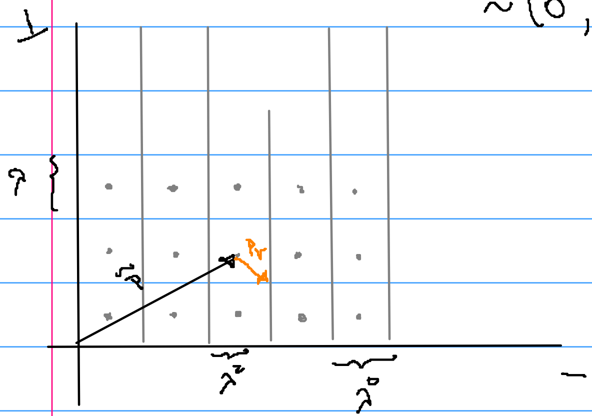
SCET Lecture II

- Separate momentum components

→ Split momenta : large piece "label"
small piece "residual"

$$p^\mu = \underbrace{\tilde{p}^\mu}_{\substack{\text{label} \\ \sim(0,1,\lambda)}} + \underbrace{p_r^\mu}_{\substack{\text{residual} \\ \sim(\lambda^2,\lambda^2,\lambda^2)}}$$



$$\left(\text{HQET} \quad p^\mu = m_b v^\mu + k^\mu \right)$$

Collinear integral : $\int d^4p \longrightarrow \sum_{\tilde{p}\neq 0} \int d^4 p_r$

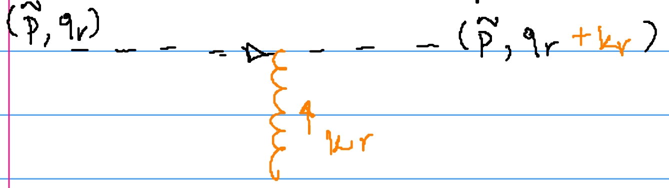
usoft integrals $\int d^4p \longrightarrow \int d^4 p_r$

Delta f'ns $\delta^{(4)}(p-q) = \int \frac{d^4x}{(2\pi)^4} \, e^{i(\tilde{p}-\tilde{q})\cdot x} \, e^{i(p_r-q_r)\cdot x}$

$$= \delta_{\tilde{p},\tilde{q}} \, \delta^{(4)}(p_r - q_r)$$

$$\hat{\xi}_n(x) \longrightarrow \tilde{\hat{\xi}}_{n,\tilde{p}}(p_r)$$

What are the implications on the Feynman rules?



- $c_n - c_n$ change label & residual momenta
- $c_n - us$ change residual
- $\cancel{c_n - c_n}$ $\rightarrow$

Momentum scale separation

$$i\partial_t \, \xi_{n,\tilde{p}}(x) \sim \lambda^2 \, \xi_{n,\tilde{p}}(x)$$

Define label momentum operator $\mathcal{P}^t$

$$\mathcal{P}^t \, \xi_{n,\tilde{p}}(x) \equiv \tilde{p}^t \, \xi_{n,\tilde{p}}(x)$$

$$\hat{\xi}_n(x) = \sum_{\tilde{p}\neq 0} \int d^4 p_r \; e^{-i\tilde{p}\cdot x} \; e^{-i p_r \cdot x} \; \xi_{n,\tilde{p}}(p_r) =$$

$$= e^{-i\mathcal{P}\cdot x} \sum_{\tilde{p}\neq 0} \xi_{n,\tilde{p}}(x) = e^{-i\mathcal{P}\cdot x} \, \xi_n(x)$$

$$i\partial_t \hat{\xi}_n(x) = e^{-i\mathcal{P}\cdot x} \left[\mathcal{P}^t + i\partial_t \right] \xi_n(x)$$

$$i\partial_t \longrightarrow \mathcal{P}^t + i\partial_t$$

Same thing A_n^t

$$\hat{A}_n^t = e^{-i\mathcal{P}\cdot x} \; A_n^t(x)$$

$$\mathcal{L} = \bar{\hat{\xi}}_n \left(iD^+ + i\not{D}_\perp \frac{1}{iD^-} i\not{D}_\perp \right) \hat{\xi}_n$$

Change $\hat{\xi}_n \to \xi_n$, $i\partial_f \to \mathcal{P}^+ + i\partial^+$

$$iD^+ = in\cdot\partial + g n\cdot A_n + g\, n\cdot A_{us} \quad \sim \lambda^2$$

$$iD_\perp^{f} = \underbrace{\mathcal{P}_\perp^{+} + g A_{n\perp}^{+}}_{\sim\lambda} + \underbrace{i\partial_\perp^{f} + g A_{us\perp}^{+}}_{\sim\lambda^2}$$

$$iD^- = \underbrace{\bar{n}\cdot\mathcal{P} + g\bar{n}\cdot A_n}_{\lambda^0} + \underbrace{i\bar{n}\cdot\partial + g\,\bar{n}\cdot A_{us}}_{\lambda^2}$$

$$\mathcal{L}^{(0)}_{n\xi} = e^{-\mathcal{P}\cdot x}\, \bar{\xi}_n \left(iD^+ + i\not{D}_{n\perp} \frac{1}{iD_n^-} i\not{D}_{n\perp} \right) \xi_n$$

$$D_{n\perp}^{+} = \mathcal{P}_\perp^{f} + g A_{n\perp}^{f}$$

$$iD_n^- = \bar{n}\cdot\mathcal{P} + g\bar{n}\cdot A_n$$

. $\mathcal{L}^{(0)}_{n\xi} \sim \lambda^4 \quad \to \quad S^{(0)} \sim \lambda^0$

From defining eq's of Wilson

$$\frac{1}{iD^-} = W_n^+ \frac{1}{\bar{n}\cdot\mathcal{P}} W_n$$

final form:

$$\mathcal{L}^{(0)}_{n\xi} = e^{-\mathcal{P}\cdot x}\, \bar{\xi}_n \left(iD^+ + i\not{D}_{n\perp} W_n^+ \frac{1}{\bar{\mathcal{P}}} W_n i\not{D}_{n\perp} \right) \xi_n$$

Collinear Gluon Lagrangian

A_n^μ, A_{us}^μ

$$\mathcal{L} = -\frac{1}{2} \mathrm{Tr}\left[G_{\mu\nu} G^{\mu\nu}\right] + \tau \,\mathrm{tr}\left[i (\partial_\mu A^\mu)^2\right] + 2\,\mathrm{tr}\left[\bar{c}\, \partial_\mu D^\mu c\right]$$

$$G_{\mu\nu} = \frac{i}{g}\left[D_\mu, D_\nu\right]$$

expand D_μ in the collinear limit

$$iD^\mu \xrightarrow[\lambda]{} iD^\mu = \frac{n^\mu}{2}\left(\bar{\mathcal{P}} + g\,\bar{n}\cdot A_n\right) + \left(\mathcal{P}_\perp^\mu + g A_{n\perp}^\mu\right) + \frac{\bar{n}^\mu}{2}\left(n\cdot\partial + g \underbrace{A_n^\mu}_{n\cdot A_n} + g \underbrace{A_{us}^\mu}_{n\cdot A_{us}}\right)$$

$$i\partial_\mu \to iD_{us}^\mu = \frac{n^\mu}{2}\bar{\mathcal{P}} + \mathcal{P}_\perp^\mu + \frac{\bar{n}^\mu}{2} in\cdot\partial + \frac{\bar{n}^\mu}{2} g\, n\cdot A_{us}$$

finally leading power collinear gluon $\mathcal{L}$:

$$\mathcal{L}_{ng}^{(0)} = \frac{1}{2g^2} \mathrm{tr}\left\{\left[iD^\mu, D_\mu\right]^2\right\} + \tau_{ng}\,\mathrm{tr}\left\{\left[iD_{us}^\mu, A_{n\mu}\right]^2\right\}$$
$$+ 2\,\mathrm{tr}\left\{\bar{c}_n\left[iD_\mu^{us}, \left[iD^\mu, c_n\right]\right]\right\}$$

Usoft quark & gluon $\mathcal{L}$

$\mathcal{L}_{us}^{(0)}$: is the same as in full QCD

$$\mathcal{L}_{us}^{(0)} = \bar{\psi}_{us}\, i\not{D}_{us}\, \psi_{us} - \frac{1}{2}\mathrm{tr}\left[G_{\mu\nu}^{us} G_{us}^{\mu\nu}\right]$$
$$+ \tau_{us}\,\mathrm{tr}\left[i(\partial_\mu A_{us}^\mu)^2\right] + 2\,\mathrm{tr}\left\{\bar{c}_{us}\, i\partial_\mu D_{us}^\mu\, c_{us}\right\}$$

with $D_{us}^\mu = i\partial^\mu + g A_{us}^\mu$

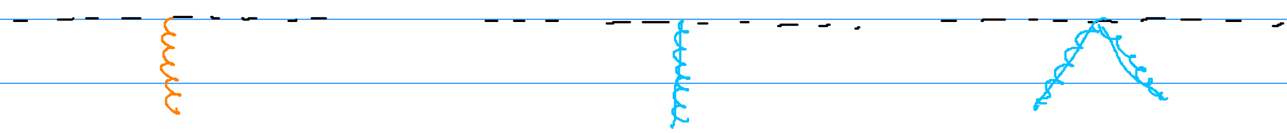
Note: τ_{n_g} and τ_{us} are different gauge fixing parameters $\longrightarrow$ Independent gauge symmetries.

Finally

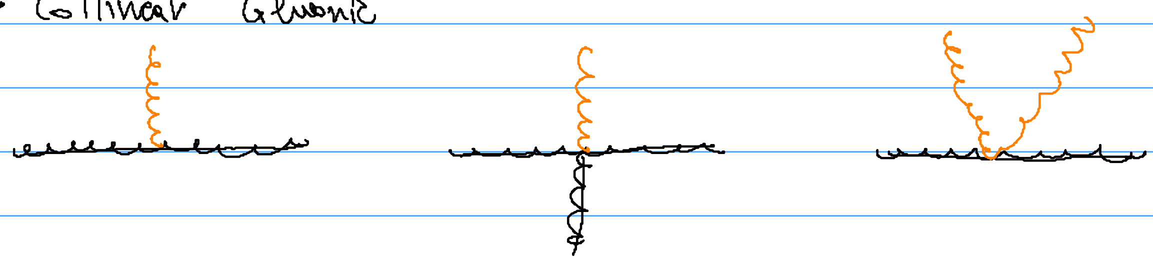
$$\mathcal{L}^{(0)}_{SCET_I} = \sum_n \left[\mathcal{L}^{(0)}_{n\xi} + \mathcal{L}^{(0)}_{ng} \right] + \mathcal{L}^{(0)}_{us}$$

Feynman rules

- Purely usoft identical to full QCD
- Purely collinear gluonic identical to QCD
- Collinear Quark



- Collinear Gluonic



Symmetries of SCET

- $\mathcal{L}^{(0)}_{SCET_I}$ derived $\mathcal{L}_{QCD}$ @ tree level
- Gauge symmetry & RPI & power counting completely fix $\mathcal{L}^{(0)}_{SCET_I}$

Gauge Symmetry

- Separate invariance c_n & us
- $U(x) = \exp\left[i\, \alpha^a(x)\, T^a \right]$

$$\partial_\mu \alpha(x) \sim Q \lambda^{\alpha}$$

$\rightsquigarrow$ We need gauge transformations that leave us inside the EFT.

- Collinear gauge transf $U_n(x)$: $\partial_\mu U_n(x) \sim (\lambda^2, 1, \lambda)\, U_n(x)$
- Usoft gauge transf $U_{us}(x)$: $\partial_\mu U_{us}(x) \sim (\lambda^2, \lambda^2, \lambda^2)\, U_{us}(x)$

<u>Collinear gauge transf.</u>

$$\xi_n(x) \longrightarrow U_n(x)\, \xi_n(x)$$
$$A_n^\mu \longrightarrow U_n(x) \left(A_n^\mu(x) + \frac{i}{g} D_{us} \right) U_n^\dagger(x)$$

$$q_{us} \rightarrow q_{us}$$
$$A_{us}^\mu \rightarrow A_{us}^\mu$$
$$W_n \rightarrow U_n(x)\, W_n$$

<u>Usoft gauge transformations</u>

$$\xi_n(x) \longrightarrow U_{us}\, \xi_n(x)$$
$$A_n^\mu(x) \longrightarrow U_{us}\, A_n^\mu(x)\, U_{us}^\dagger$$
$$q_{us} \longrightarrow U_{us}\, q_{us}$$
$$A_{us}^\mu \longrightarrow U_{us} \left(A_{us}^\mu + \frac{i}{g} \partial^\mu \right) U_{us}^\dagger$$
$$W_n \longrightarrow U_{us}\, W_n\, U_{us}^\dagger$$

e.g. heavy-to-light transition:

$$\bar{\xi}_n W_n \Gamma h_{us}$$

- Under collinear gauge transformation

$$\left(\bar{\xi}_n U_n^\dagger U_n W_n \right) \Gamma h_{us} = \bar{\xi}_n W_n \Gamma h_{us}$$

• Under usoft gauge transformation,

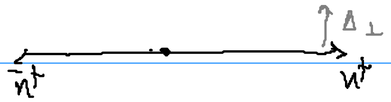
$$\to \bar{\xi}_n U_{us}^\dagger U_{us} W_n U_{us}^\dagger \Gamma U_{us} h_{us} \equiv \bar{\xi}_n W_n \Gamma h_{us}$$

<u>Reparametrization Invariance:</u>

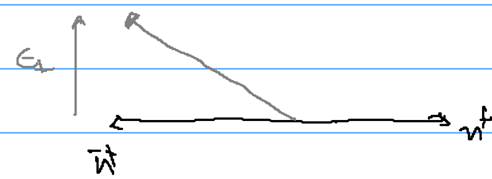
• Choice $n, \bar{n}$ "breaks" Lorentz symmetry

• Requirements $n, \bar{n}$: $n^2 = \bar{n}^2 = 0$, $n \cdot \bar{n} = 2$

(i) RPI - I : $n^\mu \to n^\mu + \Delta_\perp^\mu$
$\sim \mathcal{O}(\lambda)$



(ii) RPI - II : $\bar{n}^\mu \to \bar{n}^\mu + \epsilon_\perp^\mu$
$\sim \mathcal{O}(1)$



(iii) RPI - III : $n^\mu \to e^\alpha n^\mu$
$\bar{n}^\mu \to e^{-\alpha} \bar{n}^\mu$

(if $n^\mu = (1, 0_\perp, 1)$
$\bar{n}^\mu = (1, 0_\perp, -1)$: Boost along $\hat{z}$)

$\forall A^\mu, B^\mu$
$(n \cdot A)(\bar{n} \cdot B)$ or $\dfrac{n \cdot A}{n \cdot B}$ or $\dfrac{\bar{n} \cdot A}{\bar{n} \cdot B}$