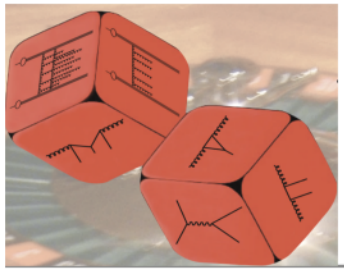
Summer School 2020

Terascale Summer School: Tutorial/Exercises - QCD and Monte Carlo techniques



Summer School 2020

- Welcome to the tutorial exercises on Monte Carlo techniques and QCD

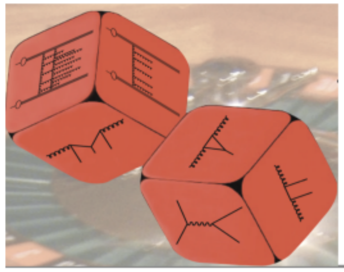
- Goal:
  - leave no one behind
  - everyone should be able to do the exercises
    - please ask questions

- additional infos under:
  - https://www.desy.de/~jung/Terascale_SummerSchool_2020_Tutorial

Summer School 2020


Terascale Summer School: Tutorial/Exercises - QCD and Monte Carlo techniques

- Structure:
  - introductory lecture (20-30 min)
  - 5-6 working groups (in breakout-rooms) with tutors:
        - Sara Taheri Monfared
        - Qun Wang
        - Armando Bermudez Martinez
        - Luis Ignacio Estevez Banos
        - Mikel Mendizabal
        - Patrick Connor
    - ask questions
    - share screen to get help



Environment

- Virtual Machine
  - running full C++ code
  - compilation, execute program

- New:
  - running python on Jupyter Notebook
  - only need web browser
  - Login with School account on Computer-Farm at DESY
    - account will be valid for 2 weeks
    - Accounts on Google Doc
      - please pick an account and mark it (and remember your account and passwd)

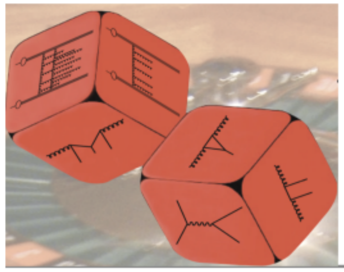
How to get started

- start VM
  - in case – passwd: terascale1234
  - perhaps change screen resolution

- compiling and running:

```
cd exercise-1
make example-1
./example-1
```

- templates are provided which include the general structure – you only have to fill the interesting – important parts

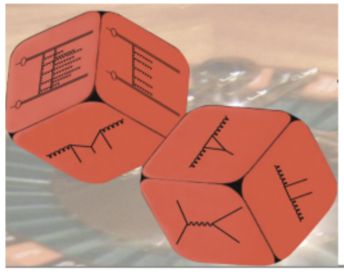


How to get started: Jupyter notebook

- open browser, and login to: https://naf-jhub.desy.de/
  - we have school accounts, valid for 2 weeks
    - Accounts on Google Doc
      - please pick an account and mark it (and remember your account and passwd)
  - (see https://confluence.desy.de/display/IS/Jupyter+on+NAF)

- to get started, logon jupyter cluster on naf
  - open terminal and copy templates and solutions:

```
wget
 https://www.desy.de/~jung/Terascale_SummerSchool_2020_Tutorial/Terascale_SummerSchool_2020_Tutorial_exericises_files/jupyter-python-all.tgz
```



Exercise 1 - Introduction

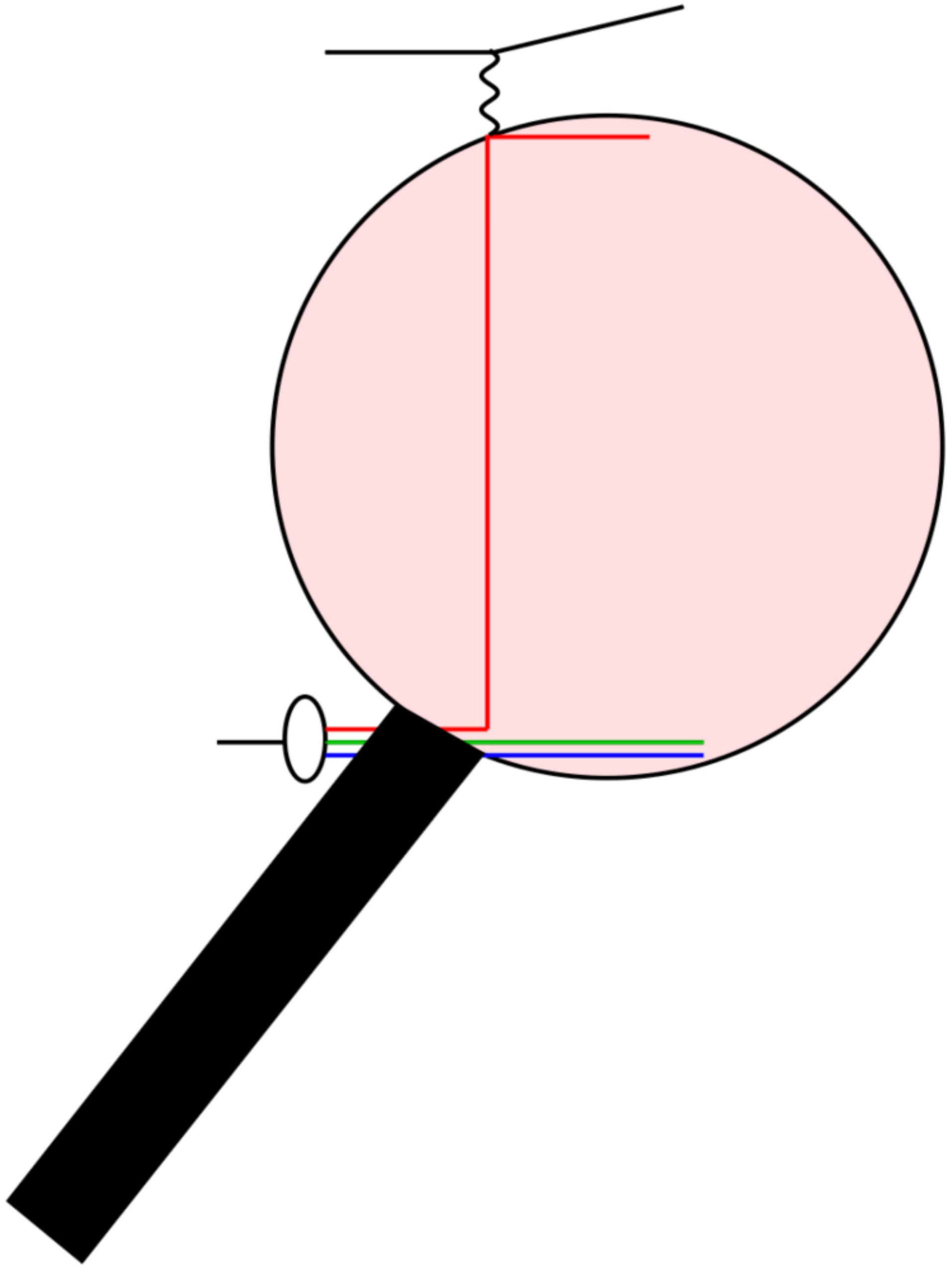
- Schedule:
  - Thursday - Exercise 1:
    - Random numbers
    - MC method
    - MC integration
  - Monday – continue Exercise 1:
    - Random numbers
    - MC method
    - MC integration

- Tuesday - Exercise 2:
  - Sudakov form factor
  - MC solution of evolution equation

- Wednesday-Thursday - Exercise 3
  - Calculation & simulation of Higgs production
  - Using MC solution of evolution equation → calculation of pt spectrum of Higgs at LHC

We can continue, if needed/wanted !

A proton in the initial state



- Deep Inelastic Scattering is a incoherent sum of

  $e^+q \to e+q$
- only 50 % of p momentum carried by quarks
- need a large gluon component
- partonic part convoluted with parton density function $f_i(x)$

$$\sigma(e^+p \to e^+X) = \sum_i f_i(x, \quad)\sigma(e^+q_i \to e^+q_i)$$

Parton distribution function

- $f_i(\xi)d\xi$ gives probability that parton *i* carries momentum fraction between ξ and $\xi + d\xi$ with $0 \leq \xi \leq 1$
- Number of partons i:

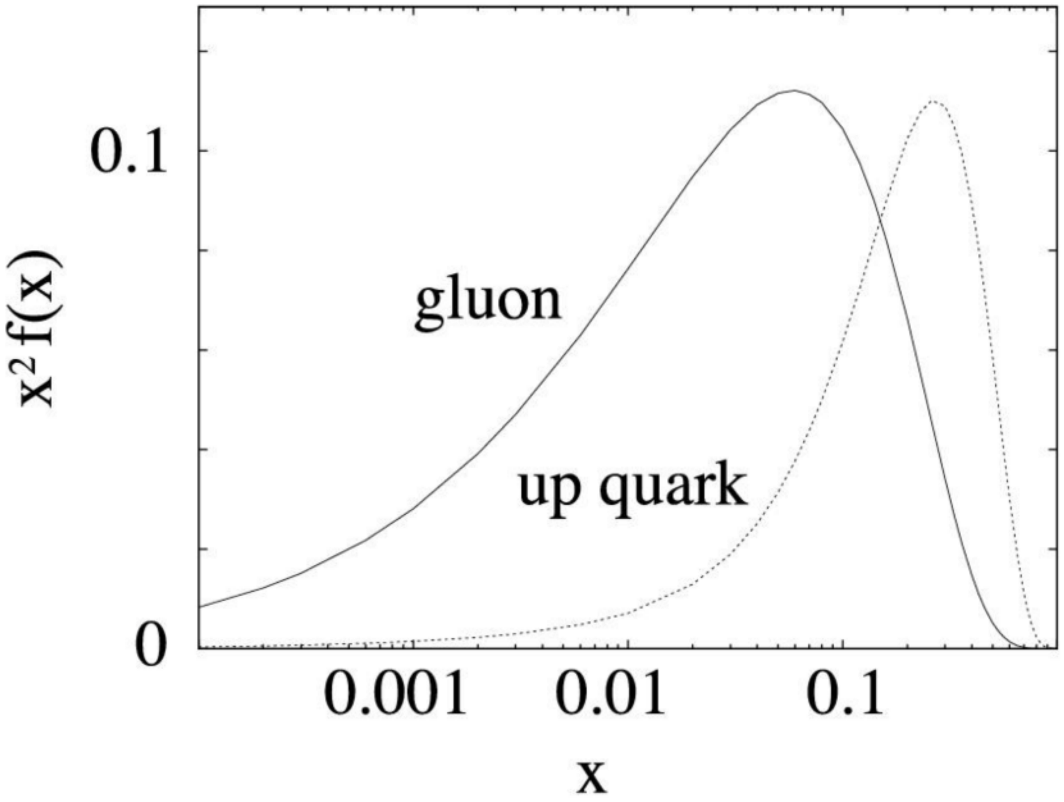
$$N_i = \int_0^1 d\xi \, f_i(\xi)$$

- Momentum fraction carried by partons i:

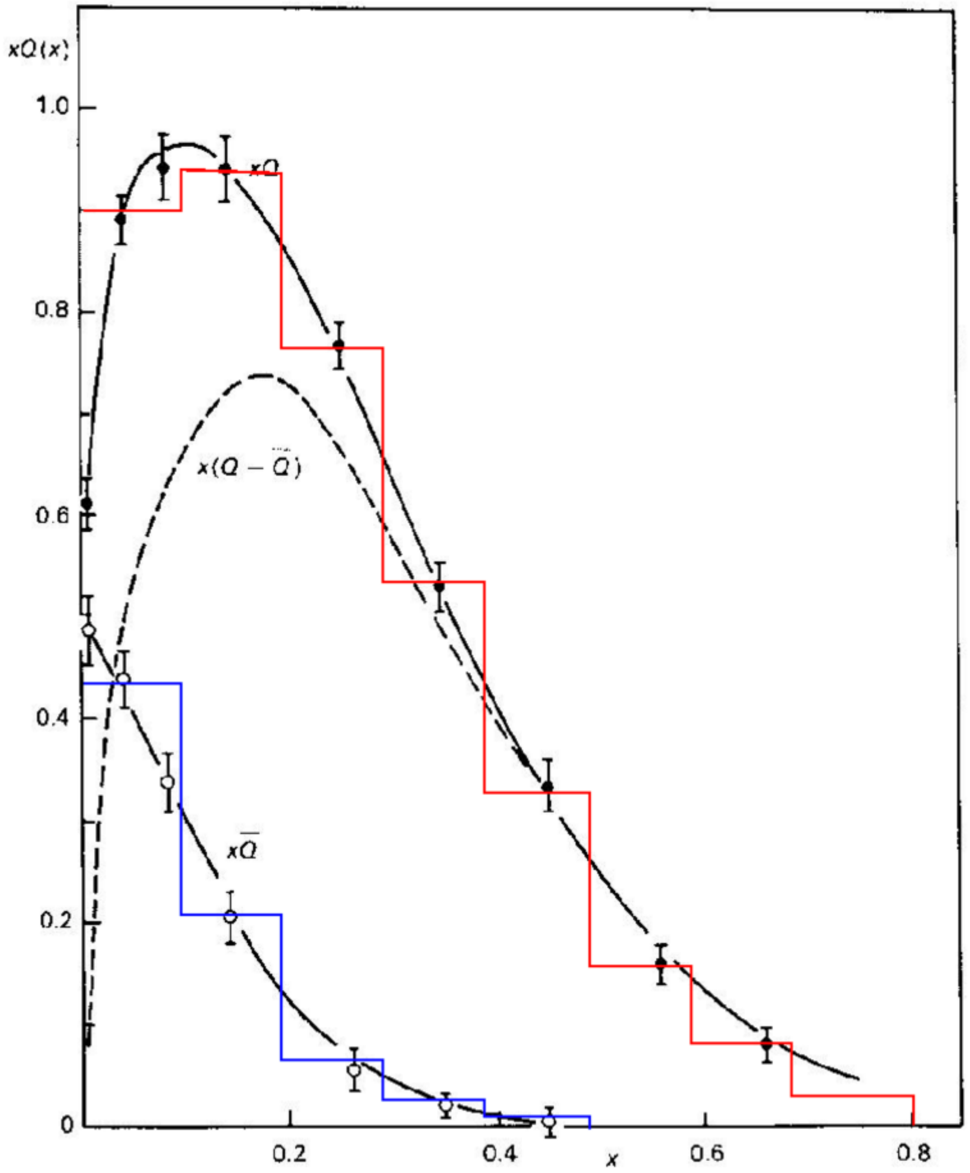
$$\frac{<p_i>}{P} = \int_0^1 d\xi \;\; \xi f_i(\xi)$$

- Define sum-rules for hadron target:
  - Number of valence partons
  - Momentum carried by partons
  - Flavor contents

From D. Soper hep-ph/9609018



Picture of the Proton



- Flavor sum rules for proton:

$$\int_0^1 dx u_V(x) = 2$$

$$\int_0^1 dx d_V(x) = 1$$

- Momentum sum of quarks:

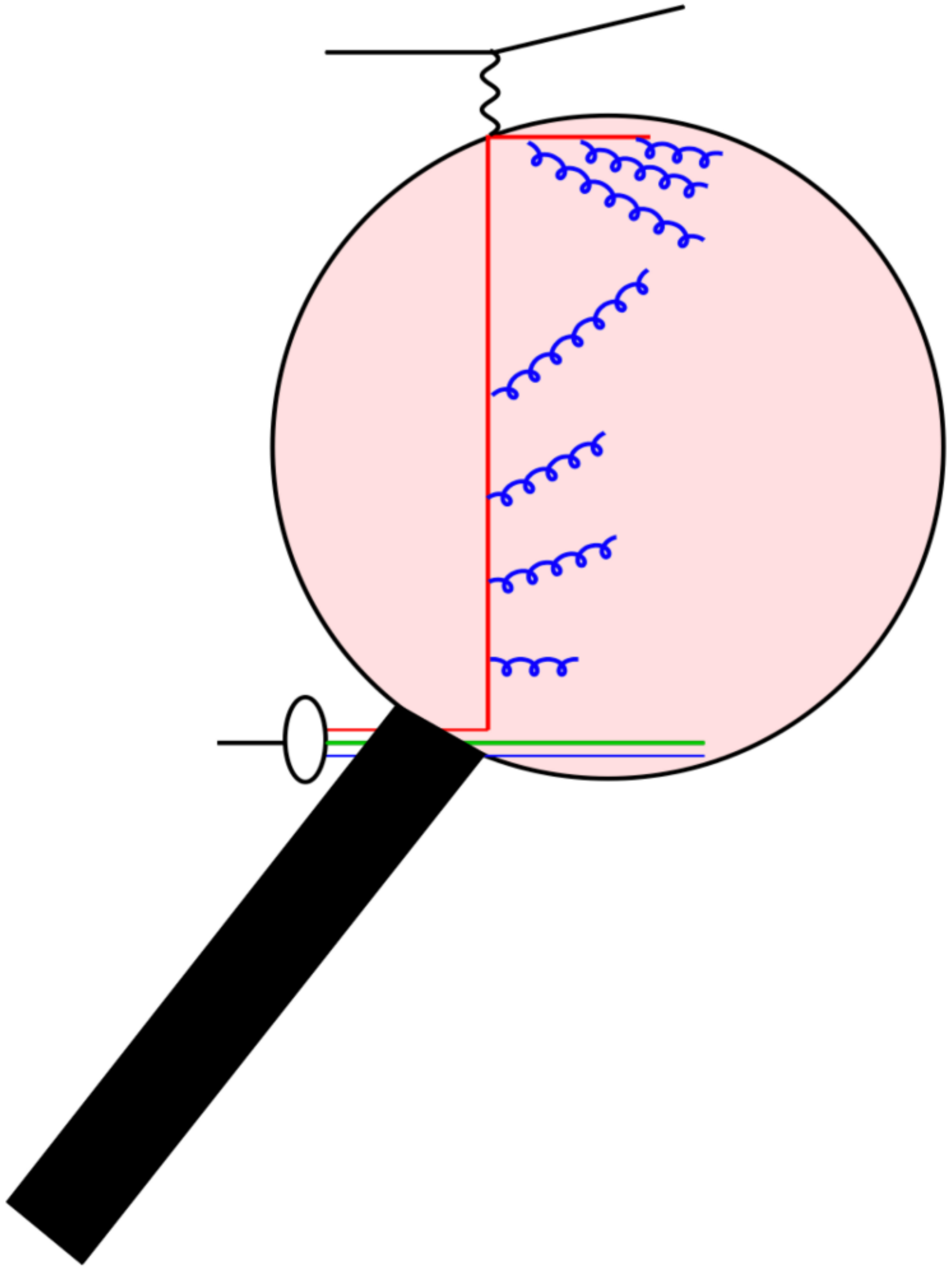
$$\sum_q \int_0^1 dx\, x\, [q(x) + \bar{q}(x)] \sim 0.5$$

- Where are the other 50 % of the proton's momentum ?

$$\int dx x q(x) \sim 0.1\,[0.9 + 0.95 + 0.85 + 0.7 + 0.35 + 0.15 + 0.1 + 0.05] = 0.1 \cdot 4.05 = 0.405$$

$$\int dx x \bar{q}(x) \sim 0.1\,[0.42 + 0.2 + 0.06 + 0.03 + 0.01] = 0.1 \cdot 0.72 = 0.072$$

A proton in the initial state



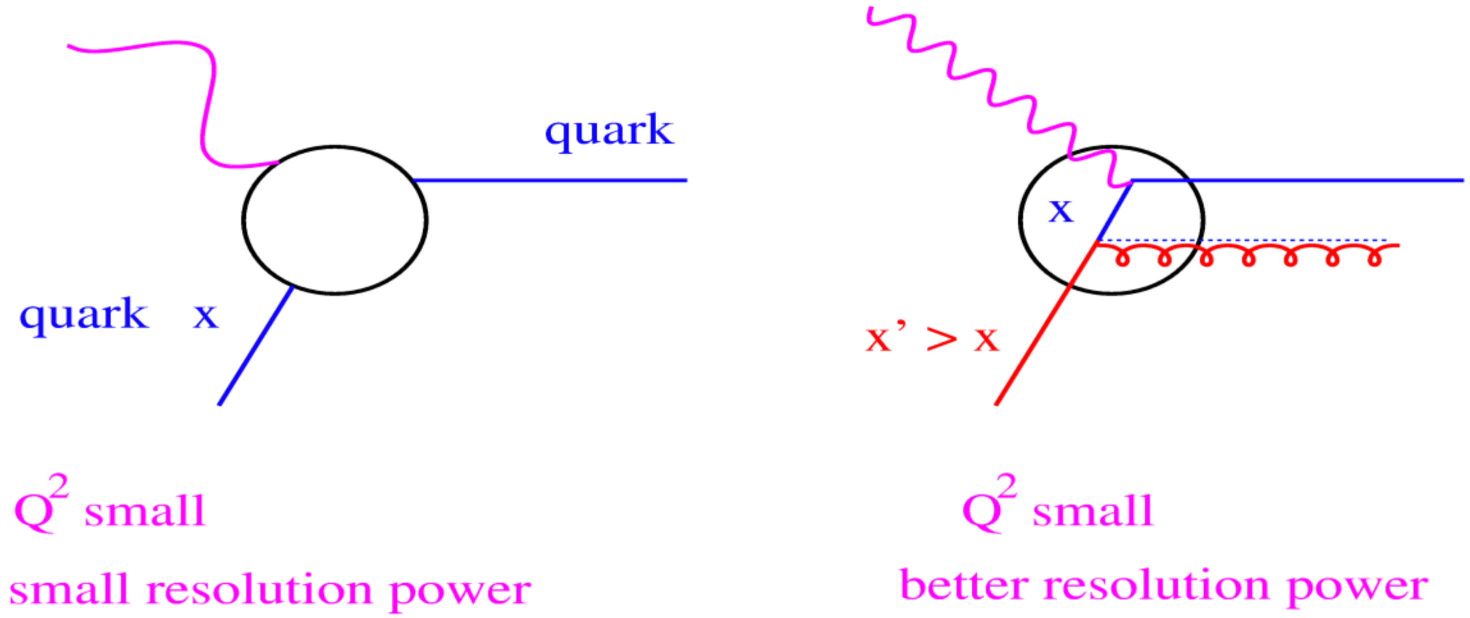
- Deep Inelastic Scattering is a incoherent sum of

  $e^+ q \to e + q$
- only 50 % of p momentum carried by quarks
- need a large gluon component
- partonic part convoluted with parton density function $f_i(x)$
- BUT we know, PDF depends on resolution scale Q^2

$$\sigma(e^+ p \to e^+ X) = \sum_i f_i(x, Q^2) \sigma(e^+ q_i \to e^+ q_i)$$

DGLAP evolution equation

- QPM: F_2 is independent of Q^2
- Q^2 dependence of structure function: **D**okshitzer**G**ribov**L**ipatov**A**ltarelli**P**arisi



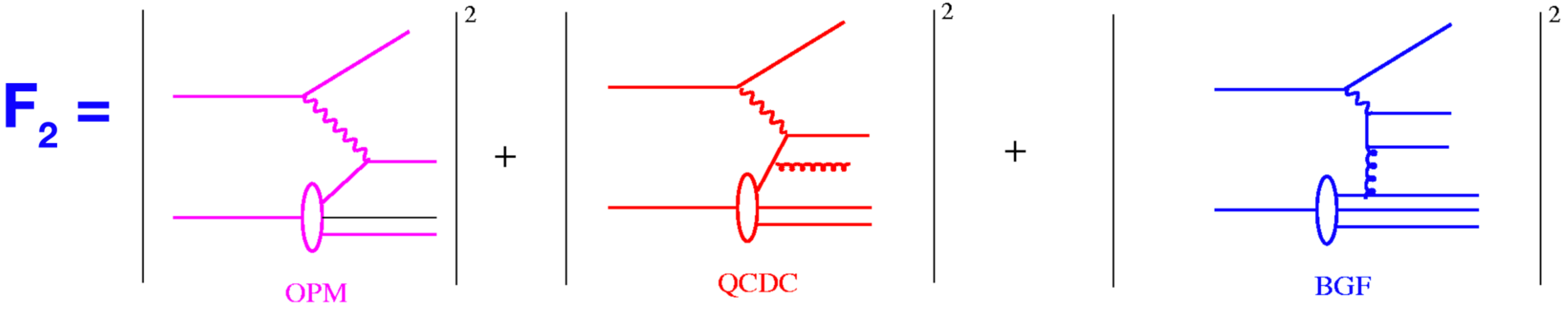
Q^2 small

small resolution power

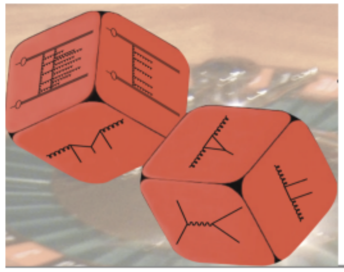
Q^2 small

better resolution power

→ Probability to find parton at small x increases with Q^2



→ Test of theory: Q^2 evolution of $F_2(x,Q^2)$!!!!!



parton density functions

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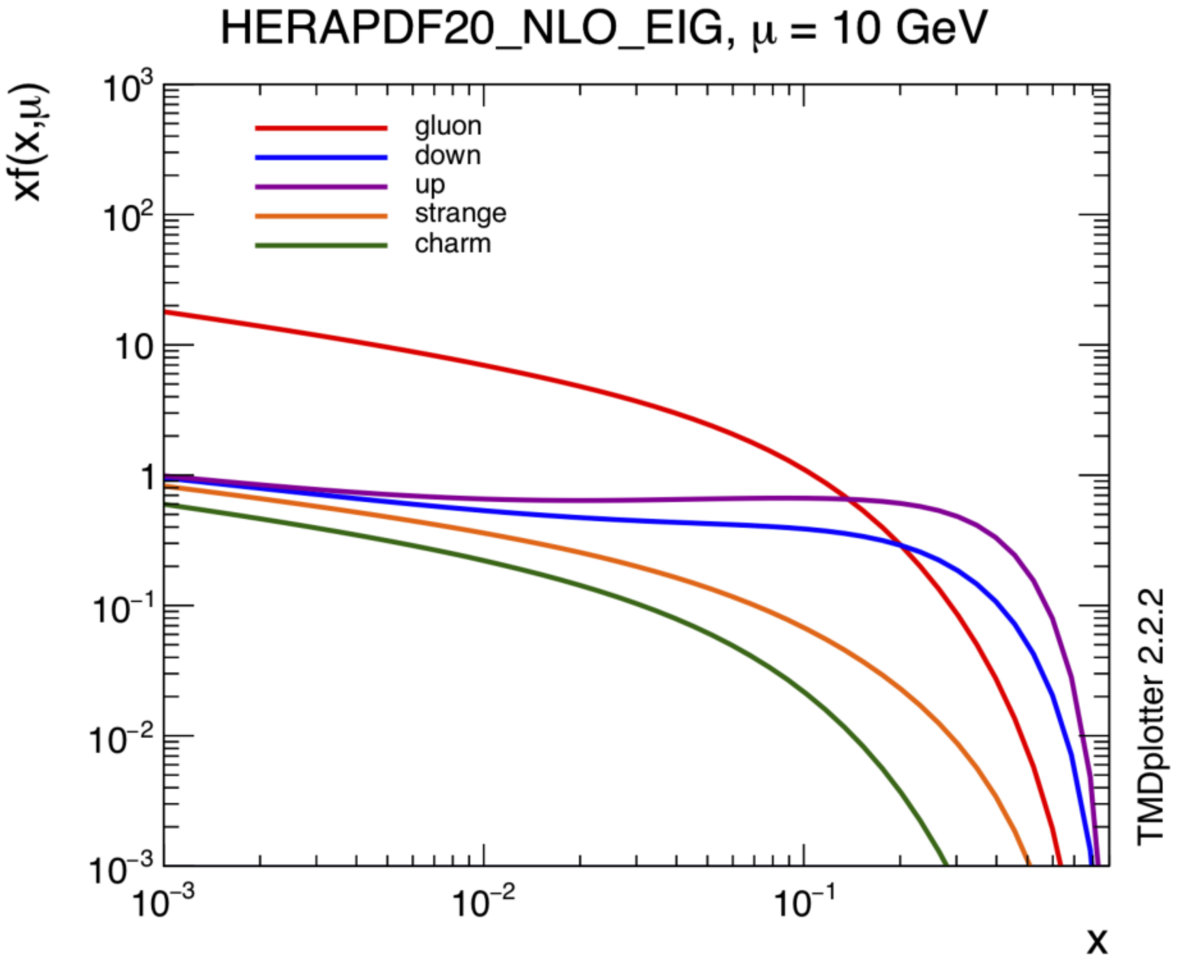
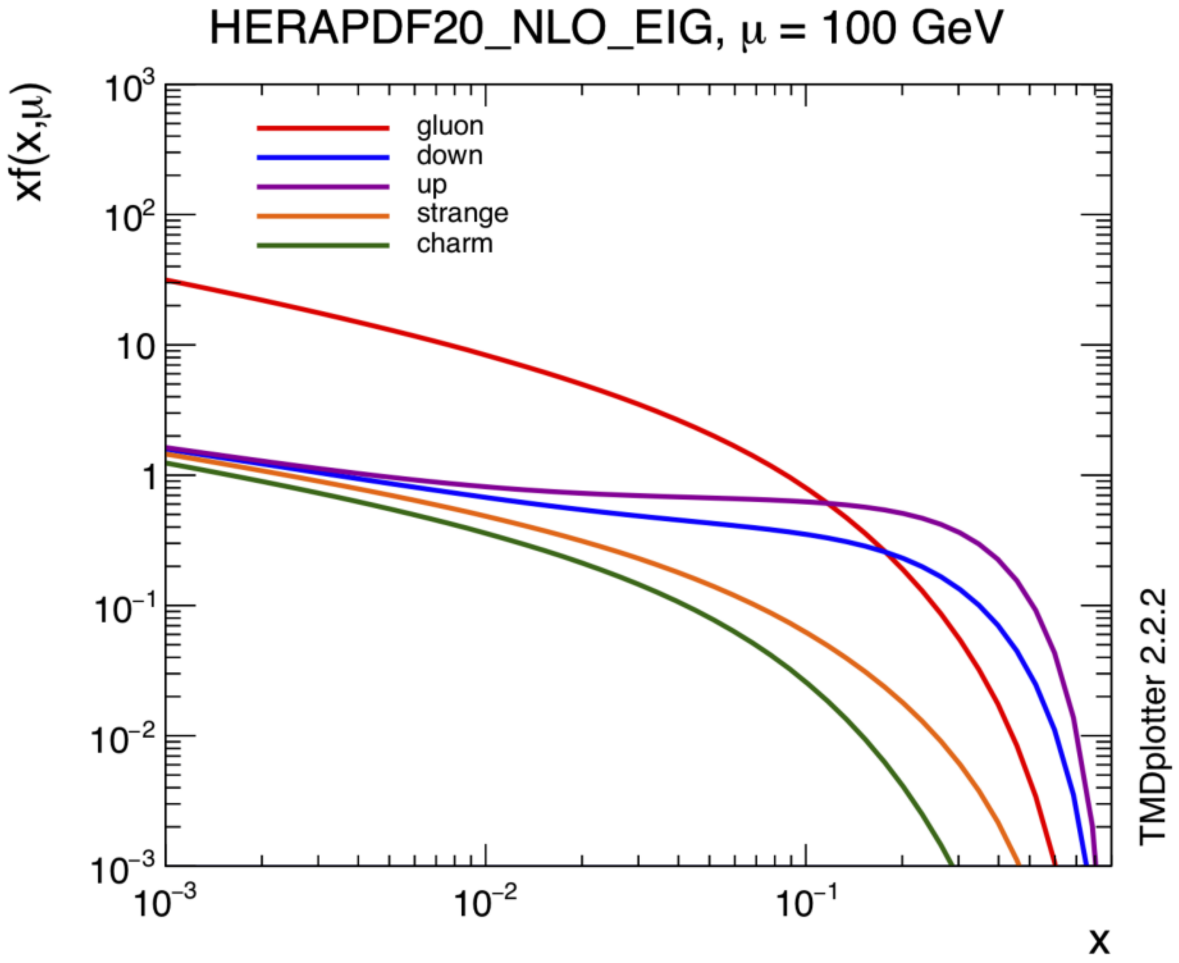
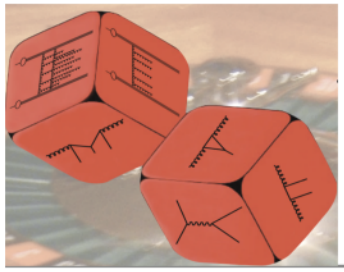






Collinear factorization: DGLAP

- introduce new scale $\mu^2 \gg \chi^2$ and include soft, non-perturbative physics into renormalized parton density:
- $$q_i(x,\mu^2) = q_i^0(x) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[q_i^0(\xi)P_{qq}\left(\frac{x}{\xi}\right) + g^0(\xi)P_{qg}\left(\frac{x}{\xi}\right)\right]\log\left(\frac{\mu^2}{\chi^2}\right)$$

- **D**okshitzer**G**ribov**L**ipatov**A**ltarelli**P**arisi equation:

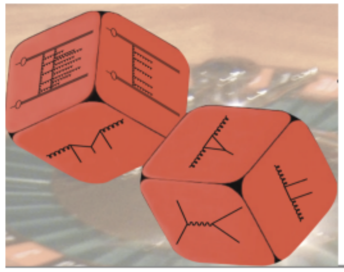
  V.V. Gribov and L.N. Lipatov Sov. J. Nucl. Phys. 438 and 675 (1972) 15, L.N. Lipatov Sov. J. Nucl. Phys 94 (1975) 20, G. Altarelli and G. Parisi Nucl.Phys.B 298 (1977) 126, Y.L. Dokshitser Sov. Phys. JETP 641 (1977) 46

$$\frac{dq_i(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[q_i(\xi,\mu^2)P_{qq}\left(\frac{x}{\xi}\right) + g(\xi,\mu^2)P_{qg}\left(\frac{x}{\xi}\right)\right]$$

- BUT there are also gluons...

$$\frac{dg(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[\sum_i q_i(\xi,\mu^2)P_{gq}\left(\frac{x}{\xi}\right) + g(\xi,\mu^2)P_{gg}\left(\frac{x}{\xi}\right)\right]$$

- DGLAP is the analogue to the beta function for running of the coupling



Divergencies again...

- collinear divergencies factored into renormalized parton distributions
- what about soft divergencies ?

treated with "plus" prescription

$$\frac{1}{1-z} \to \frac{1}{1-z}_{+} \qquad \textbf{with} \qquad \int_0^1 dz \frac{f(z)}{(1-z)_+} = \int_0^1 dz \frac{f(z)-f(1)}{(1-z)}$$

- soft divergency treated with Sudakov form factor:

$$\Delta(t) = \exp\left[- \int_{t_0}^{t} \frac{dt'}{t'} \int^{z_{max}} dz \frac{\alpha_s}{2\pi} \tilde{P}(z) \right]$$

Sudakov form factor

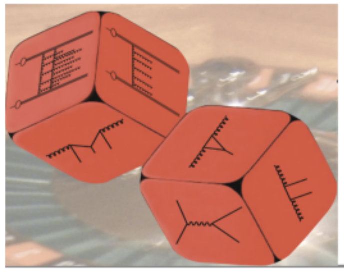
$g \to gg$ Splitting Fct $\quad \tilde{P}(z) = \frac{\bar{\alpha}_s}{1-z} + \frac{\bar{\alpha}_s}{z} + \ldots$

- Sudakov form factor .... all loop resummation

$$\Delta_S = \exp\left(-\int dz \int \frac{dq'}{q'} \frac{\alpha_s}{2\pi} \tilde{P}(z)\right)$$

$$\Delta_S = 1 + \left(-\int dz \int \frac{dq}{q} \frac{\alpha_s}{2\pi} \tilde{P}(z)\right)^1 + \frac{1}{2!}\left(-\int dz \int \frac{dq}{q} \frac{\alpha_s}{2\pi} \tilde{P}(z)\right)^2 \ldots$$

$$\tilde{P}(z)\left[1 - \int\int dz \frac{dq}{q} \frac{\alpha_s}{2\pi} \tilde{P}(z) + \frac{1}{2!}\left(-\int\int dz \frac{dq}{q} \frac{\alpha_s}{2\pi} \tilde{P}(z)\right)^2 - \ldots - \right]$$

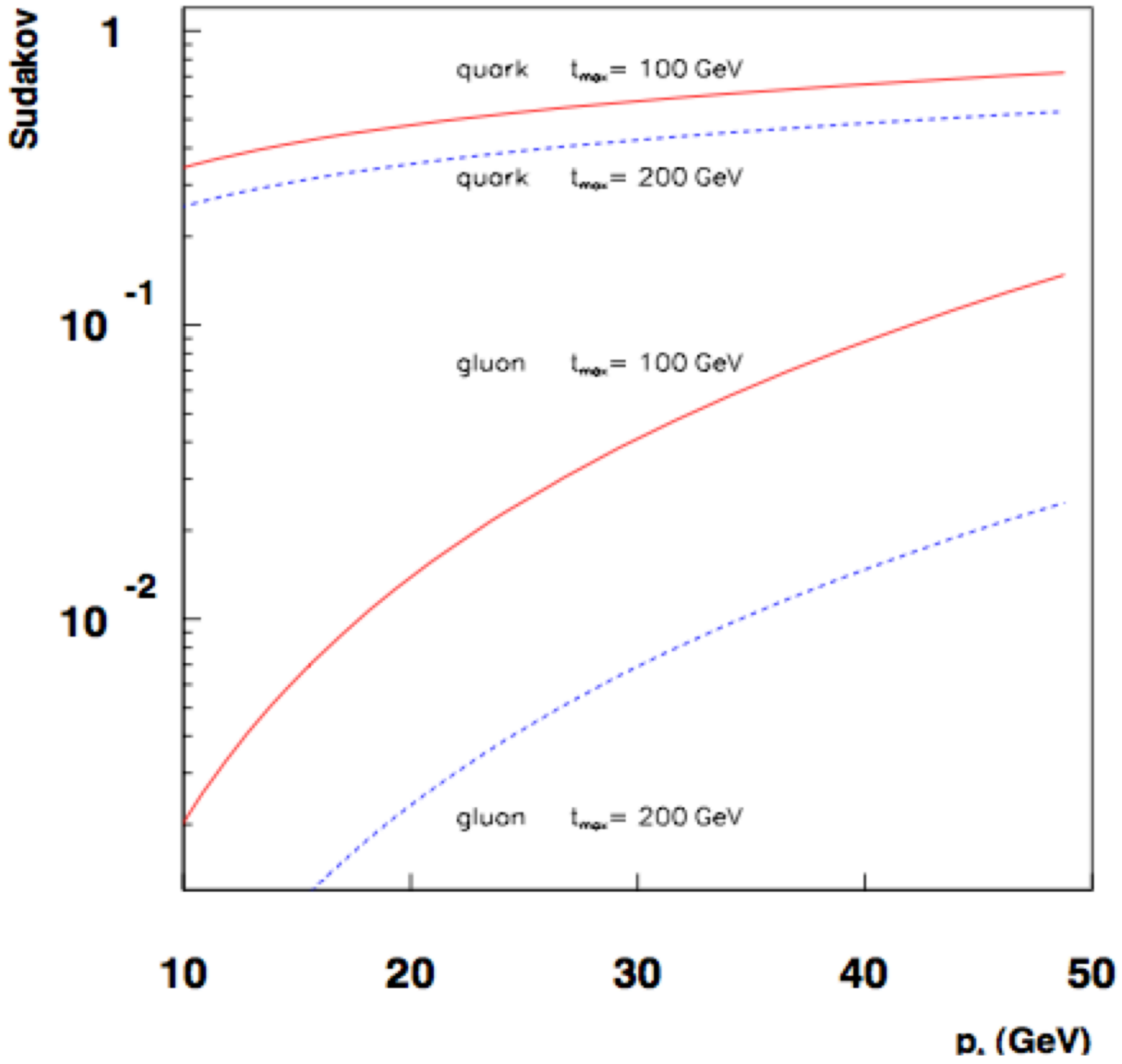


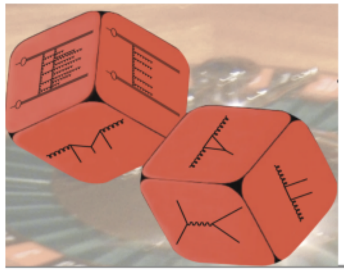
Sudakov form factor

- what is the limit on z- integration ?
  - resolvable branching ?
  - $z < 1 - \frac{Q_0^2}{Q_b^2}$ with Q_0 a soft cutoff
- probability of no -radiation between Q_a and Q_b
  -

$$
\begin{aligned}
\Pi(Q_a^2, Q_b^2) &= \frac{\Delta(Q_a^2)}{\Delta(Q_b^2)} \\
&= \exp\left[- \int_{Q_b^2}^{Q_a^2} \frac{dq^2}{q^2} \int_0^{z_{cut}} dz \frac{\alpha_s}{2\pi} P(z) \right]
\end{aligned}
$$

Sudakov form factor





Exercise 2

6. Calculate the Sudakov form factor for the scales $t_2 = 10, 100, 500$ GeV^2 as a function of t_1 and plot it as a function of t_1. Use q as the argument for α_s, and check the differences. For the z integral use $z_{min} = 0.01$ and $z_{max} = 0.99$.

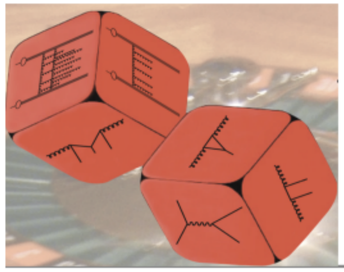
$$\log \Delta_S = -\int_{t_1}^{t_2} \frac{dt}{t} \int_{z_{min}}^{z_{max}} dz \frac{\alpha_s}{2\pi} P(z)$$

Use the gluon and also the quark splitting functions :

$$P_{gg} = 6\left(\frac{1-z}{z} + \frac{z}{1-z} + z(1-z)\right)$$

and

$$P_{qq} = \frac{4}{3}\frac{1+z^2}{1-z}$$



Breakout Room

- Now we have the tutorial for the second exercises: 30 min
  - work with your tutors in breakout rooms
    - Armando: Tutorial with C++
    - Luis Ignacio: Tutorial with Jupyter
    - Mikel: Tutorial with Jupyter

Literature for MC solution of evolution equation

- Monte Carlo and QCD lectures
  - https://www.desy.de/~jung/QCD_and_Monte_Carlo_lectures.html

- R.K. Ellis, W. J. Stirling, and B. R. Webber. QCD and collider physics. Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol., 8:1–435, 1996.
- Hautmann, F., Jung, H., Lelek, A., Radescu, V., and Zlebcik, R. (2018). Collinear and TMD quark and gluon densities from Parton Branching solution of QCD evolution equations, JHEP, 01, 070
- Bermudez Martinez, A., Connor, P., Hautmann, F., Jung, H., Lelek, A., Radescu, V., and Zlebcik, R. (2019). Collinear and TMD parton densities from fits to precision DIS measurements in the parton branching method, Phys. Rev., D99(7), 074008
- Hautmann, F., Jung, H., and Monfared, S. T. (2014). The CCFM uPDF evolution uPDFevolv, Eur. Phys. J., C74, 3082