Statistical Analysis of the $N_{\mathrm{DW}} = 1$ QCD Axion Mass Window from Topological Defects

DESY Theory Workshop 2021,
arxiv.org/abs/2108.09563

Sebastian Hoof, **Jana M. Rieß**, David J. E. Marsh

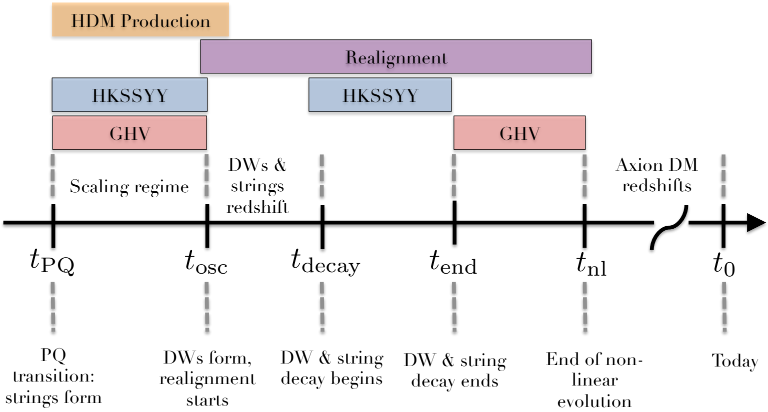
Georg-August Universität Göttingen

21. September, 2021

Motivation

- Axions are a promising dark matter candidates.
- Symmetry breaking after inflation $\Rightarrow$ axion misalignment angle varies over one Hubble patch, in principle no open parameter.
- Contribution from **topological defects** makes computation more difficult, **Issue for the last decades**.
- What QCD axion mass do we expect if it is the dark matter?
- **Goal: Provide framework for comparing different results including statistical errors.**

Different Contributions



Different Contributions

Focus on string scaling in this talk.

What are Topological Strings?

- Axion = angular degree of freedom of the PQ field: $\phi = f_a \arg \Phi$
- PQ field: "wine bottle potential"

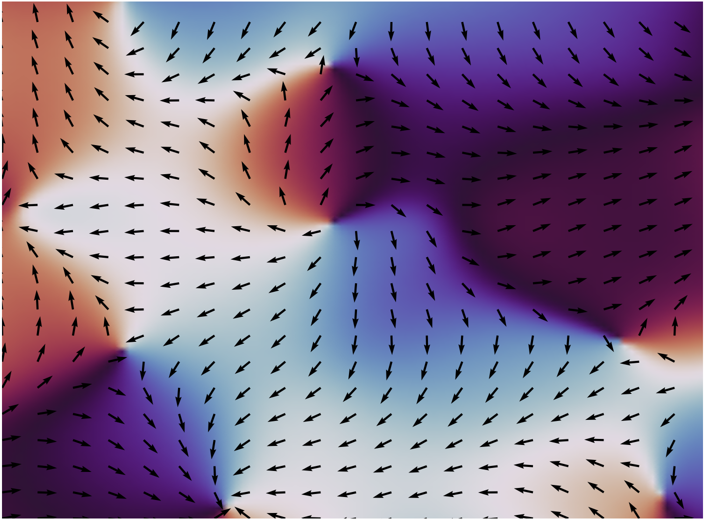
$$V(\Phi) = \lambda_r \left(|\Phi|^2 - \frac{f_a^2}{2} \right)^2 . \qquad (1)$$

- radial mode mass m_r:

$$\lambda_r \equiv m_r^2 / 2 f_a^2 \qquad (2)$$

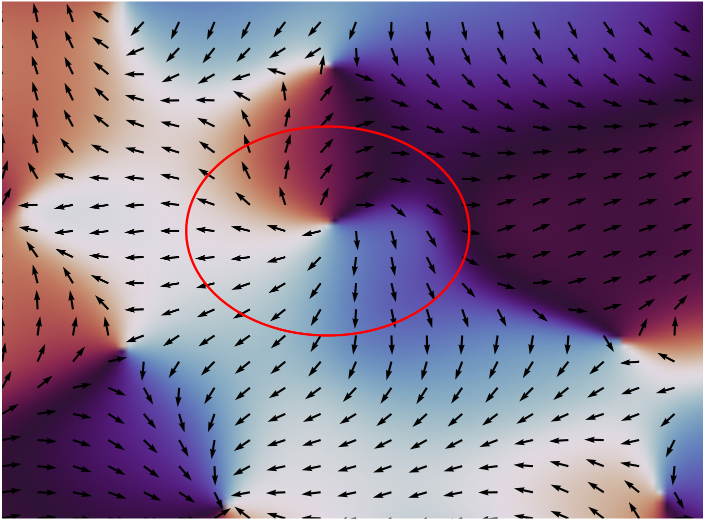
What are Topological Strings?

- In our scenario it takes random values across the Hubble patch:



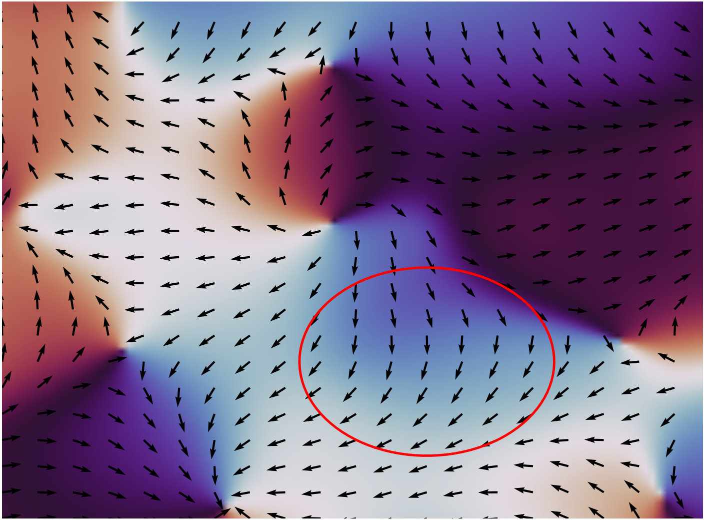
What are Topological Strings?

- Closed loops integrate to a winding number.
- This loop **does** contain a string.



What are Topological Strings?

- Closed loops integrate to a winding number.
- This loop **does not** contain a string.



What are Topological Strings?

- In 3D: a one-dimensional structure (strings)
- Change in the string energy density is converted into axions.

Difficulties in Simulations

Two scales:

- Size of the string core $\sim 1/m_r$
- Size of the Hubble patch $\sim 1/H$

$\Rightarrow l \equiv \log(m_r/H) \sim \mathcal{O}(70), m_r/H \sim 10^{30}$

Hence:

- Not feasible to simulate in the physical regime!!!
- Hopeless? No!

The Attractor Solution

The string network is thought to approach an attractor solution.
Characterized by the parameters:

- The co-moving string length per Hubble volume:

$$\xi(t) \equiv \lim_{V \to \infty} \left[\frac{L_{\mathrm{tot}}(V)\, t^2}{V} \right] , \qquad (3)$$

- The emission spectrum into axions:

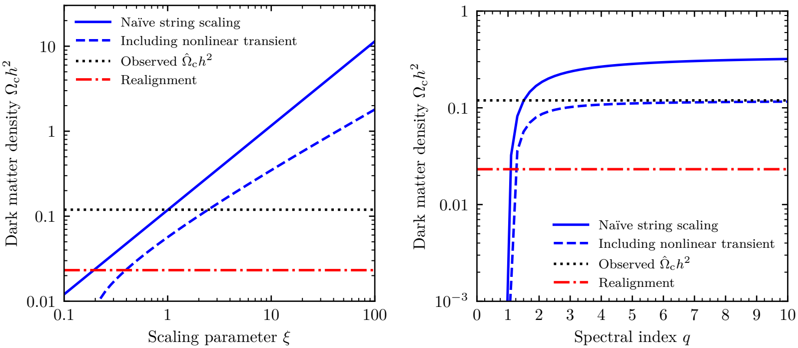
$$\dot{\rho}_s + 2H\rho_s = -\Gamma \approx -\Gamma_a \qquad (4)$$

$$\frac{\partial \Gamma_a(t,k)}{\partial k} = \frac{\Gamma_a(t)}{H(t)} F\left[\frac{k}{H(t)}, \frac{m_r}{H(t)} \right] \qquad (5)$$

$$F[x,y] \propto \frac{\left(\frac{x}{x_0}\right)^{q'} \left[1 + \Theta(x - x'') \left(\left(\frac{x''}{x} \right)^{q''-q} - 1 \right) \right]}{\left(\frac{x}{x_0}\right)^{q'+1} + 1} , \qquad (6)$$

Relic Density from Strings

Integrating $F[x, y]$ appropriately over momentum k and time t yields the axion number density and hence the relic density.

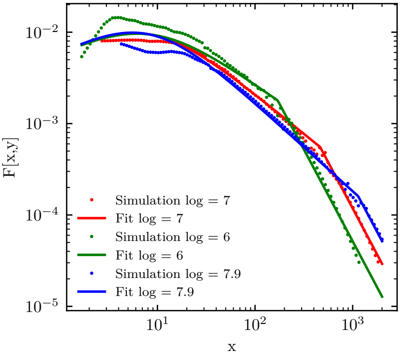


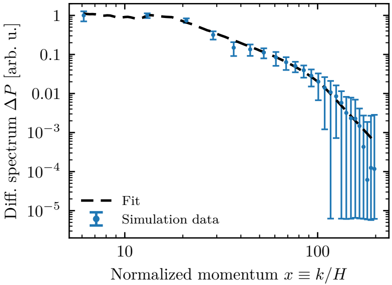
Where are the disagreements?

- What values of ξ and q does the system approach?
- $q > 1 \Rightarrow \Omega_{\rm str} > \Omega_{\rm align}$
- $q < 1 \Rightarrow \Omega_{\rm str} \ll \Omega_{\rm align}$
- Constant or scaling-violation (they change/increase with l)?
- Our work: Consider the results of two groups:
  - Takashi Hiramatsu, Masahiro Kawasaki, Toyokazu Sekiguchi, Ken'ichi Saikawa, Masahide Yamaguchi, Jun'ichi Yokoyama (HKSSYY)
  - Marco Gorghetto, Edward Hardy, Giovanni Villadoro (GHV)

[Hiramatsu, Kawasaki, Sekiguchi, Yamaguchi, Yokoyama, 2010] [Hiramatsu, Kawasaki, Saikawa, Sekiguchi, 2012] [Kawasaki, Saikawa, Sekiguchi, 2015] [Gorghetto, Hardy, Villadoro, 2018] [Gorghetto, Hardy, Villadoro, 2020]

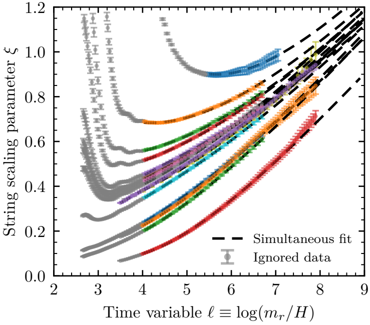
Fitting the Spectra:

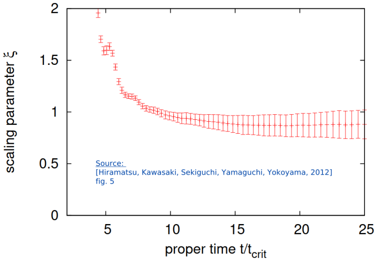




- GHV: scaling violation: $q = (0.154 \pm 0.057) + (0.1030 \pm 0.0071)\, l$
- HKSSYY: non-scaling violation: $q = 1.44 \pm 0.16$

Fitting the scaling parameter:





- GHV: scaling violation: $\xi = (-1.618 \pm 0.038) + (0.2428 \pm 0.0025)l$
- HKSSYY: non-scaling violation: $\xi = 0.87 \pm 0.14$

Other Effects included

- **Realignment**
  - Including QCD axion potential at finite T and $g_*(T)$ dependence.
  - Average over oscillations.
  - Numerical averaging over initial field values. $[-\pi, \pi)$.
- **Domain Wall Decay**
  - Simulated only by the HKSSYY group.
  - Parameters taken from their paper.
  - Only for $N_{\rm DW} = 1$.
- **Non-linear transit**
  - Pointed out and simulated by GHV.
  - Large gradients delay the point where the axion mass becomes effective.
  - Suppresses relic density.
- **Thermal Production**
  - Coupling to gluons and pions.
  - Upper-bound from $\Delta N_{\rm eff}$

Uncertainties

Included:

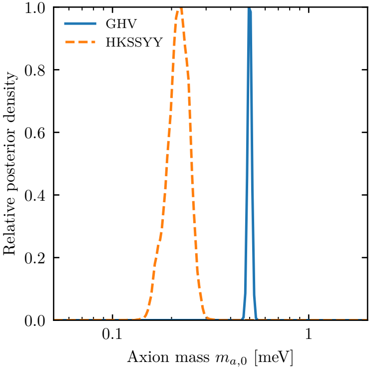
- *Statistical* errors of the fits of the string parameters ξ, q and x_0 (with bootstrapping).
- Uncertainties of the domain wall parameters σ_{DW} and ε.
- Uncertainties of the QCD axion mass inc. temperature dependence.
- Uncertainties of standard model parameters.

Not included:

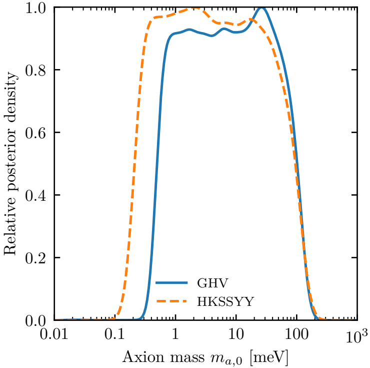
- Errors on $g_*(T)$

Results from the MCMC Scans

$\Omega_a = \Omega_{\mathrm{DM}}$



$\Omega_a < \Omega_{\mathrm{DM}}$



Summary and Conclusion

Work done:

- Statistical analysis of the axion relic density in the post-inflationary symmetry breaking scenario.
- Using common framework to analyse different simulations.
- Results:
  - $0.49\,\text{meV} < m_a < 84\,\text{meV}$ for GHV
  - $0.23\,\text{meV} < m_a < 82\,\text{meV}$ for HKSSYY

Work required:

- Do these comparisons directly with simulation data.
- Investigate systematic errors by comparing simulations.
- Requires work by / collaboration with simulation groups.

Thank you for your Attention!

Questions?

I would like to thank:

- The organizers of this workshop.
- My supervisors/co-workers Sebastian Hoof and David J. E. Marsh.
- Laura Covi for encouraging me to submit an abstract to this workshop.
- Kim W. and Melina A. for their personal support.