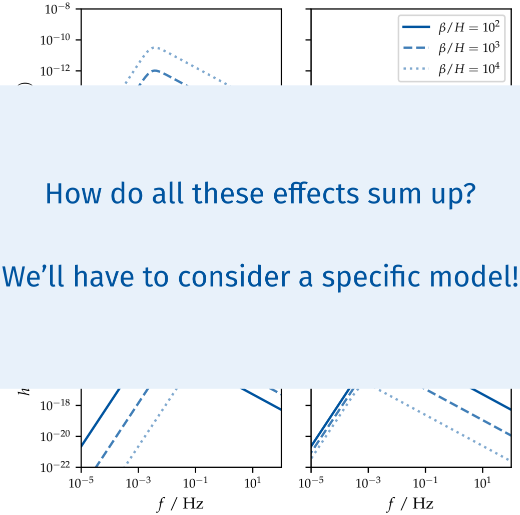
⇝ GW spectrum fixed by the transition strength α, the inverse time scale β/H, the nucleation temperature $T_{\mathrm{SM}}^{\mathrm{n}}$ and the dilution factor D

[1] This is only to get an intuition, the actually performed calculations are more involved

Parametrization of the GW signal



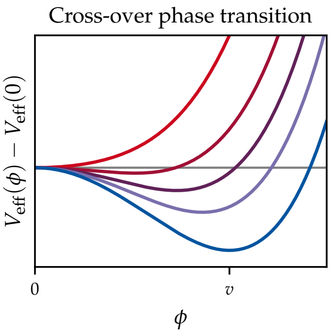
Parametrization of the GW signal

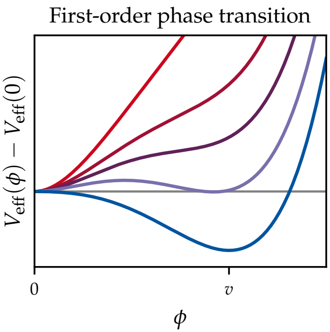


How do all these effects sum up?

We'll have to consider a specific model!

Cross-over and first-order phase transitions





The scalar field “rolls down” from $\phi = 0$ to $\phi = v$, when the bath cools from high temperatures to low temperatures.

The scalar field tunnels to the true potential minimum ($\phi \neq 0$) to minimize its action ($\sim$ free energy).

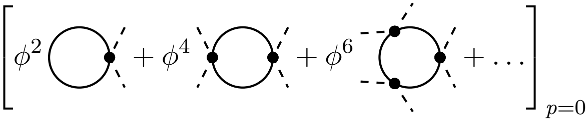
To demonstrate construction of $V_{\text{eff}}(\phi, T)$, take the toy-model Lagrangian...

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi\right)\left(\partial^\mu \phi\right) - V_{\text{tree}}(\phi)$$

$$\text{with} \quad V_{\text{tree}}(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4}\phi^4$$

... and consider all 1-loop 1-PI graphs:

$$V^{1-\text{loop}}_{\text{eff},\Phi}(\phi) =$$



And calculate 1-loop effective potential with $m^2(\phi) = \partial_\phi^2 V_{\text{tree}}(\phi) = -\mu^2 + 3\lambda\phi^2$

$$
\begin{aligned}
V_{\text{eff}}(\phi, T) &= \frac{1}{2}\int \frac{\mathrm{d}^4 k_{\text{E}}}{(2\pi)^4} \log\left[k_{\text{E}}^2 + m^2(\phi)\right] && \text{with } k_E^0 \text{ being } \frac{2\pi}{T}\text{-periodic} \\
&= \frac{T}{2}\sum_n \int_{\mathbf{k}} \log\left[\left(\frac{2\pi n}{T}\right)^2 + E_k^2\right] && \text{with } E_k = \sqrt{k^2 + m^2(\phi)} \\
&= \int_{\mathbf{k}} \left[\frac{E_k}{2} + T\log\left\{1 - e^{-E_k/T}\right\}\right] \\
&= V_{\text{CW}}(\phi) + V_{\text{T}}(\phi, T)
\end{aligned}
$$

Interpretation: V_{tree} is the classical energy density contained in a background field ϕ, $V_{\text{CW}}(+V_{\text{T}})$ is the vacuum energy density of a quantum field living in this background, which is completely analogous to the zero-point energy of a harmonic oscillator (in a thermal bath)

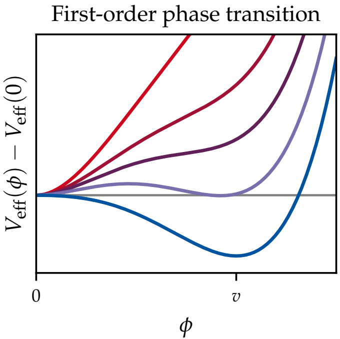
First-order phase transitions in thermal field theory

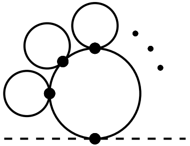
$$V_\mathsf{T} = \int_\mathbf{k} T \log \left\{ 1 - e^{-E_k/T} \right\}$$
$$= -\frac{\pi^2 T^4}{90} + \frac{T^2 m^2(\phi)}{24} - \frac{T m^3(\phi)}{12\pi} + \ldots$$

However, around T_c, V_eff is dominated by $>$ 1-loop effects. “Daisies” dominate:

$$V_\mathsf{daisy} = -\frac{T}{12\pi} \left[\left(m^2(\phi) + \Pi(T) \right)^{3/2} - m^3(\phi) \right]$$

And cancel the potential barrier in V_eff. But: Transversal gauge boson component doesn’t acquire $\Pi(T)$. $\rightsquigarrow$ Gauge bosons can save potential barrier and thus FOPTs.





First-order phase transitions in thermal field theory

Summary:

$$V_{\text{eff}}^{\text{1-loop}}(\phi, T) = V_{\text{tree}}(\phi) + V_{\text{CW}}(\phi) + V_{\text{ct}}(\phi) + V_{\text{T}}(\phi, T) + V_{\text{daisy}}(\phi, T)$$

- $V_{\text{CW}}(\phi) + V_{\text{ct}}(\phi)$: Coleman-Weinberg potential and its counter-terms
- $V_{\text{T}}(\phi, T)$: 1-loop thermal corrections
- $V_{\text{daisy}}(\phi, T)$: Daisy corrections, dominate at T_c

How to get a thermal FOPT?

- Need scalar charged under gauge group with massive gauge bosons
- Dominant $V_{\text{tree}} + V_{\text{CW}}$ contributions can always destroy potential barrier, though $\rightsquigarrow$ as in SM with too high m_h forbidding FOPT

First-order phase transitions in thermal field theory

$$V_{\text{eff}}^{1-\text{loop}}(\phi,\, T) = V_{\text{tree}} + V_{\text{CW}} + V_{\text{ct}} + V_T + V_{\text{daisy}}$$

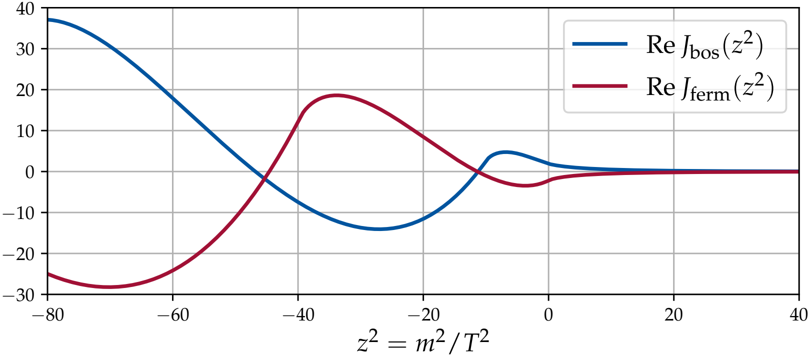
has the individual contributions

$$V_{\text{CW}}(\phi) = \sum_x \eta_x \, n_x \, \frac{m_x^4(\phi)}{64\,\pi^2} \left[\ln \frac{m_x^2(\phi)}{\Lambda^2} - C_a \right] ,$$

$$V_T(\phi,\, T) = \frac{T^4}{2\,\pi^2} \sum_x \eta_x \, n_x \, J_{\eta_x}\left(\frac{m_x^2(\phi)}{T^2} \right) ,$$

$$V_{\text{daisy}}(\phi,\, T) = -\frac{T}{12\,\pi} \sum_{\text{b}} n_b^{\text{L}} \left[\left(m^2(\phi) + \Pi(T) \right)_b^{3/2} - \left(m^2(\phi) \right)_b^{3/2} \right]$$

Thermal functions



Bubble expansion

Euclidean action of scalar field

$$S[\phi] = \int \mathrm{d}^4 x_{\mathrm{E}} \left[\frac{1}{2} \left(\frac{\partial \phi}{\partial \tau} \right)^2 + \frac{(\nabla \phi)^2}{2} + V_{\mathrm{eff}}(\phi) \right]$$

Minimizing for O(4)-case gives

$$\frac{\mathrm{d}^2 \phi}{\mathrm{d}\rho^2} + \frac{3}{\rho} \frac{\mathrm{d}\phi}{\mathrm{d}\rho} = V'_{\mathrm{eff}}(\phi)$$

At finite T and in real space:

$$\frac{\mathrm{d}^2 \phi}{\mathrm{d}r^2} + \frac{2}{r} \frac{\mathrm{d}\phi}{\mathrm{d}r} = V'_{\mathrm{eff}}(\phi, T)$$

Can be solved by overshoot-undershoot method

Bubble formation and thermal tunneling

Nucleation rate: $\Gamma = \mathcal{A} e^{-S_4}$ with

$$S_4 = \int \frac{1}{2}\left(\frac{\mathrm{d}\phi}{\mathrm{d}\tau}\right)^2 + \frac{1}{2}(\nabla\phi)^2 + V_{\text{eff}}(\phi)\,\mathrm{d}^4 x_{\text{E}}$$

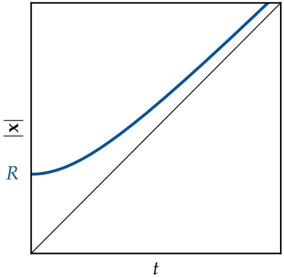
and $\mathcal{A} \sim T^4$. Extremalization yields KG equation with classical potential source:

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}\tau^2} + \Delta\phi = \frac{\mathrm{d}\,V_{\text{eff}}}{\mathrm{d}\phi}$$

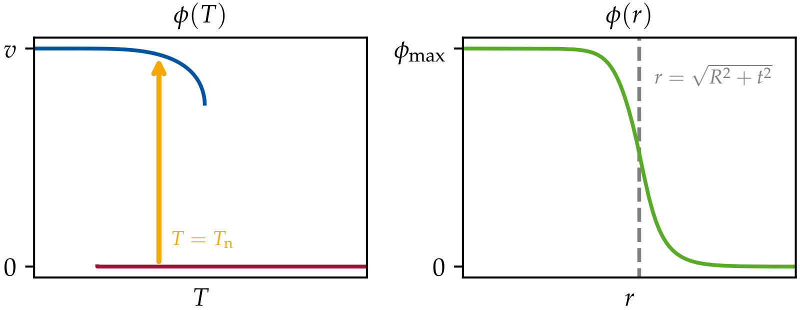
with b.c. $\phi(\rho \to \infty) \to 0$ and $\phi'(\rho = 0) = 0$ where $\rho \equiv \sqrt{\tau^2 + |\mathbf{x}|^2}$.
Solutions typically $O(4)$ symmetric:

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}\rho^2} + \frac{3}{\rho}\frac{\mathrm{d}\phi}{\mathrm{d}\rho} = \frac{\mathrm{d}\,V_{\text{eff}}}{\mathrm{d}\phi}$$

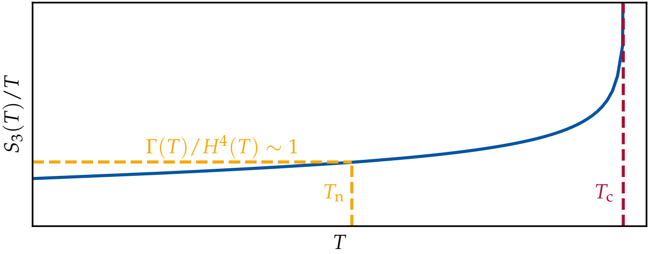
In 3-space: $r = |\mathbf{x}| = \sqrt{\rho^2 + c^2 t^2} \rightsquigarrow$
Nucleation and expansion with $v \to c$



Temperature dependence of potential minima and bubble profile



Nucleation criterion



The nucleation condition $\Gamma(T_\mathrm{n})\, H^{-4}(T_\mathrm{n}) = 1$ gives

$$\left.\frac{S_3(T)}{T}\right|_{T=T_\mathrm{n}} \sim 146 - 2\,\ln\left(\frac{g_{\mathrm{eff},\rho}^{\mathrm{tot}}(T_\mathrm{n})}{100}\right) - 4\,\ln\left(\frac{T_\mathrm{n}}{100\ \mathrm{GeV}}\right)$$

Can be solved by repeated evaluation of S_3/T and subsequent minimization.

GW parameter calculation

Radiation energy density at nucleation

$$\rho_{\mathrm{R}} = \frac{\pi^2}{30}\left(g_{\mathrm{eff},\rho}^{\mathrm{SM,n}} + g_{\mathrm{eff},\rho}^{\mathrm{DS,n}}\,\xi^4\right)\left(T_{\mathrm{SM}}^{\mathrm{n}}\right)^4$$

Transition strength

$$\alpha = \frac{1}{\rho_{\mathrm{R}}}\left(-\Delta V + T_{\mathrm{DS}}^{\mathrm{n}}\,\frac{\partial \Delta V}{\partial T}\bigg|_{T_{\mathrm{DS}}^{\mathrm{n}}}\right)$$

Inverse time scale

$$\frac{\beta}{H} = T_{\mathrm{DS}}^{\mathrm{n}}\,\frac{\mathrm{d}S_{\mathrm{E}}(T)}{\mathrm{d}T}\bigg|_{T_{\mathrm{DS}}^{\mathrm{n}}}$$

Critical transition strength for runaway bubbles

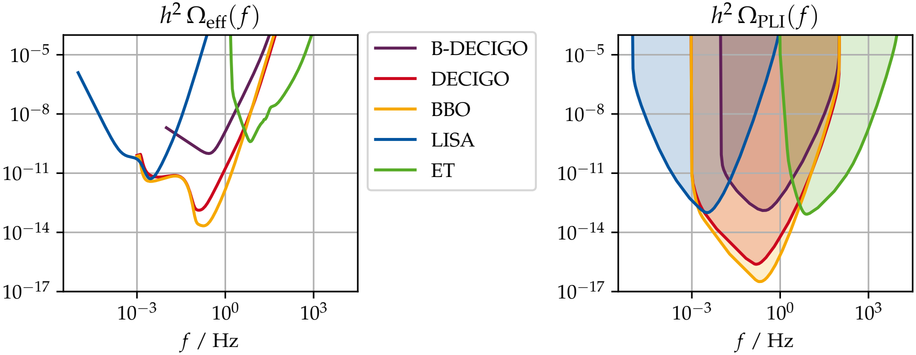
$$\alpha_\infty = \frac{\left(T_{\mathrm{DS}}^{\mathrm{n}}\right)^2}{\rho_{\mathrm{R}}}\left(\sum_{i=\mathrm{bos}} n_i\,\frac{\Delta m_i^2}{24} + \sum_{i=\mathrm{fer}} n_i\,\frac{\Delta m_i^2}{48}\right)$$

SGWB model

$$\Omega_{\mathsf{GW}}(f) = \frac{1}{\rho_c}\frac{\mathrm{d}\rho_{\mathsf{GW}}(f)}{\mathrm{d}\log f} \simeq \sum \mathcal{N}\,\Delta\left(\frac{\kappa\,\alpha}{1+\alpha}\right)^p\left(\frac{H}{\beta}\right)^q s(f)$$

	Scalar field Ω_ϕ	Sound waves Ω_{sw}	Turbulence Ω_{turb}
$\mathcal{N}$	1	$1.59\cdot 10^{-1}$	$2.01\cdot 10^{1}$
κ	κ_ϕ	κ_{sw}	$\varepsilon_{\mathsf{turb}}\kappa_{\mathsf{sw}}$
p	2	2	$\frac{3}{2}$
q	2	1	1
Δ	$\frac{0.11 v_w^3}{0.42+v_w^2}$	v_w	v_w
f_{p}	$\frac{0.62\beta}{1.8-0.1v_w+v_w^2}$	$\frac{2\beta}{\sqrt{3}v_w}$	$\frac{3.5\beta}{2v_w}$
$s(f)$	$\frac{3.8(f/f_{\mathsf{p}})^{2.8}}{1+2.8(f/f_{\mathsf{p}})^{3.8}}$	$(f/f_{\mathsf{p}})^3\left(\frac{7}{4+3(f/f_{\mathsf{p}})^2}\right)^{7/2}$	$\frac{(f/f_{\mathsf{p}})^3}{(1+f/f_{\mathsf{p}})^{11/3}[1+8\pi(f/H)]}$

Experimental sensitivities



Effective degrees of freedom

$$g^x_{\text{eff},\rho}(T_x) \equiv \frac{\rho_x(T_x)}{\rho^{\text{rel}}_{\text{bos}}(T_x)\big|_{g=1}} = g_x \frac{15}{\pi^4} \int_{z_x}^{\infty} \mathrm{d}u_x \, \frac{u_x^2 \sqrt{u_x^2 - z_x^2}}{e^{u_x} \pm 1} \, ,$$

$$g^x_{\text{eff},P}(T_x) \equiv \frac{P_x(T_x)}{P^{\text{rel}}_{\text{bos}}(T_x)\big|_{g=1}} = g_x \frac{15}{\pi^4} \int_{z_x}^{\infty} \mathrm{d}u_x \, \frac{\left(u_x^2 - z_x^2\right)^{3/2}}{e^{u_x} \pm 1} \, ,$$

$$g^x_{\text{eff},s}(T_x) = \frac{3\, g^x_{\text{eff},\rho}(T_x) + g^x_{\text{eff},P}(T_x)}{4} \, ,$$

where $u_x = \sqrt{m_x^2 + p^2}/T_x$ and $z_x = m_x/T_x$. Sum over all SM and DS species:

$$g^{\text{tot}}_{\text{eff},\rho} = g^{\text{SM}}_{\text{eff},\rho}(T_{\text{SM}}) + g^{\text{DS}}_{\text{eff},\rho}(T_{\text{SM}})\, \xi^4(T_{\text{SM}})$$
$$g^{\text{tot}}_{\text{eff},s} = g^{\text{SM}}_{\text{eff},s}(T_{\text{SM}}) + g^{\text{DS}}_{\text{eff},s}(T_{\text{SM}})\, \xi^3(T_{\text{SM}})$$

Mediator cannibalism

Conserved comoving mediator entropy $s_{\text{med}}\, a^3 = \text{const}$ gives

$$\frac{\mathrm{d}\ln s_{\text{med}}}{\mathrm{d}t} = \frac{\mathrm{d}\ln s_{\text{med}}}{\mathrm{d}\ln \rho_{\text{med}}} \frac{\dot{\rho}_{\text{med}}}{\rho_{\text{med}}} = -3\, H(t)\ ,$$

from which follows that

$$\dot{\rho}_{\text{med}} = -3\, \frac{\mathrm{d}\ln \rho_{\text{med}}}{\mathrm{d}\ln s_{\text{med}}}\, H(t)\, \rho_{\text{med}}(t)\ .$$

For $\mu_{\text{med}} = 0$, one can find function $\rho_{\text{med}}(s_{\text{med}})$, independent of particle species:

$$\frac{\mathrm{d}\ln \rho_{\text{med}}}{\mathrm{d}\ln s_{\text{med}}} = \frac{\mathrm{d}\rho_{\text{med}}}{\mathrm{d}s_{\text{med}}} \frac{s_{\text{med}}}{\rho_{\text{med}}} = \frac{\mathrm{d}\bar{\rho}_{\text{med}}}{\mathrm{d}\bar{s}_{\text{med}}} \frac{\bar{s}_{\text{med}}}{\bar{\rho}_{\text{med}}} = \frac{\mathrm{d}\ln \bar{\rho}_{\text{med}}}{\mathrm{d}\ln \bar{s}_{\text{med}}} = \frac{\mathrm{d}\ln \bar{\rho}}{\mathrm{d}\ln \bar{s}}$$

with $\bar{s}_{\text{med}} \equiv 2\,\pi^2\, s_{\text{med}}/(g_{\text{med}}\, T_{\text{DS}}^3)$ and $\bar{\rho}_{\text{med}} \equiv 2\,\pi^2\, \rho_{\text{med}}/(g_{\text{med}}\, T_{\text{DS}}^4)$.

Mediator cannibalism

That yields

$$\dot{\rho}_{\mathrm{med}} \simeq -3\,\zeta\,H\,\rho_{\mathrm{med}} - \Gamma\,\rho_{\mathrm{med}}$$

with

$$\zeta(t) = \begin{cases} \dfrac{\mathrm{d}\ln\bar{\rho}}{\mathrm{d}\ln\bar{s}}\,(\rho_{\mathrm{med}}) & \text{for} \quad \Gamma_{\mathrm{nc}}(t) \geq H(t) \\ 4/3 & \text{for} \quad \Gamma_{\mathrm{nc}}(t) < H(t), \quad t < \tilde{t} \\ 1 & \text{for} \quad \Gamma_{\mathrm{nc}}(t) < H(t), \quad t \geq \tilde{t} \end{cases},$$

where $\tilde{t} = 7\,t_{\mathrm{cd}}\,(T_{\mathrm{DS}}^{\mathrm{cd}}/m_{\mathrm{med}})^2$ denotes the time when the mediator gets non-relativistic. Number changing process rate is approximated by

$$\Gamma_{\mathrm{nc}} \simeq \Gamma_{32} \simeq \langle\sigma_{32}\,v^2\rangle\,n_{\mathrm{med}}^2$$

Mediator cannibalism

The averaged cross section reads

$$\langle \sigma_{32}\, v^2 \rangle = \frac{25\,\sqrt{5}\,\alpha_{32}^3}{3072\,\pi\, m_{\text{med}}^5} + \mathcal{O}\left(\frac{T_{\text{DS}}}{m_{\text{med}}}\right).$$

where

$$(4\,\pi\,\alpha_{32})^3 \equiv \left(\frac{\kappa_3}{m}\right)^2 \left[\left(\frac{\kappa_3}{m}\right)^2 + 3\,\kappa_4\right]^2$$

for a potential $V(\phi) = \frac{m^2}{2}\,\phi^2 + \frac{\kappa_3}{3!}\,\phi^3 + \frac{\kappa_4}{4!}\,\phi^4$. In our model: $\alpha_{32} = 2.3\,\lambda$.

Coupled set of ODEs underlying the entropy injection

$$\bar{a}' = \frac{\bar{a}}{\theta_{\mathrm{H}}} \sqrt{r + \frac{f_{\mathrm{mat}}}{\bar{a}^3} + \frac{f_{\mathrm{rad}}}{\bar{a}^4} \frac{\gamma}{\gamma_{\mathrm{cd}}} \frac{\mathcal{S}}{\mathcal{G}^{1/3}}} \,,$$

$$\mathcal{S}' = \frac{r\,\bar{a}^4}{f_{\mathrm{rad}}} \mathcal{G}^{1/3} \gamma_{\mathrm{cd}} \,,$$

$$r' = -r - 3\,\frac{\bar{a}'}{\bar{a}}\,\zeta\, r \,,$$

$$\mathcal{G}' = -\frac{3}{4} \frac{T_{\mathrm{SM}}^{\mathrm{cd}}\,\mathcal{G}\,\hat{\mathcal{G}}}{\mathcal{S}^{3/4}\,\bar{a}} \frac{4\,\mathcal{S}\,\bar{a}' - \mathcal{S}'\,\bar{a}}{T_{\mathrm{SM}}^{\mathrm{cd}}\,\hat{\mathcal{G}}\,\mathcal{S}^{1/4} + 3\,\mathcal{G}^{4/3}\bar{a}} \,,$$

$$\gamma' = \hat{\gamma}\, T_{\mathrm{SM}}^{\mathrm{cd}} \frac{3\,\mathcal{G}\,\bar{a}\,\mathcal{S}' - 12\,\mathcal{G}\,\bar{a}'\,\mathcal{S} - 4\,\mathcal{G}'\,\bar{a}\,\mathcal{S}}{12\,\mathcal{G}^{4/3}\,\mathcal{S}^{3/4}\,\bar{a}^2} \,.$$

with initial condition $\bar{a}_{\mathrm{cd}} = \mathcal{S}_{\mathrm{cd}} = r_{\mathrm{cd}} = \mathcal{G}_{\mathrm{cd}} = 1$ and γ_{cd}.

- Normalized scale factor $\bar{a} = a/a_{\mathrm{cd}}$
- Characteristic time scale $\theta_{\mathrm{H}} = \sqrt{3\,m_{\mathrm{Pl}}^2\,\Gamma^2 \rho_{\mathrm{med}}^{\mathrm{cd}}}$
- Normalized mediator energy density $r = \rho_{\mathrm{med}}/\rho_{\mathrm{med}}^{\mathrm{cd}}$
- Normalized initial DM density $f_{\mathrm{mat}} = \rho_{\mathrm{DM}}^{\mathrm{cd}}/\rho_{\mathrm{med}}^{\mathrm{cd}}$
- Normalized initial radiation energy density $f_{\mathrm{rad}} = \rho_{\mathrm{rad}}^{\mathrm{cd}}/\rho_{\mathrm{med}}^{\mathrm{cd}}$
- Normalized DOFs $\gamma = g_{\mathrm{eff},\rho}^{\mathrm{SM}}/g_{\mathrm{eff},s}^{\mathrm{SM}}$
- Normalized DOFs $\mathcal{G} = g_{\mathrm{eff},s}^{\mathrm{SM}}/g_{\mathrm{eff},s}^{\mathrm{SM,cd}}$
- Normalized SM entropy $\mathcal{S} = \left(S_{\mathrm{SM}}/S_{\mathrm{SM}}^{\mathrm{cd}}\right)^{4/3}$

Redshift and dilution of the GW background

After its emission, the GW signal gets red-shifted:

$$h^2\,\Omega_{\mathrm{GW}}(f) = \mathcal{R}h^2\;\Omega^{\mathrm{n}}_{\mathrm{GW}}\left(\frac{a_0}{a_{\mathrm{n}}}\,f\right)$$

Energy density:

$$\mathcal{R}h^2 \simeq \frac{2.4\cdot 10^{-5}}{D_{\mathrm{SM}}^{4/3}}\left(\frac{g^{\mathrm{SM},0}_{\mathrm{eff},s}}{g^{\mathrm{SM},\mathrm{n}}_{\mathrm{eff},s}}\right)^{4/3}\frac{g^{\mathrm{tot},\mathrm{n}}_{\mathrm{eff},\rho}}{2}$$

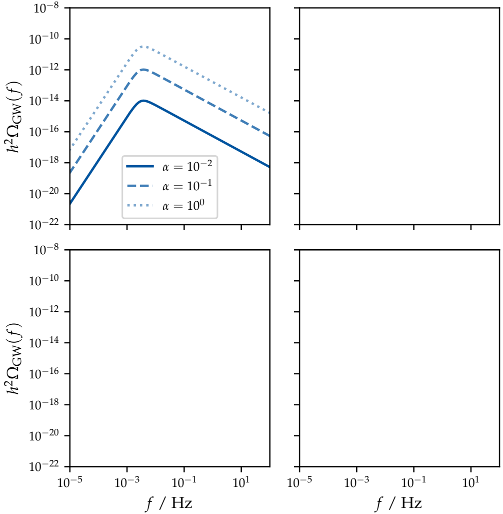
Frequency:

$$\frac{a_0}{a_{\mathrm{n}}} = D_{\mathrm{SM}}^{1/3}\left(\frac{g^{\mathrm{SM},\mathrm{n}}_{\mathrm{eff},s}}{g^{\mathrm{SM},0}_{\mathrm{eff},s}}\right)^{1/3}\frac{T^{\mathrm{n}}_{\mathrm{SM}}}{T^{0}_{\mathrm{SM}}}$$

Transition strength:

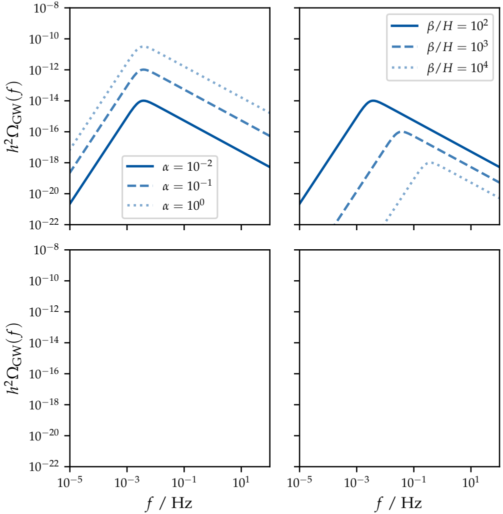
$$\alpha = \frac{\epsilon}{\rho_{\text{rad}}^{\text{n}}}$$

relates the latent heat ϵ of the transition with the energy density $\rho_{\text{rad}}^{\text{n}}$ of the surrounding heat bath. For fixed T_{DS}^{n}: $\rho_{\text{rad}}^{\text{n}} \propto \xi_{\text{n}}^{-4}$. The transition strength thus grows $\propto \xi_{\text{n}}^{4}$!



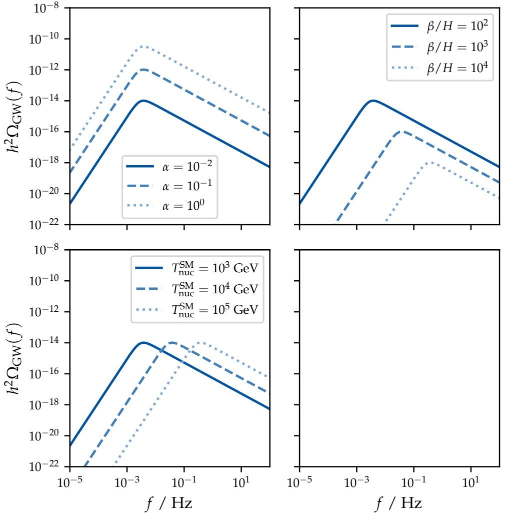
Parametrization of the GW signal

Inverse time scale:
The computation of β/H is complicated, but shows no relevant dependence of the temperature ratio between the sectors. Larger β/H indicate fast transitions. In that case, many small bubbles collide, resulting in weak signals at high frequencies.



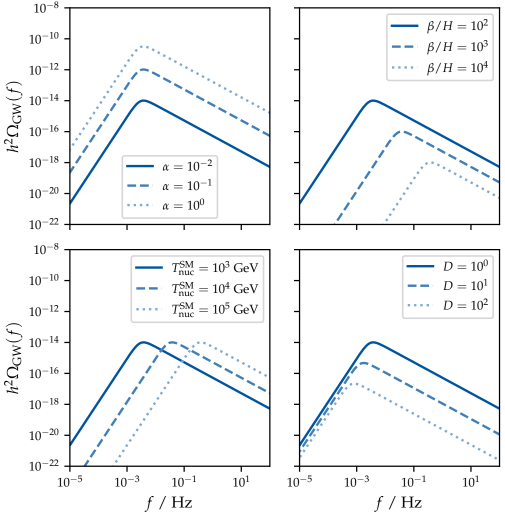
Parametrization of the GW signal

Nucleation temperature:
Keeping T_{DS}^{n} fixed, a larger temperature ratio ξ_{n} at nucleation leads to a lower T_{SM}^{n}. This corresponds to lower peak frequencies.



Dilution:

The redshift to lower frequencies and signals strengths increases with the dilution factor. D grows with the temperature ratio ξ_{n}, as more energy is injected into the SM bath from the dark sector. Unlike D_{SM}, D saturates for high temperature ratios.



The $U(1)_D$ model in detail

Lagrangian:

$$\mathcal{L} \supset |D_\mu \Phi|^2 + |D_\mu H|^2 - \frac{1}{4} B'_{\mu\nu} B'^{\mu\nu} - \frac{\epsilon}{2} B'_{\mu\nu} B^{\mu\nu} - V(\Phi, H) ,$$

$$D_\mu \Phi = \left(\partial_\mu + i\, g\, B'_\mu\right) \Phi ,$$

$$V_{\text{tree}}(\Phi, H) = -\mu^2\, \Phi^* \Phi + \lambda\, (\Phi^* \Phi)^2 - \mu_H^2\, H^\dagger H + \lambda_H\, (H^\dagger H)^2 + \lambda_p\, (\Phi^* \Phi)\, (H^\dagger H) .$$

Mass spectrum:

$$m^2_{(h,\,\phi)}(h, \phi) = \begin{pmatrix} -\mu_H^2 + 3\,\lambda_H\, h^2 + \frac{\lambda_p}{2} \phi^2 & \lambda_p\, h\, \phi \\ \lambda_p\, h\, \phi & -\mu^2 + 3\,\lambda\, \phi^2 + \frac{\lambda_p}{2} h^2 \end{pmatrix} ,$$

$$m^2_{G^0, G^+}(h, \phi) = -\mu_H^2 + \lambda_H\, h^2 + \frac{\lambda_p}{2} \phi^2 ,$$

$$m^2_\varphi(h, \phi) = -\mu^2 + \lambda\, \phi^2 + \frac{\lambda_p}{2} h^2 .$$

The $U(1)_D$ model in detail

For $\lambda_p, \epsilon \to 0$ and $\mu^2 = \lambda\, v^2$, the field-dependent dark Higgs and dark photon masses are given by

$$m_{\mathsf{DP}} = g\,\phi \overset{T=0}{=} g\,v\,, \qquad m_{\mathsf{DH}} = \sqrt{2\,\lambda}\,\phi \overset{T=0}{=} \sqrt{2\,\lambda}\,v\,.$$

The corresponding Debye masses are

$$\Pi_\Phi(T_{\mathsf{DS}}) = \left(\frac{\lambda}{3} + \frac{g^2}{4}\right) T_{\mathsf{DS}}^2\,, \qquad \Pi^{\mathsf{L}}_{A'}(T_{\mathsf{DS}}) = \frac{g^2}{3}\, T_{\mathsf{DS}}^2\,.$$

- Quartic dark Higgs coupling: λ
- $U(1)_{\mathsf{D}}$ gauge coupling: g
- Dark Higgs lifetime: τ
- Dark Higgs VEV: $v = \frac{\mu}{\sqrt{\lambda}}$
- Temperature ratio: $\xi_{\mathsf{n}} = \left.\frac{T_{\mathsf{DS}}}{T_{\mathsf{SM}}}\right|_{\mathsf{n}}$

Signal-to-noise ratios for LISA and the ET

Compute the overlap of the signals $h^2\,\Omega_{\mathsf{GW}}(f)$ and expected sensitivities $h^2\,\Omega_{\mathsf{obs}}(f)$ and weight it with the duration of the observation t_{obs} to obtain a signal-to-noise measure:

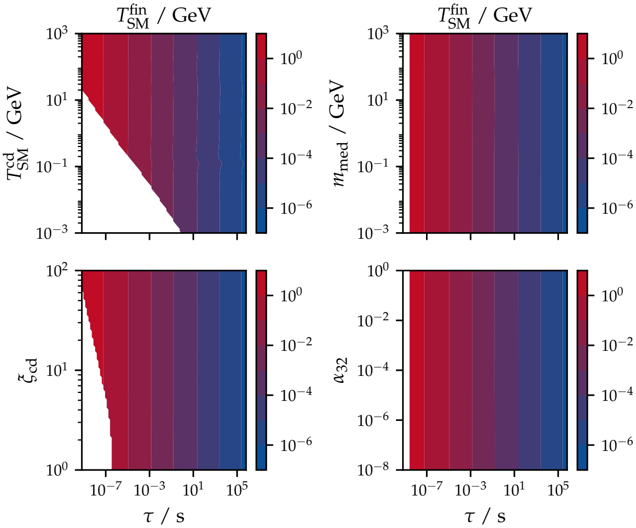
$$\rho^2 = t_{\mathsf{obs}} \int_{f_{\mathsf{min}}}^{f_{\mathsf{max}}} \mathrm{d}f \, \left[\frac{h^2\,\Omega_{\mathsf{GW}}(f)}{h^2\,\Omega_{\mathsf{obs}}(f)} \right]^2$$

If ρ exceeds a certain threshold value for a given signal, the signal is observable.

To analyze the impact of ξ_{n} and τ on the observability of the signals produced by our model, consider the benchmark points

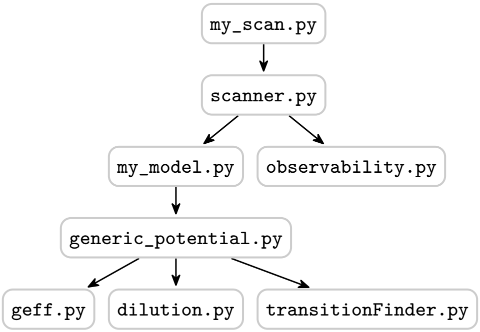
Benchmark point	λ	g	v
LISA	$1.5 \cdot 10^{-3}$	0.5	$2\,$TeV
ET	$1.5 \cdot 10^{-3}$	0.5	$10\,$PeV

Final temperature independent of all input parameters except lifetime



Our extensions to CosmoTransitions

Structure:



Example output: