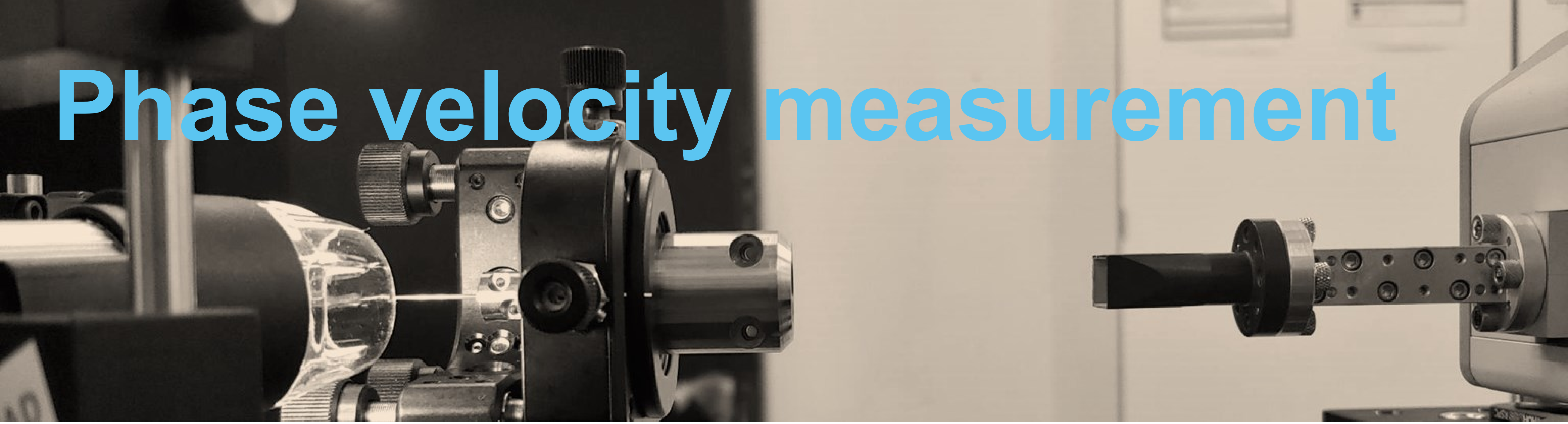

Phase velocity measurement

Calibration free characterization of integrated Terahertz waveguides

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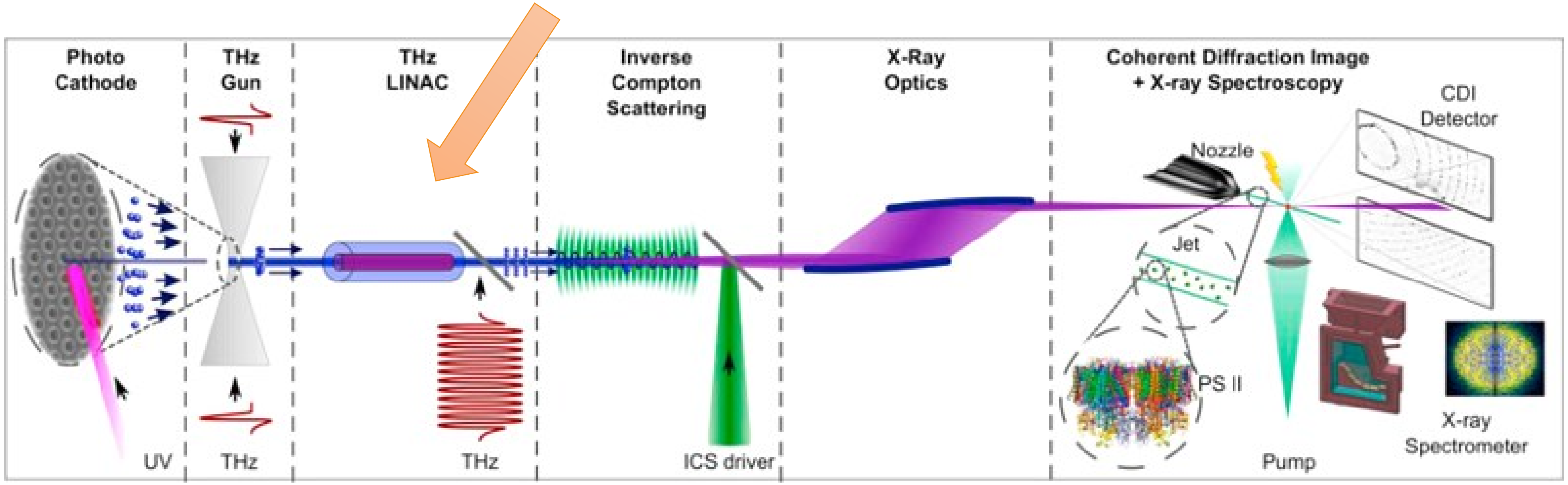
Introduction: AXSIS

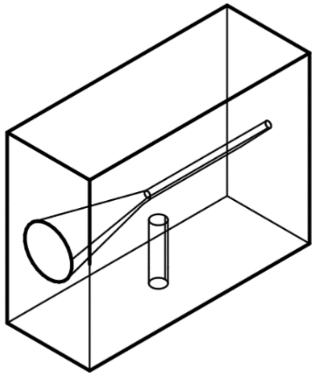
Frontiers in Attosecond X-Ray Science: Imaging and Spectroscopy

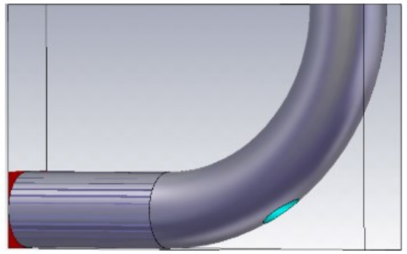
- Fully Coherent Attosecond X-ray Source
- Based on THz Acceleration and Inverse Compton Scattering



Introduction

- Emerging interest in THz waveguides in the accelerator community:
  - “Active devices” which are driven by external THz sources (streaking, accelerating)
  - “Passive devices”: Beam-driven, acts as THz source
- Due to small feature sizes → **monolithically** integrated coupler and waveguide preferential

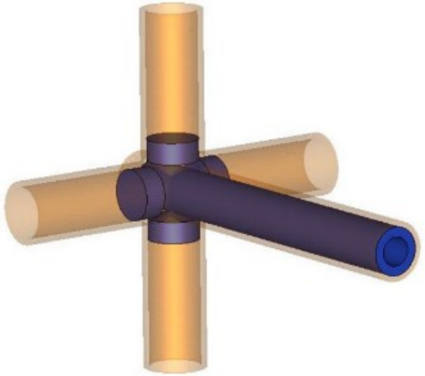


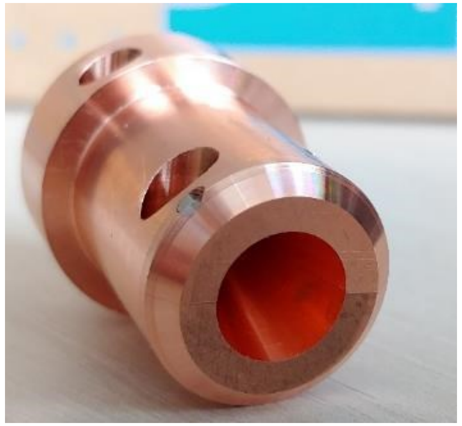


How to characterize the waveguide if you cannot separate it from the coupler?

→ Critical quantity: **phase velocity**

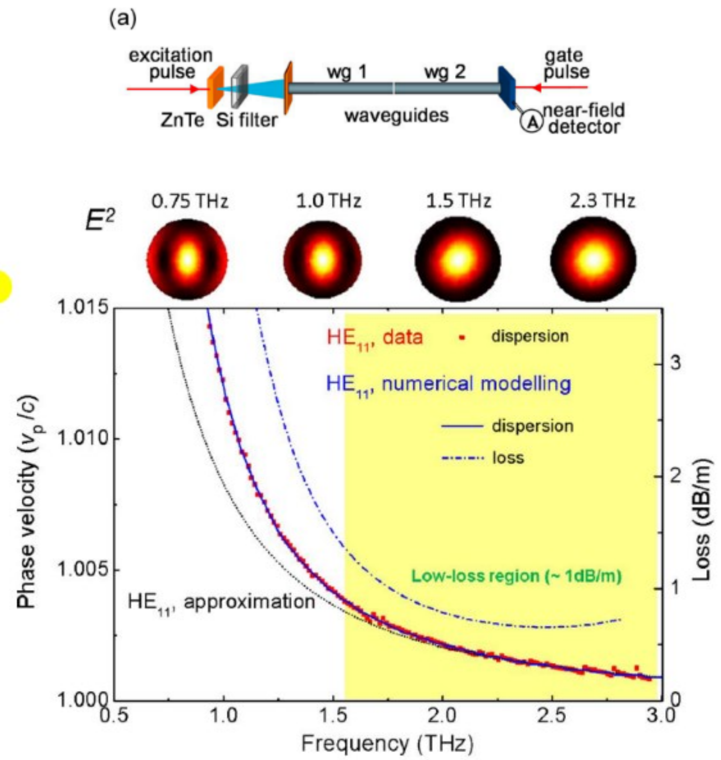
→ In case of horn coupler: Causes additional phase shift

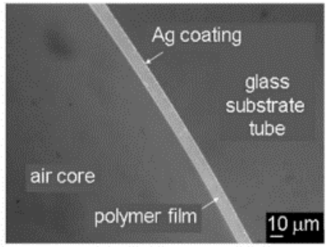




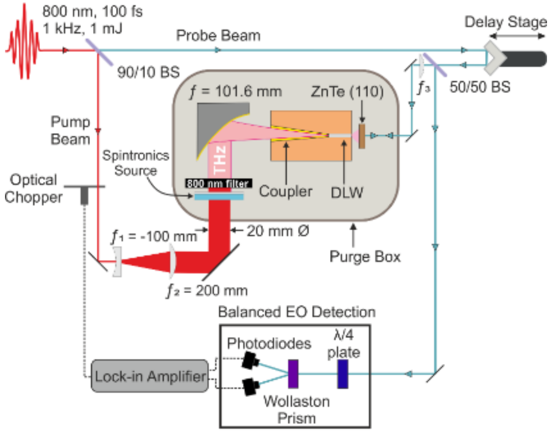
Introduction

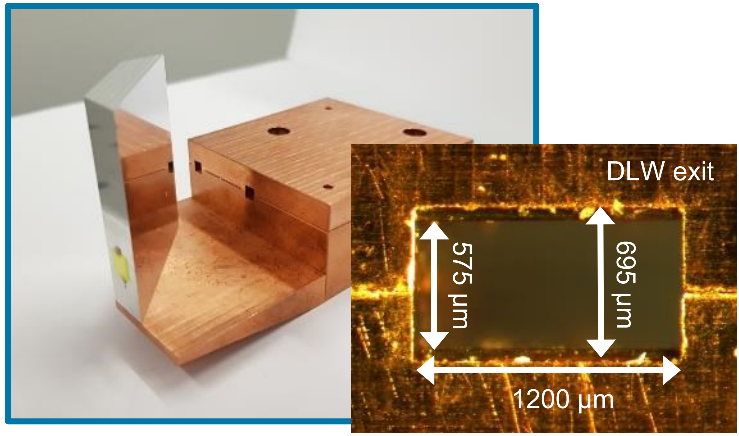
State of the art

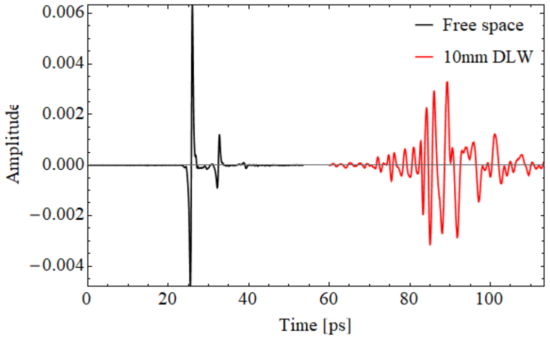


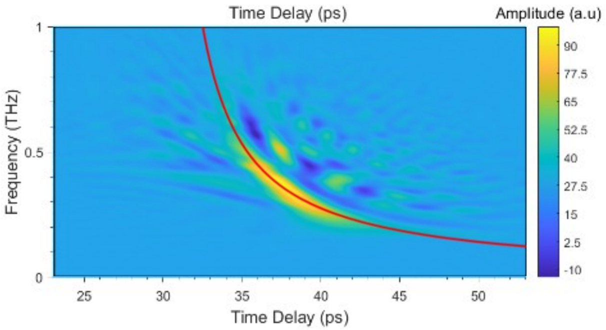


Mitrofanov, O. et al. “Reducing Transmission Losses in Hollow THz Waveguides.” *IEEE Trans. THz Sci. Tech*, **2011**.









Georgiadis, V. et al. “Characterizing the Accelerating Mode of a Dielectric-Lined Waveguide Designed for Terahertz-Driven Manipulation of Relativistic Electron Beams.” *44th IRMMW-THz*, **2019**.

S.P. Jamison, “Terahertz Driven Particle Acceleration of relativistic beams”, 4th EAAC, **2019**. (unpublished)

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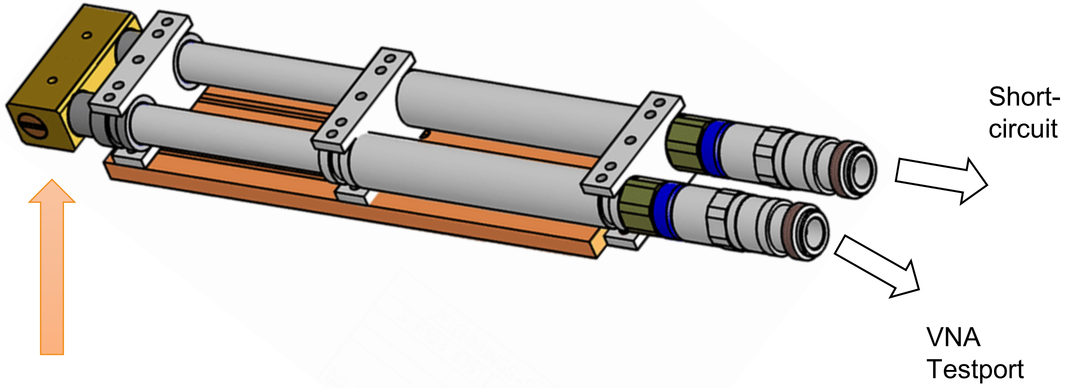
Measurement Principle

$$v_{ph} = \frac{\omega}{\beta} = f\lambda$$

- VNA: Fixed frequency → **unknown** wavenumber and wavelength in waveguide

Inspired by **lower** frequency techniques: Trombone for impedance matching (Constant impedance adjustable coaxial line)

- By varying the length of the coaxial → change circuit from “open” to “short” (180° phase)
- Measured length corresponds to half wavelength → phase velocity

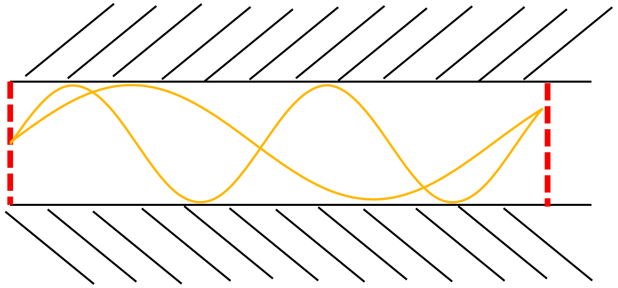


Measurement Principle

- Why not trombone?
  - Current upper frequency limit: ~ 20 GHz
  - Required feature sizes of transition infeasible
  - Connector can cause significant effect

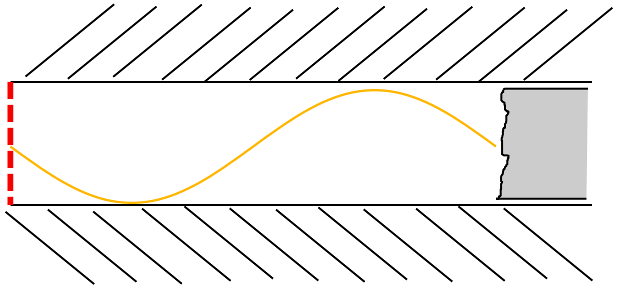
Transmission Method

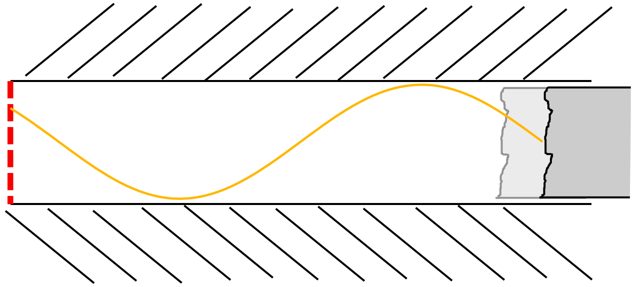
- Using phase difference between S_{21} and S_{11}
- But phase ambiguity



Reflection Method

- Circuit does not have to be closed, only a reflecting obstacle
- Obstacle movable (**subwavelength** scale)
  → measure phase difference





Four Term Error Network Model

- Issue: Waveguide **embedded** in single block with horn → can't be attached to VNA port (ref. plane)
- Additional out-coupling horn and free space
- Goal: **self-calibration** with measurements at different positions

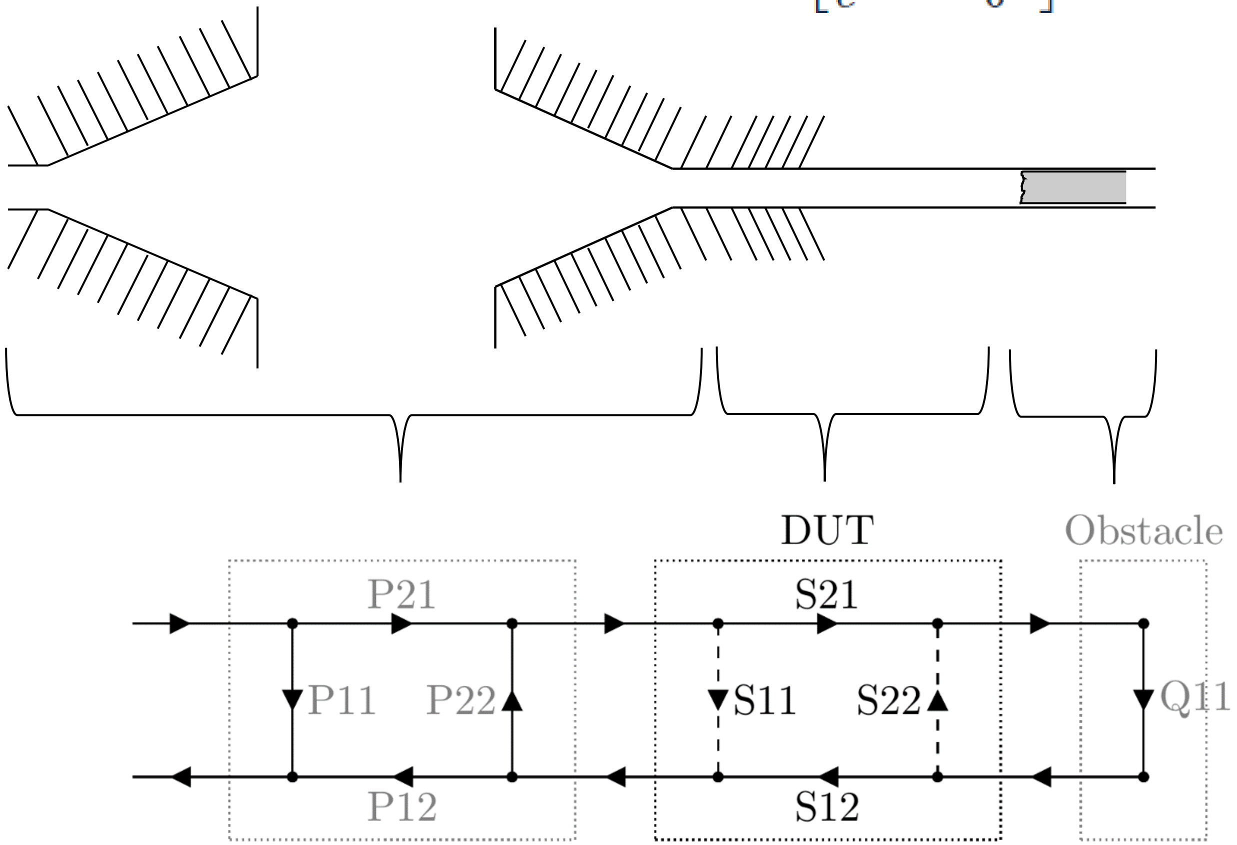
$$S_{11}^{m} = P_{11} + Q_{11}\frac{P_{21}P_{12}}{k^{-2} - P_{22}Q_{11}}$$

i-th measurement

$$S_{11}^{m,i} = a + \frac{b}{\tilde{k}^{-2l_i} - c}$$

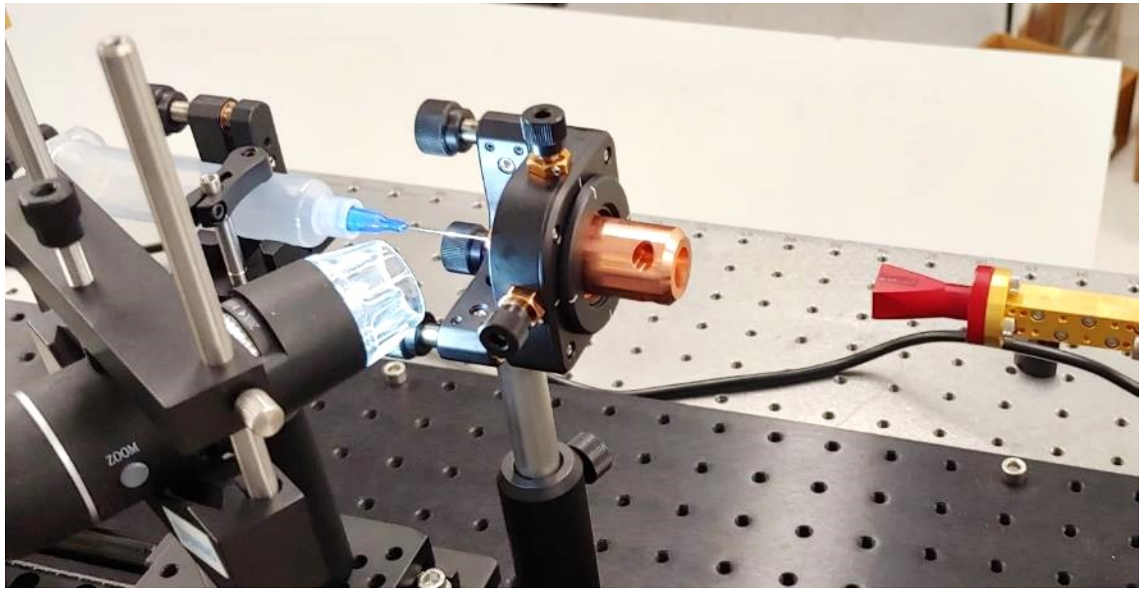
Four Term Error Model

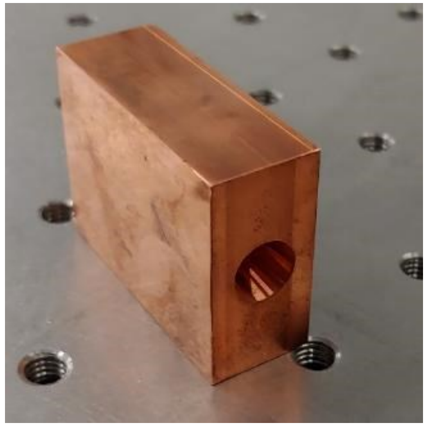
$$S = \begin{bmatrix} 0 & e^{-\gamma l} \\ e^{-\gamma l} & 0 \end{bmatrix}, \quad k := e^{-\gamma l}$$

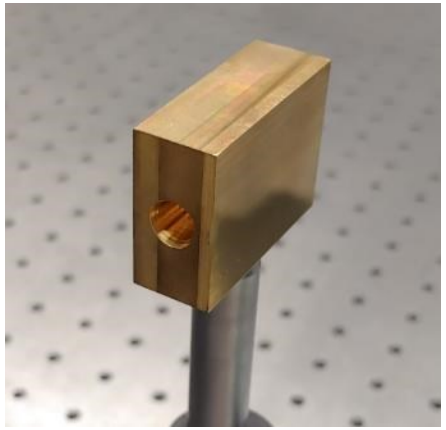


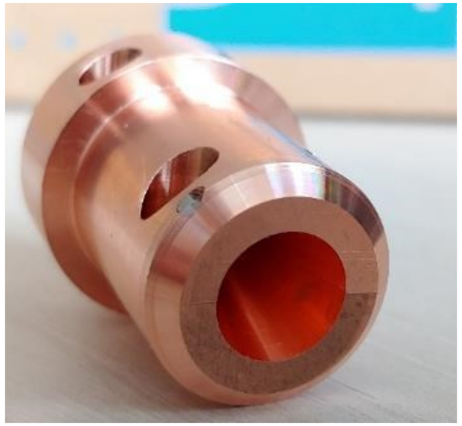
Experimental Setup and Waveguide Structure

- Frequency range: 220 GHz – 330 GHz (Streaking: 267 GHz, AXSIS: 300 GHz)
- Obstacles:
  - Syringe + Canula as obstacle (OD = 0.72 mm, ID = 0.41 mm), on motorized stage (+/- 1 µm)
  - Paper clip (OD = 0.89mm)
  - New: steel welding wire
- Three different waveguide structures:
  1. Conical horn drilled in copper monolithically
  2. Same, but in brass
  3. EDM machined in two half shelves, “split waveguide”







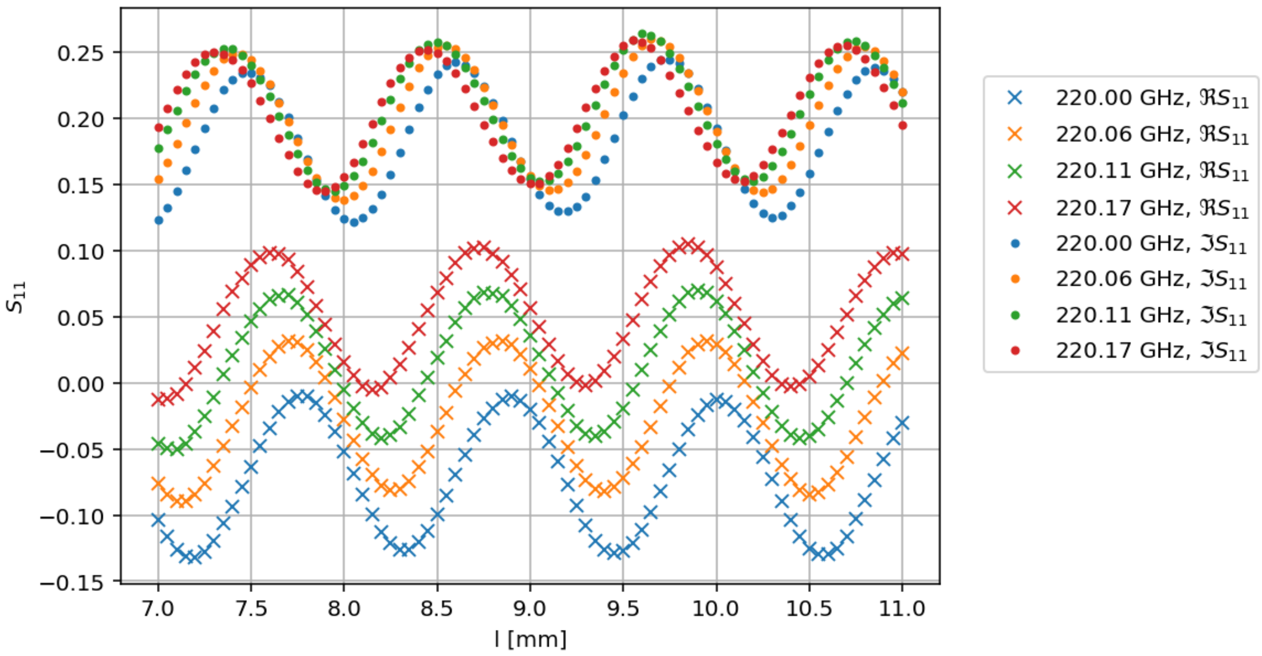


S11 depending on position in 4-term error model

Copper Waveguide 1

- Measured: $S_{11}(f,l)$
- analyzed independently for each frequency point:

$$S_{11}(l)\Big|_{f=f_0}$$



$$\mathrm{re}\left(S_{11}\right)=\mathrm{re}\left(a\right)-\frac{\left(\sin\left(2\beta l\right)+\mathrm{im}\left(c\right)\right)\mathrm{im}\left(b\right)}{-2\sin\left(2\beta l\right)\mathrm{im}\left(c\right)+2\cos\left(2\beta l\right)\mathrm{re}\left(c\right)}+\frac{\left(\cos\left(2\beta l\right)-\mathrm{re}\left(c\right)\right)\mathrm{re}\left(b\right)}{-2\sin\left(2\beta l\right)\mathrm{im}\left(c\right)+2\cos\left(2\beta l\right)\mathrm{re}\left(c\right)}$$

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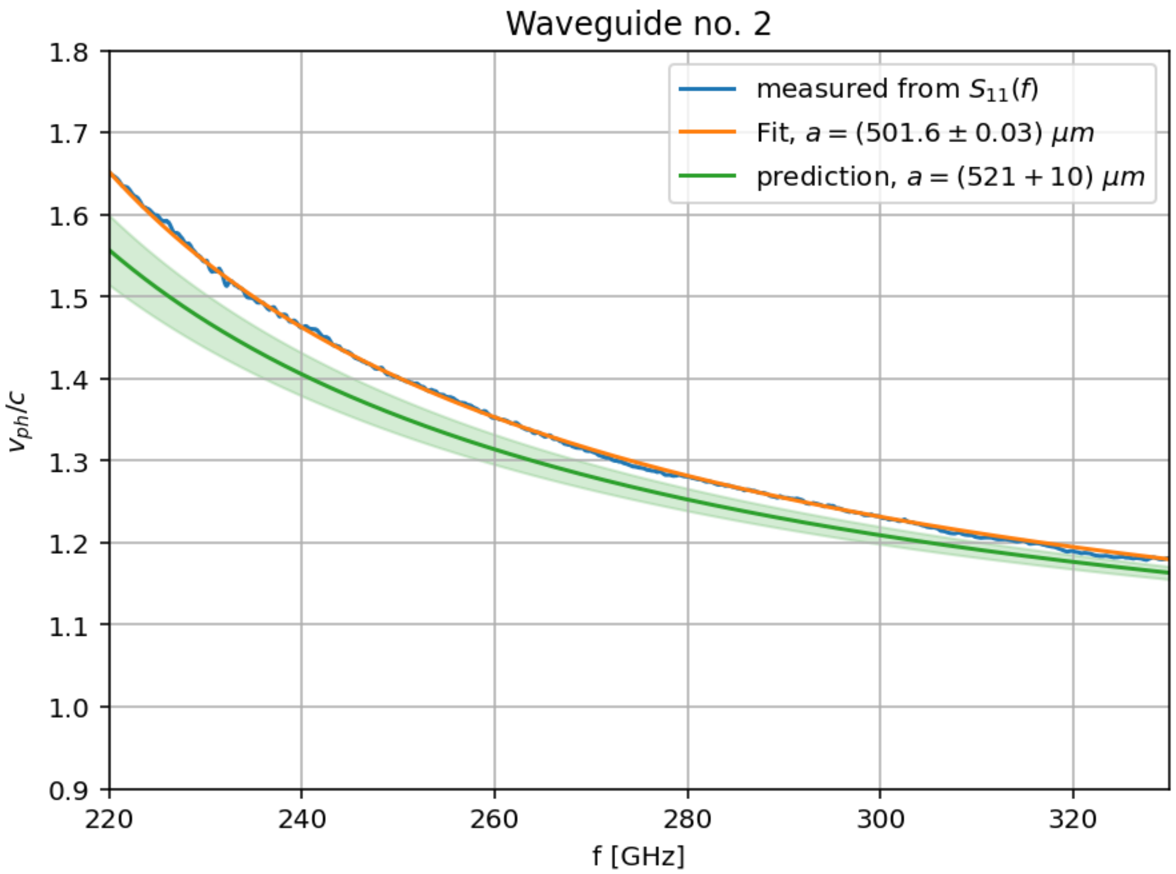
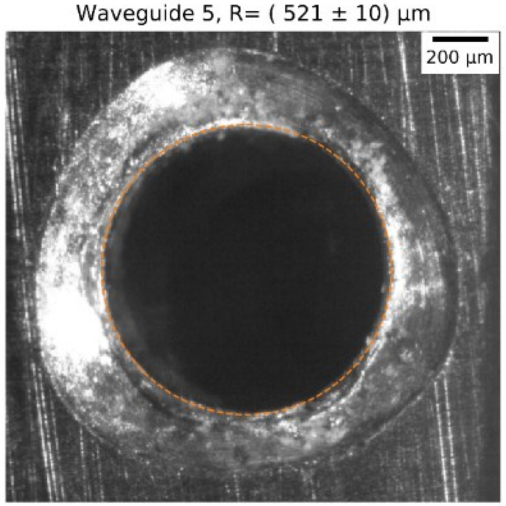
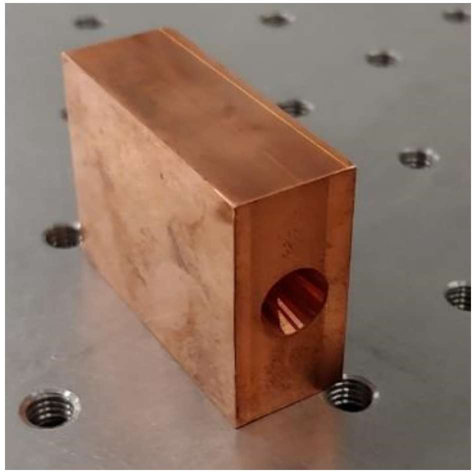
Analysis based on 4-term error model

Copper waveguide

- Tweaking initial parameters for least squares problem → Curve fit works
- Better result than harmonic analysis (FDM)



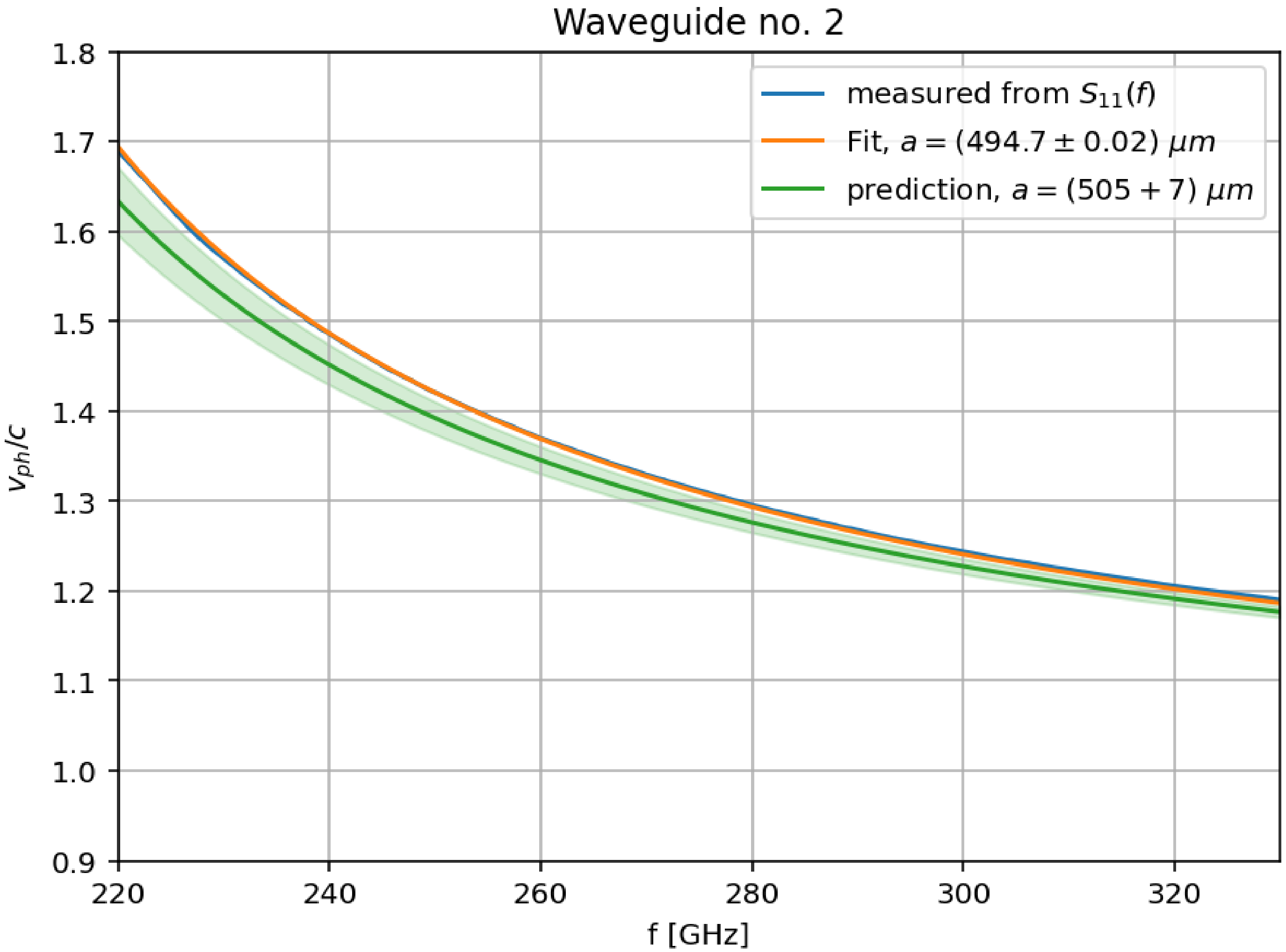


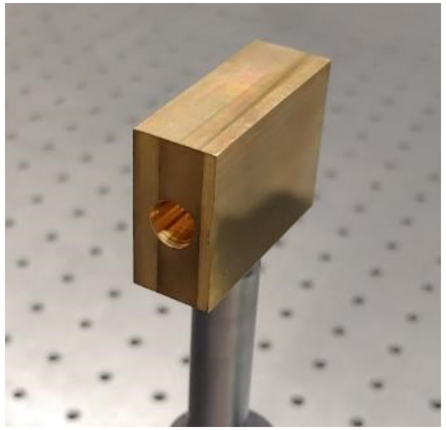


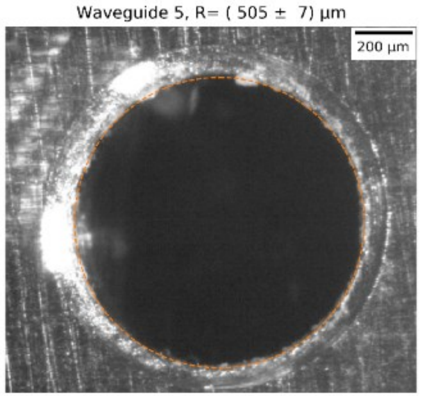
$$v_{ph} = \frac{1}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

$$f_c = \frac{c u_{11}}{2\pi a}$$

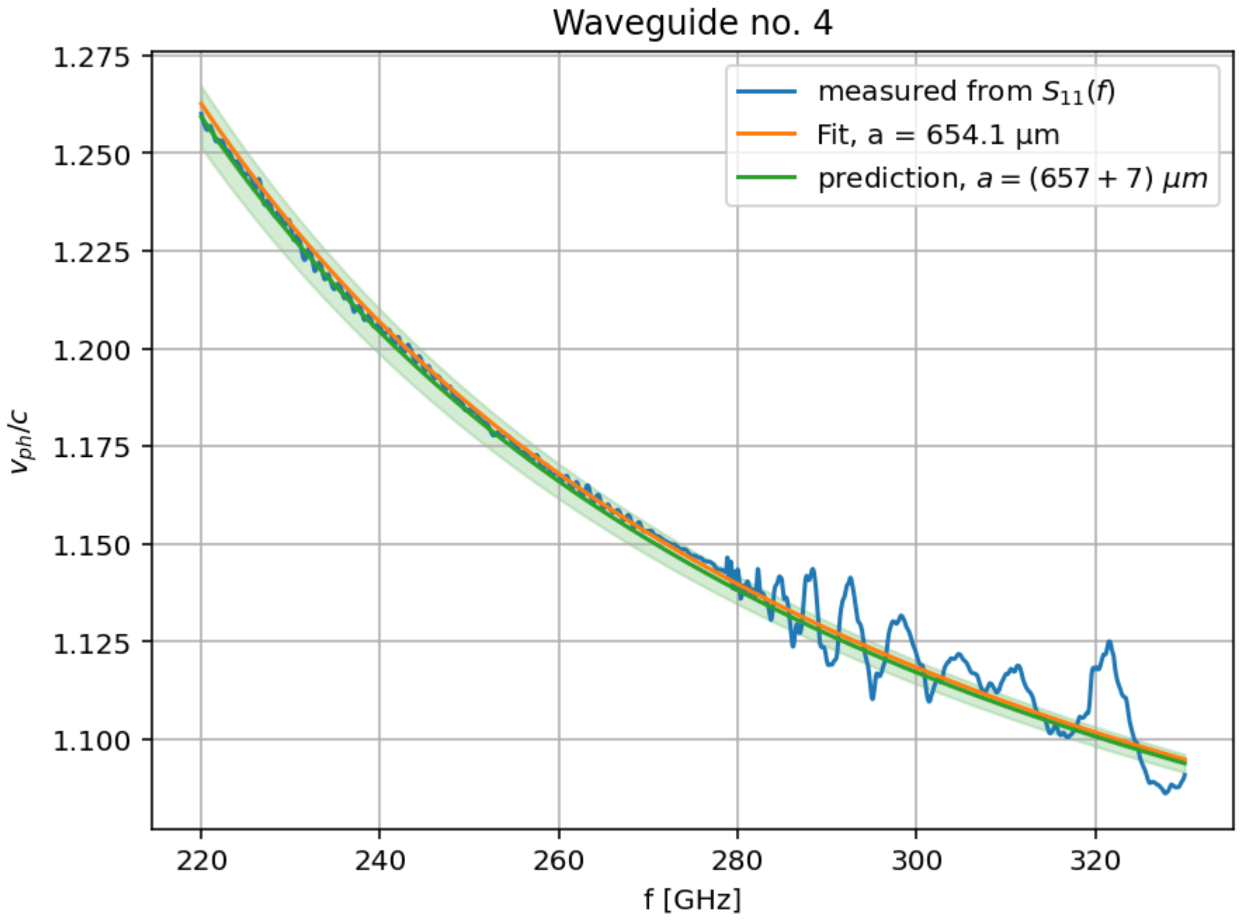
Brass waveguide

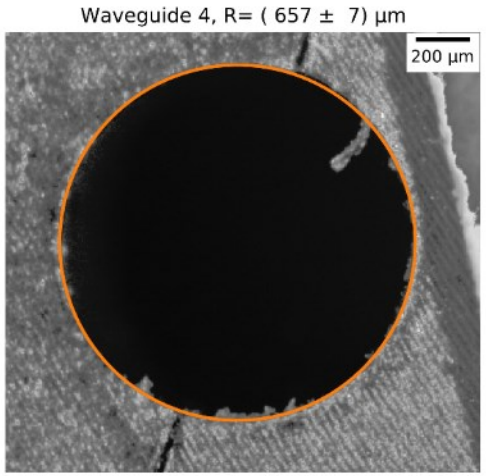


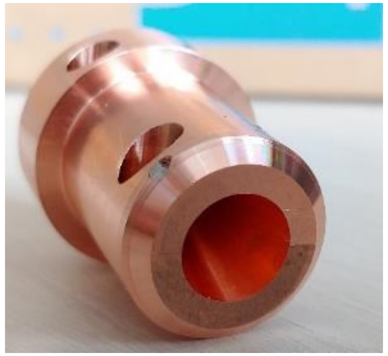


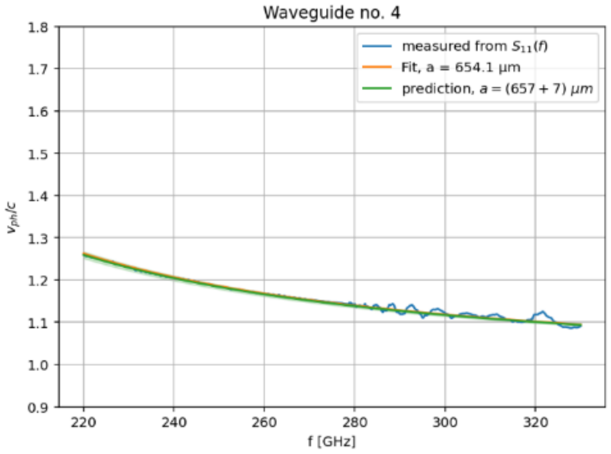


Split waveguide





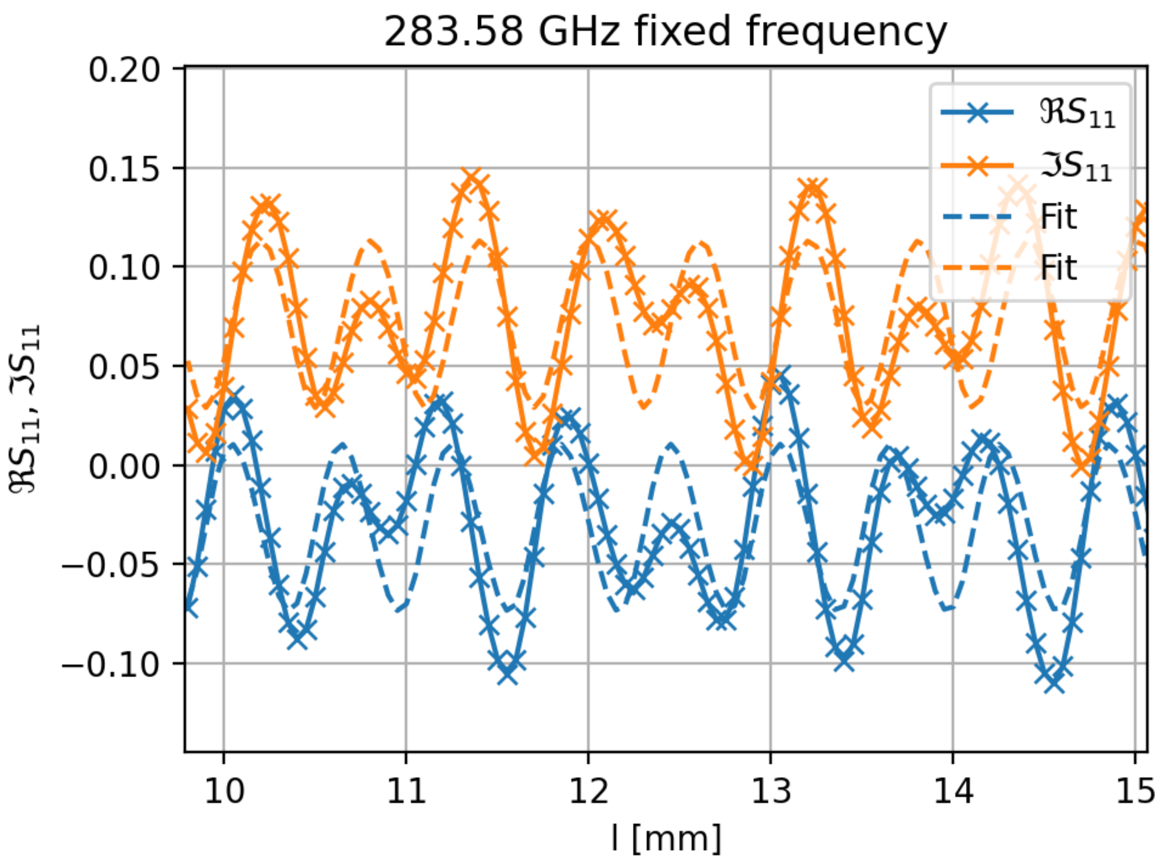


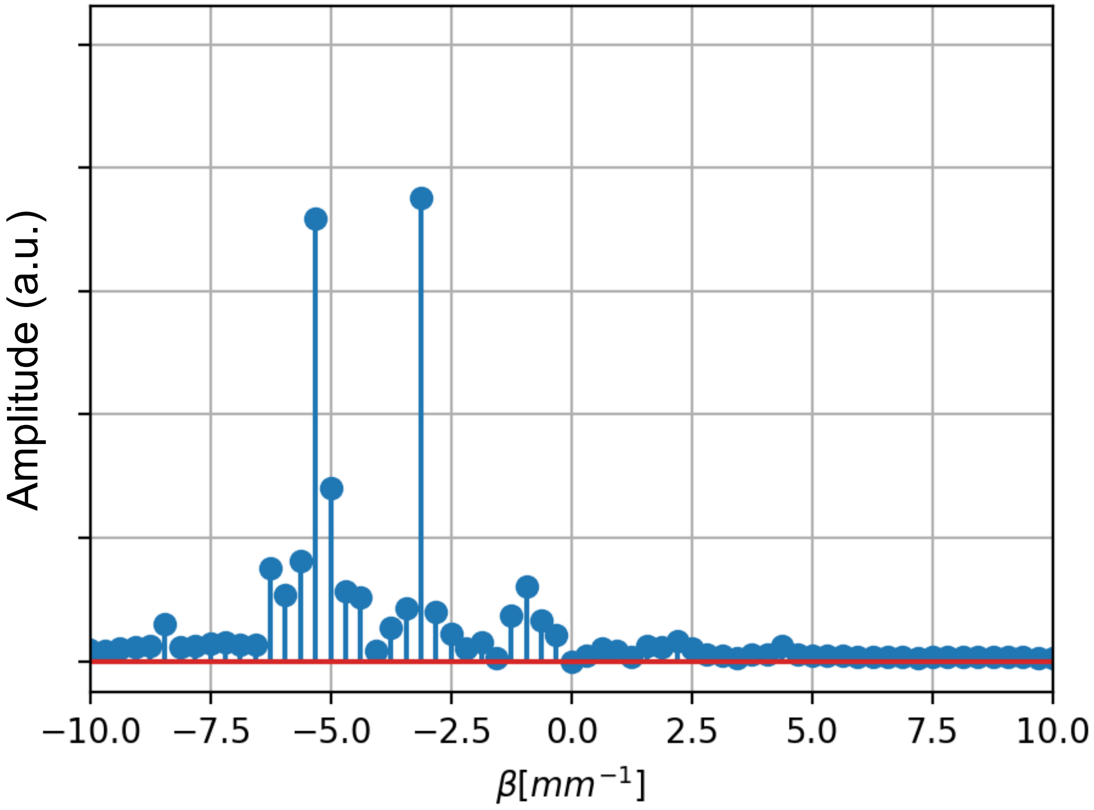


- Measured with different obstacles
- Checked reproducibility

Multimode excitation in split waveguide,

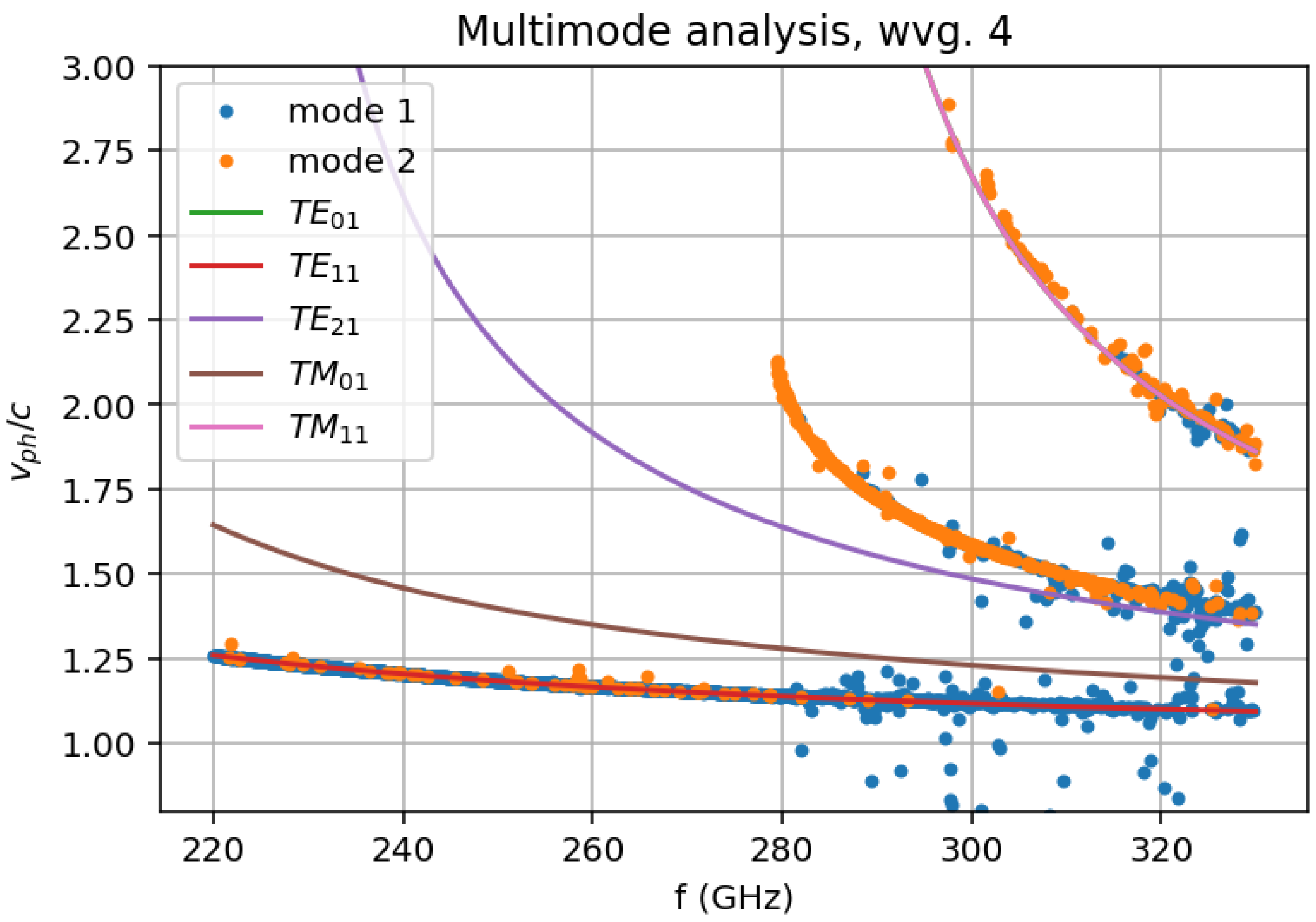
Above failing frequency





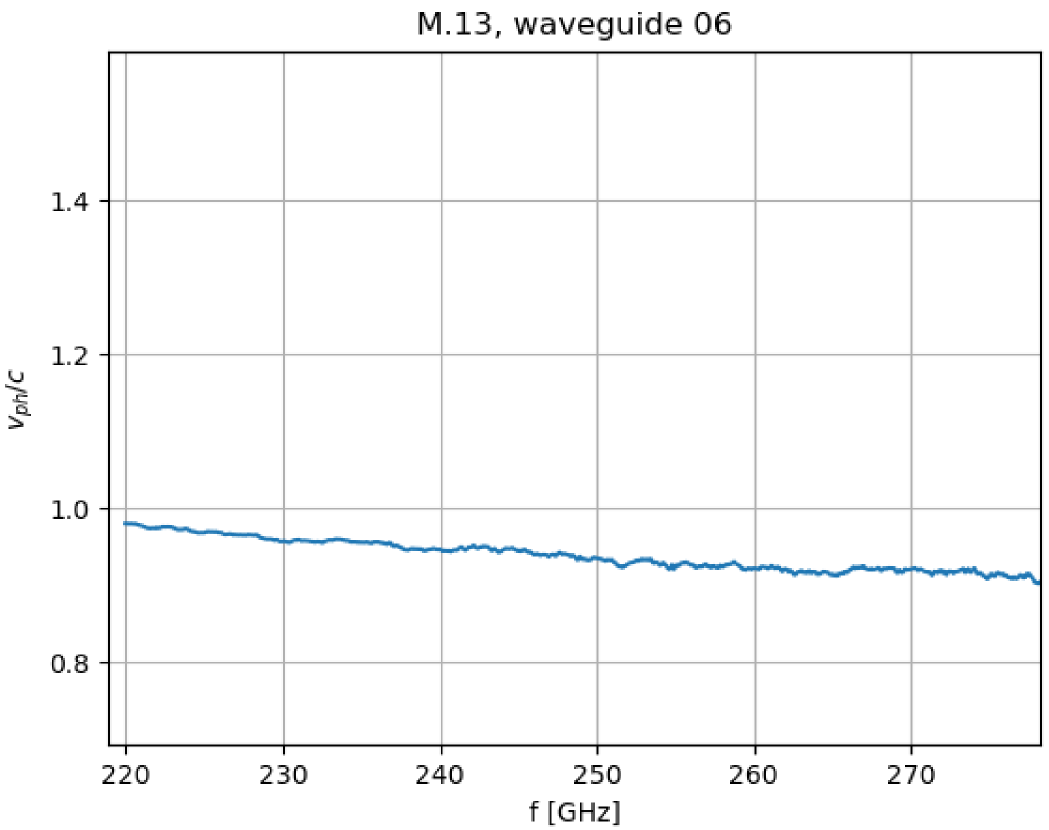
Higher order modes in metallic waveguide

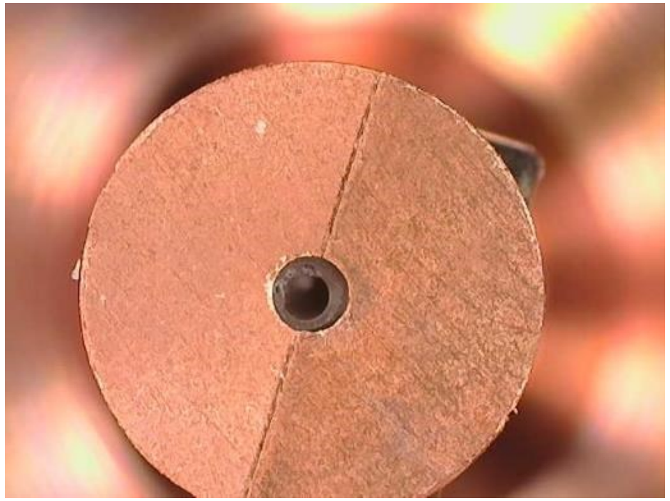
- TE_{mn}/TM_{mn} refer to analytical dispersion relations
- Using radius from microscope image: $a = 657\ \mu m$
- FDM finds a lot more modes, only selected “mode 1” and “mode 2”



Split waveguide with dielectric

Preliminary result





- Truncated $S_{11}(l)$ due to poor signal probably due to movement of the dielectric

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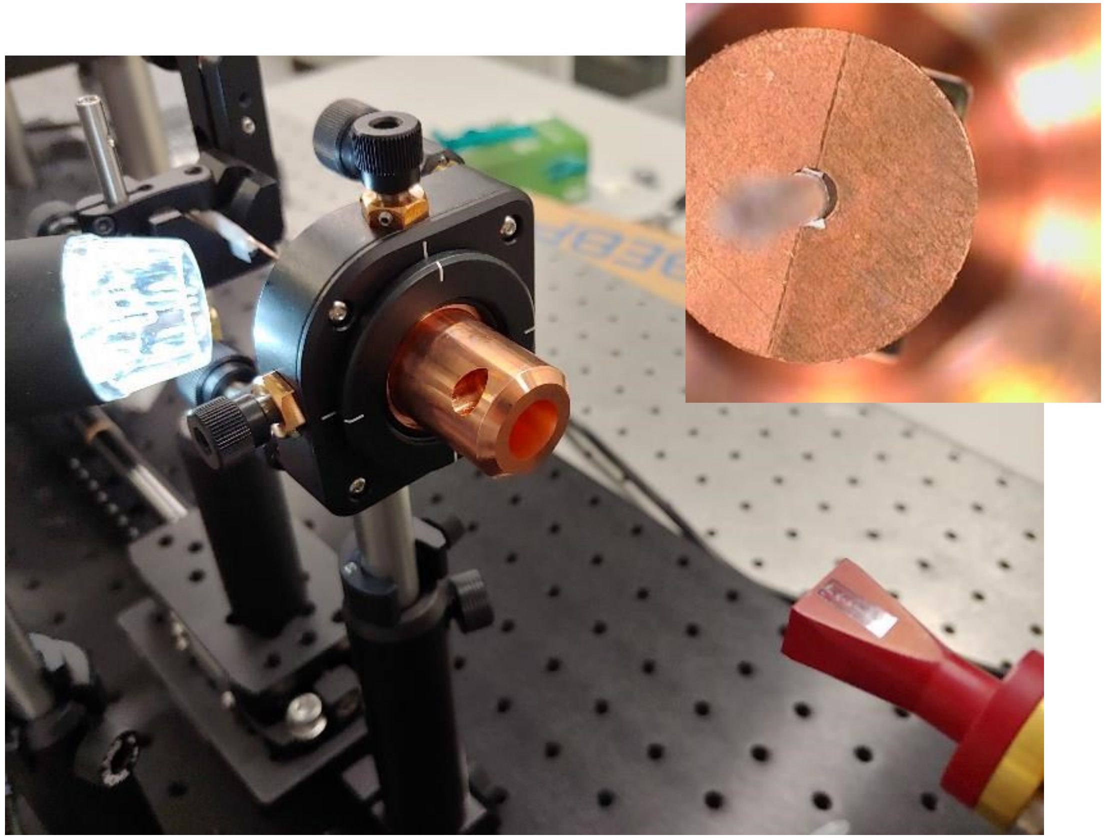
Discussion

- Uncertainty in S-parameter below 1‰ (95% confidence interval)
- 4-term error model sufficient for modelling the studied waveguides
  - Effective inner diameter for metallic waveguides
  - Phase velocity below c in dielectric loaded waveguide
- Limits of 4-term error model:
  - Effects of transitions should be small → long waveguide section
  - No reflection from behind the obstacle → obstacle not too small
    - extended model includes 9 unknown terms
  - Single mode excitation necessary
- Improving obstacle:
  - Ideal: Closed circuit, “short”, infeasible
  - Set of rods of different outer diameters
- Waveguides:
  - Imperfect Environment (oxidation of copper) and material (standard copper)
  - Dielectric layer far from acceptable

Outlook

- Improve obstacle
- Continue uncertainty evaluation
- Compare result of brass waveguide with EO sampling measurement
- Continue measurements with dielectric (better capillaries)
  → matching phase velocity

Acknowledgements:
Francois Lemery, Thomas Vinatier, Klaus Flöttmann, Wolfgang Hillert, Ralph Aßmann, Hannes Dinter, Frank Mayet, Christian Henkel, Immo Bahns, Patrick Rauer, Markus Hünning

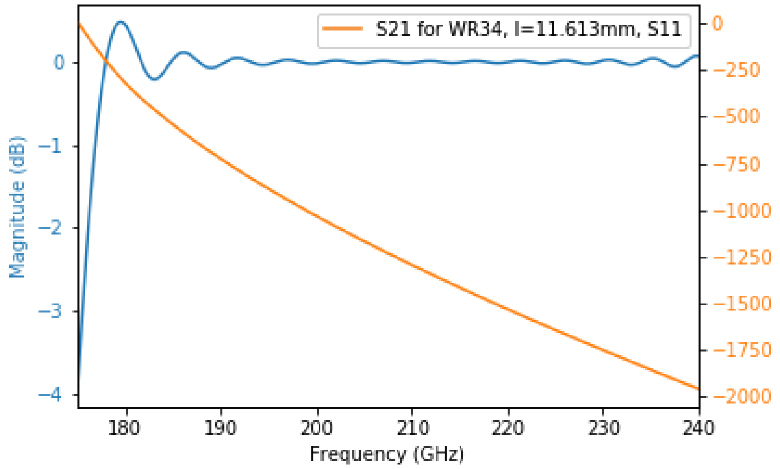


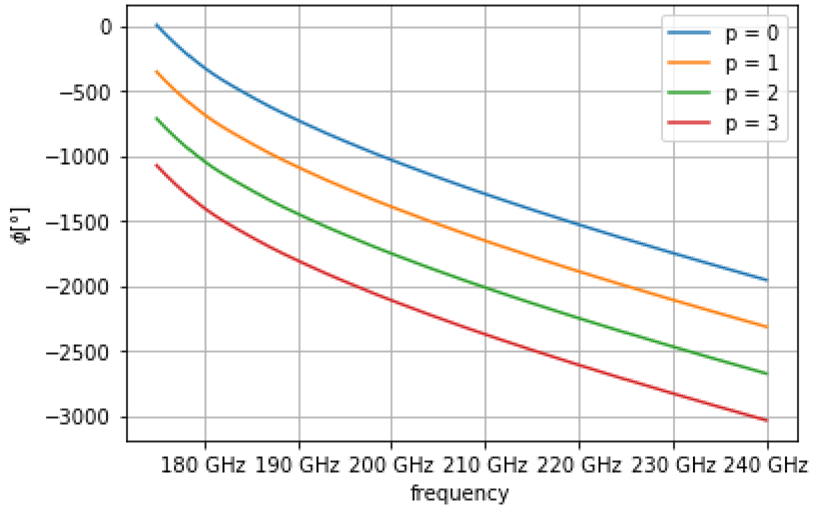
Backup

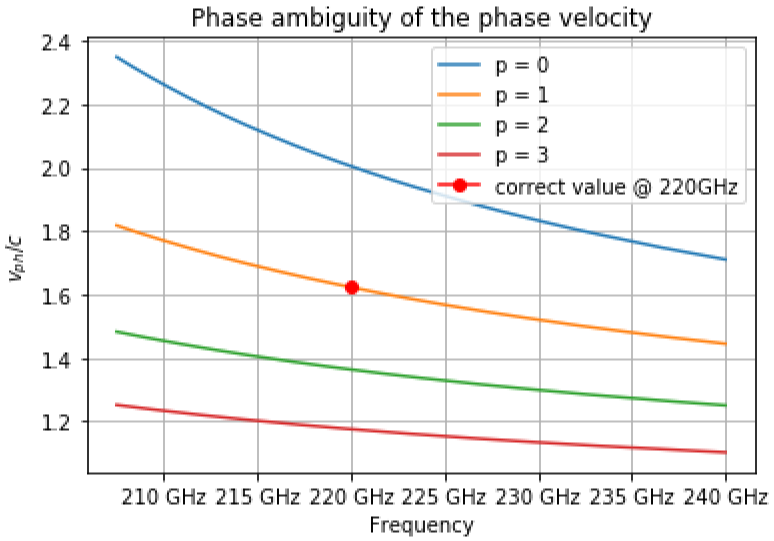
S-Parameters for Phase Velocity Measurement (Simulation)

- Simple rectangular waveguide (WR-3)
- Length: 11.613mm
- Simulated S_21
- Phase velocity from unwrapped phase

$$v_{ph} = -l f \frac{360}{\phi^{(u)}} = -l f \frac{360}{\phi + 360p}, p \in \mathbb{Z}$$





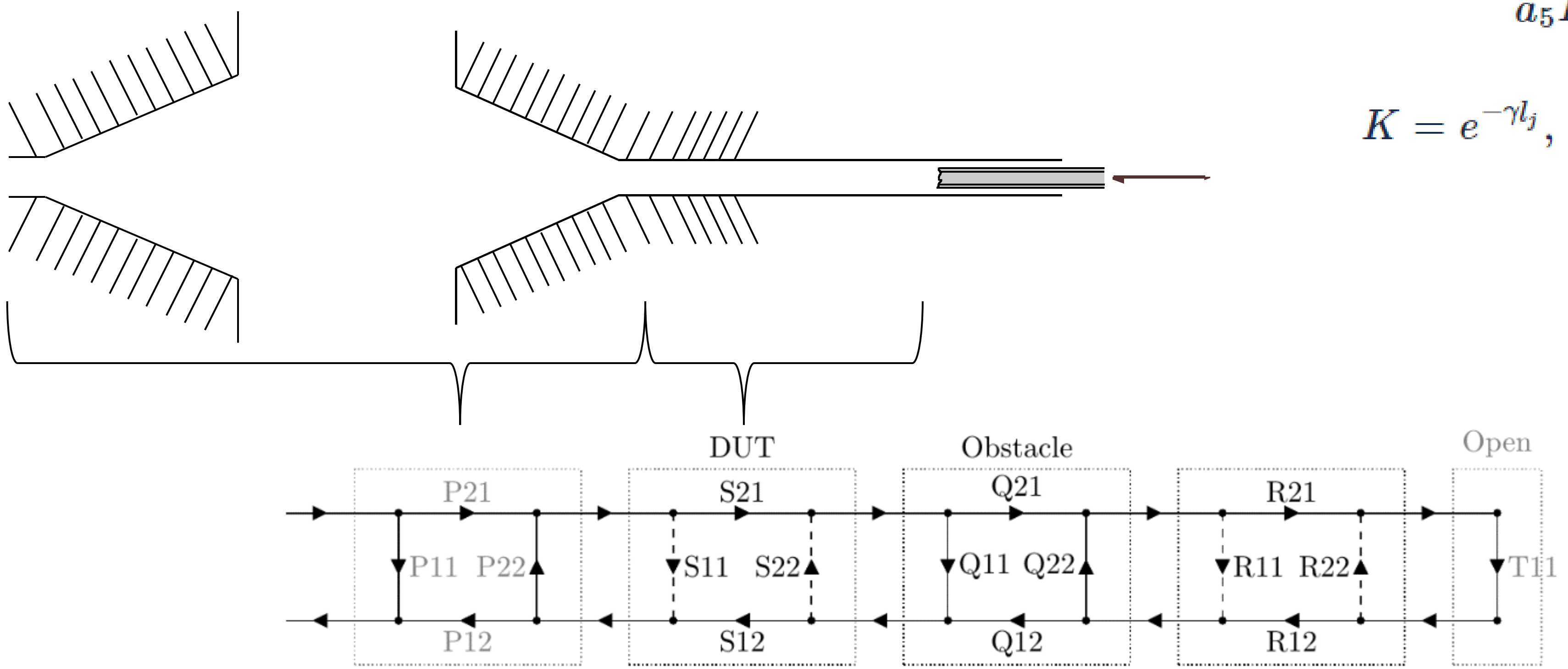


Extended 9-Term Error model

- **Issue**: canula and waveguide wall form coaxial line → additional reflection from waveguide end (interpretation after first measurements)

$$\Gamma_m^{(j)} = \frac{a_1 K\tilde{K} + a_2 K + a_3 \tilde{K} + a_4}{a_5 K\tilde{K} + a_6 K + a_7 \tilde{K} + 1},$$

$$K = e^{-\gamma l_j}, \quad \tilde{K} = e^{\tilde{\gamma} l_j}$$



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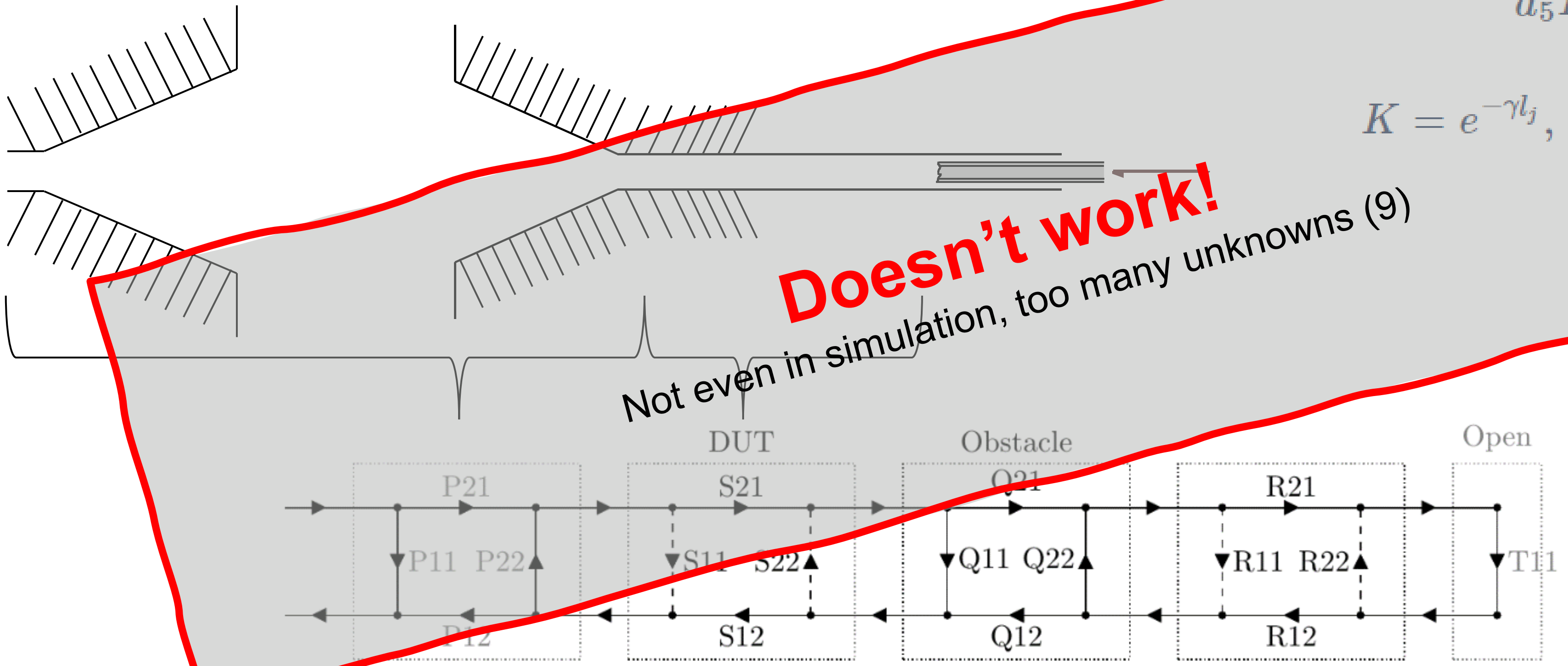


Doesn't work!

Not even in simulation, too many unknowns (9)

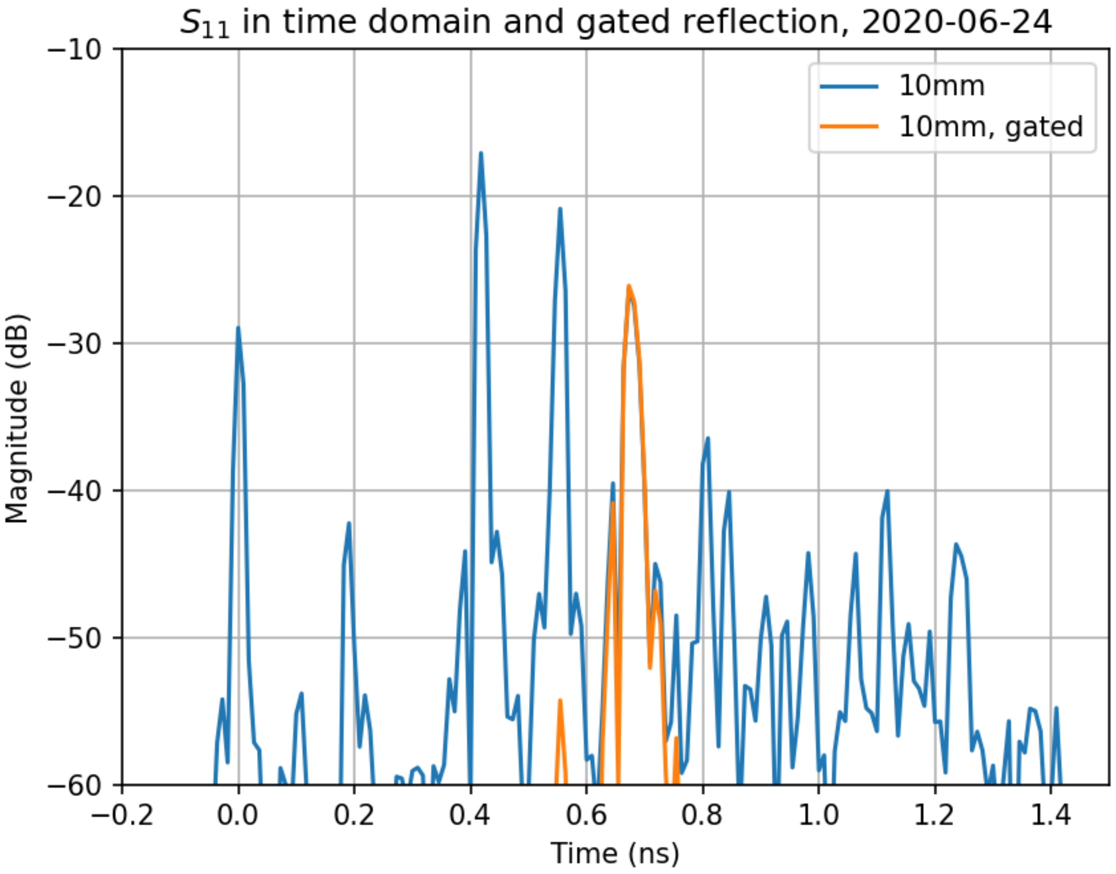
Poor man's solution: Time Gating

Time Domain Gating (without free space calibration):

1. Transform from Frequency domain to Time Domain (incomplete time domain information, but helpful tool)
2. apply window (here: Kaiser instead of Hann)
3. transform back

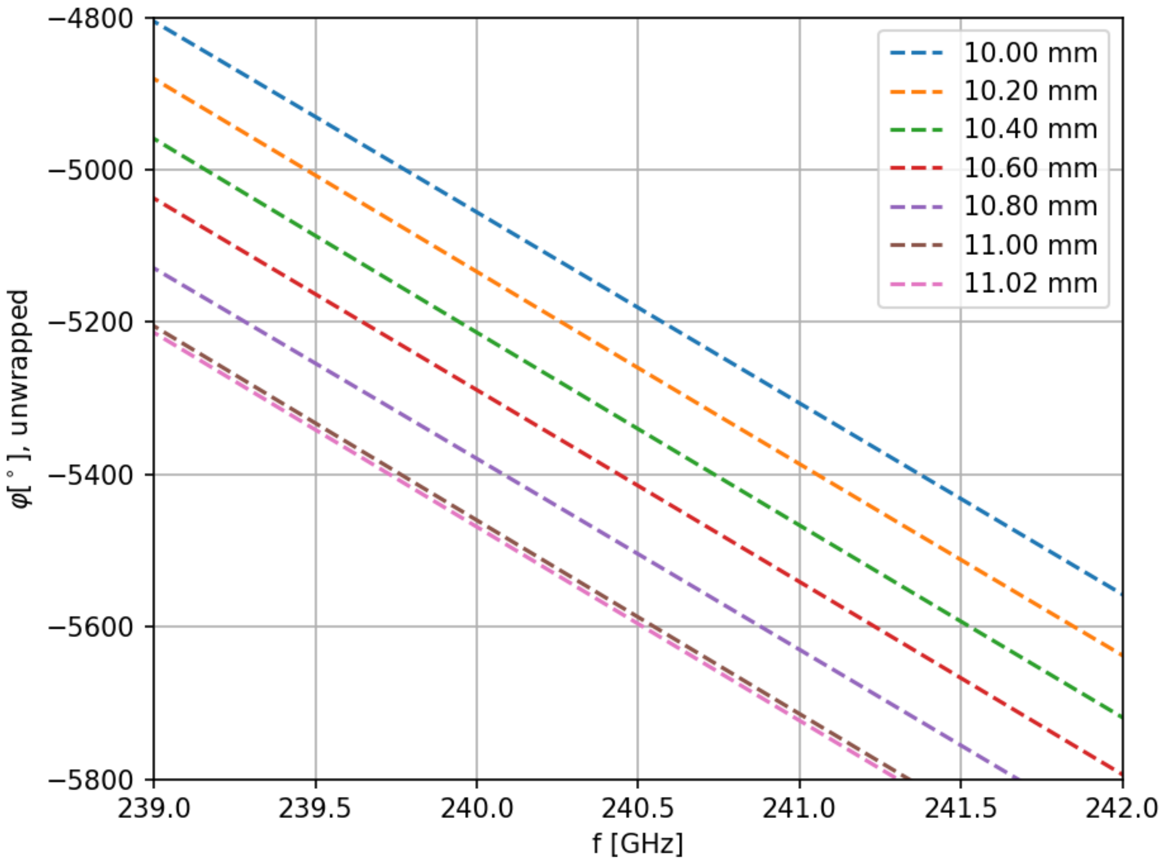
Drawbacks:

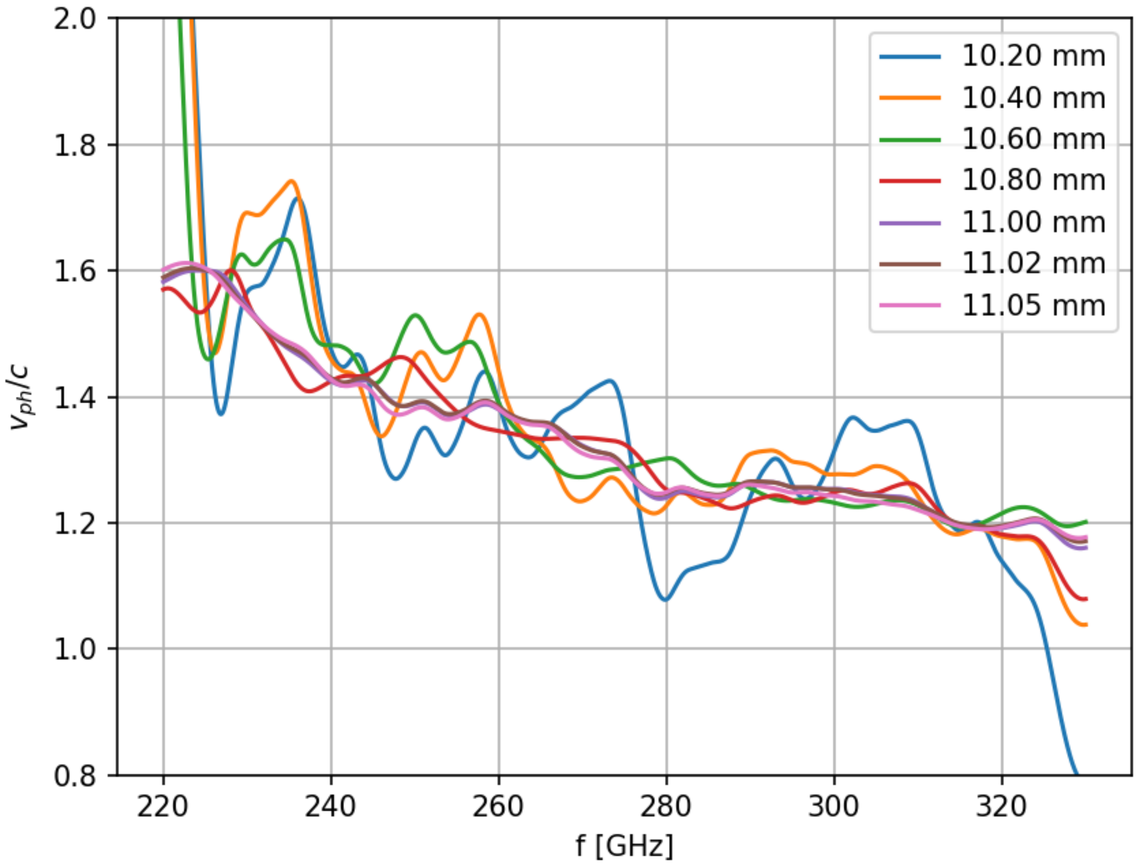
- Wiggles in frequency domain
- Result depends on window parameters (center and span)
- Frequency band must be truncated at the edges (DFT assumes a periodic signal, which the S-parameter clearly isn't)
- Phase information in bandpass transform susceptible to transform



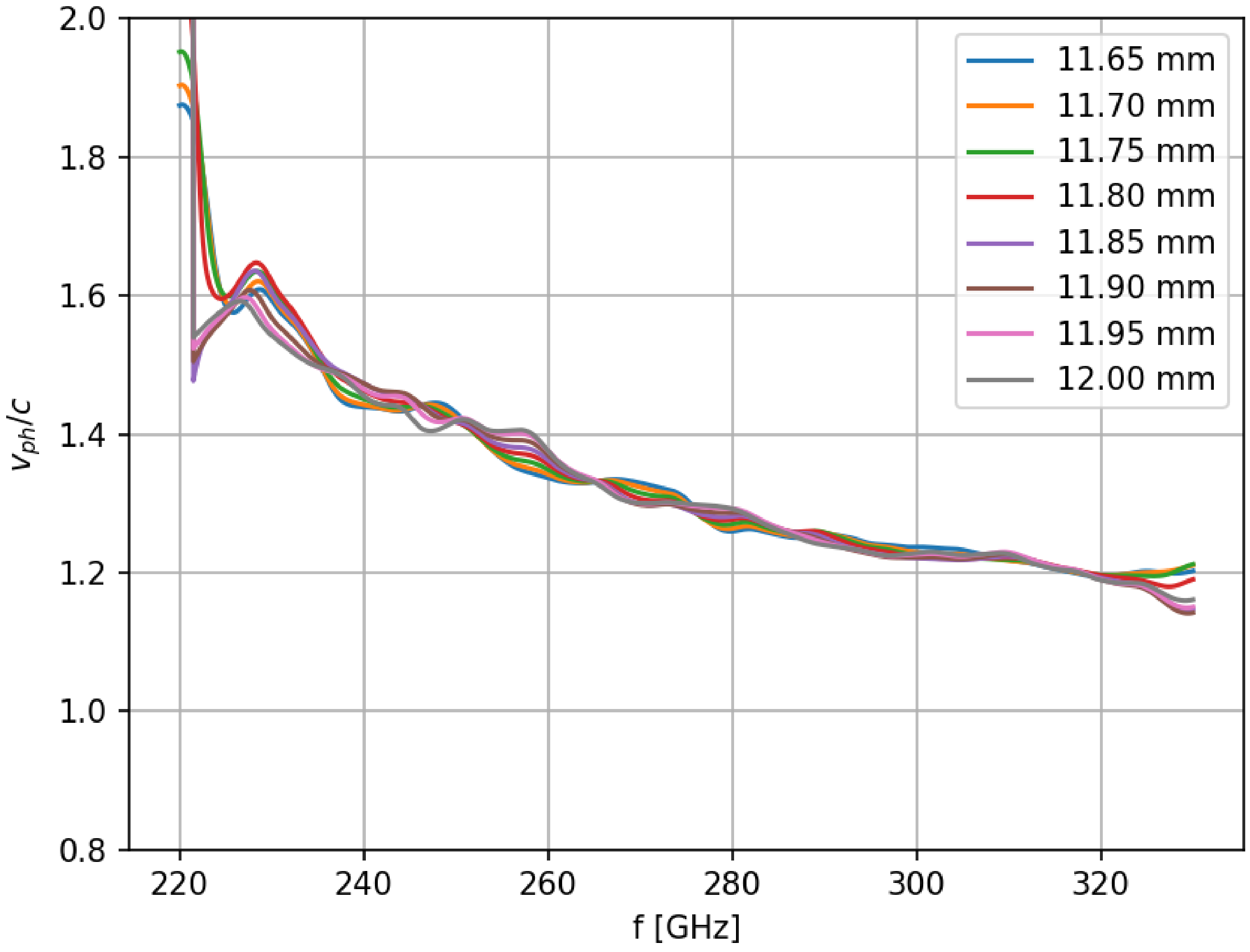
$\tau_c = 0.680\,ns, \Delta\tau = 0.252\,ns$

Measurement result with Time Domain Gating, I

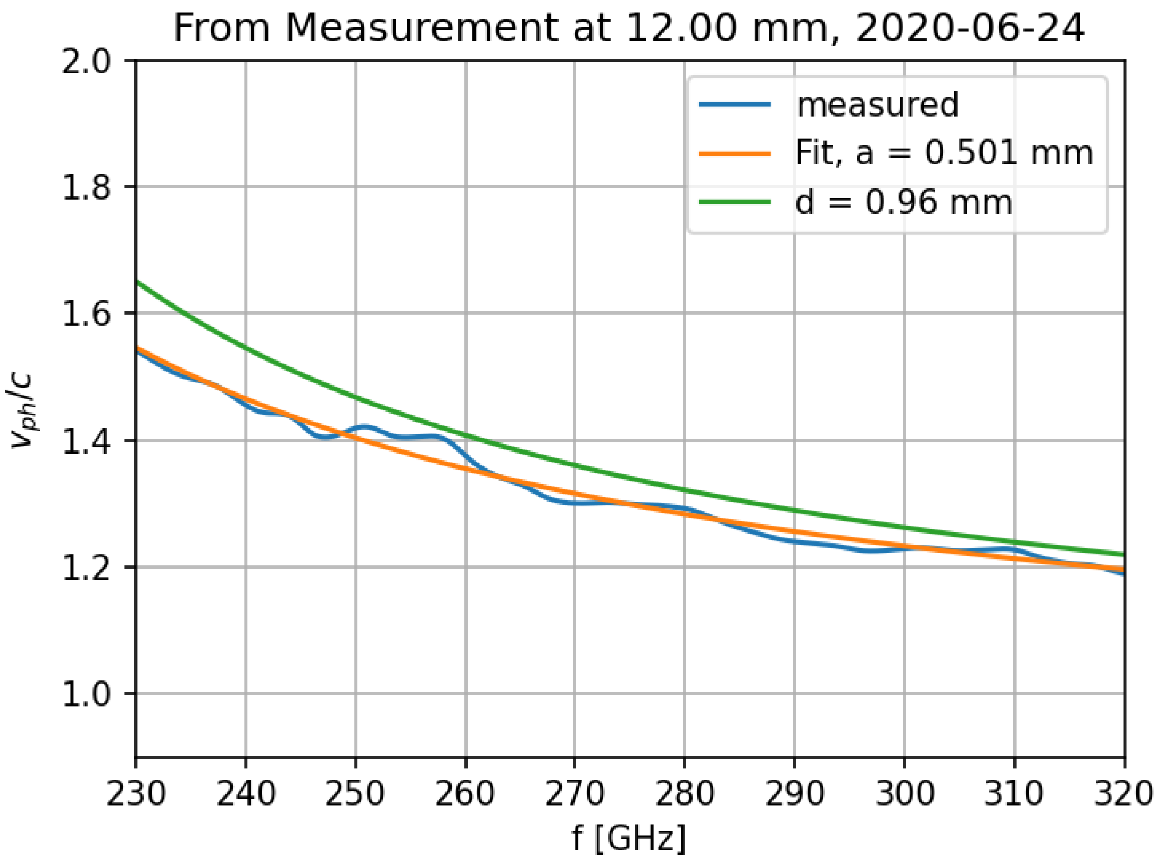




Measurement result with Time Domain Gating, II



Fit dispersion curve

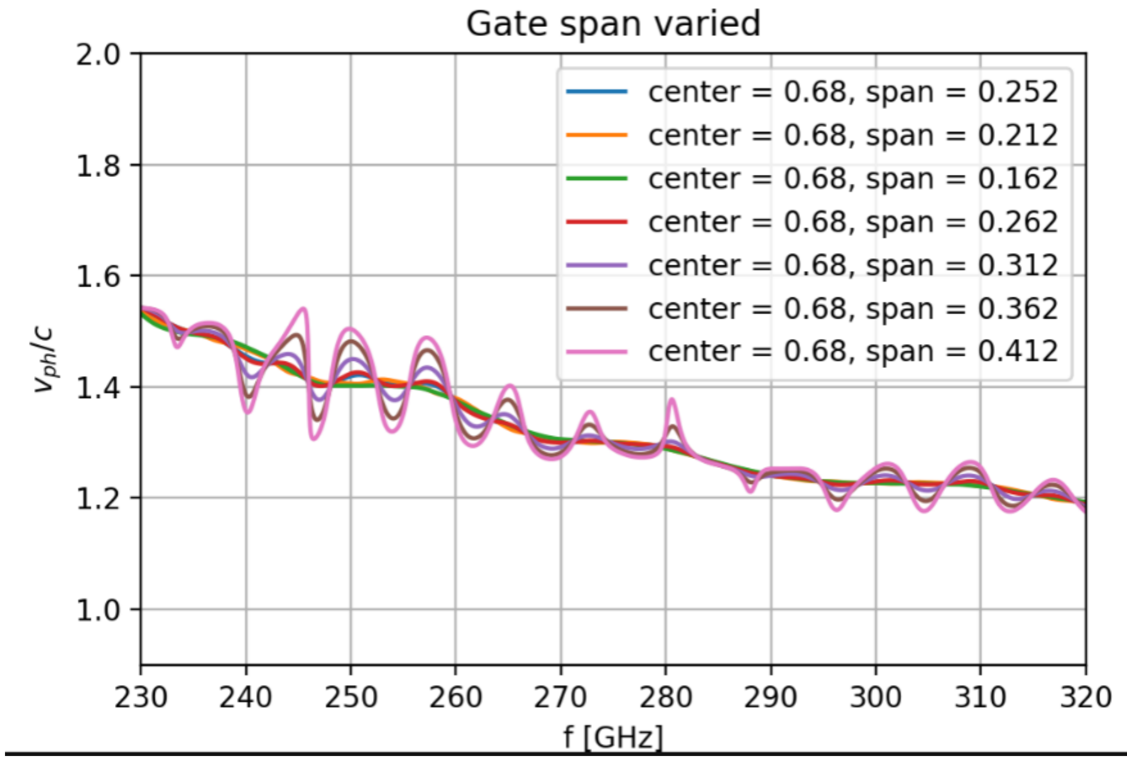


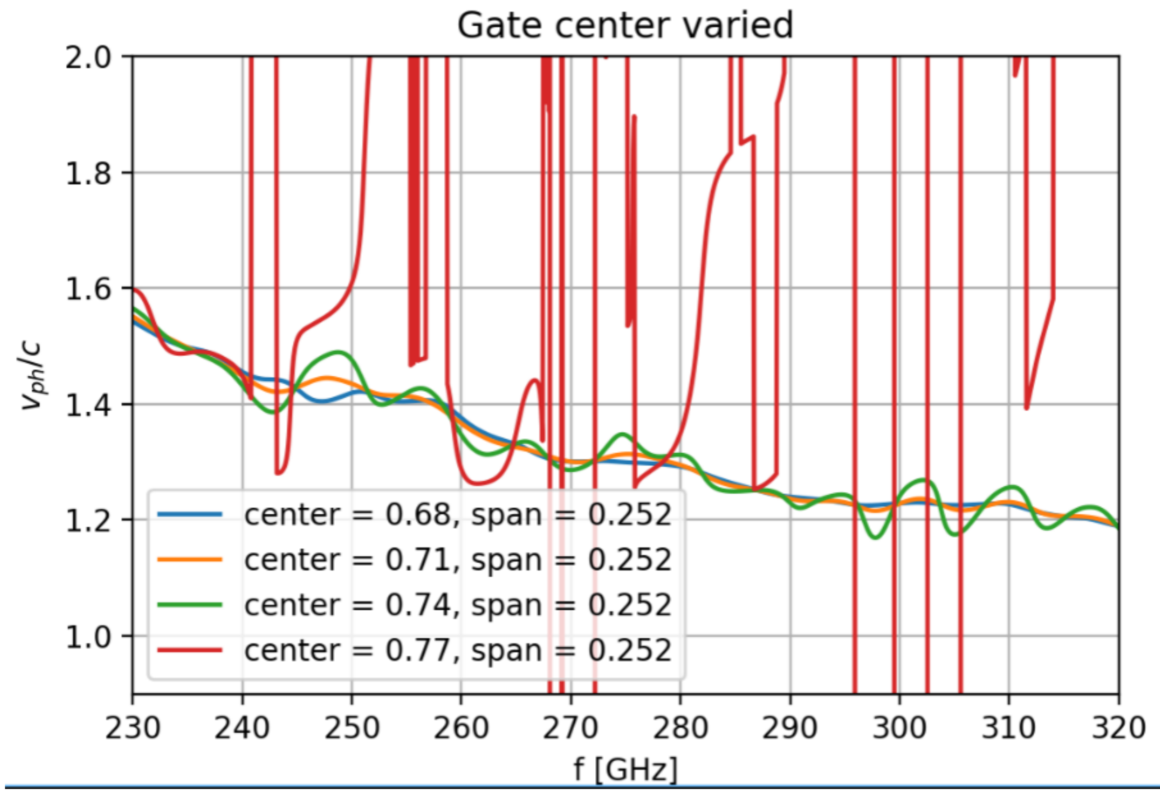
- Comparing dispersion curves
  - Measured (using time gating)
  - Fit to measured data → 0.50 mm
  - Expected from measured aperture diagonal

Fits from last 10 measurements (from 11.5 mm):

$$a = (0.500 \pm 0.002)\ mm$$

Impact of Gate parameters





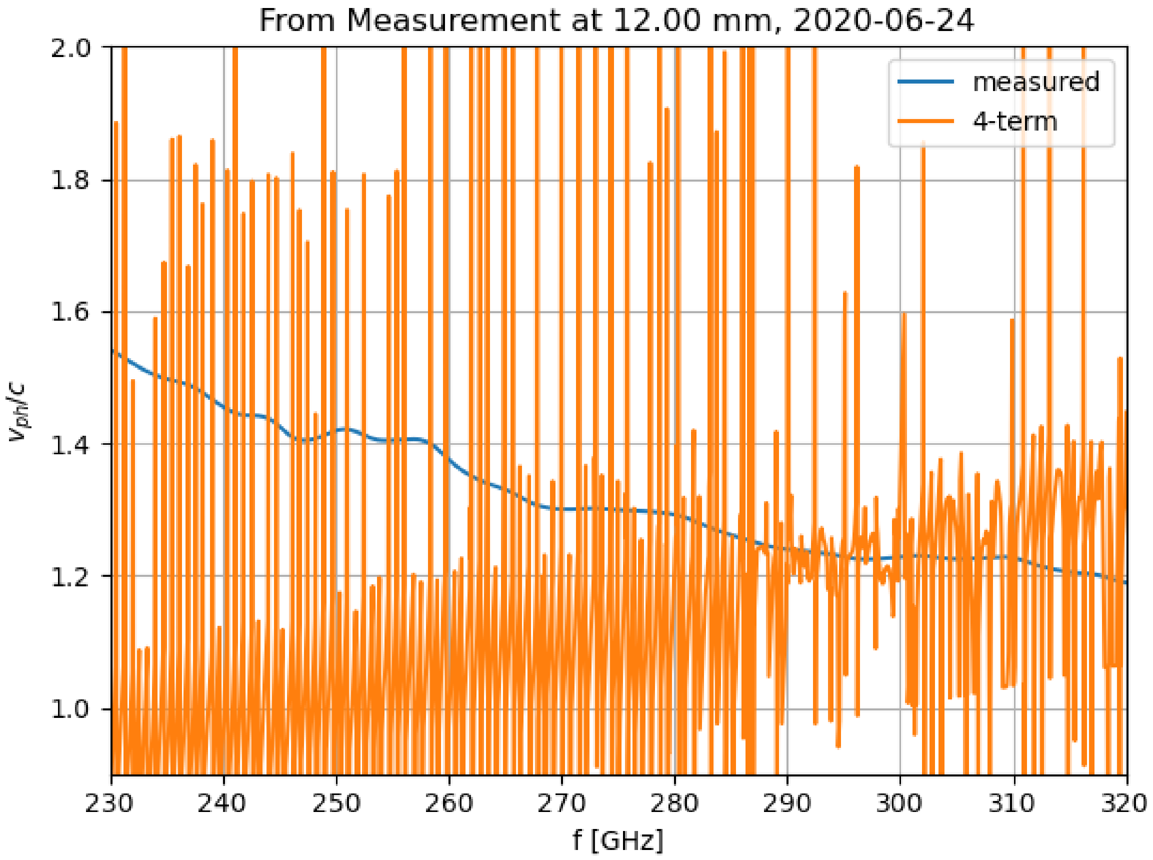
4-term error model would be better

- Per-Frequency analysis
- Does not rely on incomplete time domain information
- But:
  - error function depends on starting point
  - Complex error function
  - Sensitive to phase error

$$F = (S_{11}^{(m,j)} - a)(k^{(j)} - c) - b = 0$$

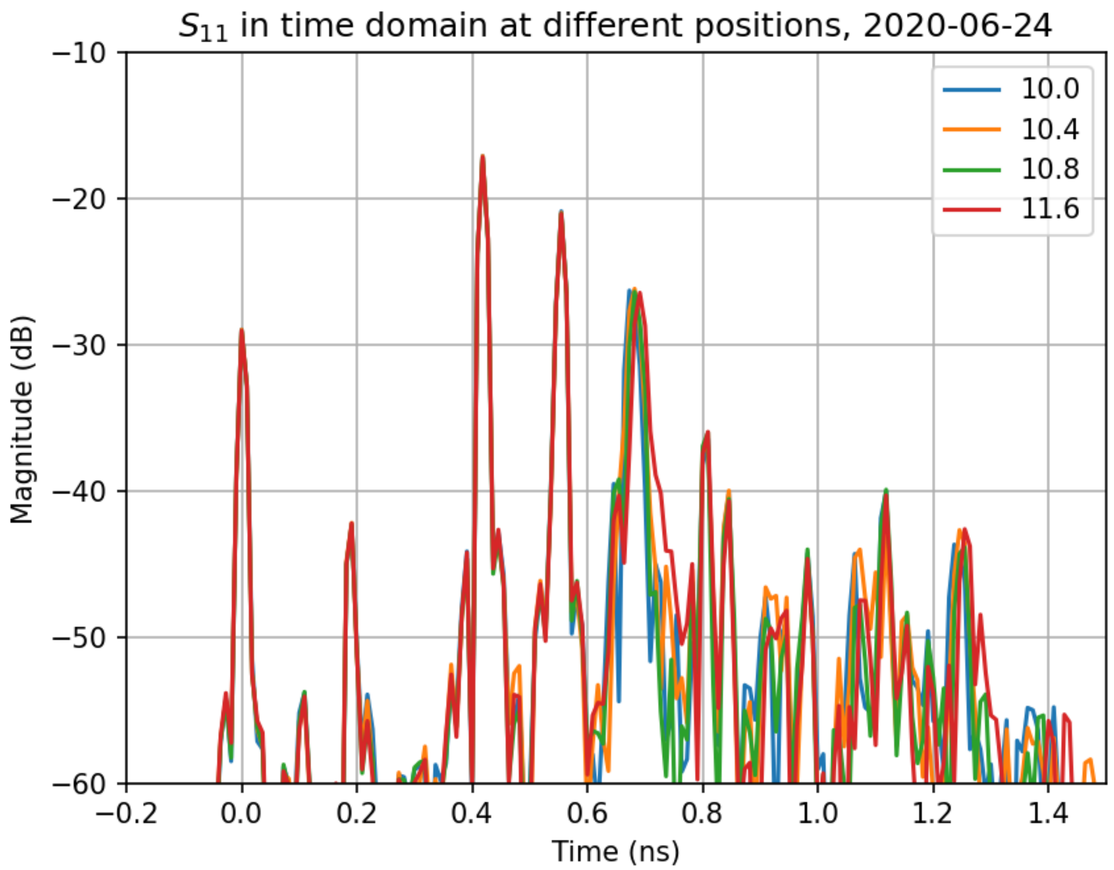
- solving for a, c, b with pairs of measurements and back-substitution
- Root finding problem

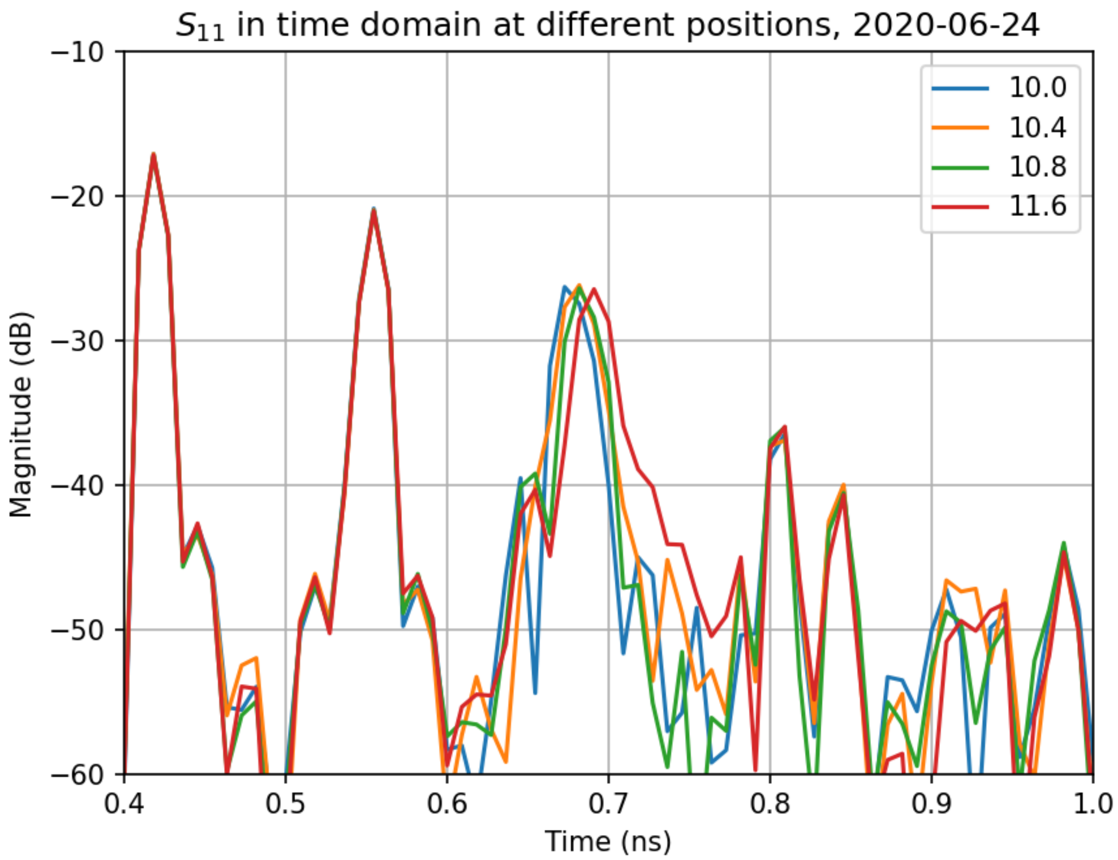
Four-term error model applied



- Using measurement at 10.00, 10.50, 11.00, 11.50 mm

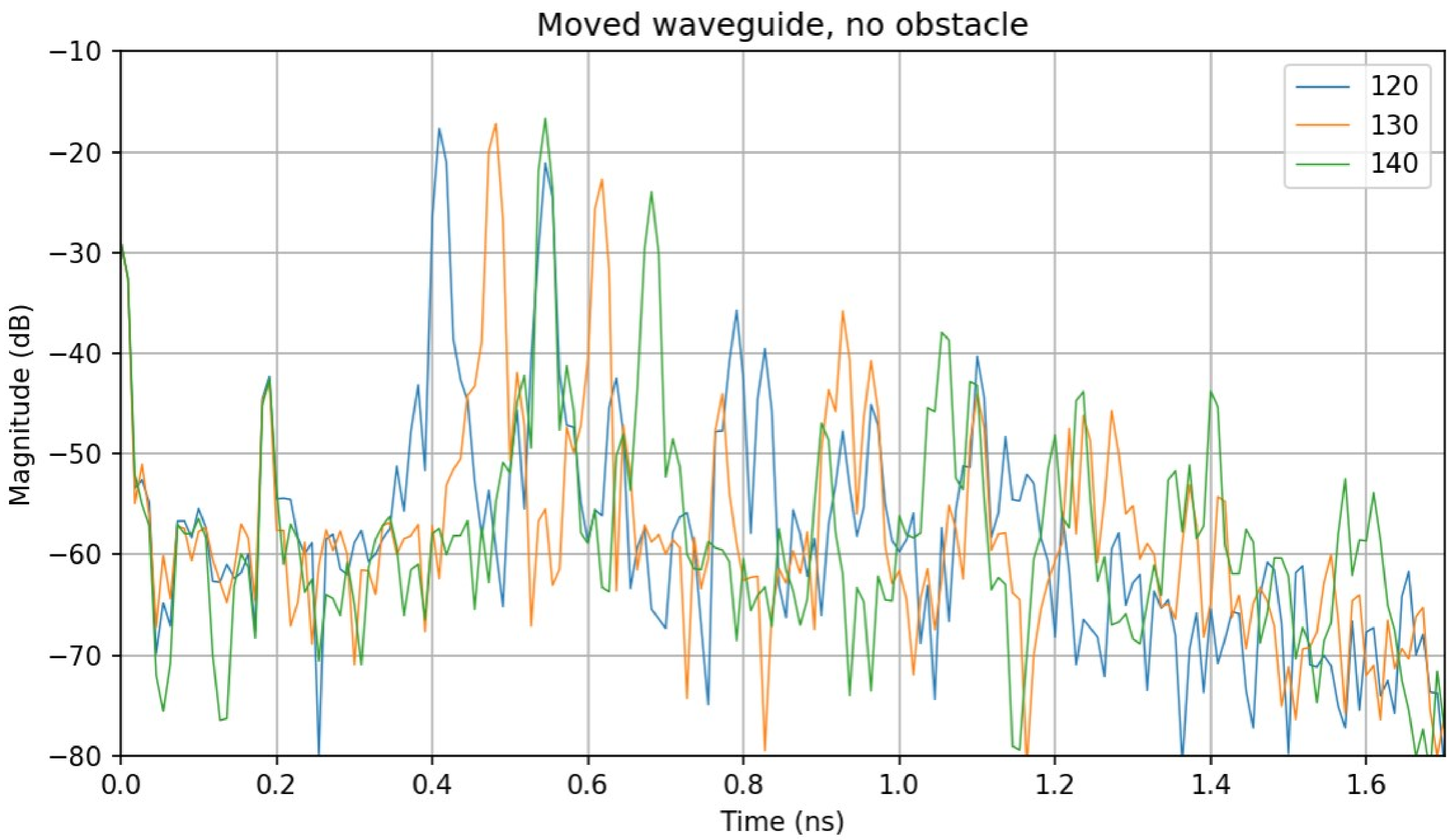
Time domain transform at different obstacle position





Time domain transform of obstacle free measurements

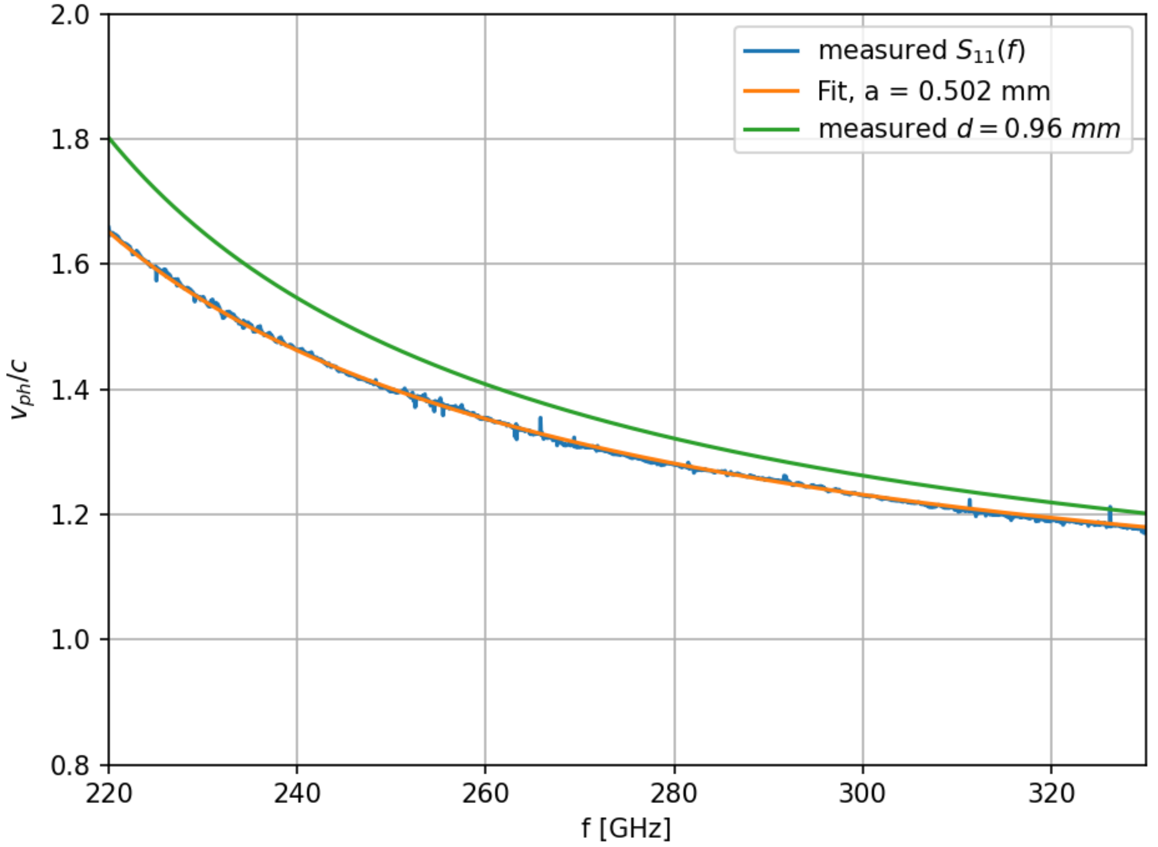
Measurement from 2020.07.02



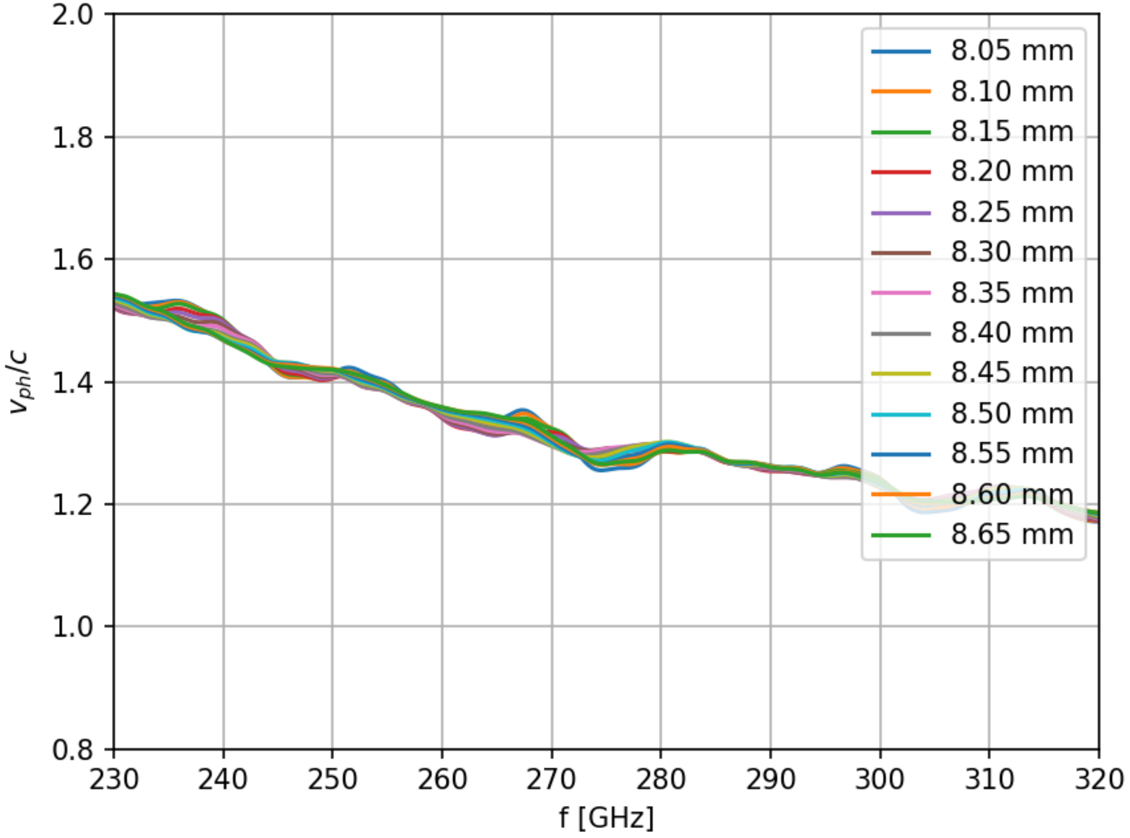
Harmonic analysis

Comparison to time domain gating

Based on harmonic analysis

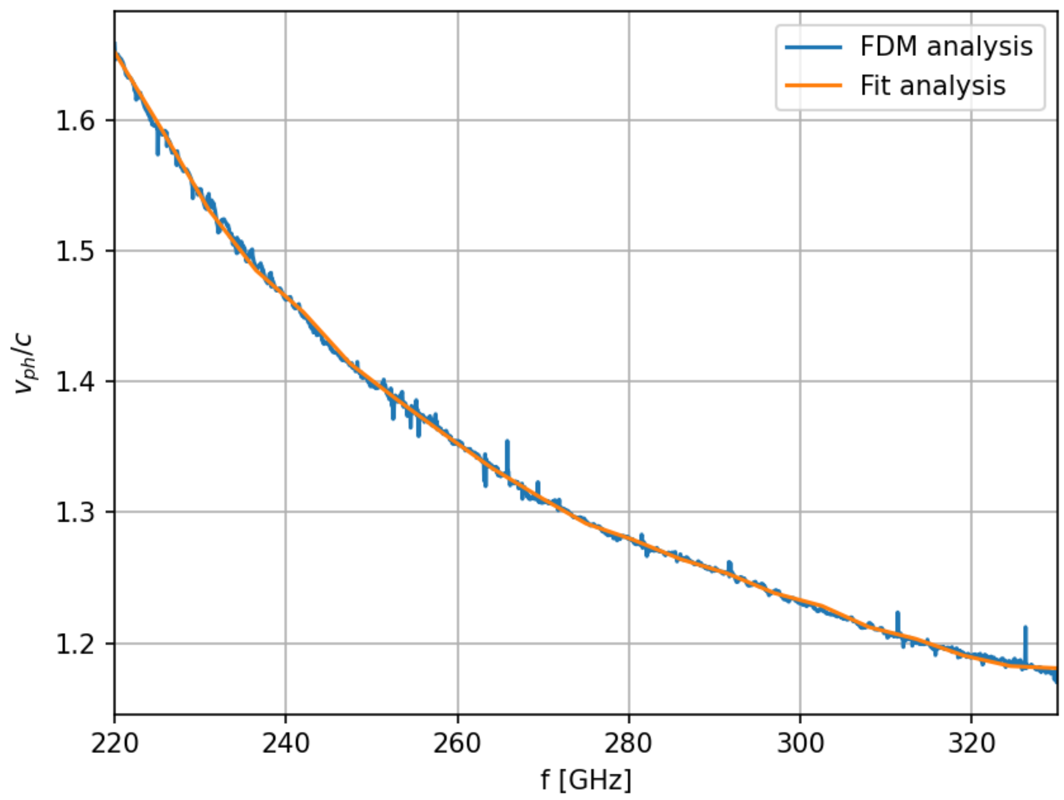


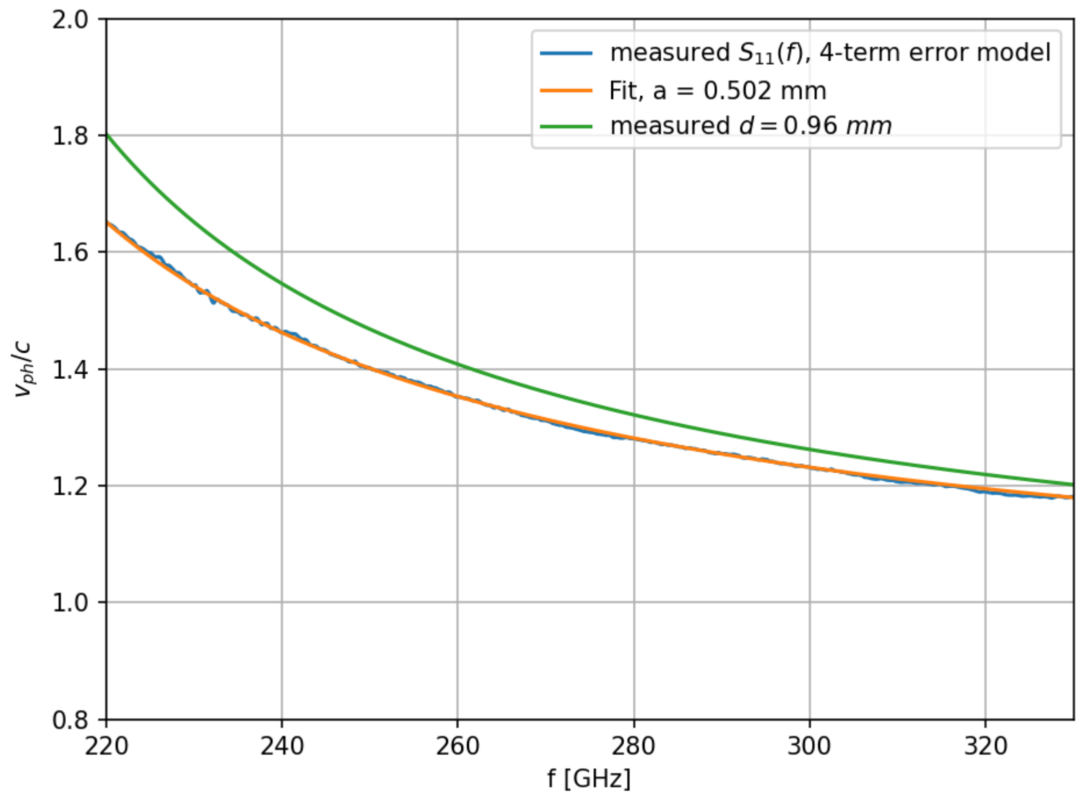
Based on time domain gating



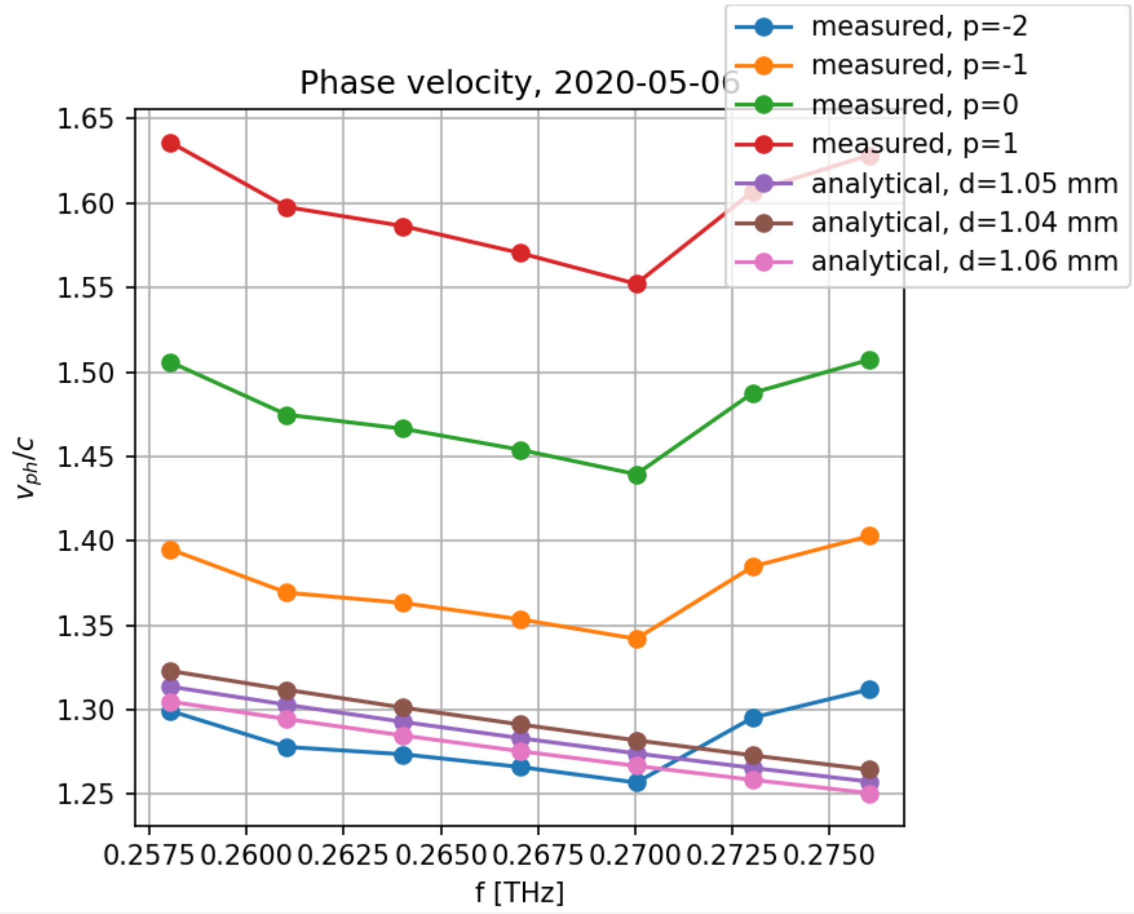
Analysis based on 4-term error model

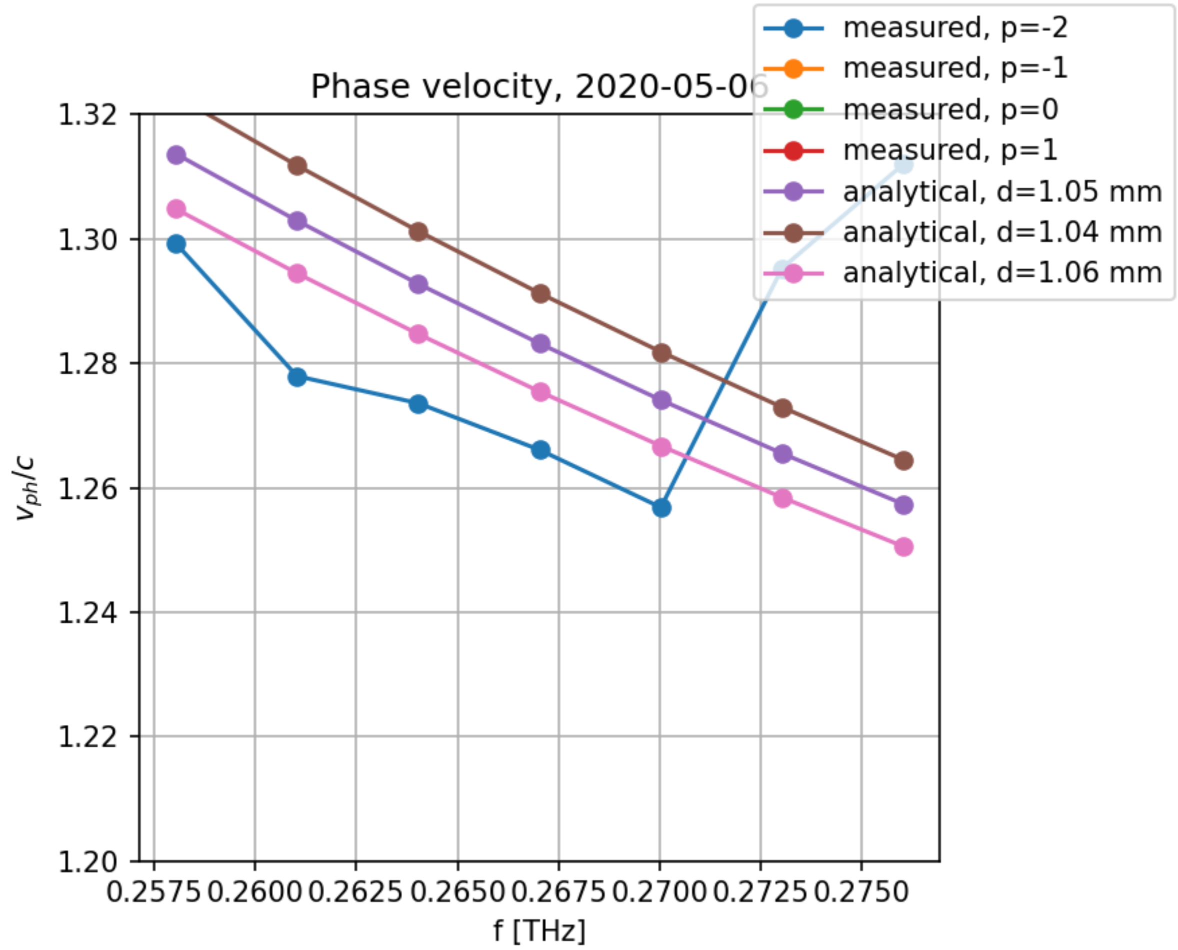
- While tweaking initial parameters for least squares problem: Found sign error
- → Curve fit works
- Better result than harmonic analysis



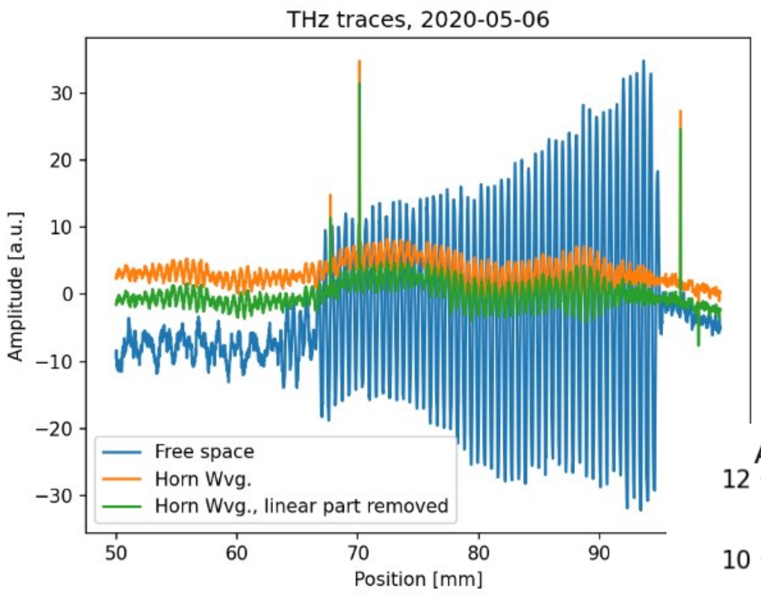


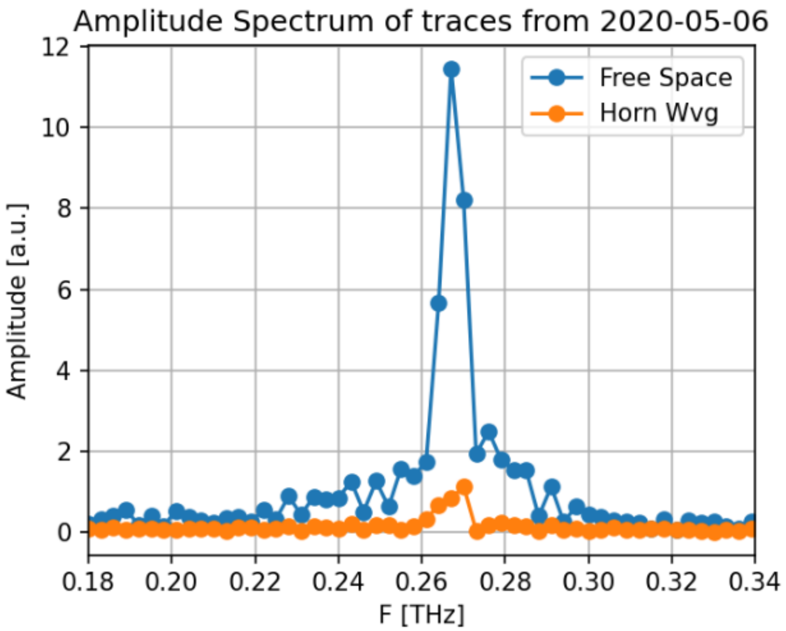
Phase velocity measured with EO sampling

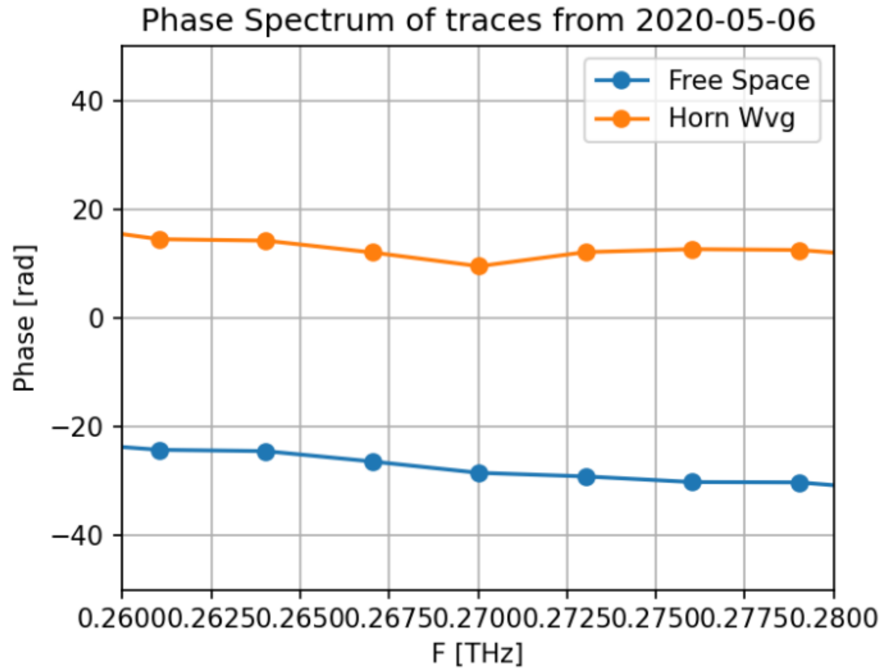




THz traces with EO sampling

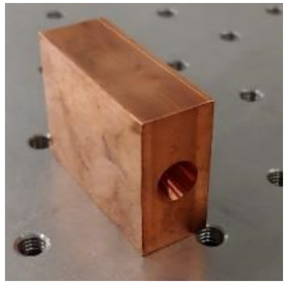






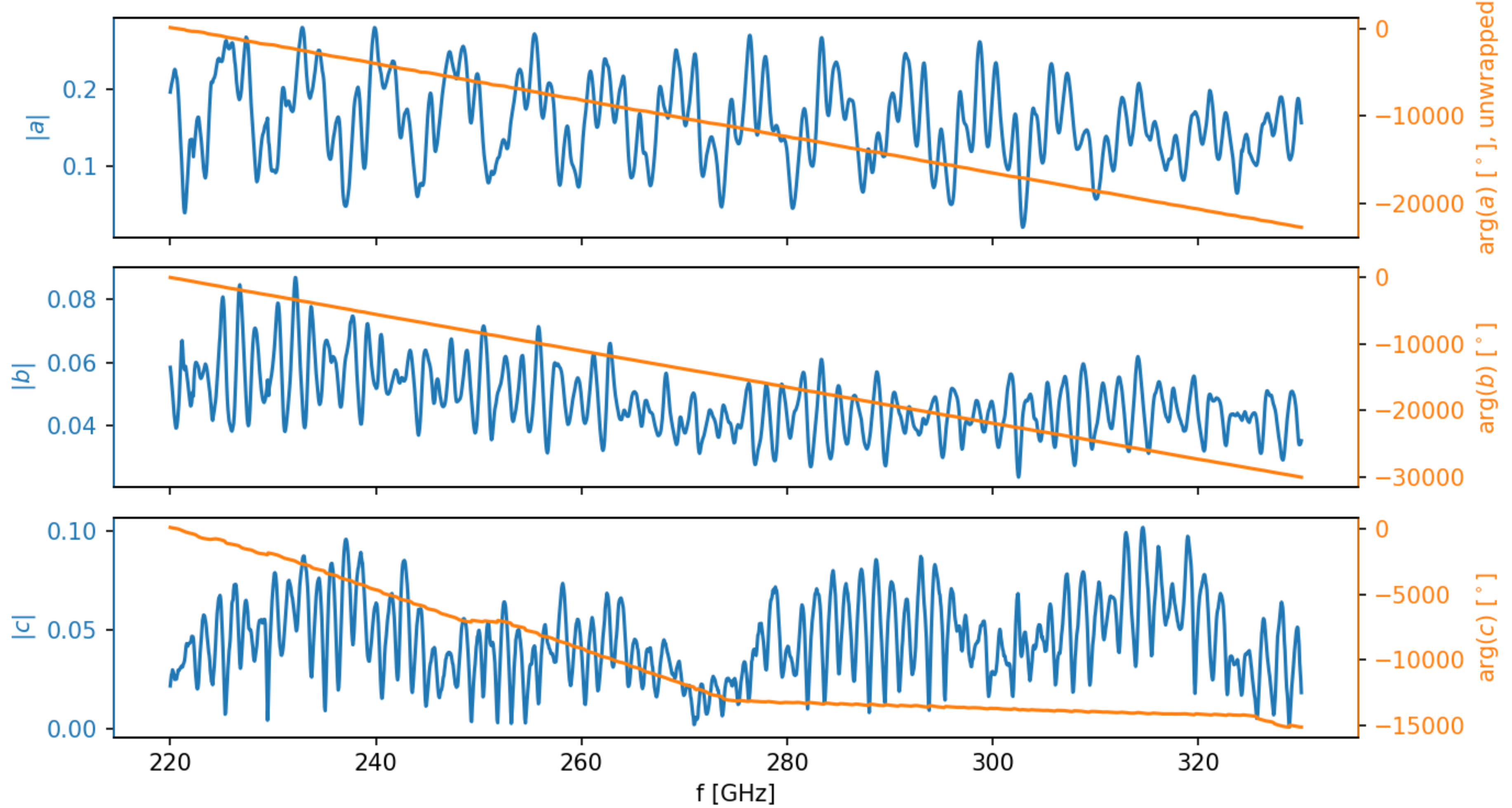
Calibration coefficients

Magnitude and unwrapped phase



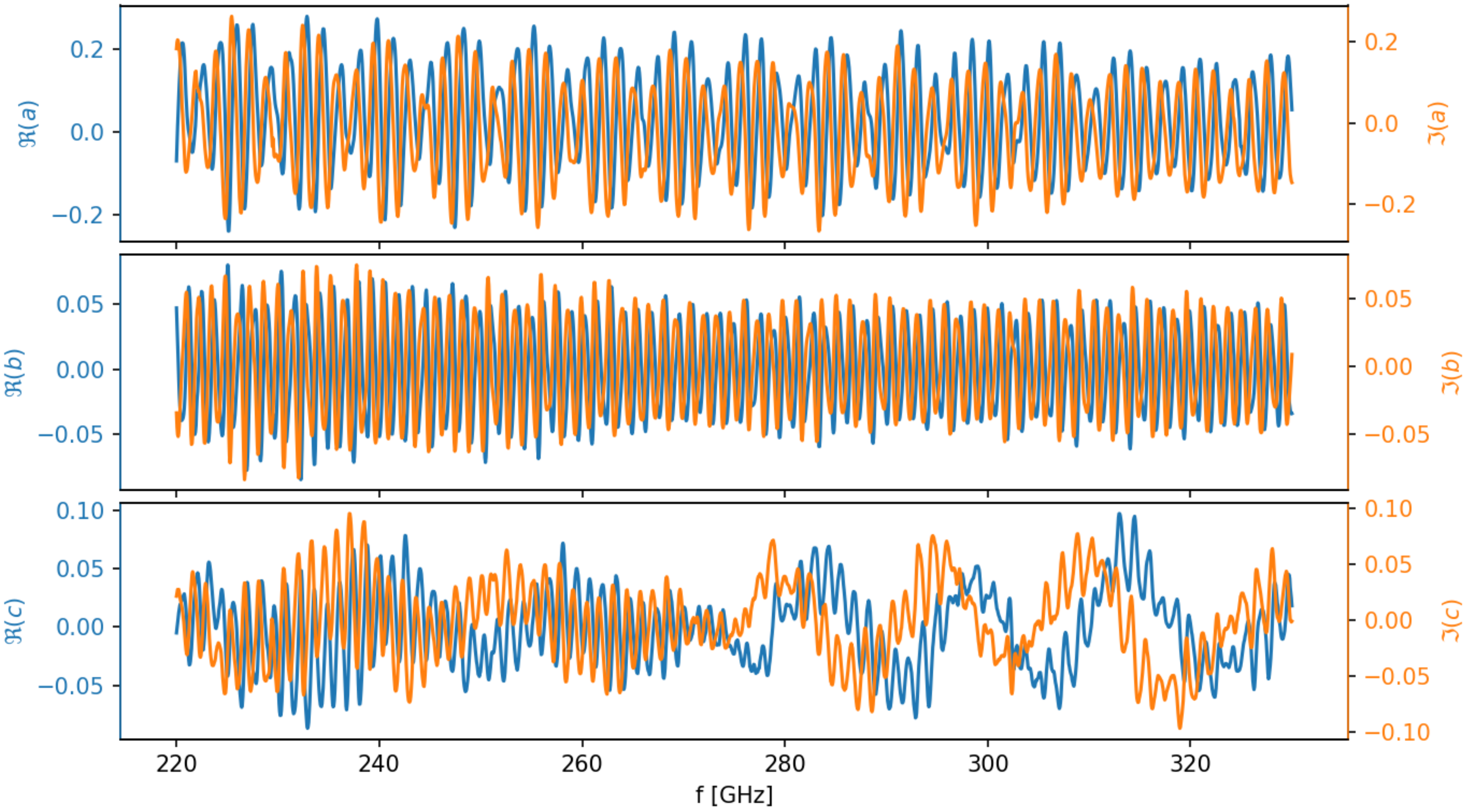
$$S_{11}^{m} = P_{11} + Q_{11} \frac{P_{21} P_{12}}{k^{-2} - P_{22} Q_{11}}$$

$$S_{11}^{m,i} = a + \frac{b}{\tilde{k}^{-2l_i} - c}$$



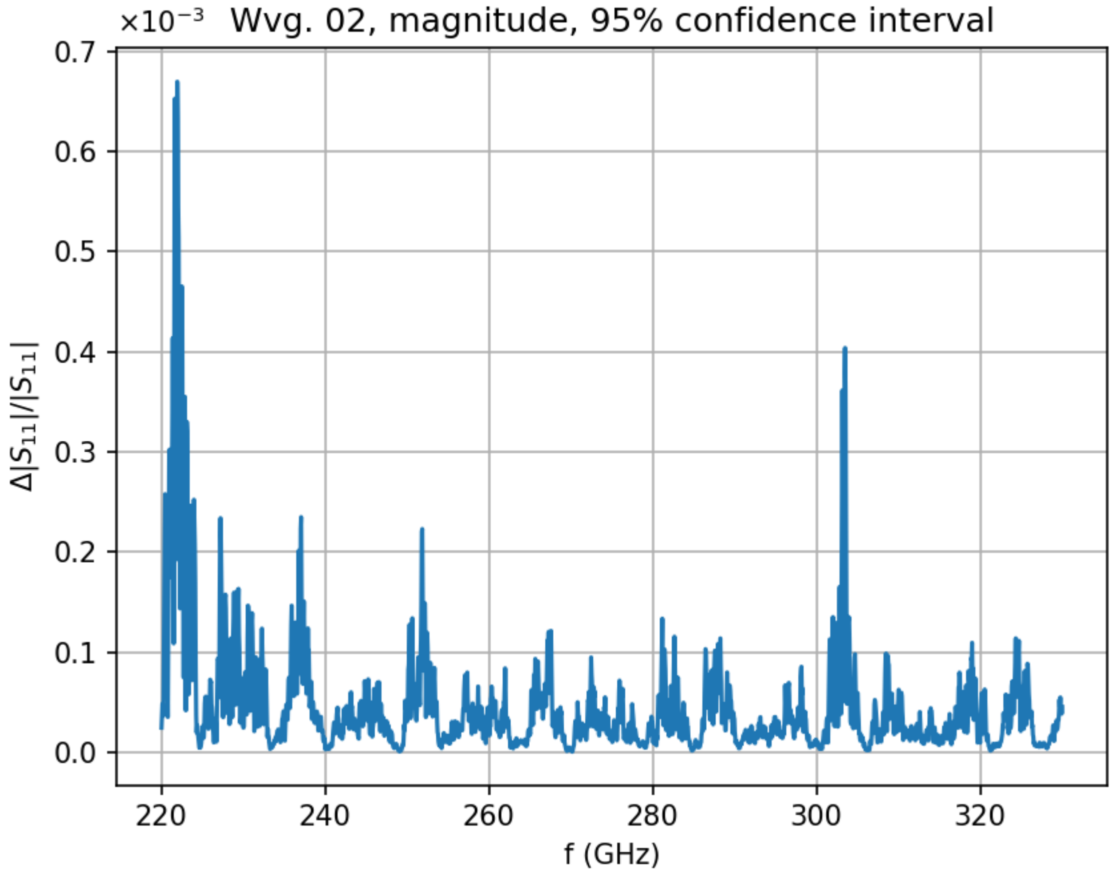
Calibration coefficients

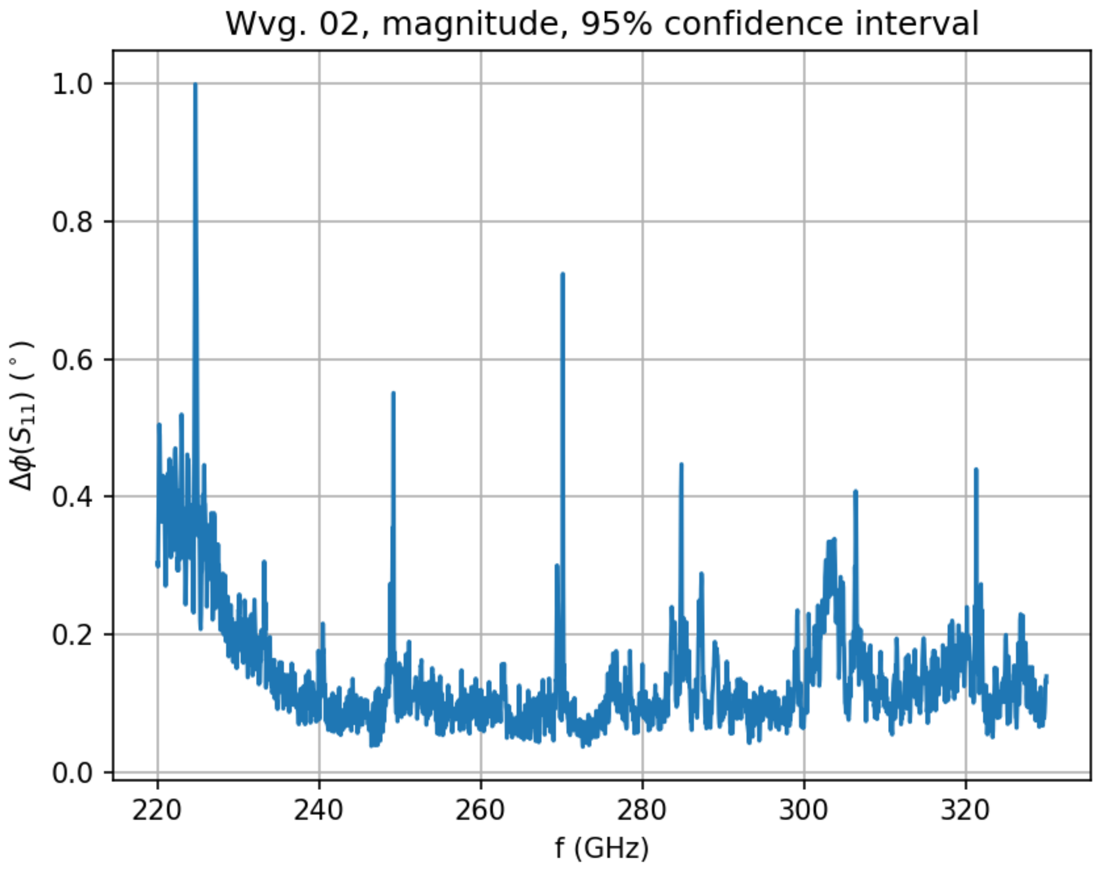
Real and Imaginary



Uncertainty in S-parameter

Waveguide 2 (brass)

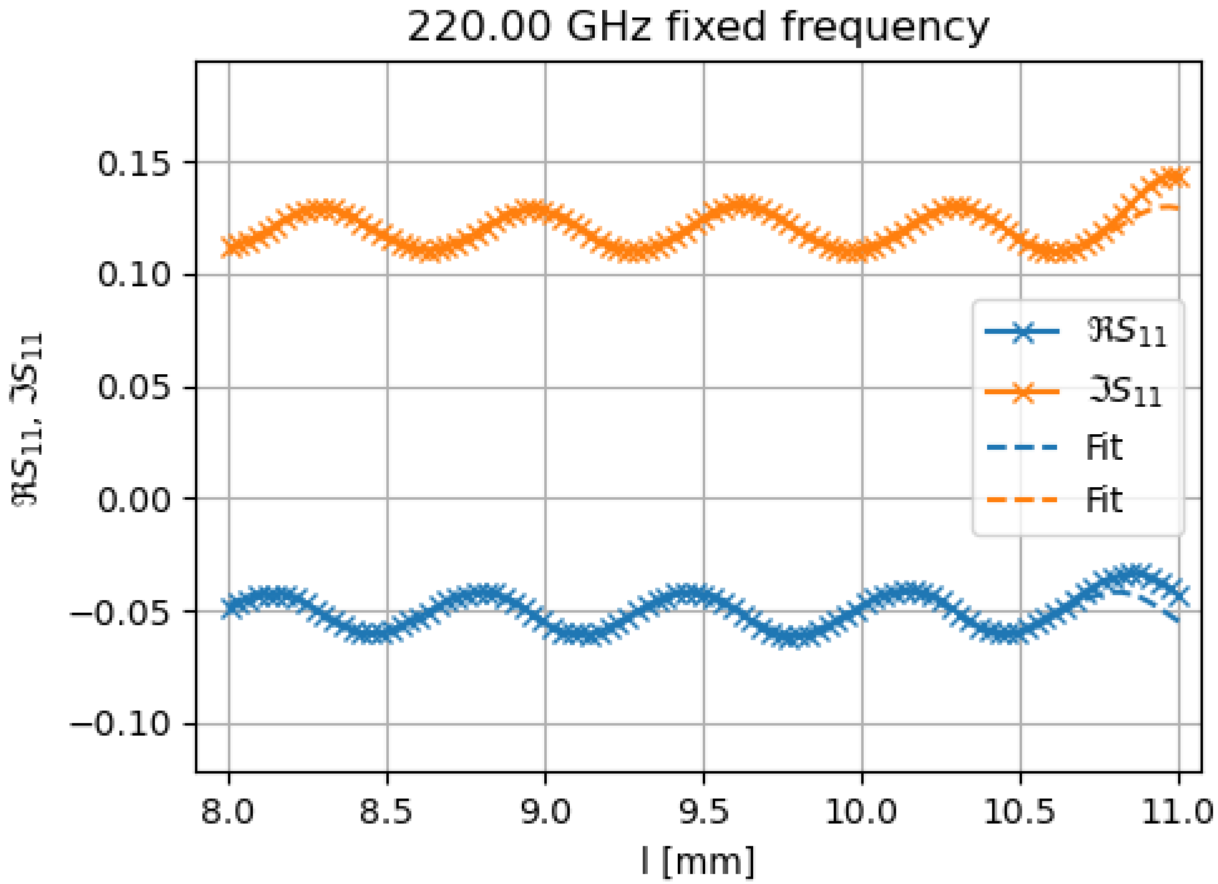


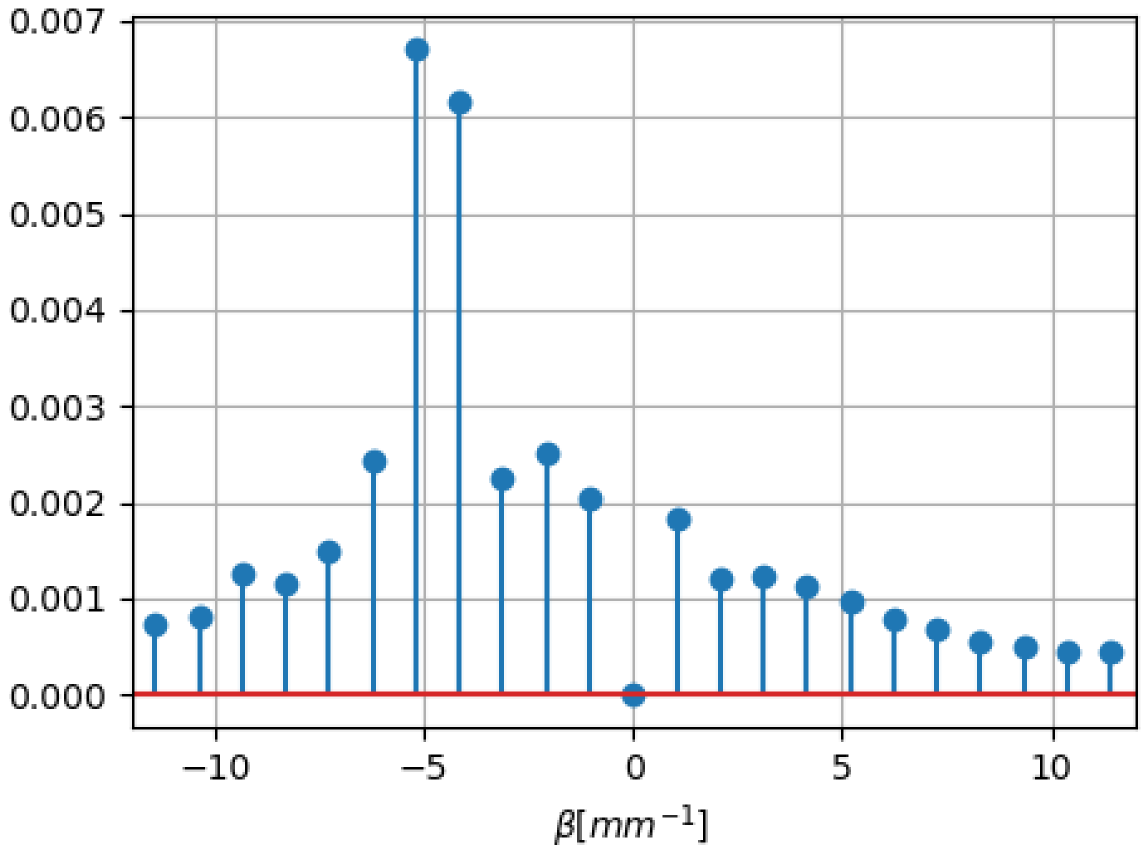


At a fixed position of the obstacle

Split waveguide with dielectric, S_{11}

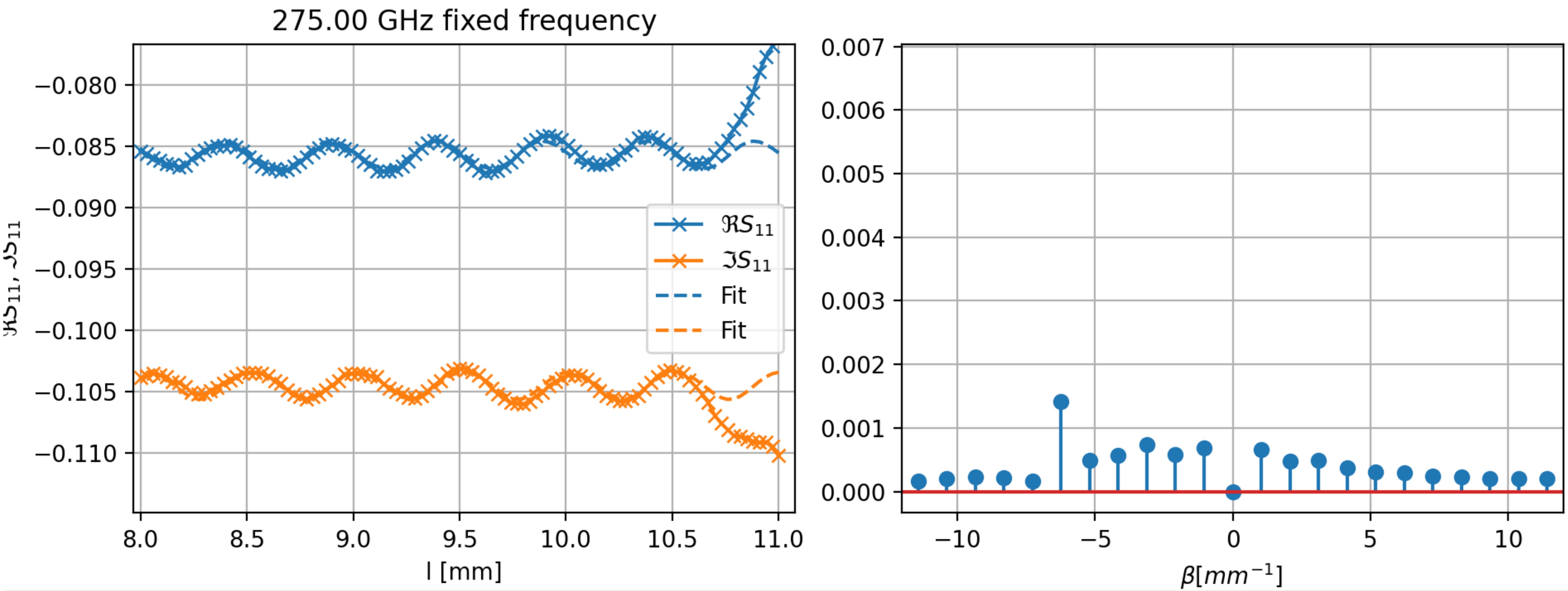
S_{11} at low frequency





Split waveguide with dielectric, S_{11}

S_{11} at medium frequency



Split waveguide with dielectric, S_{11}

S_{11} at high frequency