Dark Matter and Electroweak Phase Transition in the Inert Doublet Model

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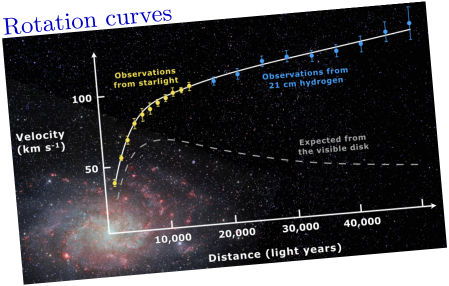
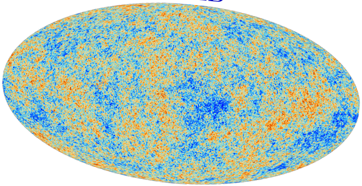
talk based on JCAP09(2021)011 by Florian Goertz, Yun Jiang, and SF

Motivation

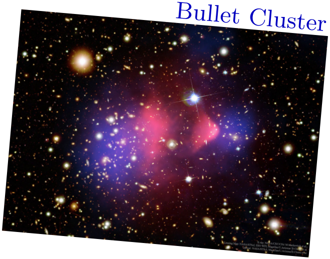
Rotation curves



CMB



Bullet Cluster



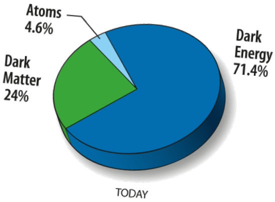
What is the new form of matter?

Why is there matter at all?

Standard Model cannot explain → New Physics required

Background – Dark Matter

- 'ordinary' matter only a small fraction of entire energy content
- WIMP (*weakly interacting massive particle*) as a possible candidate among others



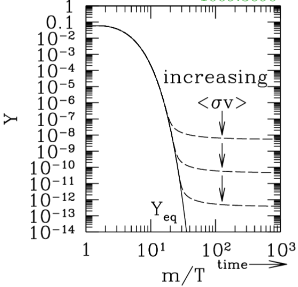
- number density n and entropy density s give rise to comoving DM abundance

$$Y\left(\frac{m}{T}\right) \stackrel{\text{def}}{=} n/s$$

- relic abundance after freeze-out:

$$\Omega h^2 = 0.1200(12)$$

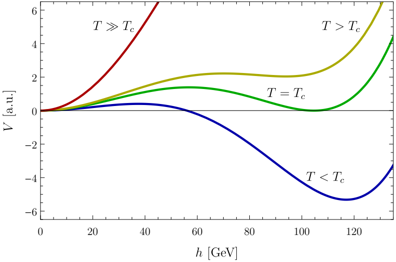
1009.3690

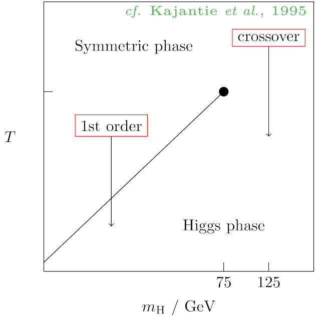


Background – Electroweak baryogenesis

- Sakharov conditions for successful EWBG:
  (1) violation of baryon number conservation
  (2) C and CP violation
  (3) departure from thermal equilibrium

- problems of the SM
  (1) lacking sufficient CP violation in CKM matrix
  (2) SM Higgs being too heavy





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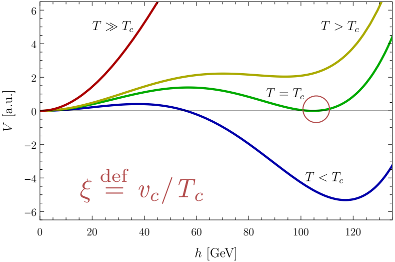

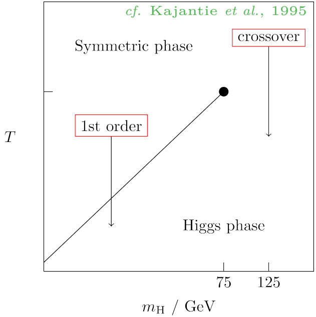

Background – Electroweak baryogenesis

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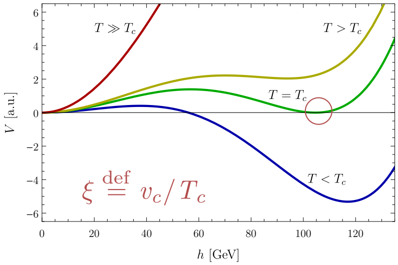
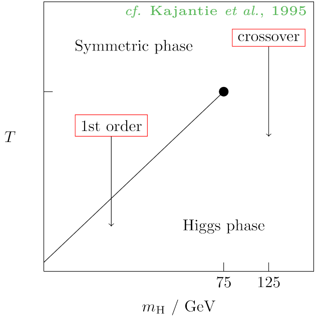
SM cannot be the final answer!





Inert Doublet Model

further reading: hep-ph/0612275, 1204.4722, 1207.0084, 1504.05949, 1612.00511, ...

- extended scalar sector of the SM:

$$H_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}\sqrt{2}\phi^+ \\ h + i\phi\end{pmatrix} \quad , \quad H_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}\sqrt{2}H^+ \\ H + iA\end{pmatrix}$$

- DM particle H stable for $\mathbb{Z}_2$ symmetry

$\mathbb{Z}_2$ symmetry:
$H_1 \to H_1$
$H_2 \to -H_2$

$$\begin{aligned} V_{\text{tree}} = & \mu_1^2 |H_1|^2 + \mu_2^2 |H_2|^2 + \lambda_1 |H_1|^4 + \boxed{\lambda_2} |H_2|^4 \\ & + \boxed{\lambda_3} |H_1|^2 |H_2|^2 + \boxed{\lambda_4} \left|H_1^\dagger H_2\right|^2 + \frac{\boxed{\lambda_5}}{2}\left[\left(H_1^\dagger H_2\right)^2 + \text{h.c.}\right] \end{aligned}$$

- free parameters: $\{\lambda_2, \underbrace{\boxed{\lambda_{345}}}_{= \lambda_3 + \lambda_4 + \lambda_5}, m_H, m_{H^\pm}, m_A\}$

Theoretical and experimental constraints

vacuum stability

perturb. unitarity

EW precision test

LEP data analysis

exotic Higgs decay

requiring stability:

$$\lambda_1 > 0 \quad \textbf{fine}$$
$$\lambda_2 > 0 \quad \textbf{by choice}$$
$$\lambda_3 > -2\sqrt{\lambda_1 \lambda_2}$$
$$\lambda_3 + \lambda_4 - |\lambda_5| > -2\sqrt{\lambda_1 \lambda_2}$$

avoiding charge-breaking vacuum:

$$\lambda_4 - |\lambda_5| < 0$$

1612.00511

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perturbativity & unitarity:

$|c_i| < 8\pi \quad (\Rightarrow |\lambda_i| < 4\pi)$

$$c_{1,2} = \lambda_3 \pm \lambda_4$$

$$c_{3,4} = -3\lambda_1 - 3\lambda_2 \pm \sqrt{9\left(\lambda_1 - \lambda_2\right)^2 + \left(2\lambda_3 + \lambda_4\right)^2}$$

$$c_{5,6} = \lambda_3 \pm \lambda_5$$

$$c_{7,8} = -\lambda_1 - \lambda_2 \pm \sqrt{\left(\lambda_1 - \lambda_2\right)^2 + \lambda_5^2}$$

$$c_{9,10} = \lambda_3 + 2\lambda_4 \pm 3\lambda_5$$

$$c_{11,12} = -\lambda_1 - \lambda_2 \pm \sqrt{\left(\lambda_1 - \lambda_2\right)^2 + \lambda_4^2}$$

1612.00511

Theoretical and experimental constraints

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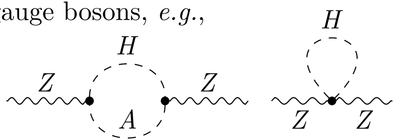
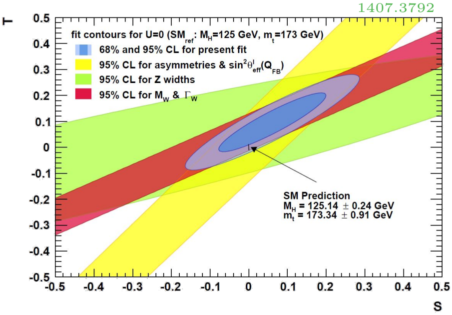
contributions to self-energies of EW gauge bosons, *e.g.*,





$$S|_{U=0} = 0.06 \pm 0.09\ , \quad T|_{U=0} = 0.10 \pm 0.07$$

Theoretical and experimental constraints

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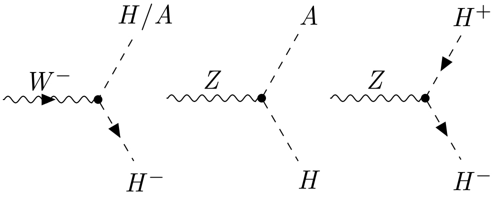

$$\Rightarrow \begin{cases} m_{H,A} + m_{H^\pm} > m_{W^\pm} \\ m_H + m_A > m_Z \\ 2m_{H^\pm} > m_Z \end{cases}$$

reinterpretation of LEP-II data:

$$m_H > 80 \text{ GeV} \cup m_A > 100 \text{ GeV} \cup \boxed{\overset{m_A - m_H}{\Delta \mathrm{m}}} < 8 \text{ GeV}$$

and in addition $m_{H^\pm} > 70$ GeV

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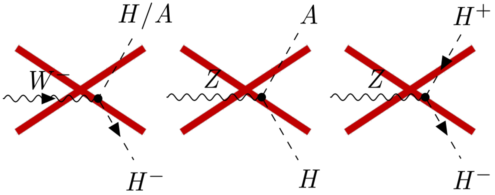

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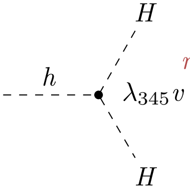
Theoretical and experimental constraints

- vacuum stability
- perturb. unitarity
- EW precision test
- LEP data analysis
- **exotic Higgs decay**

decays of the SM Higgs to lighter, $\mathbb{Z}_2$-odd scalars, *e.g.*,



$m_h = 125$ GeV

results from collider experiments:

$$\mathrm{BR}\,(h \to \mathrm{inv.}) \leq \begin{cases} 0.26 \text{ from ATLAS} \\ 0.19 \text{ from CMS} \end{cases}$$

Dark Matter in the IDM

- Boltzmann equation for DM number density n_H (freeze-out):

$$\frac{\mathrm{d}n_H}{\mathrm{d}t} = -3Hn_H - \langle\sigma_{\text{eff}} v\rangle \left[n_H^2 - \left(n_H^{\text{eq}}\right)^2\right]$$

$\rightarrow$ solve numerically with MICROMEGAs

$\Delta m \stackrel{\text{def}}{=} m_{A,H^{\pm}} - m_H$

- viable parameter spaces with significant amount of DM
- strong direct-detection limits from XENON1T experiment

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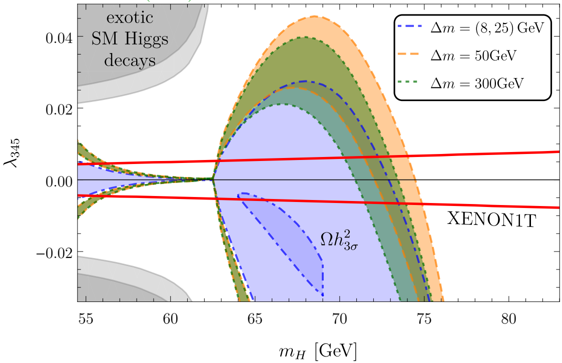
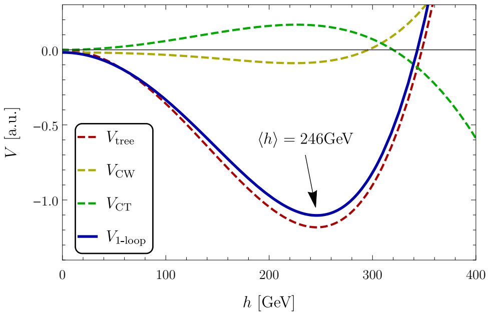
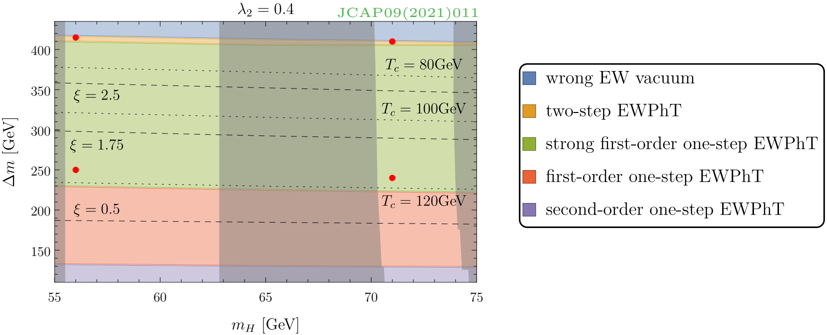


Finite-temperature effective potential

$$V_{\text{eff}}(h, H, T) = V_{\text{tree}} + V_{\text{CW}}(h, H) + V_{\text{CT}}(h, H) + V_T(h, H, T)$$

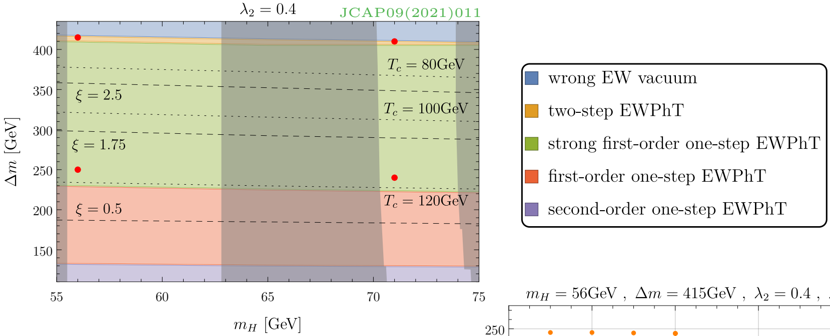


EWPhT with Dark Matter



correct relic abundance & strong 1st-order EWPhT

EWPhT with Dark Matter



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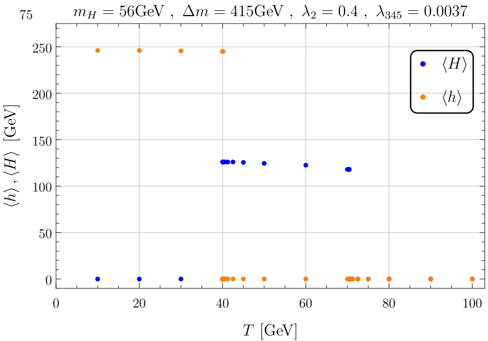

Conclusions & Outlook

- investigation of the IDM as a simple extension of the SM
- IDM can account both for DM and for strong 1st-order EWPhT
- both one- and two-step strong 1st-order EWPhT

  $\rightarrow$ testable gravitational wave signatures?
- IDM is lacking sufficient CP violation

  $\rightarrow$ induce additional CP violation by adding

$$\mathcal{L}_{\not{C}\mathrm{P}} = \frac{c_1}{2} \left|H_2\right|^2 \widetilde{F}_{\mu\nu} F^{\mu\nu} + c_2 \left(H_1^\dagger H_2\right) \left(\overline{Q}_L H_2 d_R + \overline{Q}_L \widetilde{H}_2 u_R\right) + \text{h.c.}$$

with $c_{1,2} \sim \text{TeV}^{-2}$

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with $c_{1,2} \sim \mathrm{TeV}^{-2}$

Thanks for your attention

Feel free to contact me for discussions: *fabian@mpi-hd.mpg.de*

Sources for images

Sec. *Motivation*:

- Rotation curves

  By Mario De Leo - Own work, CC BY-SA 4.0, `https://commons.wikimedia.org/w/index.php?curid=74398525`

- CMB

  `https://www.esa.int/ESA_Multimedia/Images/2013/03/Planck_CMB`

- Bullet Cluster

  `https://science.nasa.gov/matter-bullet-cluster`

Sec. *Background – Dark Matter*

- Universe's energy budget

  `https://map.gsfc.nasa.gov/universe/uni_matter.html`

Last access of each webpage: September 20th, 2021 at 17:45 CEST

Backup slides

Backup: field-dependent masses

$$\hat{m}_V^2\,(h,H) = \frac{h^2 + H^2}{v^2}\,m_V^2 \quad , \quad \hat{m}_f^2\,(h) = \frac{h^2}{2}\,y_f^2\,,$$

$$\widehat{M}_S^2 = \frac{1}{2}\begin{pmatrix} 6\lambda_1 h^2 - 2\lambda_1 v^2 + \lambda_{345} H^2 & 2hH\lambda_{345} \\ 2hH\lambda_{345} & 6\lambda_2 H^2 + \lambda_{345} h^2 + 2\mu_2^2 \end{pmatrix}$$

$$\widehat{M}_P^2 = \frac{1}{2}\begin{pmatrix} 2\lambda_1 h^2 - 2\lambda_1 v^2 + \bar{\lambda}_{345} H^2 & 2hH\lambda_5 \\ 2hH\lambda_5 & 2\lambda_2 H^2 + \bar{\lambda}_{345} h^2 + 2\mu_2^2 \end{pmatrix}$$

$$\widehat{M}_\pm^2 = \frac{1}{2}\begin{pmatrix} 2\lambda_1 h^2 - 2\lambda_1 v^2 + \lambda_3 H^2 & hH\,(\lambda_4 + \lambda_5) \\ hH\,(\lambda_4 + \lambda_5) & 2\lambda_2 H^2 + \lambda_3 h^2 + 2\mu_2^2 \end{pmatrix}$$

Backup: renormalization conditions

renormalization conditions:

$$\left.\frac{\partial V_{\mathrm{CT}}}{\partial h}\right|_{\mathrm{vev}} = -\left.\frac{\partial V_{\mathrm{CW}}}{\partial h}\right|_{\mathrm{vev}}$$

$$\left.\frac{\partial^2 V_{\mathrm{CT}}}{\partial h^2}\right|_{\mathrm{vev}} = -\left.\left(\frac{\partial^2 \left.V_{\mathrm{CW}}\right|_{n_{\phi(\pm)}=0}}{\partial h^2} + \frac{1}{32\pi^2}\sum_{i=\phi,\phi^\pm} n_i \left(\frac{\partial \hat{m}_i^2\,(h,H)}{\partial h}\right)^2 \ln\frac{m_{\mathrm{IR}}^2}{Q^2}\right)\right|_{\mathrm{vev}}$$

$$\left.\frac{\partial^2 V_{\mathrm{CT}}}{\partial H^2}\right|_{\mathrm{vev}} = -\left.\left(\frac{\partial^2 \left.V_{\mathrm{CW}}\right|_{n_{\phi(\pm)}=0}}{\partial H^2} + \frac{1}{32\pi^2}\sum_{i=\phi,\phi^\pm} n_i \left(\frac{\partial \hat{m}_i^2\,(h,H)}{\partial H}\right)^2 \ln\frac{m_{\mathrm{IR}}^2}{Q^2}\right)\right|_{\mathrm{vev}}$$

Remove the Goldstone modes from the CW potential in the second and third line and adding instead the regular sums over Goldstone modes on the right-hand sides.

Backup: scalars' tree-level masses

$$M_S^2 = \begin{pmatrix} 2\lambda_1 v^2 & 0 \\ 0 & \mu_2^2 + \lambda_{345} v^2/2 \end{pmatrix}$$

$$M_P^2 = \frac{1}{2}\begin{pmatrix} 0 & 0 \\ 0 & \bar{\lambda}_{345} v^2 + 2\mu_2^2 \end{pmatrix}$$

$$M_\pm^2 = \frac{1}{2}\begin{pmatrix} 0 & 0 \\ 0 & \lambda_3 v^2 + 2\mu_2^2 \end{pmatrix}$$

$$m_H^2 = \mu^2 + \frac{\lambda_{345}}{2} v^2 \quad , \quad m_A^2 = \mu^2 + \frac{\bar{\lambda}_{345}}{2} v^2 \quad , \quad m_{H^\pm}^2 = \mu^2 + \frac{\lambda_3}{2} v^2$$

$$m_h^2 = 2\lambda_1 v^2 \quad , \quad \text{with } \overset{(-)}{\lambda}_{345} \overset{\text{def}}{=} \frac{\lambda_3 + \lambda_4 \pm \lambda_5}{2}$$

Backup: EW precision tests

$$S = \frac{x_2^6 f_a(x_2) - x_1^6 f_a(x_1) + 9x_1^2 x_2^2 \left(x_2^2 f_b(x_2) - x_1^2 f_b(x_1)\right)}{72\pi \left(x_2^2 - x_1^2\right)^3}$$

$$T = \frac{f_c\left(m_{H^\pm}^2, m_A^2\right) + f_c\left(m_{H^\pm}^2, m_H^2\right) - f_c\left(m_A^2, m_H^2\right)}{32\pi^2 \alpha v^2}$$

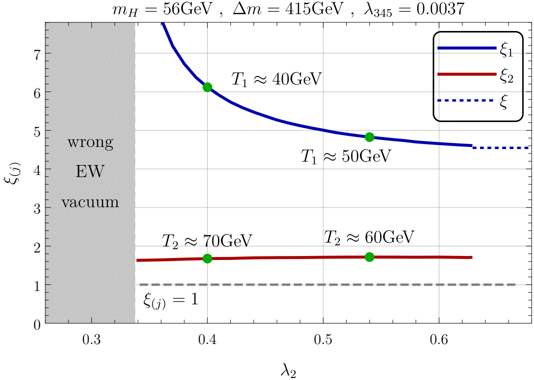
$$f_a(x) \stackrel{\text{def}}{=} -5 + 12\ln x \quad , \quad f_b(x) \stackrel{\text{def}}{=} 3 - 4\ln x$$

$$f_c(x, y) \stackrel{\text{def}}{=} \begin{cases} \frac{x+y}{2} - \frac{xy}{x-y}\ln\frac{x}{y} & \text{for } x \neq y \\ 0 & \text{for } x = y \end{cases}$$

with

$$x_1 \stackrel{\text{def}}{=} \frac{m_H}{m_{H^\pm}} \quad , \quad x_2 \stackrel{\text{def}}{=} \frac{m_A}{m_{H^\pm}} \; .$$

Backup: λ_2-dependence



Backup: T-contributions to EW gauge boson masses

$$\tilde{m}^2_{W_L} = \frac{h^2 + H^2}{v^2}\, m^2_W + 2 g^2_W\, T^2$$

$$\tilde{m}^2_{Z_L,\gamma_L} = \frac{h^2 + H^2 + 8\, T^2}{8} \left(g^2_W + g'^2\right) \pm \Delta$$

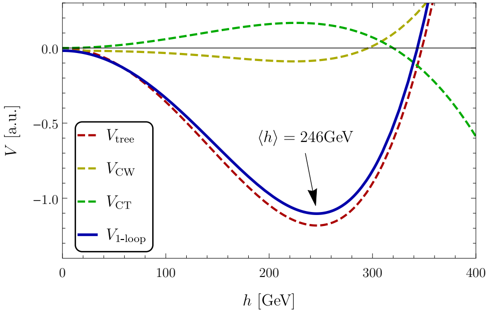
$$\text{with } \Delta^2 \stackrel{\text{def}}{=} \frac{\left(h^2 + H^2 + 8\, T^2\right)^2}{64} \left(g^2_W + g'^2\right)^2 - g^2_W\, g'^2\, T^2 \left(h^2 + H^2 + 4\, T^2\right)$$

Backup: Finite-temperature effective potential

$$V_{\text{eff}}(h, H, T) = V_{\text{tree}} + V_{\text{CW}}(h, H) + V_{\text{CT}}(h, H) + V_T(h, H, T)$$

$$V_{\text{CW}}(h, H) = \sum_i \frac{n_i}{64\pi^2} \hat{m}_i^4(h, H) \left[\ln\left(\frac{\hat{m}_i^2}{Q^2} \right) - C_i \right]$$

$$V_{\text{CT}}(h, H) = \delta m_h^2 h^2 + \delta m_H^2 H^2 + \delta\lambda_1 h^4$$



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$$V_{\text{eff}}(h, H, T) = V_{\text{tree}} + V_{\text{CW}}(h, H) + V_{\text{CT}}(h, H) + V_T(h, H, T)$$

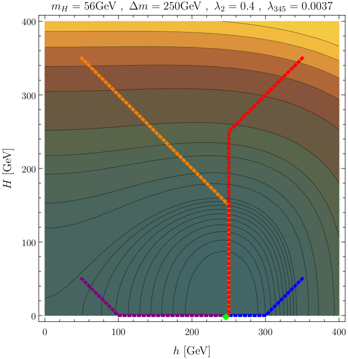
$$V_T(h, H) = \frac{T^4}{2\pi^2}\left[\sum_i n_i^{\text{B}} J_{\text{B}}\left(\frac{\tilde{m}_i^2(h, H, T)}{T^2}\right) + \sum_i n_i^{\text{F}} J_{\text{F}}\left(\frac{\hat{m}_i^2(h, H)}{T^2}\right)\right]$$

with T-dependent boson masses $\tilde{m}_i$ and

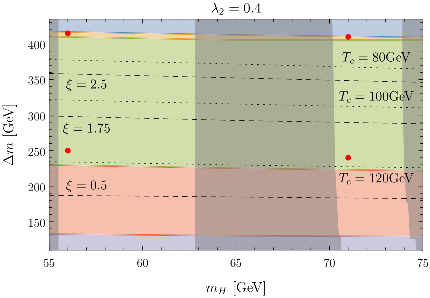
$$J_{\text{B/F}}(x) \stackrel{\text{def}}{=} \pm \int_0^\infty \mathrm{d}t\; t^2 \ln\left[1 \mp e^{-\sqrt{t^2+x}}\right]$$

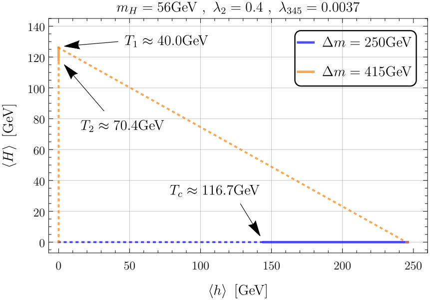
$$= \lim_{N\to\infty} \mp \sum_{l=1}^{N} \frac{(\pm 1)^l x}{l^2} K_2\left(\sqrt{x}\, l\right)$$

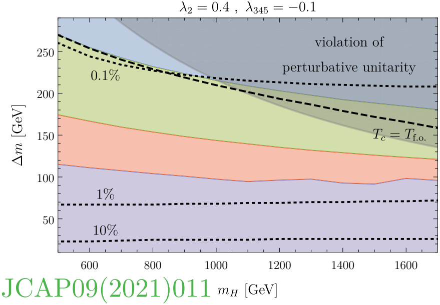
- masses of longitudinal components $W_L^{\pm}$, Z_L, γ_L get affected by finite T

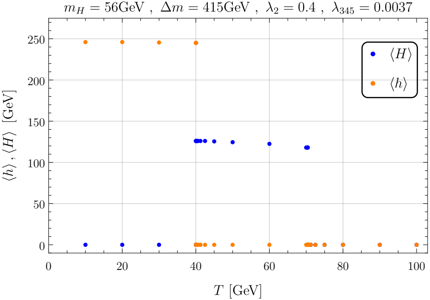


Backup: EWPhT with Dark Matter









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Backup: Effective annihilation cross-section

$$\langle \sigma_{\text{eff}} v \rangle \stackrel{\text{def}}{=} \sum_{j=1}^{N} \langle \sigma v \rangle_{Hj} \frac{n_H^{\text{eq}} n_j^{\text{eq}}}{\left(n^{\text{eq}}\right)^2} \text{ with } n^{\text{eq}} \stackrel{\text{def}}{=} \sum_i n_i^{\text{eq}}$$