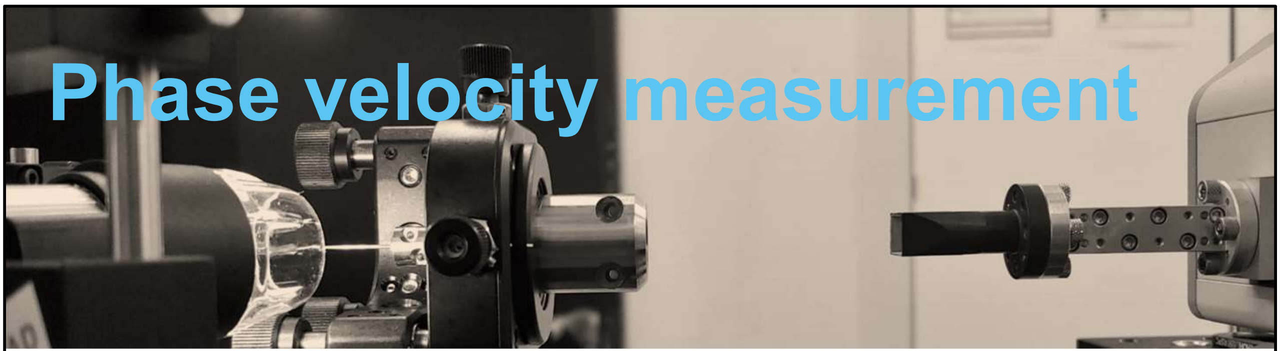
Phase velocity measurement



Calibration free characterization of integrated Terahertz waveguides

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HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

Universität Hamburg DER FORSCHUNG | DER LEHRE | DER BILDUNG

AXSIS

DESY.

Agenda

Introduction

Measurement Principle and Four term error model

Current measurement results

Discussion and Outlook

- Why?
- What and how?
- Show results
- Limits and outlook

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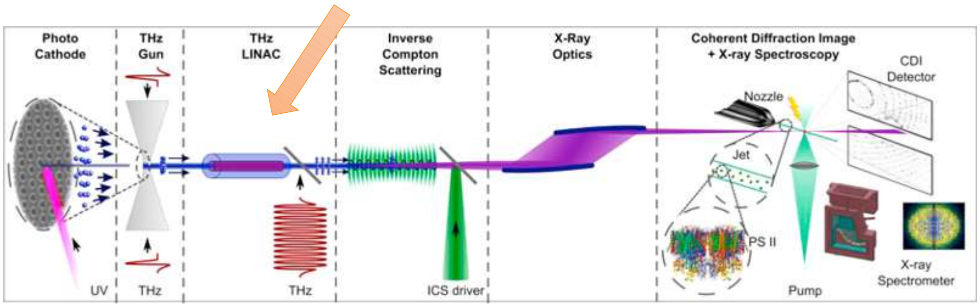
Introduction: AXSIS

Frontiers in Attosecond X-Ray Science: Imaging and Spectroscopy

- Fully Coherent Attosecond X-ray Source
- Based on THz Acceleration and Inverse Compton Scattering



- to outrun radiation damage effects due to the necessary high X-ray irradiance
- Phase velocity here not refering to the accelerating mode but the fundamental mode
- Still useful insights about the waveguide

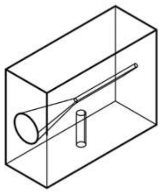
Introduction

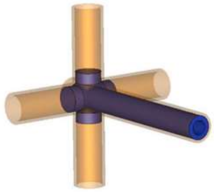
- Emerging interest in THz waveguides in the accelerator community:
  - “Active devices” which are driven by external THz sources (streaking, accelerating)
  - “Passive devices”: Beam-driven, acts as THz source
- Due to small feature sizes → **monolithically** integrated coupler and waveguide preferential

How to characterize the waveguide if you cannot separate it from the coupler?

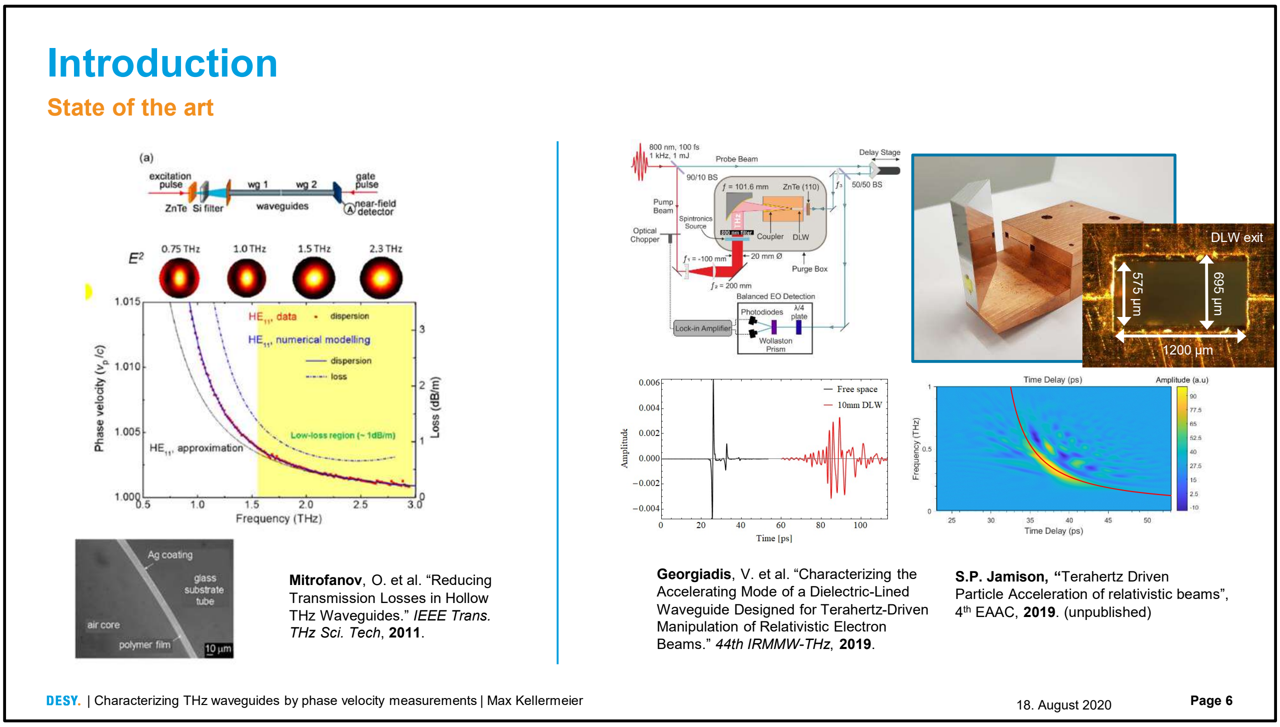
→ Critical quantity: **phase velocity**

→ In case of horn coupler: Causes additional phase shift





- A more general introduction
- Waveguides:
  - Simple funnel and tube in a solid block
  - More advanced couplers like the one from Francois proposal for THz generation
  - Or this fancy side couplers foreseen by our CFEL collegues for AXSIS
- Phase velocity: keep phase slippage as small as possible
- Relatively short waveguides → phase shift at horn matters



- Mitrofanov:
  - Optically generated THz
  - First waveguide used as luncher, no coupler included
  - Long waveguide (120 cm) → phase shift from coupling negligible
  - 2.2 mm diameter Ag/PS waveguide
  - Single cycle pulse, using spectrogram, i.e. frequency depending on time → limited by capabilities of the transform
- Group at CLARA in Daresbury applied same optical setup:
  - Traces shown → Phase and power spectra extracted but haven't seen a dispersion line so far
  - My guess: Horn contribution can't be taken out

Agenda

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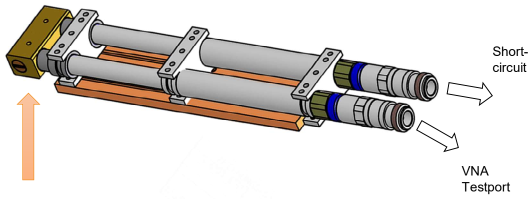
Measurement Principle

$$v_{ph} = \frac{\omega}{\beta} = f\lambda$$

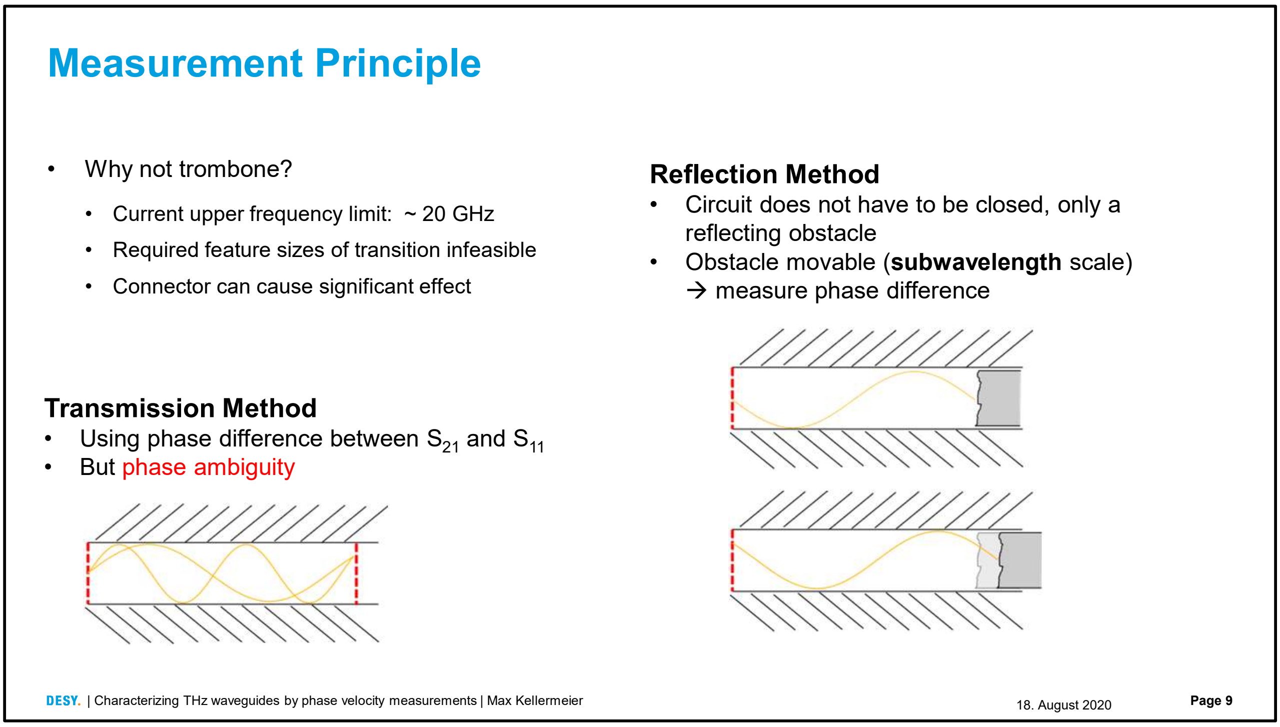
- VNA: Fixed frequency → **unknown** wavenumber and wavelength in waveguide

Inspired by **lower** frequency techniques: Trombone for impedance matching (Constant impedance adjustable coaxial line)

- By varying the length of the coaxial → change circuit from “open” to “short” (180° phase)
- Measured length corresponds to half wavelength → phase velocity



- In RF field: wavevector usually beta instead of k
- VNA sends out CW signal
- Trombone: same principle applied in music instrument
    - RF device: Denoted as phase adjusters, line stretchers
- One port to VNA, one short-circuited



- Two main aspects:
  - Feature sizes required for a trombone mechanism infeasible at these high frequencies
  - Connector can cause significant contribution, similar to horn
    - At low frequencies the impedances of the connectors are matched
- Generalization:
  - Why short-circuit? Any reflection exploited
  - Length of device fixed → change position of the origin of the reflection
- Comment on Transmission technique:
  - S-parameters are applied for Linear Time invariant systems → no time information
  - Even though both reflection and transmission → phase ambiguity (30° vs. 390°)

Four Term Error Network Model

- Issue: Waveguide **embedded** in single block with horn → can't be attached to VNA port (ref. plane)
- Additional out-coupling horn and free space
- Goal: **self-calibration** with measurements at different positions

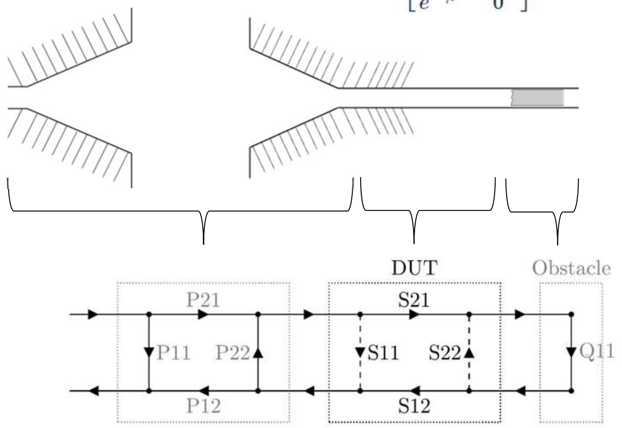
$$\Longrightarrow \quad S_{11}^{m} = P_{11} + Q_{11}\frac{P_{21}P_{12}}{k^{-2} - P_{22}Q_{11}}$$

i-th measurement

$$\Longrightarrow \quad S_{11}^{m,i} = a + \frac{b}{\tilde{k}^{-2l_i} - c}$$

Four Term Error Model

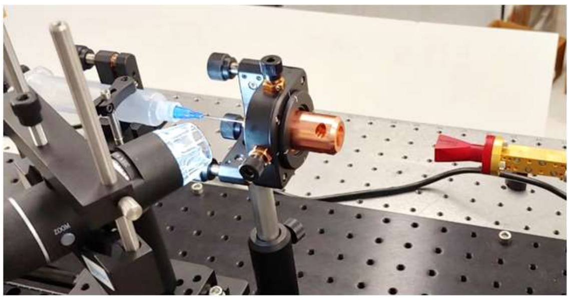
$$S = \begin{bmatrix} 0 & e^{-\gamma l} \\ e^{-\gamma l} & 0 \end{bmatrix}, \quad k := e^{-\gamma l}$$

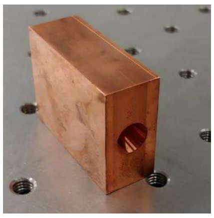


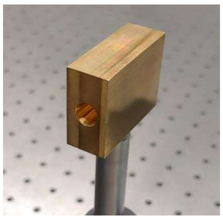
- But: Principle does not account for couplers and freespace propagation → multiple reflections at interfaces and interference
- → Self-calibration method, denoted as 4-term error model
- Illustrated here: horns , freespace, waveguide, obstacle
- S-matrix for a waveguide, gamma includes the wavevector beta, k denoted as propagation factor
- Doing the math:
  - Reflection from each network interface is a Möbius transform
  - arrive at S-parameter measured at test port
  - Terms similar to a standard Open-Short-Match calibration (directivity, source match, reflection tracking)

Experimental Setup and Waveguide Structure

- Frequency range: 220 GHz – 330 GHz (Streaking: 267 GHz, AXSIS: 300 GHz)
- Obstacles:
  - Syringe + Canula as obstacle (OD = 0.72 mm, ID = 0.41 mm), on motorized stage (+/- 1 µm)
  - Paper clip (OD = 0.89mm)
  - New: steel welding wire
- Three different waveguide structures:
  1. Conical horn drilled in copper monolithically
  2. Same, but in brass
  3. EDM machined in two half shelves, “split waveguide”





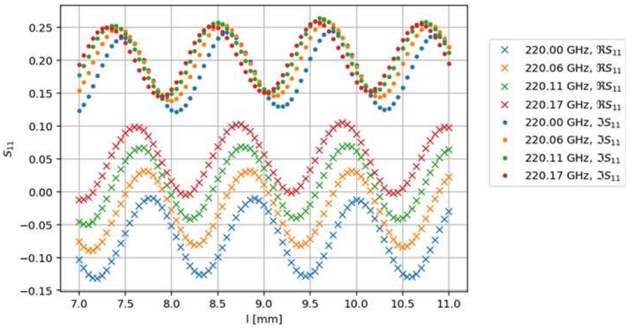


S11 depending on position in 4-term error model

Copper Waveguide 1

- Measured: $S_{11}(f, l)$
- analyzed independently for each frequency point:

$$S_{11}(l)\Big|_{f=f_0}$$



$$\mathrm{re}\left(S_{11}\right) = \mathrm{re}\left(a\right) - \frac{\left(\sin\left(2\beta l\right) + \mathrm{im}\left(c\right)\right)\mathrm{im}\left(b\right)}{-2\sin\left(2\beta l\right)\mathrm{im}\left(c\right) + 2\cos\left(2\beta l\right)\mathrm{re}\left(c\right)} + \frac{\left(\cos\left(2\beta l\right) - \mathrm{re}\left(c\right)\right)\mathrm{re}\left(b\right)}{-2\sin\left(2\beta l\right)\mathrm{im}\left(c\right) + 2\cos\left(2\beta l\right)\mathrm{re}\left(c\right)}$$

- After measurement set: data set of S_11(f,l)
- Analyzed separately for each frequency point by fitting the 4-term error model to it
- Example: Real and Imaginary part versus obstacle position, for selected frequency point
- Nice sine wave, but does not have to be the case in the 4-term error model
  - Contributions from error term b and c can significantly distort the shape of the waveform
  - Here: b and c small

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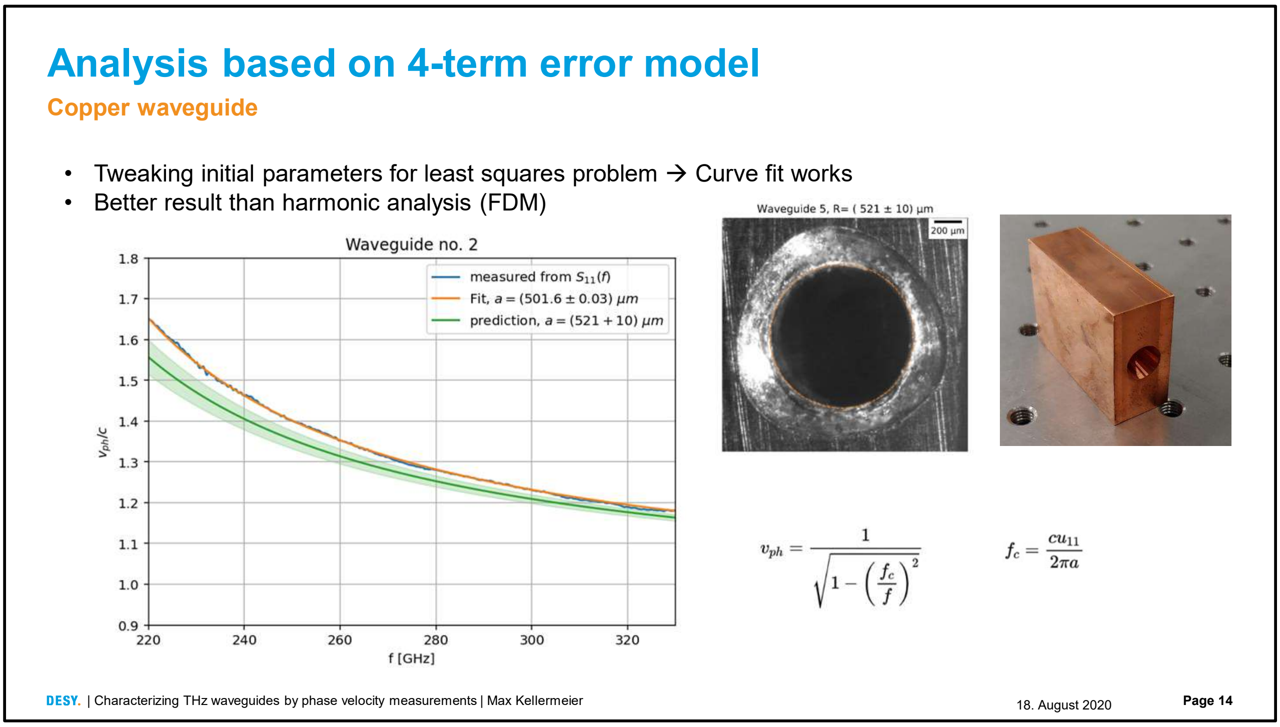
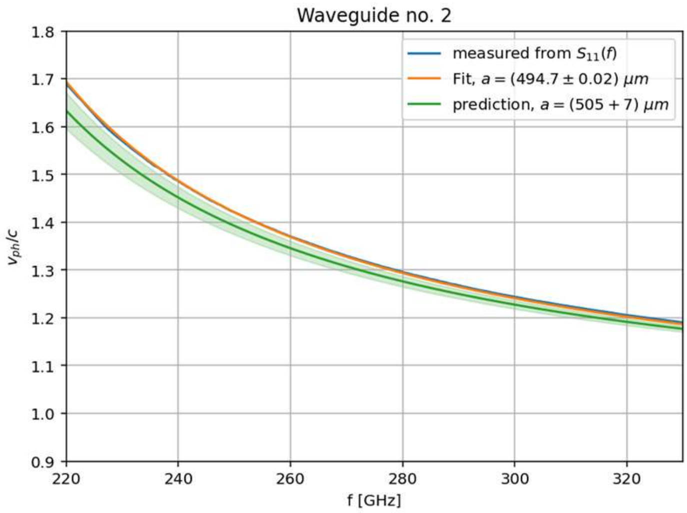
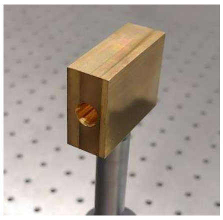
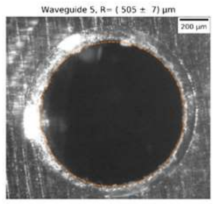


- Measurement 05
- What do you see here?
- In blue (behind the orange one): phase velocity computed from the 4-term error model (for each frequency point independently!)
- In orange: fitting the dispersion line of the analytical solution for a metallic waveguide
- In green: prediction based on the aperture measured under the microscope → a circle detection algorithm applied to the microscope image (Hough transform + accumulator)
- Measurement does not fall into the error band of the prediction
  - Uncertainty in radius might be too optimistic
  - Hole drilled → might be larger at the outside than inside
  - → fit result more trusted than the microscope image

Brass waveguide

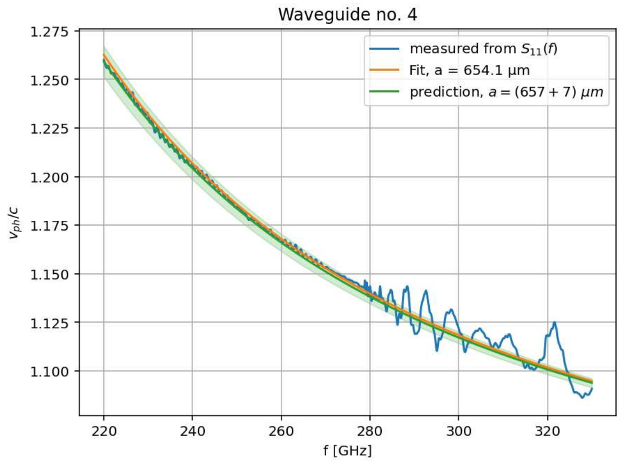


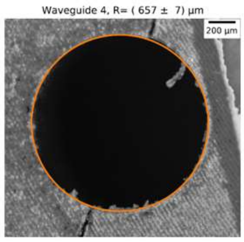


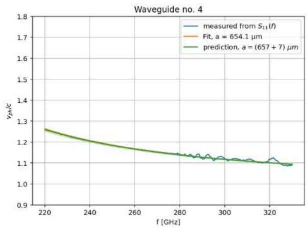


- Measurement 7
- Same as before, with brass wvg
- Measured dispersion line smoother than the copper one
- Microscope image looks already cleaner

Split waveguide





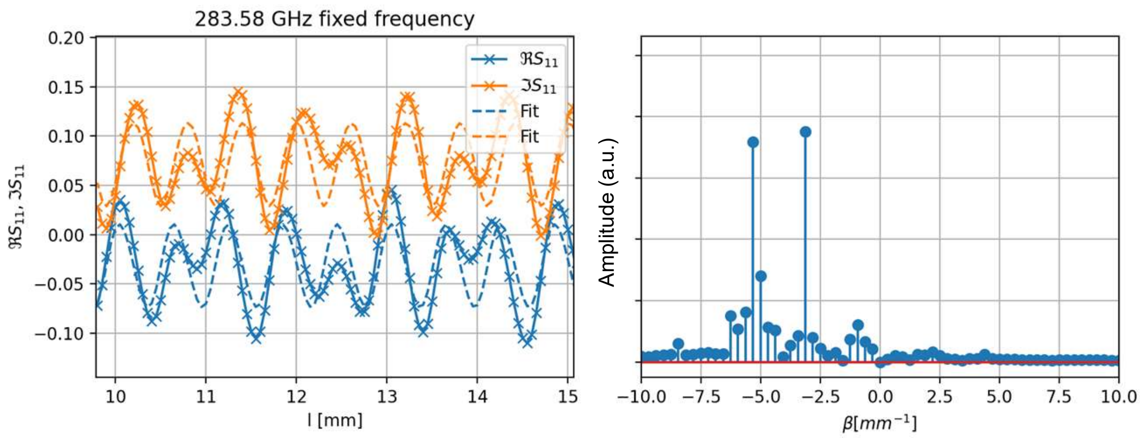


- Measured with different obstacles
- Checked reproducibility

- Measurement 12
- Third Last metallic waveguide
- Cross section at exit and inside are more likely to be the same as EDM machined, not drilled
- Prediction and Measurement match well up to a certain frequency of 280 GHz
- What happened there? Verified by another measurement

Multimode excitation in split waveguide,

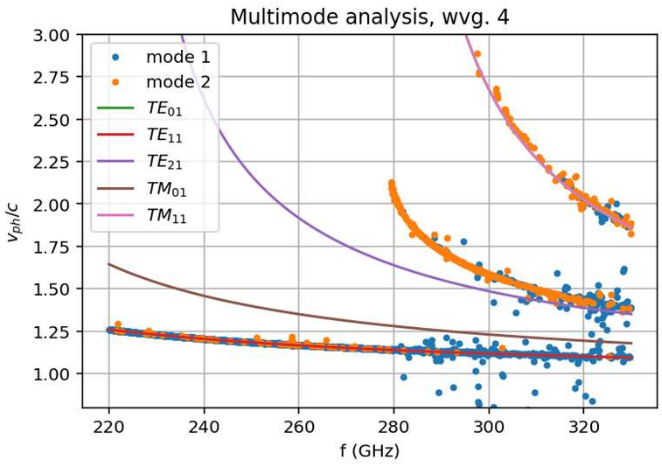
Above failing frequency



- At single frequency point: S_11(l)
- Beat of two different wavelengths directly visible → Fit fails, 4-term error model does not include multiple modes
- Transform to Fourier Domain (k-space) confirms the interpretation: Two wavevector components with almost equal amplitude

Higher order modes in metallic waveguide

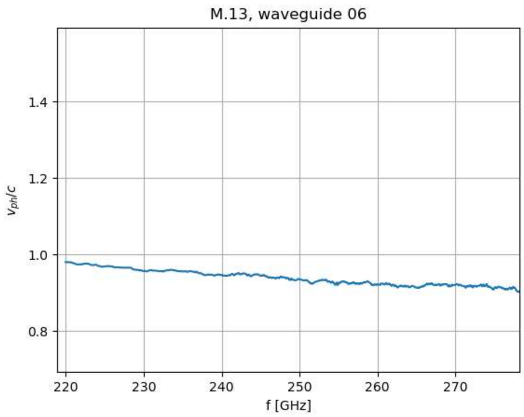
- TE_{mn}/TM_{mn} refer to analytical dispersion relations
- Using radius from microscope image: $a = 657\ \mu m$
- FDM finds a lot more modes, only selected “mode 1” and “mode 2”

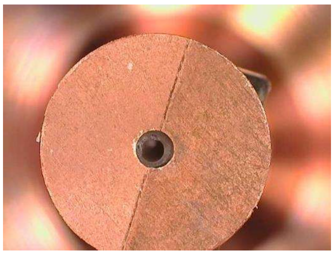


- Measurement 12
- Applying a Fourier Domain analysis using FDM → multiple dispersion lines found
- FDM useful but can’t describe error terms so not as accurate as 4-term error model in general
- Solid lines: Analytical modes expected from radius
- TE_01 hidden behind TM_11
- Apart from fundamental mode:
  - Dispersion line close to TE21 but clearly separate
  - TM11 matches very well
- Still unclear why TE_21 is so far off

Split waveguide with dielectric

Preliminary result





- Truncated $S_{11}(l)$ due to poor signal probably due to movement of the dielectric

- M.13
- Capillary inserted in waveguide
- 3d printed capillary → far from perfect (not centered, sprues on the outside distorting the waveguide inside)
- Dispersion line got pulled down (in fact, pulled down too far)
- We would like to cross the speed of light

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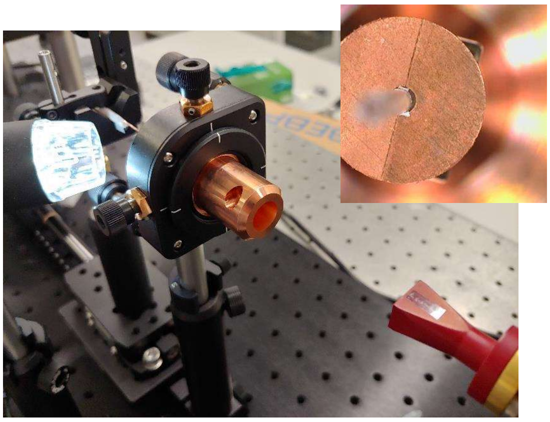
Discussion

- Uncertainty in S-parameter below 1‰ (95% confidence interval)
- 4-term error model sufficient for modelling the studied waveguides
  - Effective inner diameter for metallic waveguides
  - Phase velocity below c in dielectric loaded waveguide
- Limits of 4-term error model:
  - Effects of transitions should be small → long waveguide section
  - No reflection from behind the obstacle → obstacle not too small
    - extended model includes 9 unknown terms
  - Single mode excitation necessary
- Improving obstacle:
  - Ideal: Closed circuit, “short”, infeasible
  - Set of rods of different outer diameters
- Waveguides:
  - Imperfect Environment (oxidation of copper) and material (standard copper)
  - Dielectric layer far from acceptable

Outlook

- Improve obstacle
- Continue uncertainty evaluation
- Compare result of brass waveguide with EO sampling measurement
- Continue measurements with dielectric (better capillaries)
  → matching phase velocity



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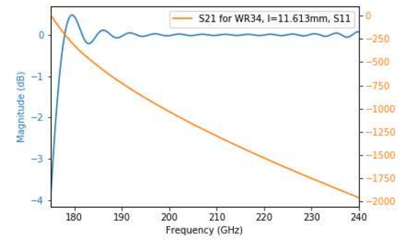
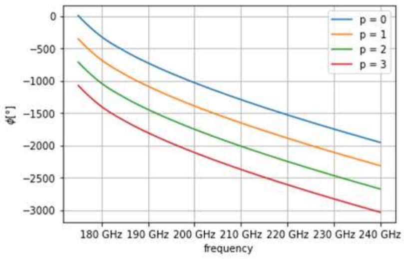
Backup

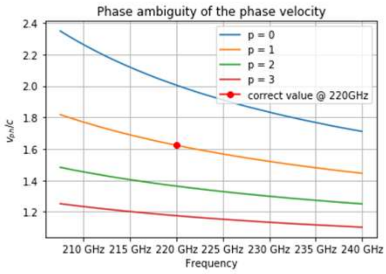
S-Parameters for Phase Velocity Measurement (Simulation)

- Simple rectangular waveguide (WR-3)
- Length: 11.613mm
- Simulated S_21
- Phase velocity from unwrapped phase

$$v_{ph} = -l f \frac{360}{\phi^{(u)}} = -l f \frac{360}{\phi + 360p}, p \in \mathbb{Z}$$





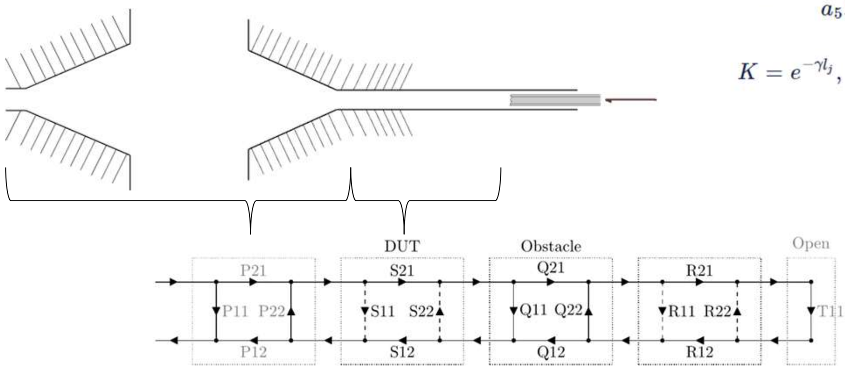


Extended 9-Term Error model

- **Issue**: canula and waveguide wall form coaxial line → additional reflection from waveguide end (interpretation after first measurements)

$$\Gamma_m^{(j)} = \frac{a_1 K\tilde{K} + a_2 K + a_3 \tilde{K} + a_4}{a_5 K\tilde{K} + a_6 K + a_7 \tilde{K} + 1},$$

$$K = e^{-\gamma l_j}, \quad \tilde{K} = e^{\tilde{\gamma} l_j}$$



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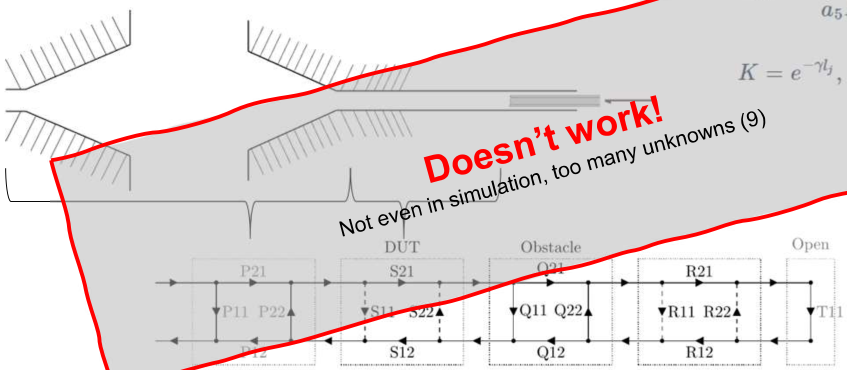
$$K = e^{-\gamma l_j}, \quad \tilde{K} = e^{\tilde{\gamma} l_j}$$



Doesn't work!

Not even in simulation, too many unknowns (9)

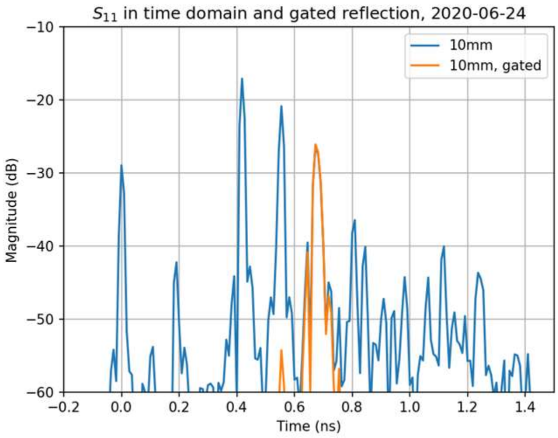
Poor man's solution: Time Gating

Time Domain Gating (without free space calibration):

1. Transform from Frequency domain to Time Domain (incomplete time domain information, but helpful tool)
2. apply window (here: Kaiser instead of Hann)
3. transform back

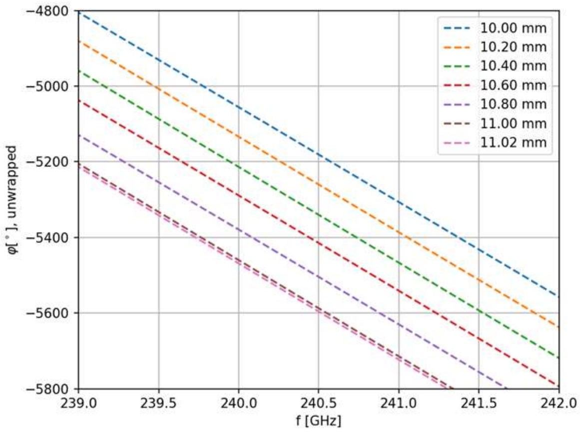
Drawbacks:

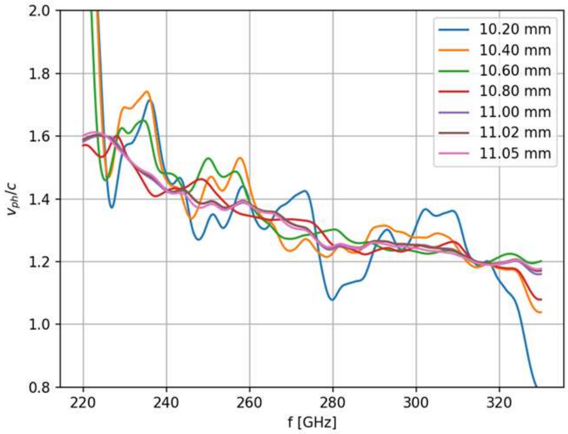
- Wiggles in frequency domain
- Result depends on window parameters (center and span)
- Frequency band must be truncated at the edges (DFT assumes a periodic signal, which the S-parameter clearly isn't)
- Phase information in bandpass transform susceptible to transform



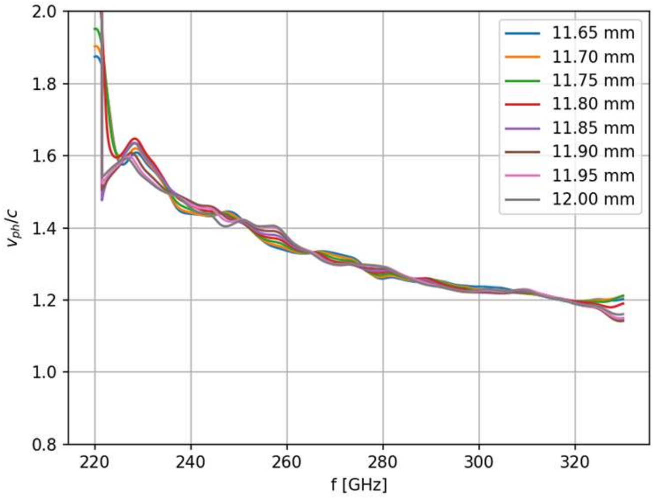
$\tau_c = 0.680\ ns, \Delta\tau = 0.252\ ns$

Measurement result with Time Domain Gating, I

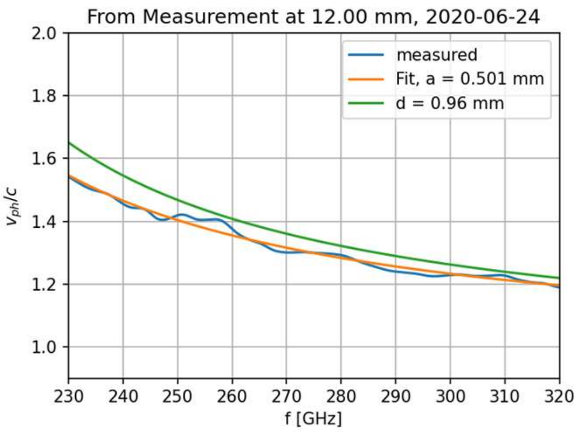




Measurement result with Time Domain Gating, II



Fit dispersion curve

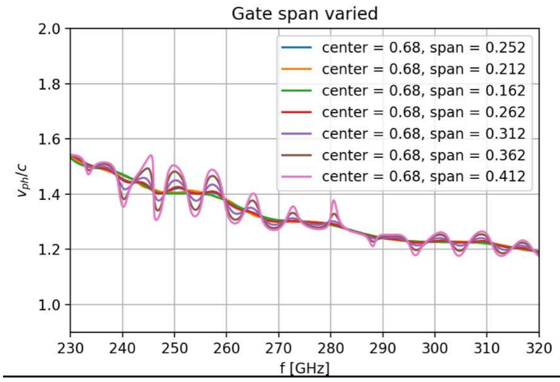


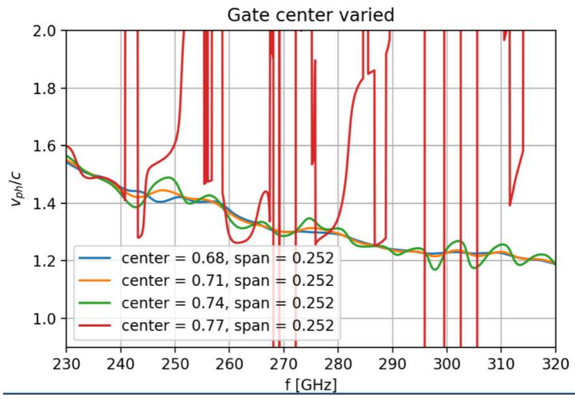
- Comparing dispersion curves
  - Measured (using time gating)
  - Fit to measured data → 0.50 mm
  - Expected from measured aperture diagonal

Fits from last 10 measurements (from 11.5 mm):

$$a = (0.500 \pm 0.002)\ mm$$

Impact of Gate parameters





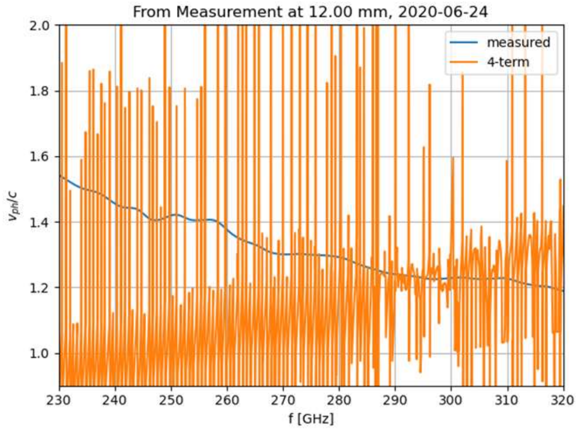
4-term error model would be better

- Per-Frequency analysis
- Does not rely on incomplete time domain information
- But:
  - error function depends on starting point
  - Complex error function
  - Sensitive to phase error

$$F = (S_{11}^{(m,j)} - a)(k^{(j)} - c) - b = 0$$

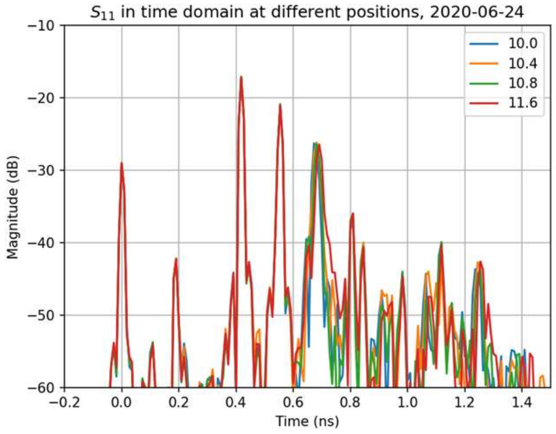
- solving for a, c, b with pairs of measurements and back-substitution
- Root finding problem

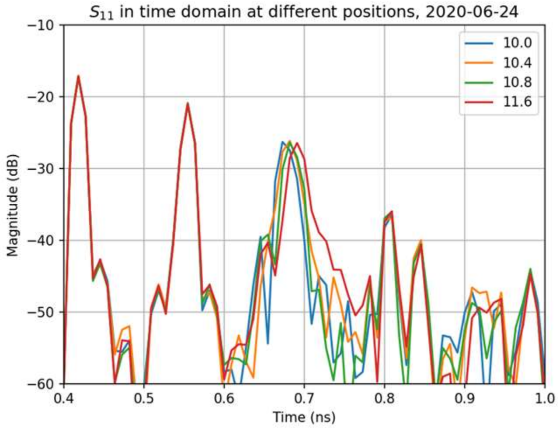
Four-term error model applied



- Using measurement at 10.00, 10.50, 11.00, 11.50 mm

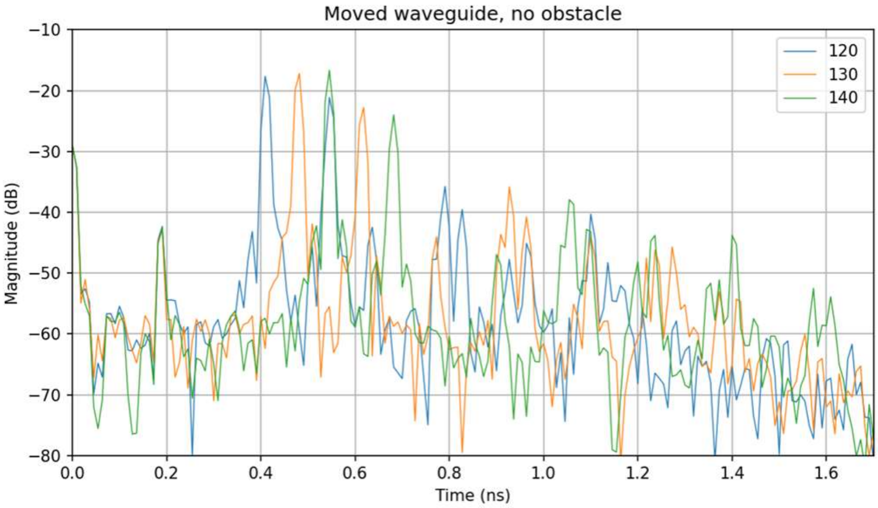
Time domain transform at different obstacle position





Time domain transform of obstacle free measurements

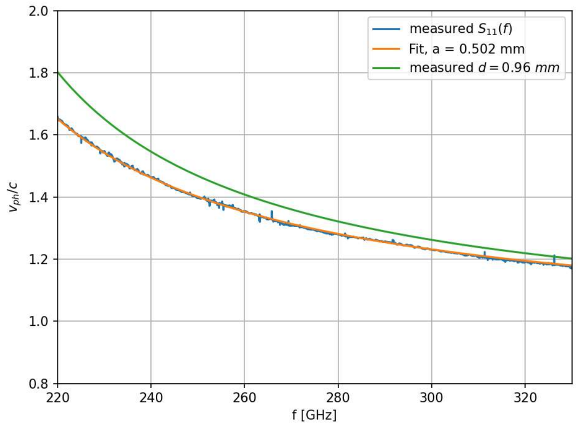
Measurement from 2020.07.02



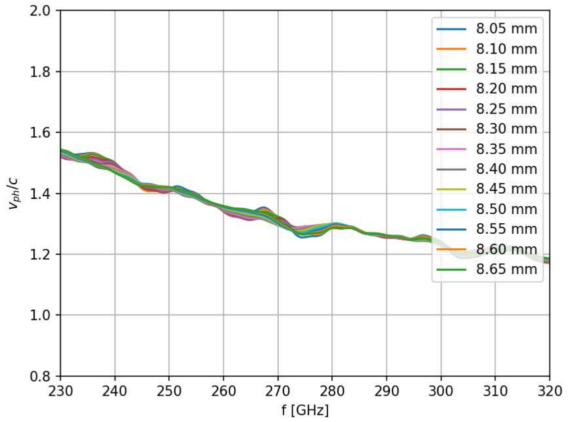
Harmonic analysis

Comparison to time domain gating

Based on harmonic analysis

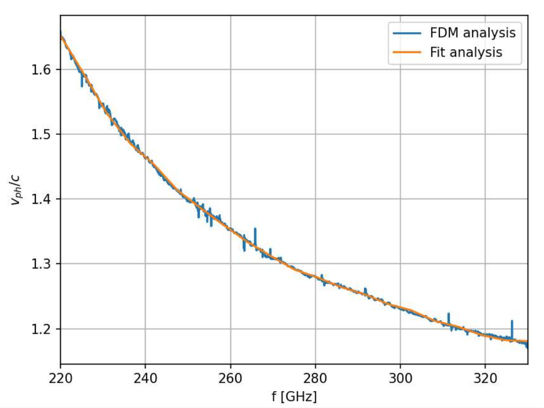


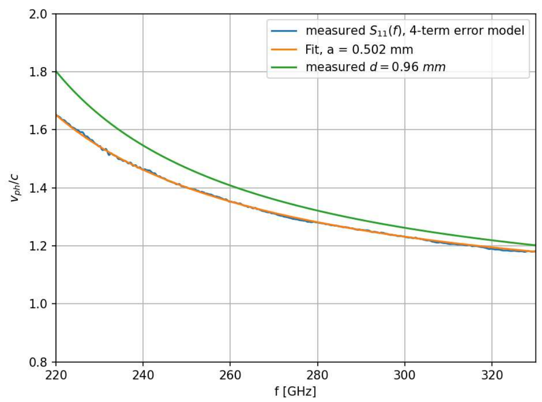
Based on time domain gating



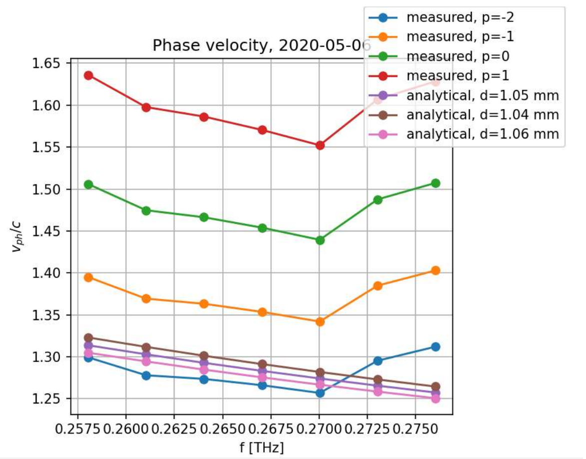
Analysis based on 4-term error model

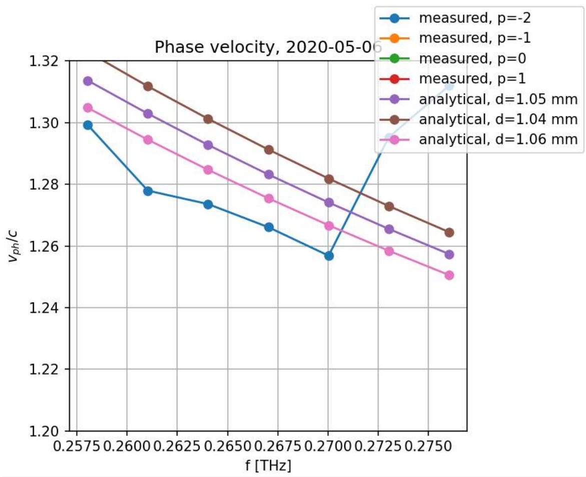
- While tweaking initial parameters for least squares problem: Found sign error
- → Curve fit works
- Better result than harmonic analysis



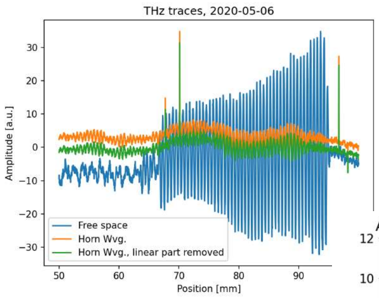


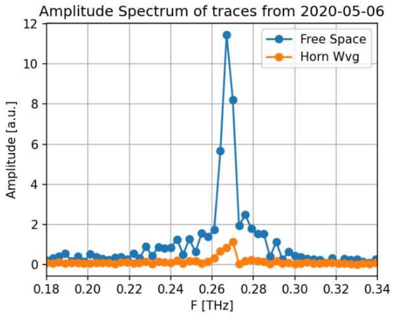
Phase velocity measured with EO sampling

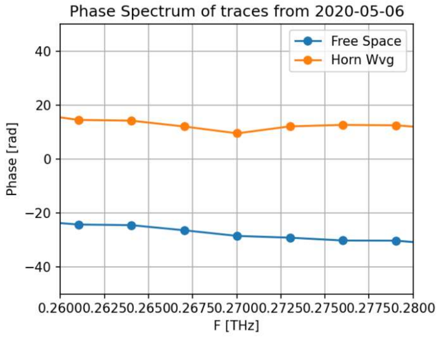




THz traces with EO sampling

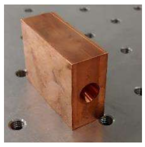






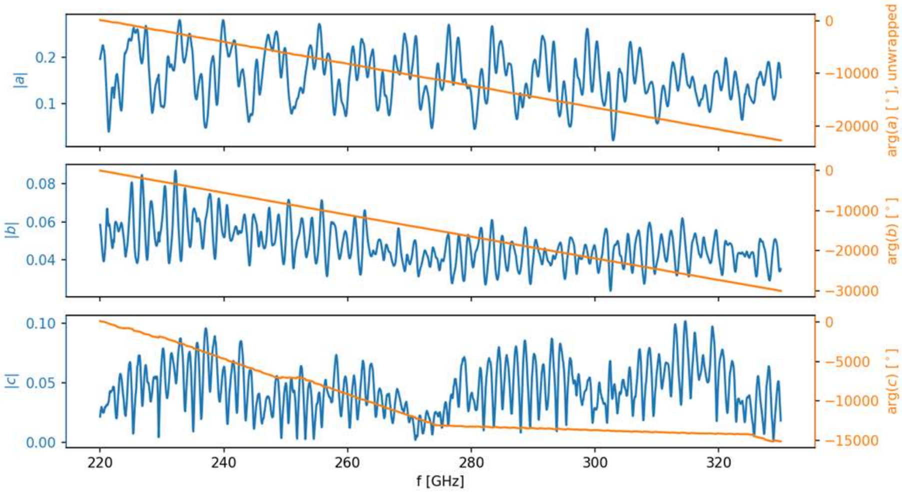
Calibration coefficients

Magnitude and unwrapped phase



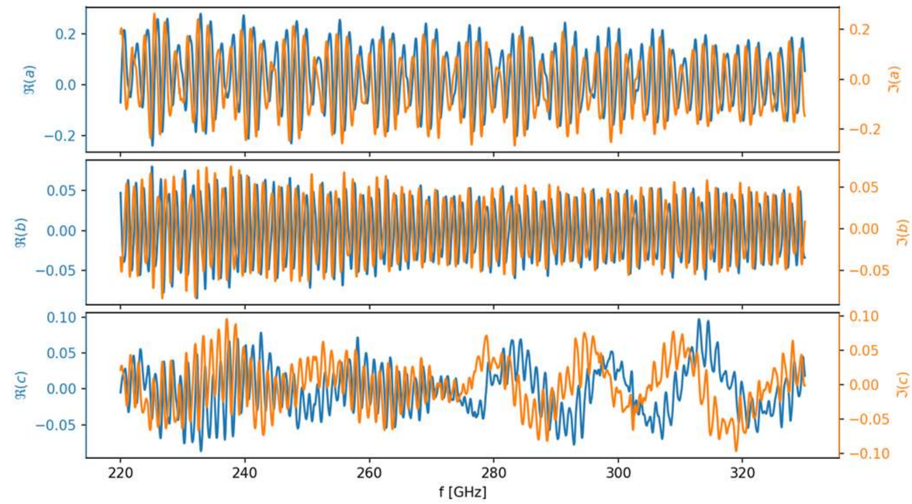
$$S_{11}^{m} = P_{11} + Q_{11}\frac{P_{21}P_{12}}{k^{-2} - P_{22}Q_{11}}$$

$$S_{11}^{m,i} = a + \frac{b}{\tilde{k}^{-2l_i} - c}$$



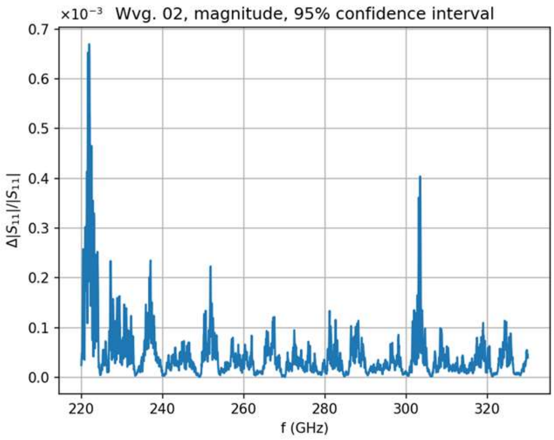
Calibration coefficients

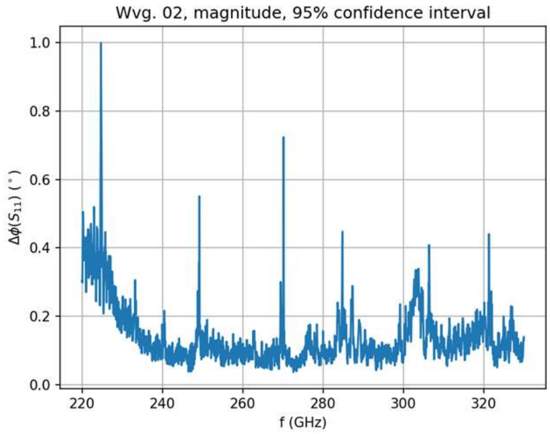
Real and Imaginary



Uncertainty in S-parameter

Waveguide 2 (brass)



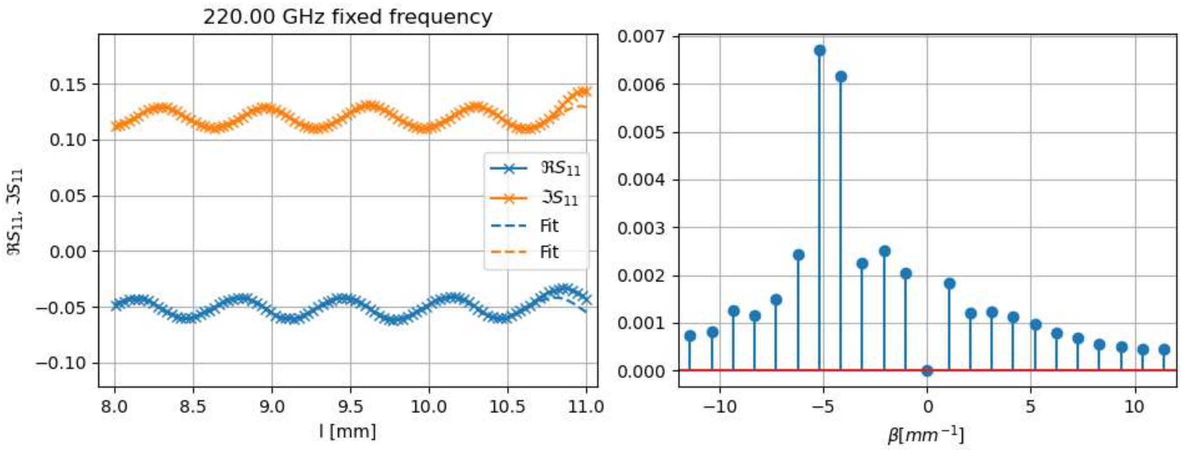


At a fixed position of the obstacle

- Measurement 17

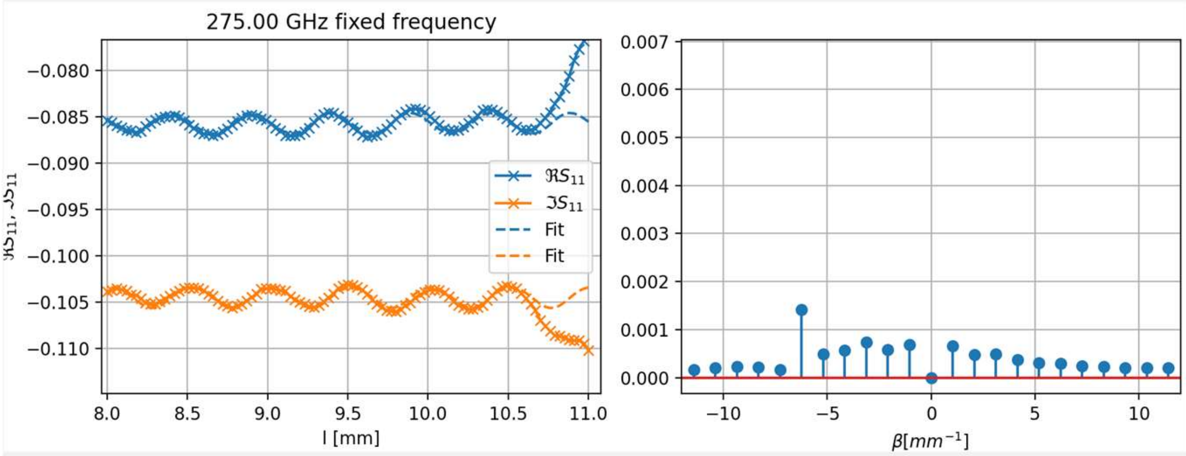
Split waveguide with dielectric, S_{11}

S_{11} at low frequency



Split waveguide with dielectric, S_{11}

S_{11} at medium frequency



Split waveguide with dielectric, S_{11}

S_{11} at high frequency