SUSY QCD Corrections to Higgs Production via Gluon Gluon Fusion

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In collaboration with
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ggh/QQh

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Outline

◇ **Introduction**

◇ **QCD corrections to squark loops for $M_{\tilde{g}} \longrightarrow \infty$**

◇ **Full SUSY-QCD corrections**

◇ **Decoupling of $M_{\tilde{g}}$ contributions for $M_{\tilde{g}} \longrightarrow \infty$**

◇ **Conclusions**

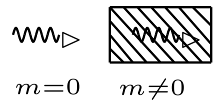
Higgs Physics

Higgs physics at future colliders:
Establish experimentally the Higgs Mechanism

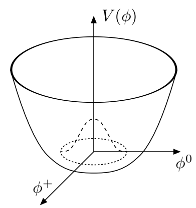
The Higgs mechanism:
Creation of particle masses in a gauge-invariant way

Test of the Higgs mechanism

- Discovery – m
- Spin and CP properties – J^{PC}
- Interaction with the scalar Higgs with $v = 246$ GeV$\neq 0$ $\rightsquigarrow$ $g_{HXX} \sim m_X^{(2)}$



- EWSB requires Higgs potential $\leftrightarrow$ $\lambda_{HHH}, \lambda_{HHHH}$



The MSSM Higgs Sector

MSSM Higgs sector – supersymmetry & anomaly free theory $\Rightarrow$ 2 complex Higgs doublets

$\stackrel{\text{EWSB}}{\longrightarrow}$ neutral, CP-even h, H neutral, CP-odd A charged H^+, H^-

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Higgs masses

$$M_h \lesssim 140 \text{ GeV}$$

$$M_{A,H,H^\pm} \sim \mathcal{O}(v)...1 \text{ TeV}$$

Ellis et al;Okada et al;Haber,Hempfling;
Hoang et al;Carena et al;Heinemeyer et al;
Zhang et al;Brignole et al;...

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Decoupling limit:

$M_A \sim M_H \sim M_{H^\pm} \gtrsim v$

$M_h \rightarrow$ max. value, $\tan\beta$ fixed; h becomes SM-like

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Modified couplings with respect to the SM: (decoupling limit Gunion,Haber)

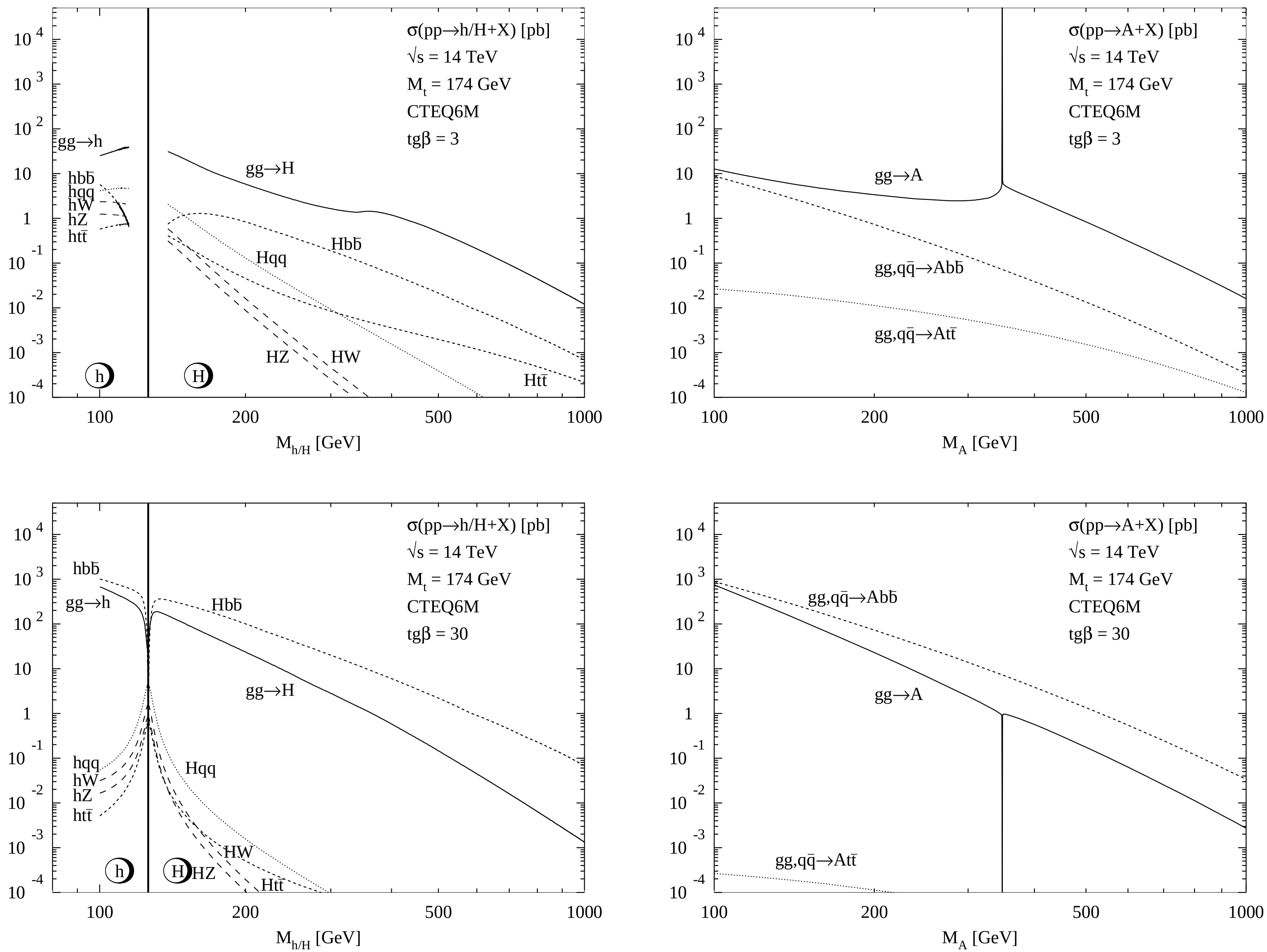
Φ	$g_{\Phi u\bar{u}}$	$g_{\phi d\bar{d}}$	$g_{\Phi VV}$
h	$c_\alpha/s_\beta \to 1$	$-s_\alpha/c_\beta \to 1$	$s_{\beta-\alpha} \to 1$
H	$s_\alpha/s_\beta \to 1/\text{tg}\beta$	$c_\alpha/c_\beta \to \text{tg}\beta$	$c_{\beta-\alpha} \to 0$
A	$1/\text{tg}\beta$	$\text{tg}\beta$	0

$\tan\beta \uparrow \quad \Rightarrow g_{\Phi uu} \downarrow$

$g_{\Phi dd} \uparrow$

$g_{\Phi VV}^{MSSM} \lesssim g_{\Phi VV}^{SM}$

MSSM Higgs Boson Production at the LHC

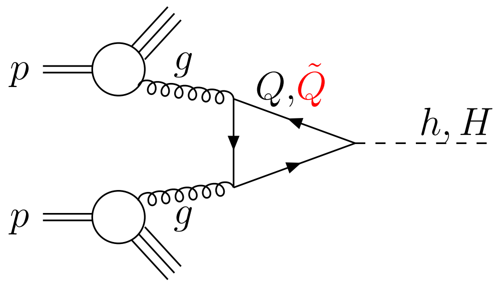


Spira

gg → H, h at leading order

Lowest order - 1 loop

Georgi,...;Gamberini,...



$$\sigma(pp \to \Phi + X) = \sigma_0^\Phi \tau_\Phi \frac{d\mathcal{L}^{gg}}{d\tau_\Phi}$$

$$\sigma_0^{h/H} = \frac{G_F \alpha_S^2(\mu_R)}{288\sqrt{2}\pi} \left| \sum_Q g_Q^{h/H} F_Q^{h/H}(\tau_Q) + \sum_{\tilde{Q}} g_{\tilde{Q}}^{h/H} F_{\tilde{Q}}^{h/H}(\tau_{\tilde{Q}}) \right|^2 \qquad \sigma_0^A = \frac{G_F \alpha_s^2}{128\sqrt{2}\pi} \left| \sum_Q g_Q^A F_Q^A(\tau_Q) \right|^2$$

$$F_Q^{h/H}(\tau_Q) = \frac{3}{2}\tau_Q \Big[1 + (1-\tau_Q) f(\tau_Q) \Big] \qquad F_Q^A(\tau_Q) = \tau_Q f(\tau_Q)$$

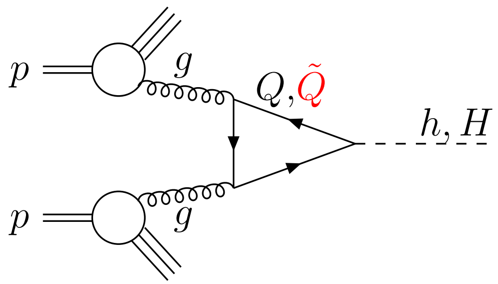
$$F_{\tilde{Q}}^{h/H}(\tau_{\tilde{Q}}) = -\frac{3}{4}\tau_{\tilde{Q}} \Big[1 - \tau_{\tilde{Q}} f(\tau_{\tilde{Q}}) \Big]$$

$$f(\tau) = \begin{cases} \arcsin^2 \frac{1}{\sqrt{\tau}} & \tau \geq 1 \\ -\frac{1}{4}\left[\log\left(\frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}} \right) - i\pi \right]^2 & \tau < 1 \end{cases} \qquad \tau_\Phi = \frac{M_\Phi^2}{s}, \quad \tau_{Q,\tilde{Q}} = \frac{4m_{Q,\tilde{Q}}^2}{M_\Phi^2}$$

$gg \to H, h$ at leading order

Lowest order - 1 loop

Georgi,...;Gamberini,...



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Remarks:

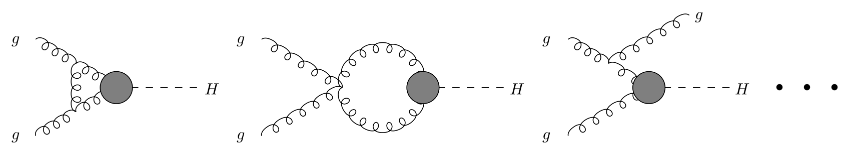
- $gg \to A$ no $\tilde{Q}$ contribution at LO
- MSSM: $\tan\beta \uparrow \Rightarrow b|\tilde{b} \uparrow$, and $t|\tilde{t} \downarrow$
- 3rd generation dominant, $\tilde{t}, \tilde{b}$ contributions important for $m_{\tilde{q}} \lesssim 400$ GeV.

Comments

QCD corrections to top & bottom loops (2-loop)

◇ NLO (SM, MSSM): increase σ by $\sim$ 10...100% — Spira,Djouadi,Graudenz,Zerwas; Dawson;Kauffman,Schaffer

◇ SM; tg$\beta \lesssim 5$: $M_\Phi \ll m_t$ approximation for K-factor [$\Delta \lesssim$ 25%] — Krämer,Laenen,Spira



◇ NNLO @ $M_\Phi \ll m_t \Rightarrow$ further increase by 20-30%
scale dependence: $\Delta \lesssim 10 - 15\%$ — Harlander,Kilgore; Anastasiou,Melnikov; Ravindran,Smith,van Neerven

◇ Mass effects on NNLO corrections: small in interm. mass region — Marzani,Ball,DelDuca,Forte, Vicini,Harlander,Ozeren; Pag,Rogat,Steinhauser

◇ Estimate of NNNLO effects: $\rightsquigarrow$ improved convergence
scale dependence: $\Delta \lesssim 10 - 15\%$ — Moch,Vogt; Ravindran

◇ Soft gluon resummation: $\sim 10\%$ — Catani,de Florian,Grazzini,Nason

Comments

EW/QCD corrections

◇ EW 2-loop effects $\sim -4 - 6$ % enhancement — Aglietti eal; Degrassi,Maltoni;Actis eal

◇ mixed EW-QCD corrections — Anastasiou,Boughezal,Petriello

NLO corrections to squark loops

◇ heavy squark limit — Dawson,Djouadi,Spira

◇ full SUSY-QCD corrections in heavy mass limit — Harlander,Steinhauser; Harlander,Hofmann

$\mathbf{m_{\tilde{Q}} \lesssim 400}$ **GeV:** squarks play a significant role $\rightsquigarrow$

◇ NLO squark mass effects — Anastasiou,Beerli,Bucherer, Daleo,Kunszt;Aglietti,Bonciani, Degrassi,Vicini;MMM,Spira

◇ full NLO SUSY QCD calculation — Anastasiou,Beerli,Daleo; MMM,Rzehak,Spira

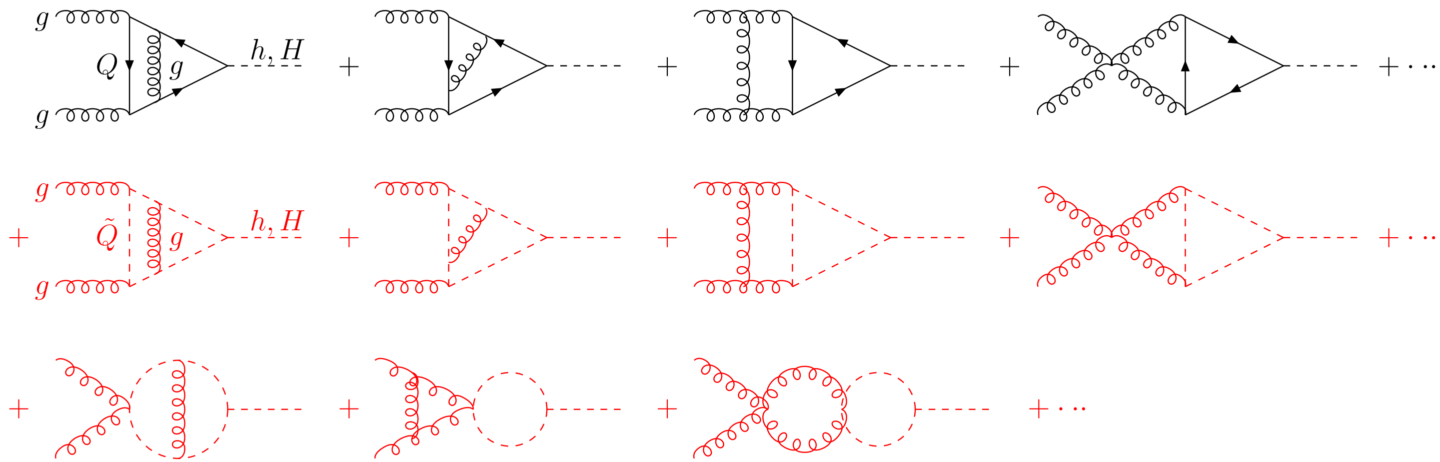
$\mathcal{F}$irst $\mathcal{S}$tep: QCD corrections

$$\Delta\hat{\sigma}_{ij} = \sigma_0 \Big\{ C_{ij}\delta(1-\hat{\tau}) + D_{ij}\Theta(1-\hat{\tau}) \Big\} \frac{\alpha_s}{\pi} \qquad \hat{\tau} = \frac{M_\Phi^2}{\hat{s}}$$

↗ virtual+soft corrections

↑ real corrections

Virtual corrections [2 loops, first step: no gluino contributions]



UV-,IR-,Coll-singularities in $n = 4 - 2\epsilon$ dimensions.

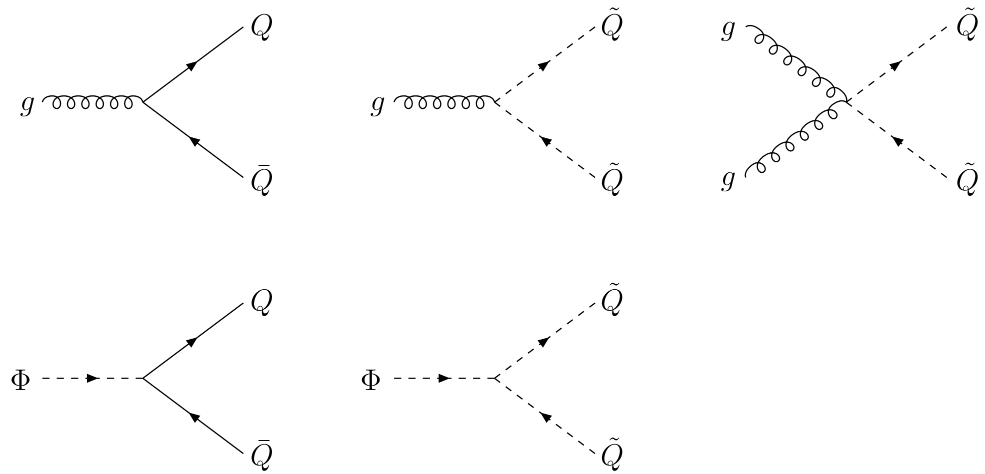
Renormalization

$\mathcal{L}$agrangian separates gluon and gluino exchange contributions in a renormalizable way

$$\mathcal{L} = -\frac{1}{4}G^{a\mu\nu}G^a_{\mu\nu} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}(\partial_\mu\Phi)^2 - \frac{M_\Phi^2}{2}\Phi^2$$
$$+ \sum_Q \left[\bar{Q}(i\not{D} - m_Q)Q - g_Q^\Phi \frac{m_Q}{v}\bar{Q}Q\Phi\right] + \sum_{\tilde{Q}}\left[|D_\mu\tilde{Q}|^2 - m_{\tilde{Q}}^2|\tilde{Q}|^2 - g_{\tilde{Q}}^\Phi \frac{m_{\tilde{Q}}^2}{v}|\tilde{Q}|^2\Phi\right]$$

$$iD_\mu = i\partial_\mu - g_S G^a_\mu T^a - eA_\mu \mathcal{Q}$$

Gluon, $\Phi = \mathrm{H/h}$ interaction vertices:



Renormalization - cont'd

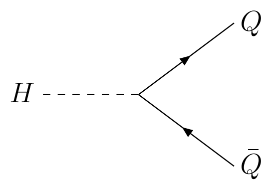
◇ **Quark/Squark mass $m_{Q,\tilde{Q}}$:** on-shell

◇ **$g^H_{\tilde{Q}}$ not renormalized** [MSSM: $g^{h/H}_{\tilde{Q}} = \frac{m_Q^2}{m_{\tilde{Q}}^2} g_Q^{h,H}$ + mixing terms + D-terms]

◇ **α_S $\overline{\text{MS}}$ (5 active flavours)**

◇ **$HQ\bar{Q}$ vertex:**

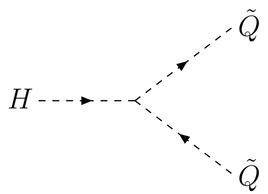
$$\mathcal{L}_{\text{int}} = -g^H_Q \frac{m_{Q_0}}{v} \bar{Q}_0 Q_0 H = -g^H_Q \frac{m_Q}{v} \bar{Q} Q H \underbrace{\left[Z_2 - \frac{\delta m_Q}{m_Q} \right]}_{Z_{HQQ}} + \mathcal{O}(\alpha_S^2)$$



$$\Gamma_{H\bar{Q}Q}(q^2 = 0) \neq Z_{HQQ}$$ Braaten, Leveille

◇ **$H\tilde{Q}\tilde{Q}$ vertex:**

$$\mathcal{L}_{\text{int}} = -g^H_{\tilde{Q}} \frac{m^2_{\tilde{Q}_0}}{v} \tilde{Q}_0^* \tilde{Q}_0 H = -g^H_{\tilde{Q}} \frac{m^2_{\tilde{Q}}}{v} \tilde{Q}^* \tilde{Q} H \underbrace{\left[Z_2^{\tilde{Q}} - \frac{\delta m^2_{\tilde{Q}}}{m^2_{\tilde{Q}}} \right]}_{Z_{H\tilde{Q}\tilde{Q}}} + \mathcal{O}(\alpha_S^2)$$



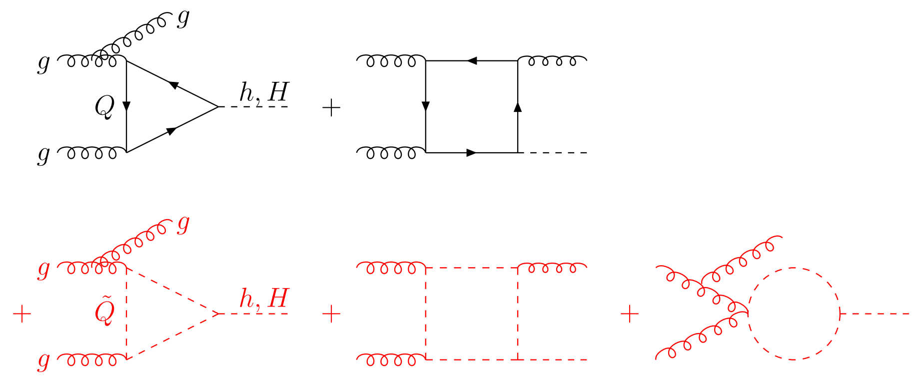
$$\Gamma_{H\tilde{Q}\tilde{Q}}(q^2 = 0) \neq Z_{H\tilde{Q}\tilde{Q}}$$ disregard renorm. of $g^H_{\tilde{Q}}$!

Real Corrections

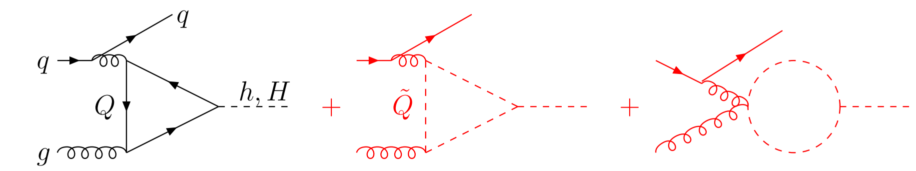
After renormalization: IR & coll. singularities $\rightsquigarrow$ real corrections have to be added.

3 incoherent processes:

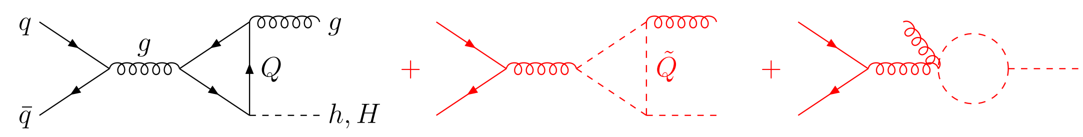
gg → Hg:



gq → Hq:



q$\bar{q}$ → Hg:



Phase space integration in $n = 4 - 2\epsilon$ dimensions $\rightsquigarrow$ IR, Coll. singularities: poles in ϵ

Result

- α_S: $\overline{\text{MS}}$ scheme, 5 active flavours - μ=Ren. scale, Q=Fact. scale, $\mu^2 = Q^2 = M_\phi^2$

$$
\begin{aligned}
\sigma(pp \to \phi + X) &= \sigma_0^\phi [1 + C^\phi \tfrac{\alpha_S}{\pi}] \tau_\phi \tfrac{d\mathcal{L}_{gg}}{d\tau_\phi} + \Delta\sigma_{gg}^\phi + \Delta\sigma_{gq}^\phi + \Delta\sigma_{q\bar{q}}^\phi \\
C^\phi(\tau_Q, \tau_{\tilde{Q}}) &= \pi^2 + C_1^\phi(\tau_Q, \tau_{\tilde{Q}}) + \tfrac{33-2N_F}{6} \log \tfrac{\mu^2}{M_\phi^2} \\
\Delta\sigma_{gg}^\phi &= \int_{\tau_\phi}^1 d\tau \tfrac{d\mathcal{L}_{gg}}{d\tau} \tfrac{\alpha_S}{\pi} \sigma_0^\phi \Big\{ -\hat{\tau} P_{gg}(\hat{\tau}) \log \tfrac{Q^2}{\hat{s}} + d_{gg}^\phi(\hat{\tau}, \tau_Q, \tau_{\tilde{Q}}) \\
&\quad + 12 \Big[\Big(\tfrac{\log(1-\hat{\tau})}{1-\hat{\tau}} \Big)_+ - \hat{\tau}[2 - \hat{\tau}(1-\hat{\tau})] \log(1-\hat{\tau}) \Big] \Big\} \\
\Delta\sigma_{gq}^\phi &= \int_{\tau_\phi}^1 d\tau \sum_{q,\bar{q}} \tfrac{d\mathcal{L}_{gq}}{d\tau} \tfrac{\alpha_S}{\pi} \sigma_0^\phi \Big\{ -\tfrac{\hat{\tau}}{2} P_{gq}(\hat{\tau}) \Big[\log \tfrac{Q^2}{\hat{s}(1-\hat{\tau})^2} \Big] d_{gq}^\phi(\hat{\tau}, \tau_Q, \tau_{\tilde{Q}}) \Big\} \\
\Delta\sigma_{q\bar{q}}^\phi &= \int_{\tau_\phi}^1 d\tau \sum_q \tfrac{d\mathcal{L}_{q\bar{q}}}{d\tau} \tfrac{\alpha_S}{\pi} \sigma_0^\phi \, d_{q\bar{q}}^\phi(\hat{\tau}, \tau_Q, \tau_{\tilde{Q}})
\end{aligned}
$$

- $\tau_{Q,\tilde{Q}} = \frac{4m_{Q,\tilde{Q}}^2}{M_\Phi^2}$, $\hat{\tau} = \frac{m_\phi^2}{\hat{s}}$

The Scenario

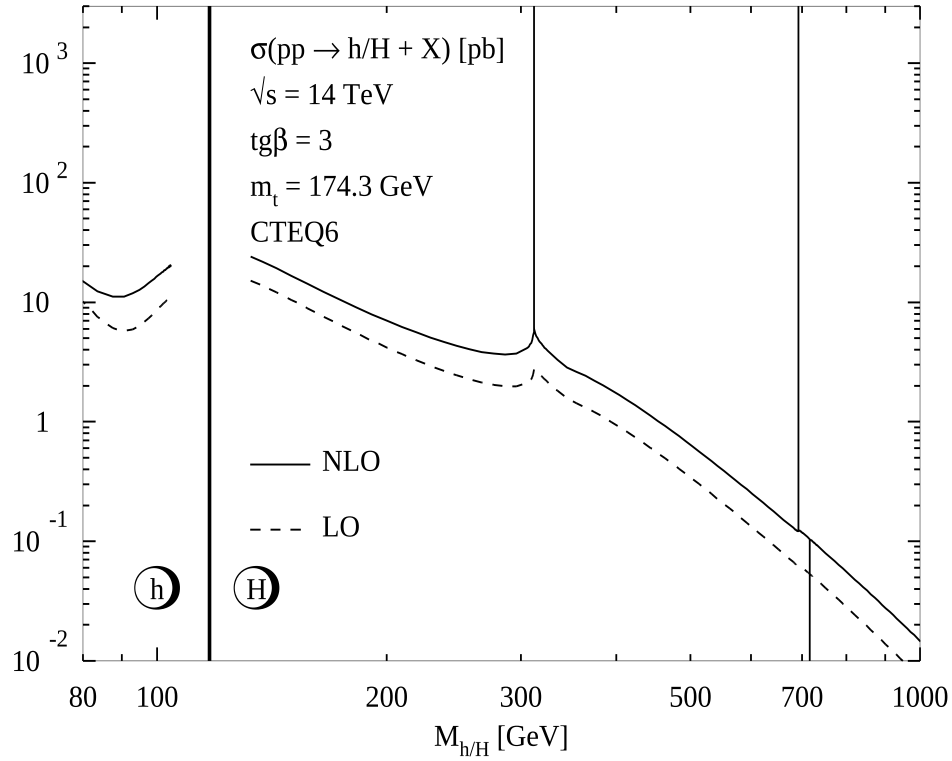
The gluophobic Higgs scenario $[m_t = 174.3$ GeV$]$ Carena,Heinemeyer,Wagner,Weiglein

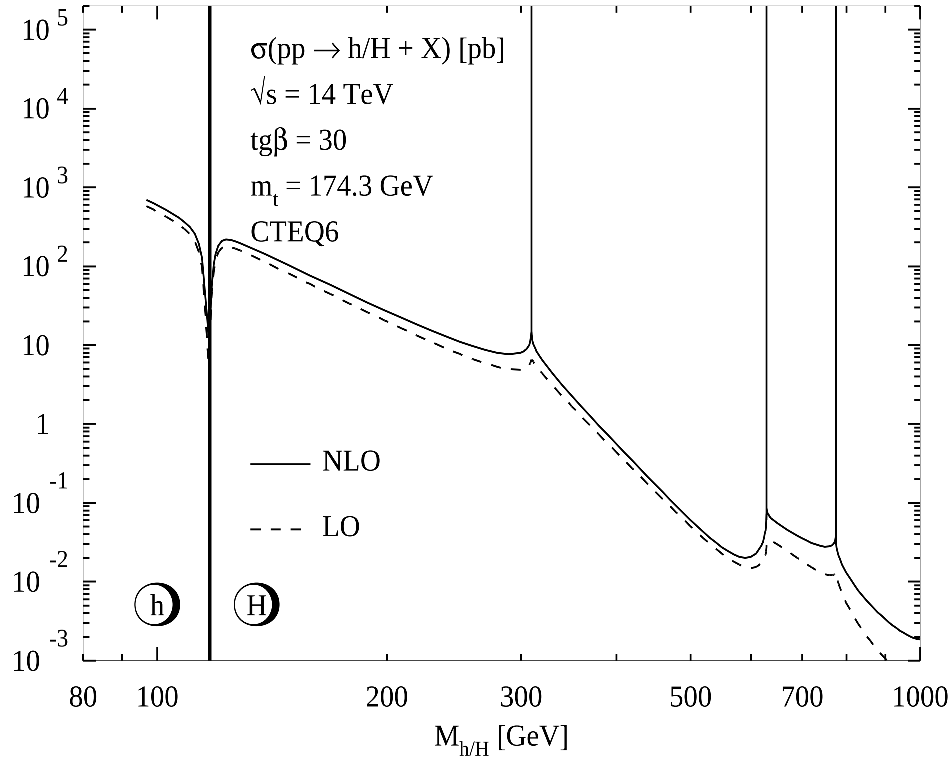
$$M_{SUSY} = 350 \text{ GeV},\ \mu = M_2 = 300 \text{ GeV},\ X_t = -770 \text{ GeV},\ A_b = A_t,\ m_{\tilde{g}} = 500 \text{ GeV}$$

$\tan\beta = 3$		$\tan\beta = 30$	
$m_{\tilde{t}_1} = 156$ GeV	$m_{\tilde{t}_2} = 517$ GeV	$m_{\tilde{t}_1} = 155$ GeV	$m_{\tilde{t}_2} = 516$ GeV
$m_{\tilde{b}_1} = 346$ GeV	$m_{\tilde{b}_2} = 358$ GeV	$m_{\tilde{b}_1} = 314$ GeV	$m_{\tilde{b}_2} = 388$ GeV

NLO cross section $\longrightarrow$

The LO and NLO cross section w/ Squarks





$\Delta \sim 20 - 100\%$

Kinks, bumps, spikes: $\tilde{t}_1\bar{\tilde{t}}_1, \tilde{b}_1\bar{\tilde{b}}_1, \tilde{b}_2\bar{\tilde{b}}_2$ thresholds in consecutive order with rising Higgs mass.

Coulomb singularities

$\tilde{Q}\bar{\tilde{Q}}$ thresholds: Formation of 0^{++} states $\rightsquigarrow$ Coulomb singularities

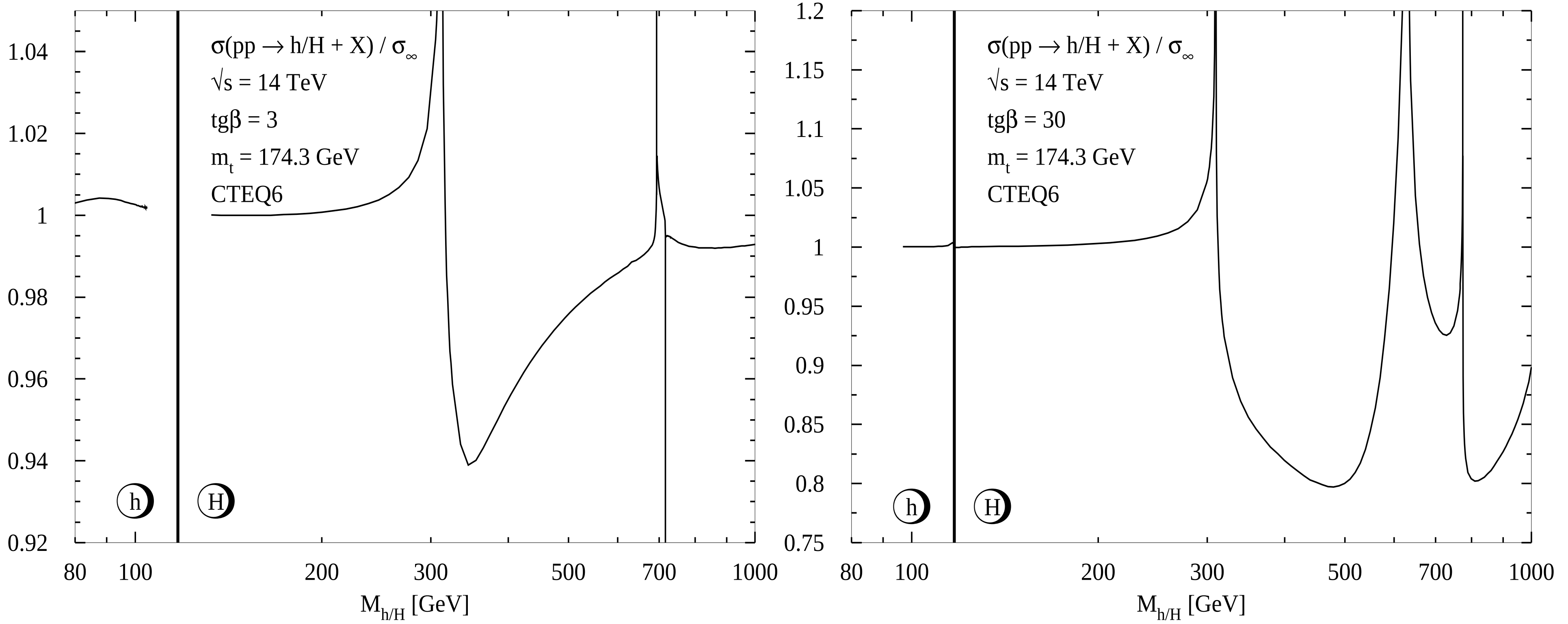
Singular behaviour can be derived from the Sommerfeld rescattering corrections $\rightsquigarrow$

At each specific $\tilde{Q}_0\bar{\tilde{Q}}_0$ threshold:

$$C_1(\tau_Q, \tau_{\tilde{Q}}) \to \text{Re}\left\{ \frac{g^\Phi_{\tilde{Q}_0} \tilde{F}(\tilde{Q}_0) \frac{16\pi^2}{3(\pi^2-4)} \left[-\ln\left(\tau^{-1}_{\tilde{Q}_0} - 1\right) + i\pi + const \right]}{\sum_Q g^\Phi_Q F(\tau_Q) + \sum_{\tilde{Q}} g^\Phi_{\tilde{Q}} \tilde{F}(\tau_{\tilde{Q}})} \right\}$$

Agrees quantitatively with numerical results.

σ_{NLO} w/ full squark mass dependence / σ_{NLO} in the heavy squark limit

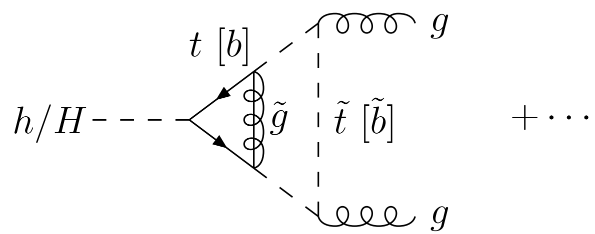


$\sigma(pp \to h/H + X)/\sigma_\infty$ up to 20%

Kinks, bumps, spikes: $\tilde{t}_1\bar{\tilde{t}}_1, \tilde{b}_1\bar{\tilde{b}}_1, \tilde{b}_2\bar{\tilde{b}}_2$ thresholds in consecutive order with rising Higgs mass.

Genuine SUSY-QCD corrections

- **Limit heavy SUSY masses** $\rightarrow \mathcal{O}(10\ \%)$ Harlander,Steinhauser,Hofmann



- **Numerical analysis:** $F_Q^{h/H}(\tau_Q) \rightarrow F_Q^{h/H}(\tau_Q)[1 + C_{SUSY}^Q \frac{\alpha_S}{\pi}]$

- $m^2_{Q/\tilde{Q}} \rightarrow m^2_{Q/\tilde{Q}}(1 - i\epsilon)$

- **5-dimensional Feynman integral** $\rightarrow$ **endpoint substractions:**

$$\int_0^1 dx\, \frac{f(x)}{x(1-x)} \rightarrow \int_0^1 dx \left\{ \frac{f(x)}{x(1-x)} - \frac{f(0)}{x} - \frac{f(1)}{(1-x)} \right\}$$

$\Rightarrow$ isolation of singularities

Genuine SUSY-QCD corrections

- **Thresholds for $M_H > 2m_Q$** → numerical instabilities → partial integration

$$\int_0^1 dz \frac{f(z)}{(a+bz)^2} = -\frac{f(z)}{b(a+bz)}\Big|_0^1 + \int_0^1 dz\, \frac{f'(z)}{b(a+bz)}$$
$$\int_0^1 dz \frac{f(z)}{a+bz} = \frac{f(z)}{b}\ln(a+bz)\Big|_0^1 - \int_0^1 dz\, \frac{f'(z)}{b}\ln(a+bz)$$

⇒ thresholds in arguments of logs ⇒ stabilization

[more involved for quadratic polynomials]

- **Renormalization**

α_S :	$\overline{\text{MS}}$ scheme [5 flavours]
$m_Q, m_{\tilde{Q}}, A_t$:	on-shell
A_b :	on-shell ↔ $\overline{\text{MS}}$
A_t, A_b :	anomalous SUSY-restoring counter-terms

$$A_b = \frac{\sin 2\theta_b}{2m_b}(m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2) + \mu\tan\beta$$
$$\delta\theta_b = \frac{1}{2}\text{Re}\frac{\Sigma_{12}(m_{\tilde{b}_1}^2)+\Sigma_{12}(m_{\tilde{b}_2}^2)}{m_{\tilde{b}_2}^2 - m_{\tilde{b}_1}^2}$$

The Scenario

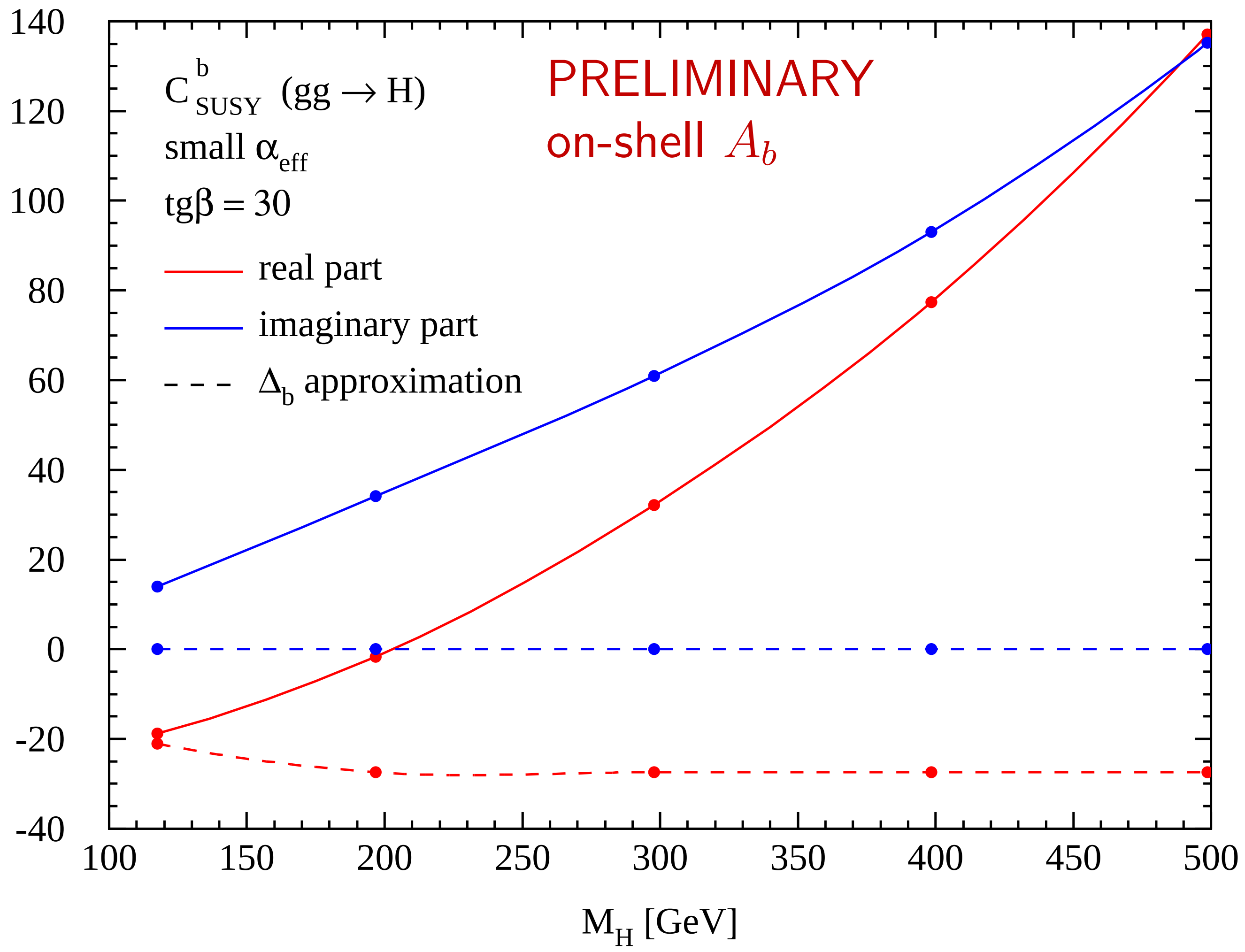
Small α_{eff} scenario [modified]

$$
\begin{aligned}
\tan\beta &= 30 \\
M_{\tilde{Q}} &= 800 \text{ GeV} \\
M_{\tilde{g}} &= 1000 \text{ GeV} \qquad \longleftarrow \\
M_2 &= 500 \text{ GeV} \\
A_b = A_t &= -1.133 \text{ TeV} \\
\mu &= 2 \text{ TeV}
\end{aligned}
$$

$$
m_{\tilde{t}_1} = 679 \text{ GeV} \qquad m_{\tilde{t}_2} = 935 \text{ GeV}
$$

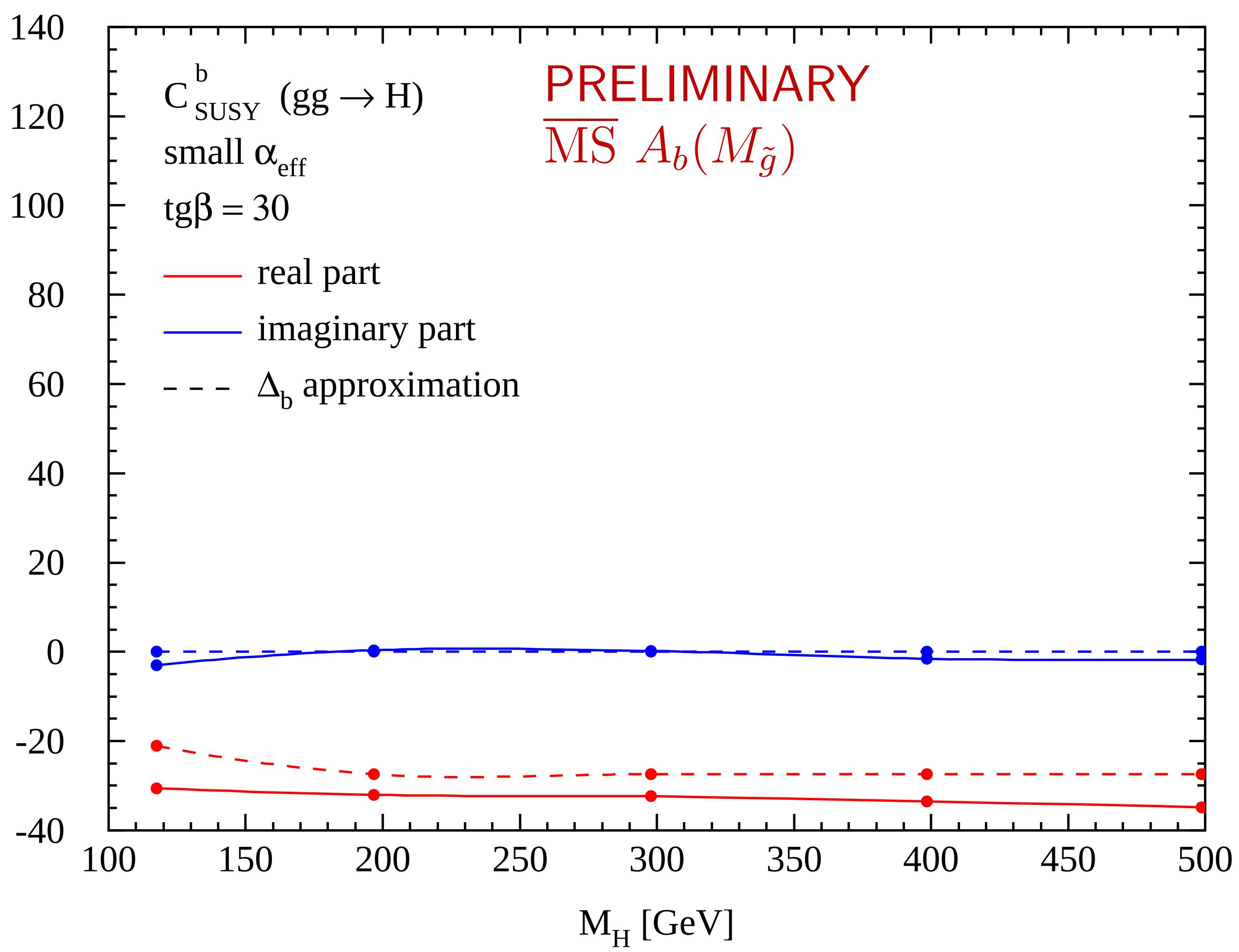
$$
m_{\tilde{b}_1} = 601 \text{ GeV} \qquad m_{\tilde{b}_2} = 961 \text{ GeV}
$$

Preliminary results



MMM, Rzehak, Spira

Preliminary results



MMM, Rzehak, Spira

Heavy loop particle mass limit

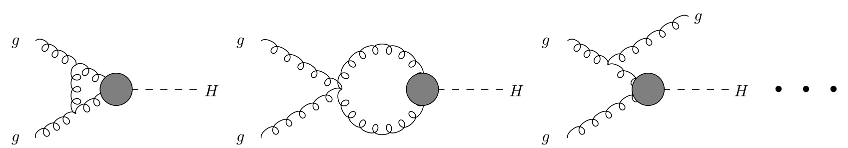
- **Heavy quarks/squarks and very heavy gluinos** $[m_{\tilde{g}}^2 \gg m_{Q,\tilde{Q}}^2 \gg M_\phi^2]$

$$c_Q^\phi \to \frac{11}{2} \qquad d_{gg}^\phi \to -\frac{11}{2}(1-\hat{\tau})^3$$

$$d_{gq}^\phi \to -1 + 2\hat{\tau} - \frac{\hat{\tau}^2}{3} \qquad d_{q\bar{q}}^\phi \to \frac{32}{27}(1-\hat{\tau})^3$$

$$c_{\tilde{Q}}^\phi \to 9 \qquad c_{SUSY}^\phi \to \frac{10}{3}$$

- c_{SUSY}^ϕ**: proper decoupling of gluinos** $\to$ non-supersymmetric $\mathcal{L}_{eff}$ MMM,Rzehak,Spira



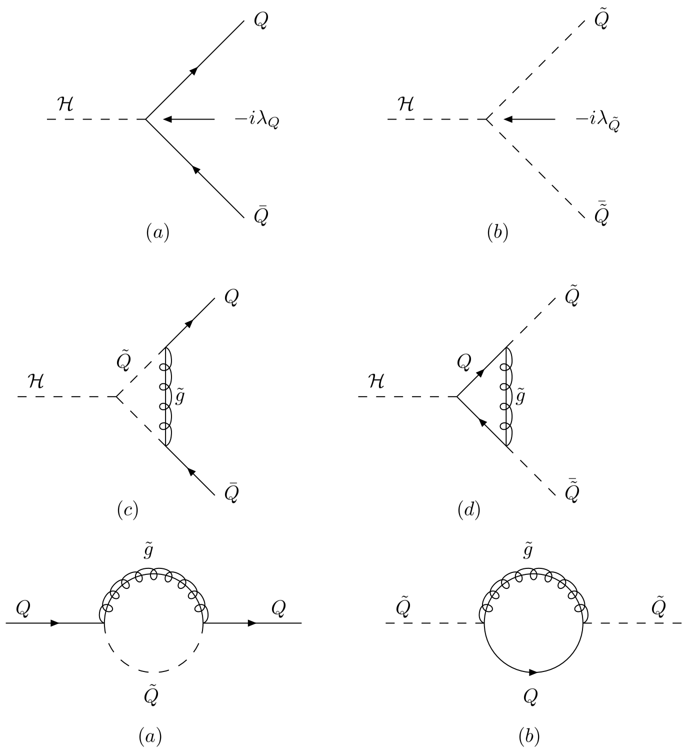
- Harlander,Steinhauser: **mass degenerate squarks, no mixing, supersymmetric renormalization**

$M_{\tilde{g}} \gg m_{\tilde{Q}}, m_Q$:

$$C_{SQCD}^{HS} = \frac{11}{2} - \frac{4}{3}\ln\frac{M_{\tilde{g}}^2}{m_{\tilde{Q}}^2} - 2\ln\frac{m_{\tilde{Q}}^2}{m_Q^2} \qquad \left[\text{SUSY: } g_{\tilde{Q}}^{\mathcal{H}} = 2 g_Q^{\mathcal{H}} \frac{m_Q^2}{m_{\tilde{Q}}^2}\right]$$

Heavy loop particle mass limit

- $M_{\tilde{g}} \gg m_{\tilde{Q}}, m_Q$: Supersymmetry lost due to decoupled gluino→ integrate gluinos out



- **No mixing at LO:**

$$\lambda_Q = g_Q^{\mathcal{H}} \frac{m_Q}{v} \qquad \lambda_{\tilde{Q}} = 2 g_Q^{\mathcal{H}} \frac{m_Q^2}{v} = \kappa \lambda_Q^2 \qquad \kappa = 2 \frac{v}{g_Q^{\mathcal{H}}}$$

Heavy loop particle mass limit

- **SUSY beyond LO:** $\overline{\text{MS}}$ couplings $[\mu_R > M_{\tilde g}]$

$$\bar\lambda_{\tilde Q}(\mu_R) = \kappa \bar\lambda_Q^2(\mu_R)$$

- $\mu_R < M_{\tilde g}$: (i) threshold corrections

  (ii) different RGEs [decoupled $\tilde g$]

- $\mu_R < M_{\tilde g}$: momentum-substracted coupling $\to$ threshold correction:

  (i) threshold correction:

$$\bar\lambda_{Q,MO}(M_{\tilde g}) = \bar\lambda_Q(M_{\tilde g})\left\{1 - \frac{3}{8} C_F \frac{\alpha_S(M_{\tilde g})}{\pi}\right\}$$

  (ii) different RGEs:

$$\mu_R^2 \frac{\partial \bar\lambda_Q(\mu_R)}{\partial \mu_R^2} = -\frac{C_F}{2}\frac{\alpha_S(\mu_R)}{\pi}\bar\lambda_Q(\mu_R) \qquad [\mu_R > M_{\tilde g}]$$

$$\mu_R^2 \frac{\partial \bar\lambda_{Q,MO}(\mu_R)}{\partial \mu_R^2} = -\frac{3}{4} C_F \frac{\alpha_S(\mu_R)}{\pi}\bar\lambda_{Q,MO}(\mu_R) \qquad [\mu_R < M_{\tilde g}]$$

Heavy loop particle mass limit

- **analogously for $\lambda_{\tilde{Q}}$:**

  (i) threshold correction:

$$\bar{\lambda}_{\tilde{Q},MO}(M_{\tilde{g}}) = \bar{\lambda}_{\tilde{Q}}(M_{\tilde{g}})\left\{1 + \frac{3}{4}C_F\frac{\alpha_S(M_{\tilde{g}})}{\pi}\right\}$$

  (ii) different RGEs:

$$\mu_R^2\frac{\partial\bar{\lambda}_{\tilde{Q}}(\mu_R)}{\partial\mu_R^2} = -C_F\frac{\alpha_S(\mu_R)}{\pi}\bar{\lambda}_{\tilde{Q}}(\mu_R) \qquad [\mu_R > M_{\tilde{g}}]$$

$$\mu_R^2\frac{\partial\bar{\lambda}_{\tilde{Q},MO}(\mu_R)}{\partial\mu_R^2} = -\frac{C_F}{2}\frac{\alpha_S(\mu_R)}{\pi}\bar{\lambda}_{\tilde{Q},MO}(\mu_R) \qquad [\mu_R < M_{\tilde{g}}]$$

- **Relation to quark pole mass:** Gray,Broadhurst,Grafe,Schilcher

$$g_Q^\phi\frac{m_Q}{v} = \bar{\lambda}_{Q,MO}(m_Q)\left\{1 + C_F\frac{\alpha_S(m_Q)}{\pi}\right\}$$

Heavy loop particle mass limit

$$2g_Q^{\mathcal{H}}\frac{m_Q^2}{v} = \bar\lambda_{\tilde Q,MO}(m_{\tilde Q})\Big\{1 + C_F\frac{\alpha_S}{\pi}\Big(\ln\frac{M_{\tilde g}^2}{m_{\tilde Q}^2} + \frac{3}{2}\ln\frac{m_{\tilde Q}^2}{m_Q^2} + \frac{1}{2}\Big)\Big\}$$

$$\mathcal{L}_{eff} = \frac{\alpha_S}{12\pi}G^{a\mu\nu}G^a_{\mu\nu}\frac{\mathcal{H}}{v}\Big\{\sum_Q g_Q^{\mathcal{H}}\Big[1 + \frac{11}{4}\frac{\alpha_S}{\pi}\Big] + \sum_{\tilde Q}\frac{g_{\tilde Q}^{\mathcal{H}}}{4}\Big[1 + C_{SQCD}\frac{\alpha_S}{\pi}\Big]\Big\}$$

$$g_{\tilde Q}^{\mathcal{H}} = v\frac{\bar\lambda_{\tilde Q,MO}(m_{\tilde Q})}{m_{\tilde Q}^2}$$

$$\Delta C_{SQCD} = \frac{4}{3}\ln\frac{M_{\tilde g}^2}{m_{\tilde Q}^2} + 2\ln\frac{m_{\tilde Q}^2}{m_Q^2} + \frac{2}{3} \quad \Longrightarrow \quad \boxed{C_{SQCD} = \frac{37}{6}}$$

- **Solution to RGEs** $[\beta_0 = (33 - 2N_F - N_{\tilde F})/12]$

$$\boxed{\bar\lambda_{\tilde Q,MO}(m_{\tilde Q}) = 2g_Q^{\mathcal{H}}\frac{m_Q^2}{v}\frac{1+\frac{3}{2}C_F\frac{\alpha_S(M_{\tilde g})}{\pi}}{1+2C_F\frac{\alpha_S(m_Q)}{\pi}}\left(\frac{\alpha_S(M_{\tilde g})}{\alpha_S(m_{\tilde Q})}\right)^{\frac{C_F}{\beta_0}}\left(\frac{\alpha_S(m_{\tilde Q})}{\alpha_S(m_Q)}\right)^{\frac{3C_F}{2\beta_0}}}$$

- **No $\tilde Q$ loops to $gg \to A$ at LO** $\Rightarrow$ no $\ln M_{\tilde g}$ Harlander,Hofmann

Conclusions

◇ SUSY QCD corrections at NLO to $gg \to h/H$ including the full squark mass dependence.

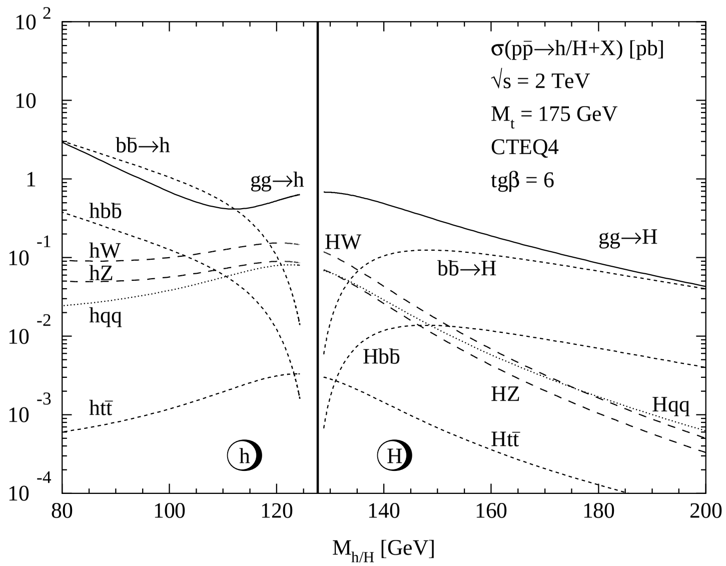
◇ Inclusion of full squark mass dependence has significant effects on the K-factor compared to the heavy squark mass limit. The deviation can be as large as $\mathcal{O}(20\%)$ for $gg \to h/H$.

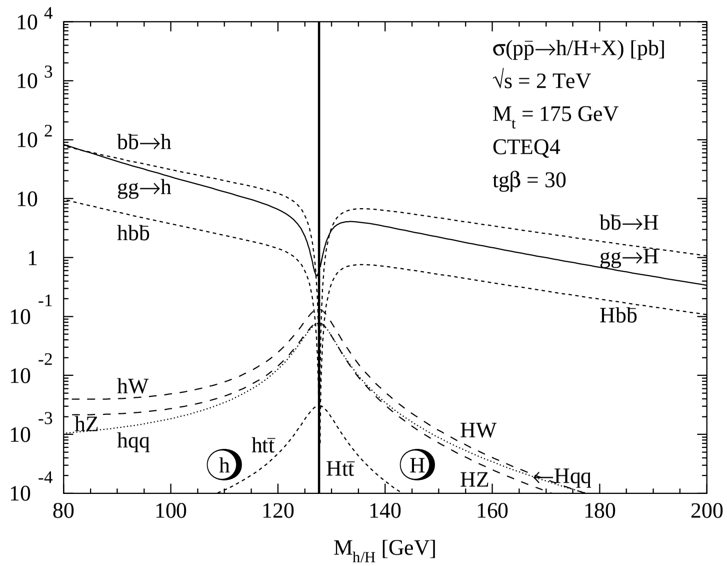
◇ Large QCD corrections + large genuine SUSY QCD corrections for large $\tan\beta$ in MSSM
$\leftarrow \Delta_b$ approximation for $\overline{\text{MS}}$ A_b.

◇ $\mathcal{H}gg$ coupling: decoupling of gluinos for large $M_{\tilde{g}}$: consistent with Appelquist-Carazzone theorem if properly renormalized $\to$ effective Lagrangian.

MSSM Higgs Boson Production at the Tevatron

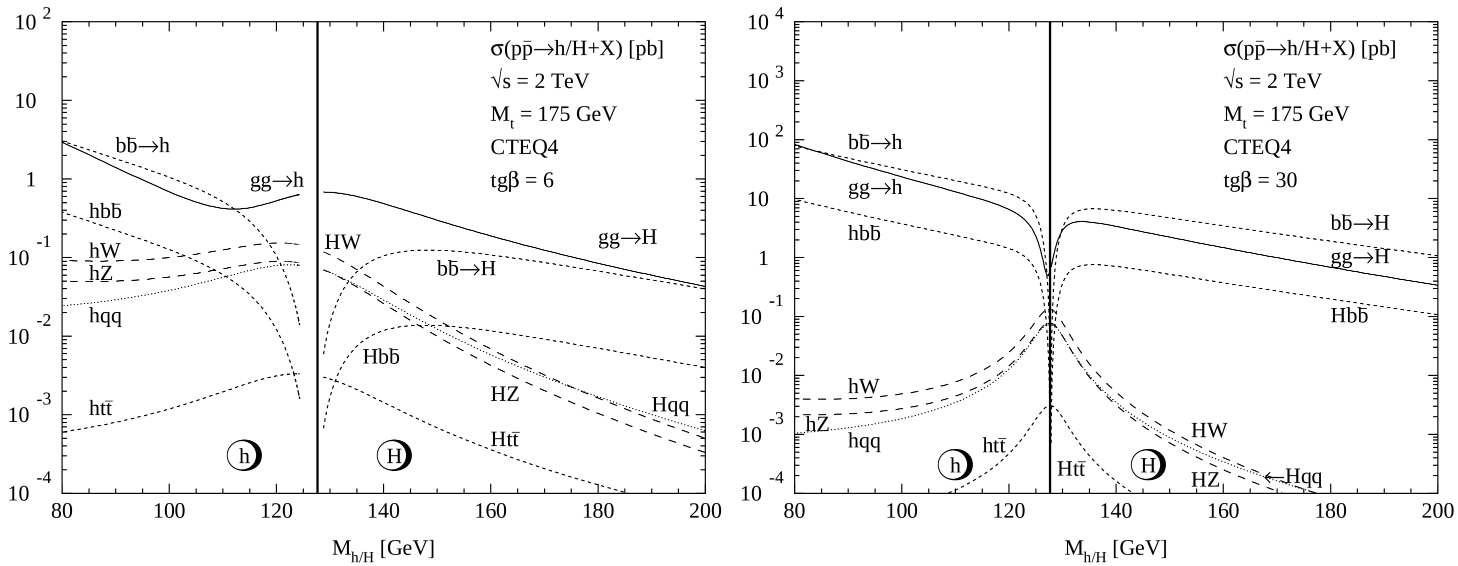
Spira



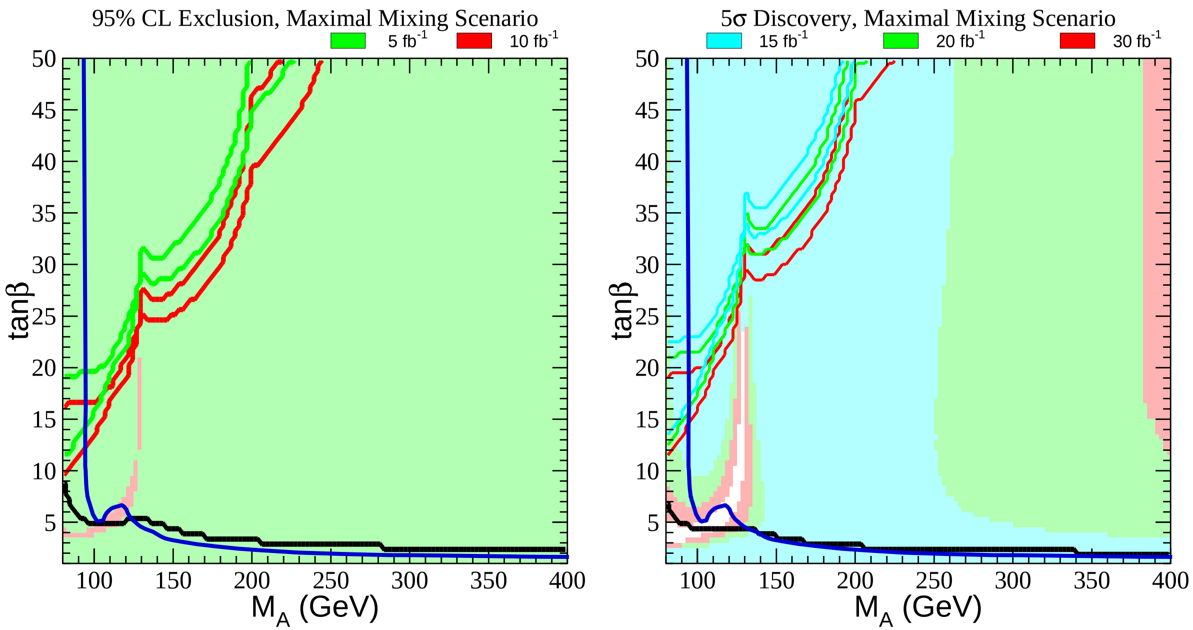


MSSM Higgs Boson Production at the Tevatron

Spira

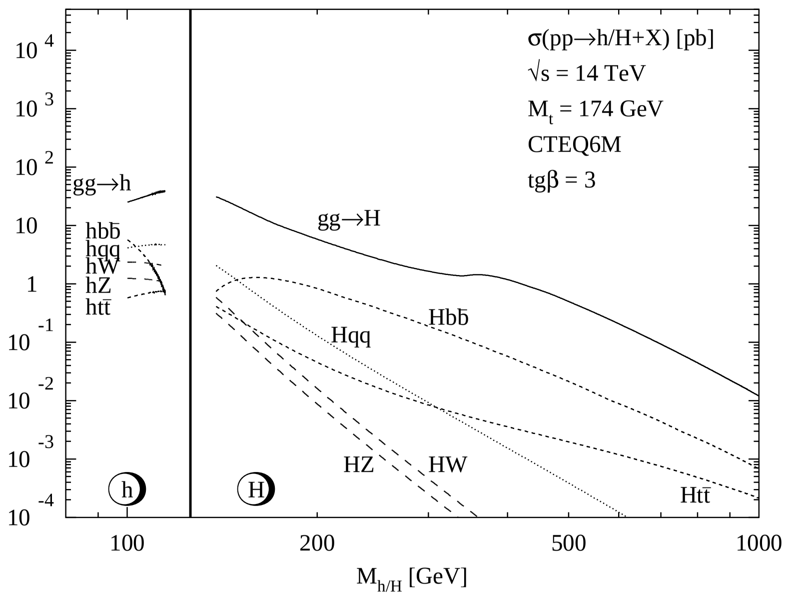


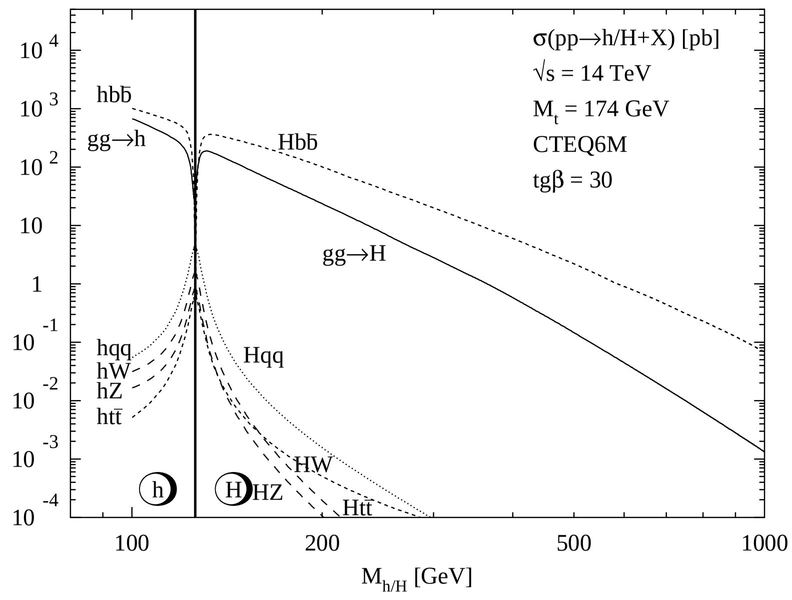
Carena et al.



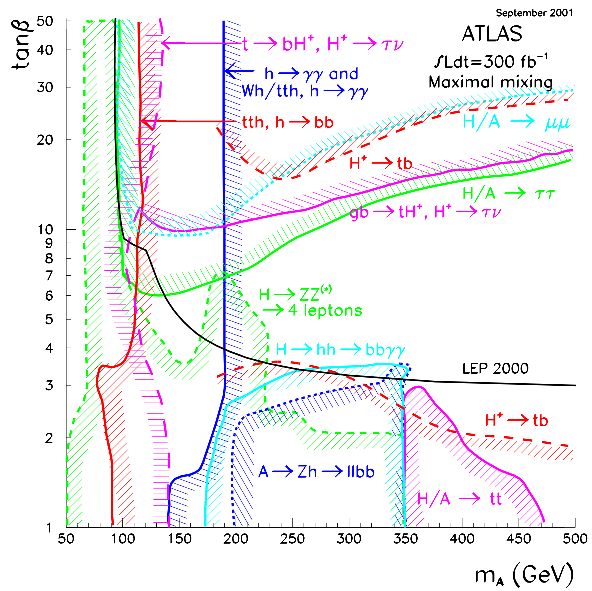
MSSM Higgs Boson Production at the LHC

Spira

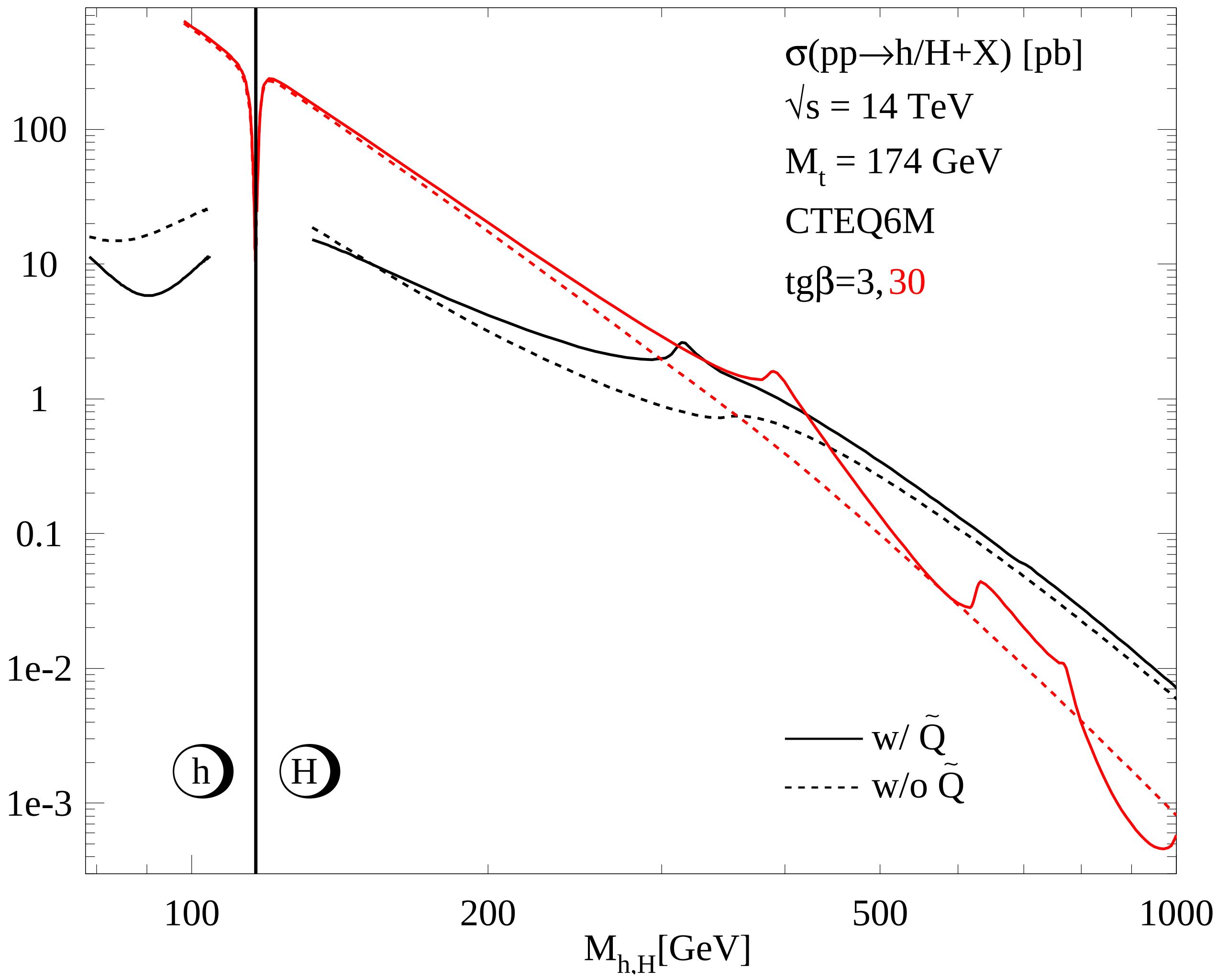




ATLAS



The LO cross section w/ and w/o Squarks



Virtual corrections - heavy loop particle mass limit

Total virtual correction [heavy squark/quark limit]:

$$C_{\rm virt} = \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left(\frac{4\pi\mu^2}{M_\Phi^2}\right)^{\epsilon}\left\{-\frac{3}{\epsilon^2} - \frac{33-2N_F}{6\epsilon}\left(\frac{\mu^2}{M_\Phi^2}\right)^{-\epsilon} + \pi^2 + \frac{11}{2} + \frac{7}{2}\mathrm{Re}\left\{\frac{\sum_{\tilde{Q}} g_{\tilde{Q}}^{\Phi}\tilde{F}(\tau_{\tilde{Q}})}{\sum_Q g_Q^\Phi F(\tau_Q)+\sum_{\tilde{Q}} g_{\tilde{Q}}^{\Phi}\tilde{F}(\tau_{\tilde{Q}})}\right\}\right\}$$

↑ IR ↑ Coll

[without squark loops only $\frac{11}{2}$]

To get a finite cross section the real corrections have to be added.

Real corrections - heavy loop particle mass limit

Total real corrections [heavy squark/quark limit]:

$$\begin{aligned}
C_{\text{real}} &= \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left(\frac{4\pi\mu^2}{m_\Phi^2}\right)^\epsilon \left\{\frac{3}{\epsilon^2} + \frac{33-2N_F}{6\epsilon}\right\} \\
D_{gg} &= -\frac{\hat\tau}{\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left(\frac{4\pi\mu^2}{\hat s}\right)^\epsilon P_{gg}(\hat\tau) - \frac{11}{2}(1-\hat\tau)^3 \\
&\quad +12\left\{\left(\frac{\log(1-\hat\tau)}{1-\hat\tau}\right)_+ - \hat\tau[2-\hat\tau(1-\hat\tau)]\log(1-\hat\tau)\right\} \\
D_{gq} &= -\left\{\frac{1}{2\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left(\frac{4\pi\mu^2}{\hat s}\right)^\epsilon - \log(1-\hat\tau)\right\}\hat\tau P_{gq}(\hat\tau) - 1 + 2\hat\tau - \frac{\hat\tau^2}{3} \\
D_{q\bar q} &= \frac{32}{27}(1-\hat\tau)^3
\end{aligned}$$

- IR, Coll. poles in C_{real} subtract the corresponding ones of the virtual corrections.
- Coll. poles in the real corrections (Altarelli-Parisi kernels as coefficients)
  $\rightsquigarrow$ absorbed in NLO structure functions.

Result - heavy loop particle mass limit

$$
\begin{aligned}
\sigma(pp \to \Phi + X) &= \sigma_0 [1 + C \tfrac{\alpha_S}{\pi}] \tau_\Phi \frac{d\mathcal{L}_{gg}}{d\tau_\Phi} + \Delta\sigma_{gg} + \Delta\sigma_{gq} + \Delta\sigma_{qq} \\
C &= \pi^2 + \tfrac{11}{2} + \tfrac{7}{2} \mathrm{Re} \left\{ \frac{\sum_{\tilde{Q}} g_{\tilde{Q}}^\Phi \tilde{F}(\tau_{\tilde{Q}})}{\sum_Q g_Q^\Phi F(\tau_Q) + \sum_{\tilde{Q}} g_{\tilde{Q}}^\Phi \tilde{F}(\tau_{\tilde{Q}})} \right\} + \frac{33-2N_F}{6} \log \frac{\mu^2}{M_\Phi^2} \\
\Delta\sigma_{gg} &= \int_{\tau_\Phi}^1 d\tau \frac{d\mathcal{L}_{gg}}{d\tau} \frac{\alpha_S}{\pi} \sigma_0 \Big\{ -\hat{\tau} P_{gg}(\hat{\tau}) \log \frac{Q^2}{\hat{s}} - \frac{11}{2}(1-\hat{\tau})^3 \\
&\quad + 12 \Big[\Big(\frac{\log(1-\hat{\tau})}{1-\hat{\tau}} \Big)_+ - \hat{\tau}[2 - \hat{\tau}(1-\hat{\tau})] \log(1-\hat{\tau}) \Big] \Big\} \\
\Delta\sigma_{gq} &= \int_{\tau_\Phi}^1 d\tau \sum_{q,\bar{q}} \frac{d\mathcal{L}_{gq}}{d\tau} \frac{\alpha_S}{\pi} \sigma_0 \Big\{ -\frac{\hat{\tau}}{2} P_{gq}(\hat{\tau}) \Big[\log \frac{Q^2}{\hat{s}} - 2 \log(1-\hat{\tau}) \Big] \\
&\qquad - 1 + 2\hat{\tau} - \frac{\hat{\tau}^2}{3} \Big\} \\
\Delta\sigma_{q\bar{q}} &= \int_{\tau_\Phi}^1 d\tau \sum_q \frac{d\mathcal{L}_{q\bar{q}}}{d\tau} \frac{\alpha_S}{\pi} \sigma_0 \frac{32}{27} (1-\hat{\tau})^3
\end{aligned}
$$

[μ=Ren. scale, Q=Fact. scale]

natural scales: $\mu^2 = Q^2 = M_\Phi^2$