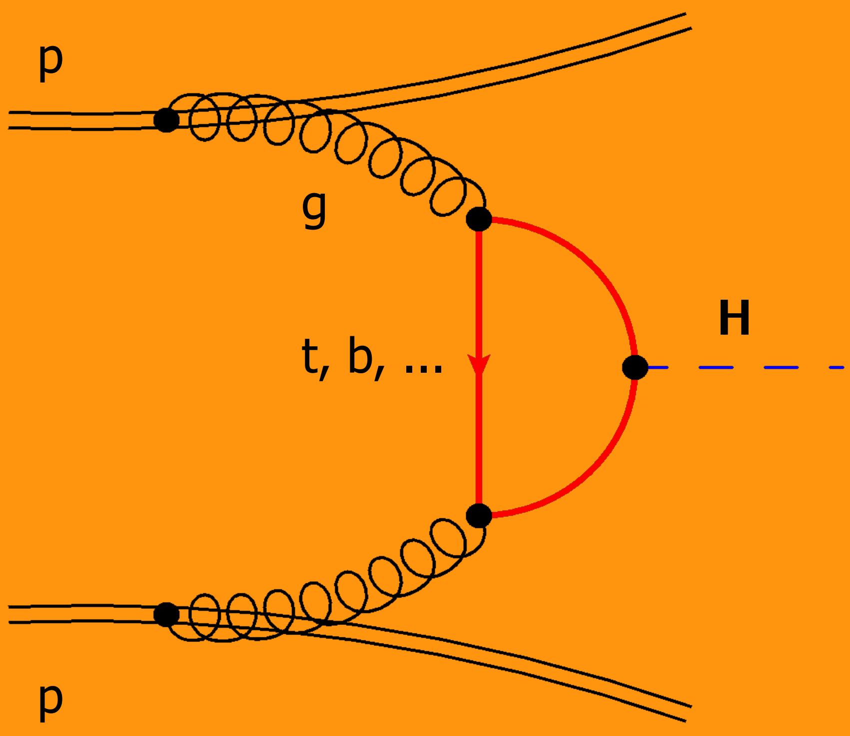
Top mass effects in gluon fusion

Alexey Pak, TTP Karlsruhe

work done in collaboration with
Matthias Steinhauser and Mikhail Rogal
JHEP 1002:025,2010

Higgs boson production at the LHC: pp→H+X



Dominant channel in SM (for intermediate m_h):

gg→H via a top-quark loop

Very well studied process!

Relevant scales:

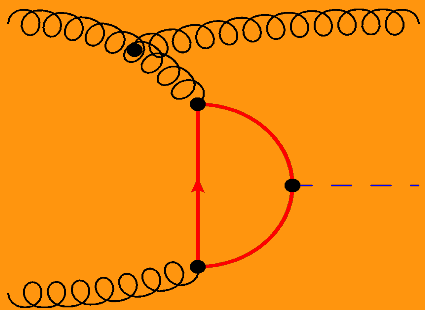
$$\sqrt{S} \sim 14\,\mathrm{TeV}\,(\mathit{protons})$$
$$\sqrt{s} \sim 100-14000\,\mathrm{GeV}\,(\mathit{partons})$$
$$m_h \sim 100-300\,\mathrm{GeV}$$
$$m_t \sim 170\,\mathrm{GeV}$$

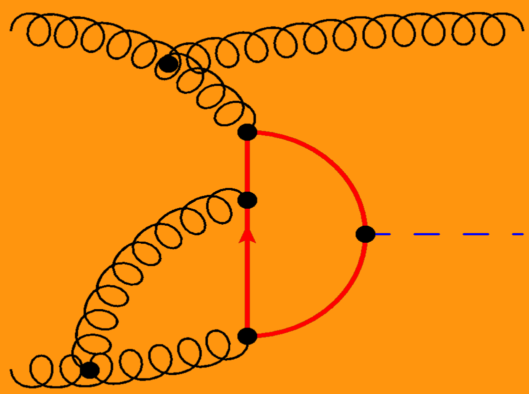
Leading order: **[Geordi, Glashow, Machacek, Nanopoulos '78]**
(full dependence on m_h , m_t)

Theoretical predictions

Inclusive cross-section @ $O(\square_s)$: ~O(70%)

[Dawson; Djouadi, Spira, Zerwas '91]
[Spira et al '95] (exact)





Cross-section @ $O(\square_s^2)$: ~O(10%), scale dep. O(%)

[Harlander, Kilgore '02] (soft expansion)
[Anastasiou, Melnikov '02],
[Ravindran, Smith, van Neerven '03]

Beyond fixed order PT:
NNLO + NNLL
N^3LO threshold-enhanced
π^2-resummation, ...

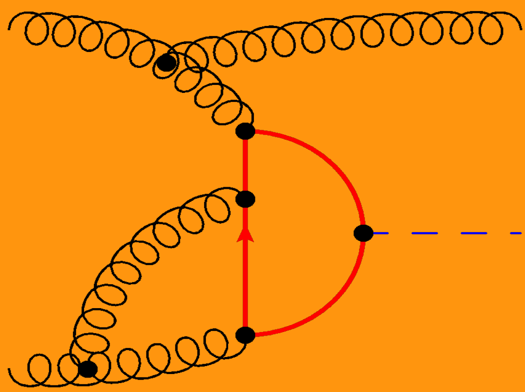
Fully differential:
NLO (exact), **NNLO**

Also: EW, QCD-EW, ...

Catani, de Florian, Grazzini, Nason; Ahrens, Becher, Neubert, Yang; Actis, Passarino, Sturm, Uccirati; Anastasiou, Boughezal, Petriello; ...

Most results assume $m_t \square \infty$, and use heavy top effective theory (EFT)

Beyond the m_t□∞ limit at the NNLO



NNLO large s leading logs (m_t/s, m_h/s □ 0):

[Marzani et al '08]: gg channel (quark channels in [Harlander,Mantler,Marzani,Ozeren '09])

1/m_t expansion near heavy top limit (s/m_t, m_h/m_t □ 0):

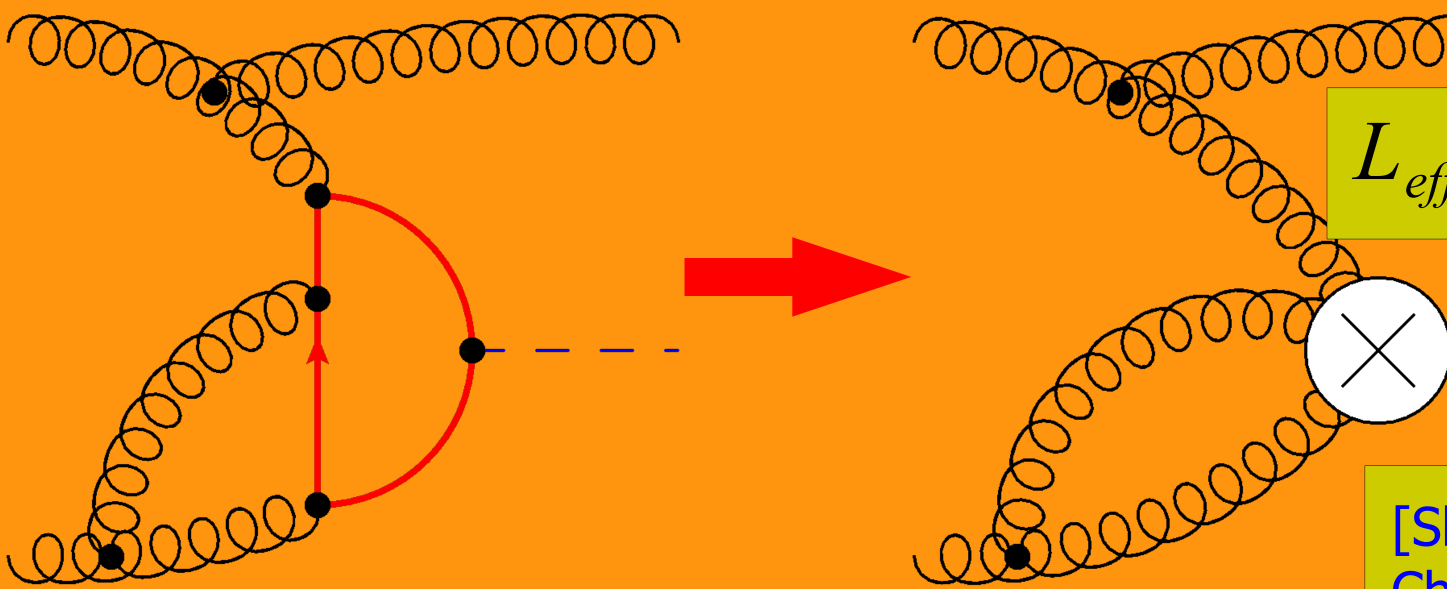
[Harlander, Ozeren; Pak, Steinhauser, Rogal '09]: virtual corrections

[Harlander, Ozeren '09]: full NNLO cross-section (soft expansion)

This work: full NNLO cross-section, confirmation by a different method

Heavy top limit: effective theory

Formally integrate top quark out: effective ggH, gggH, ... vertices remain



$$L_{eff} = C \cdot H\, G_{\mu\nu} G^{\mu\nu}$$

[Shroeder, Steinhauser; Chetyrkin, Kuehn, Sturm; Spira et al]

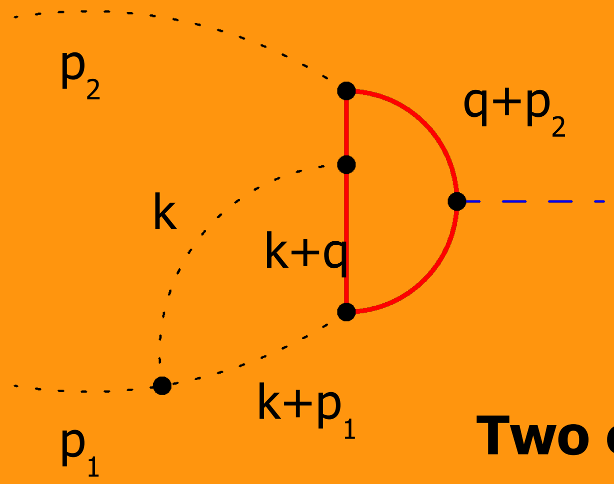
Top quarks in FS prohibited: $\frac{m_h^2}{4 m_t^2} \ll 1$

Assume: $s = (p_1 + p_2)^2 \ll m_t^2$ **(???)**

Simple leading order; for $O(1/m_t^n)$ terms need to consider higher order operators, Feynman rules... (e.g. [Neill, 09])

Alternative to EFT: asymptotic expansion

Multiple-scale integral:

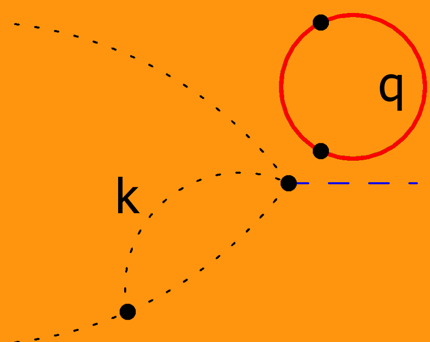


loop momenta k, q can be "large" or "small"

$$\sim \int \frac{d^D k \, d^D q}{\left((q+k)^2+m_t^2\right)(k+p_1)^2 \ldots}$$

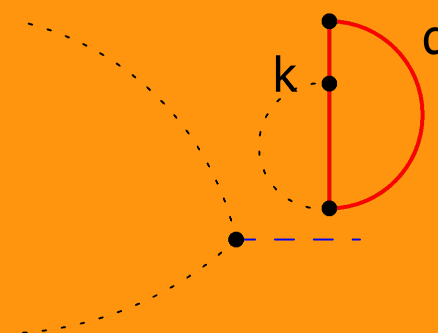
Two contributing expansion "regions":

$$\frac{1}{(q+k)^2+m_t^2} = \frac{1}{q^2+m_t^2} - \frac{2qk+k^2}{(q^2+m_t^2)^2} + \ldots$$



+

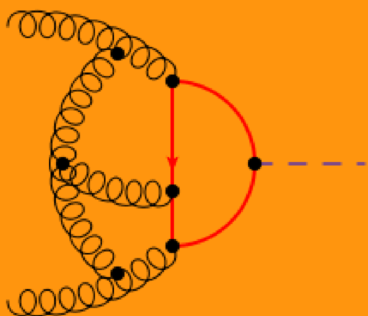
$$\frac{1}{(k+p_1)^2} = \frac{1}{k^2} - \frac{2k\,p_1}{(k^2)^2} + \ldots$$



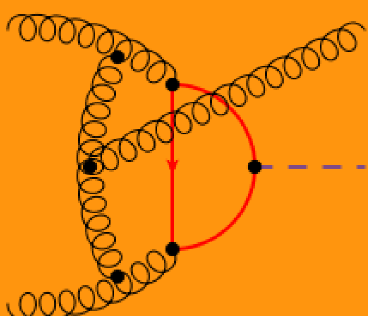
Same limitations and predictive power as EFT, but easier automation for higher order m_t-suppressed terms

Calculation techniques

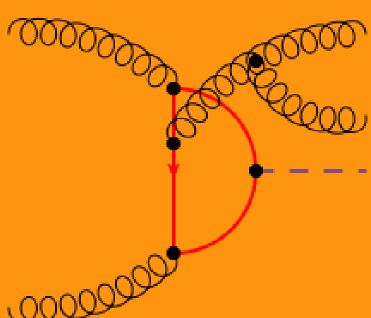
Contributions to consider at NNLO:



Virtual:
3-loop, 3-point



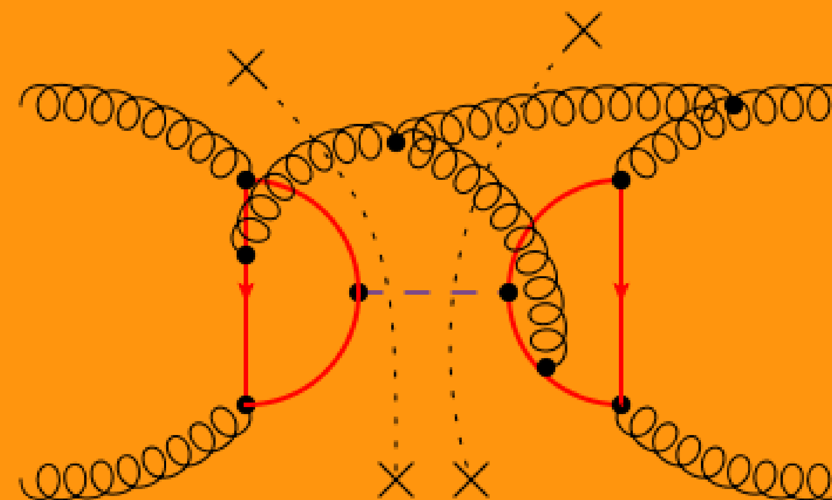
Single real emission:
2-loop, 4-point



Double real emission:
1-loop, 5-point

Squared matrix element:
sum over colour, Lorentz indices, combinatorics, signs, ...

Optical theorem:
avoid summation over final states and integration over phase space



4-loop, imaginary part only, forward scattering

Many diagrams: ~ 20000

But: can apply multi-loop methods to phase space integration!

Workflow of the calculation

- Diagrams generation: **QGRAF** (~10^6 4-loop diagrams) + Perl scripts that sort out zeros

- Asymptotic expansion, mapping on pre-defined topologies: **Q2E/EXP** and custom Perl/C++ program (more general expansion algorithm)

- Calculation: **FORM** programs: **MATAD** setup and independent program

- IBP reduction: Laporta algorithm, C++ program **rows** (internally uses **FERMAT**)

- Convolutions with splitting functions: done in Mellin space with **FORM**

- Master integrals: by differential equations with a **Mathematica** program, use **HPL.m** and by soft expansion

- Many steps have independent cross-checks, analytic and numerical

Partonic cross-sections

After adding all contributions, renormalization, cancellation of collinear singularities, etc:

$$\sigma_{gg} \sim A^{(0)}\delta(1-x) + \frac{\alpha_s}{\pi}\left(A^{(1)}\delta(1-x) + B_+^{(1)}(x) + C^{(1)}(x)\right)$$

$$+\left(\frac{\alpha_s}{\pi}\right)^2\left(A^{(2)}\delta(1-x) + B_+^{(2)}(x) + C^{(2)}(x)\right)$$

$$x = \frac{m_h^2}{s}$$

$$s = (p_1 + p_2)^2$$

$$\rho = \frac{m_h^2}{m_t^2}$$

Purely virtual contributions (no real radiation)

Partonic cross-sections

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$$x = \frac{m_h^2}{s}$$

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$$\rho = \frac{m_h^2}{m_t^2}$$

Plus-distributions, radiation of very soft gluons (important near x=1)

Partonic cross-sections

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$$+\left(\frac{\alpha_s}{\pi}\right)^2\left(A^{(2)}\delta(1-x) + B_+^{(2)}(x) + C^{(2)}(x)\right)$$

$$x = \frac{m_h^2}{s}$$

$$s = (p_1 + p_2)^2$$

$$\rho = \frac{m_h^2}{m_t^2}$$

Hard real radiation, important for small x. Expressed in terms of HPLs

Partonic cross-sections

After adding all contributions, renormalization, cancellation of collinear singularities, etc:

$$\sigma_{gg} \sim A^{(0)}\delta(1-x) + \frac{\alpha_s}{\pi}\left(A^{(1)}\delta(1-x) + B_+^{(1)}(x) + C^{(1)}(x)\right)$$
$$+\left(\frac{\alpha_s}{\pi}\right)^2\left(A^{(2)}\delta(1-x) + B_+^{(2)}(x) + C^{(2)}(x)\right)$$

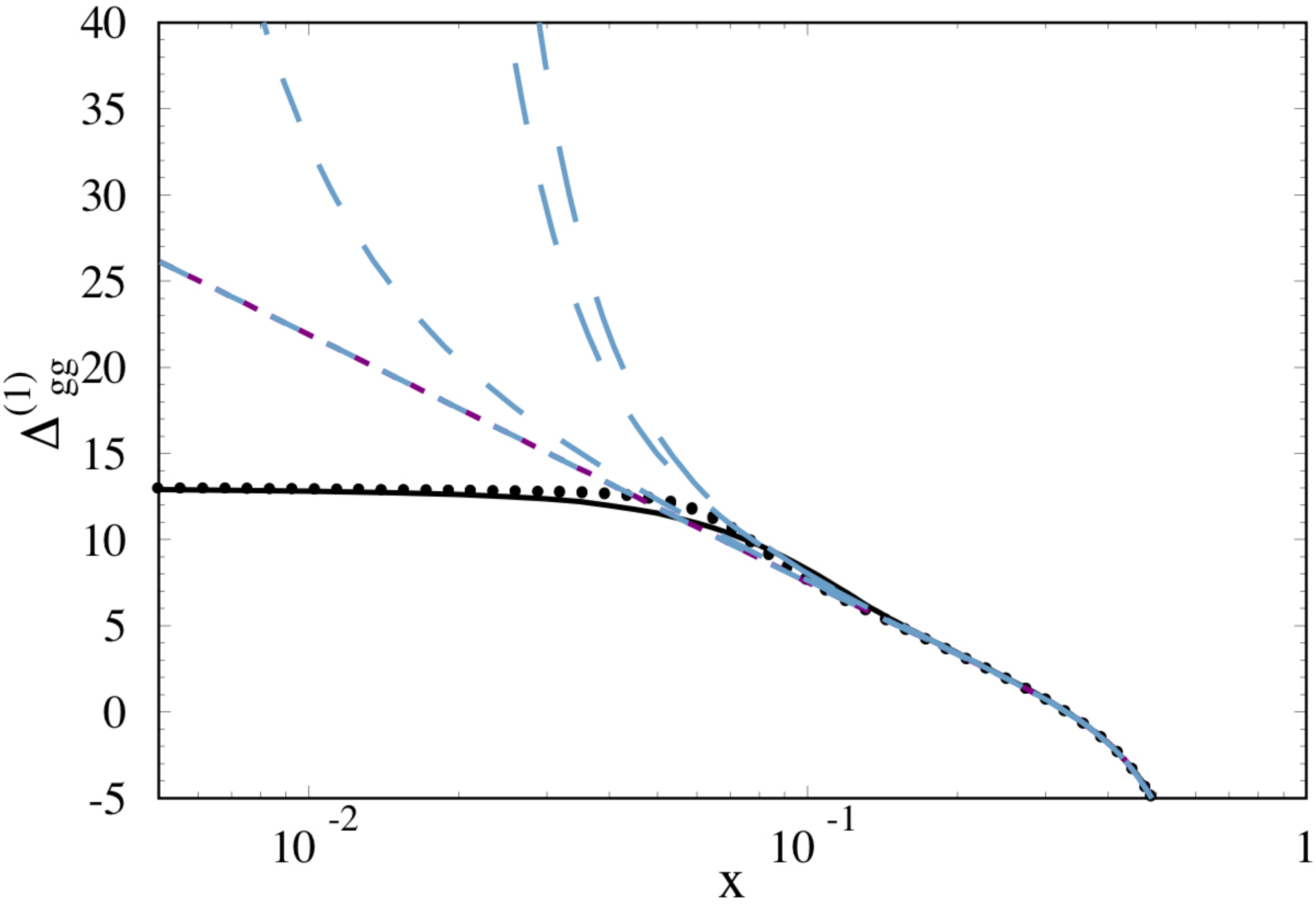
$$x = \frac{m_h^2}{s}$$
$$s = (p_1 + p_2)^2$$
$$\rho = \frac{m_h^2}{m_t^2}$$

Coefficients A, B, C – series in □

We obtained O($1/m_t^4$) terms for gg, and O($1/m_t^6$) terms for the quark channels.

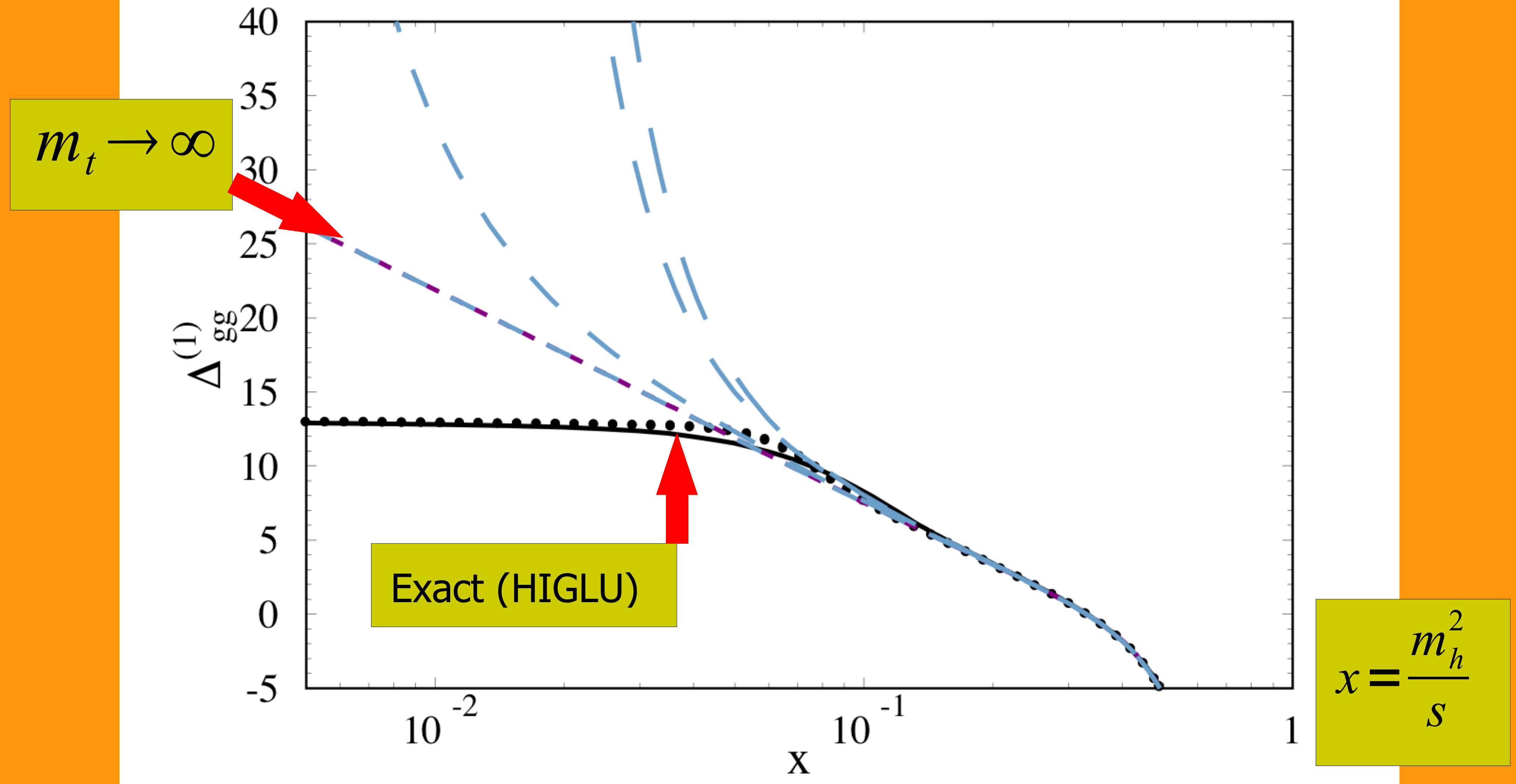
After expanding in (1-x), we find complete agreement with the result by Harlander and Ozeren!

NLO top mass effects, gg channel (non-singular)



$$x = \frac{m_h^2}{s}$$

NLO top mass effects, gg channel



NLO top mass effects, gg channel

Singularities due to:

$$\frac{s}{m_t^2} = \frac{m_h^2}{m_t^2} \cdot \frac{1}{x}$$

$m_t \to \infty$

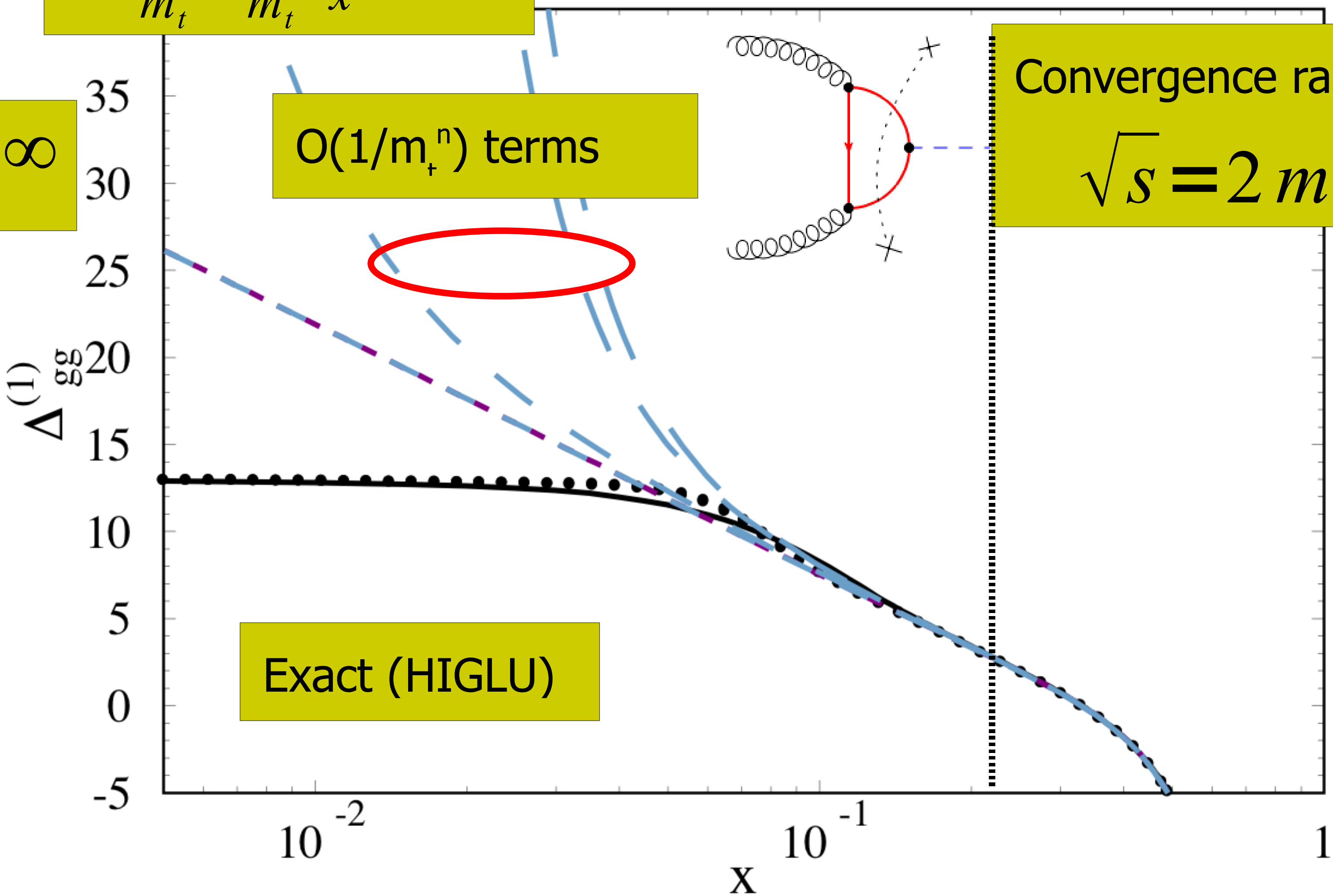
O(1/m_t^n) terms

Convergence radius

$$\sqrt{s} = 2\, m_t$$

Exact (HIGLU)

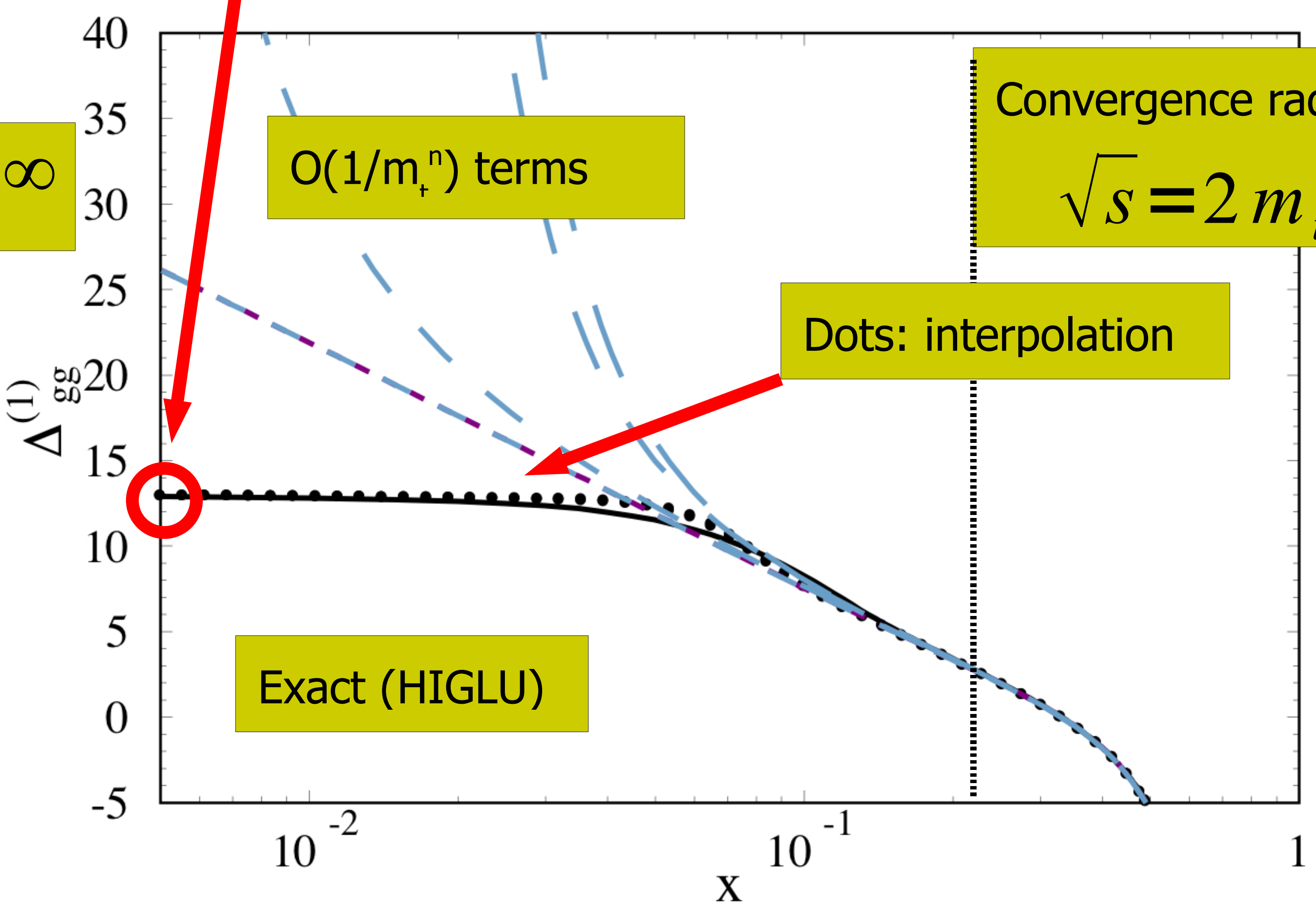
$$x = \frac{m_h^2}{s}$$



NLO top mass effects, gg channel

NLO asymptotics: limiting value
[Marzani, Ball, Del Duca, Forte, Vicini '08]

$m_t \to \infty$



O(1/m_t^n) terms

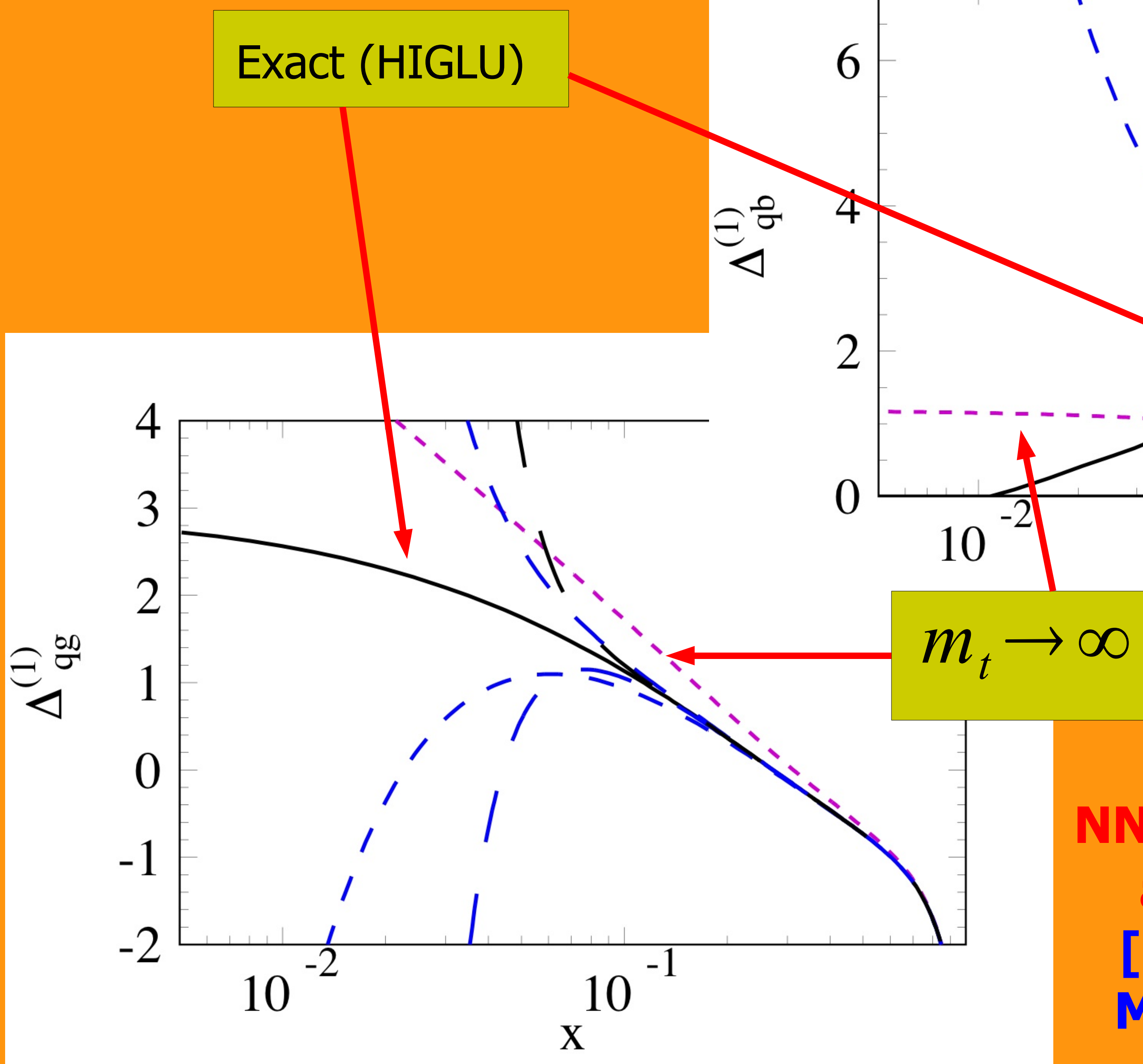
Convergence radius $\sqrt{s} = 2\,m_t$

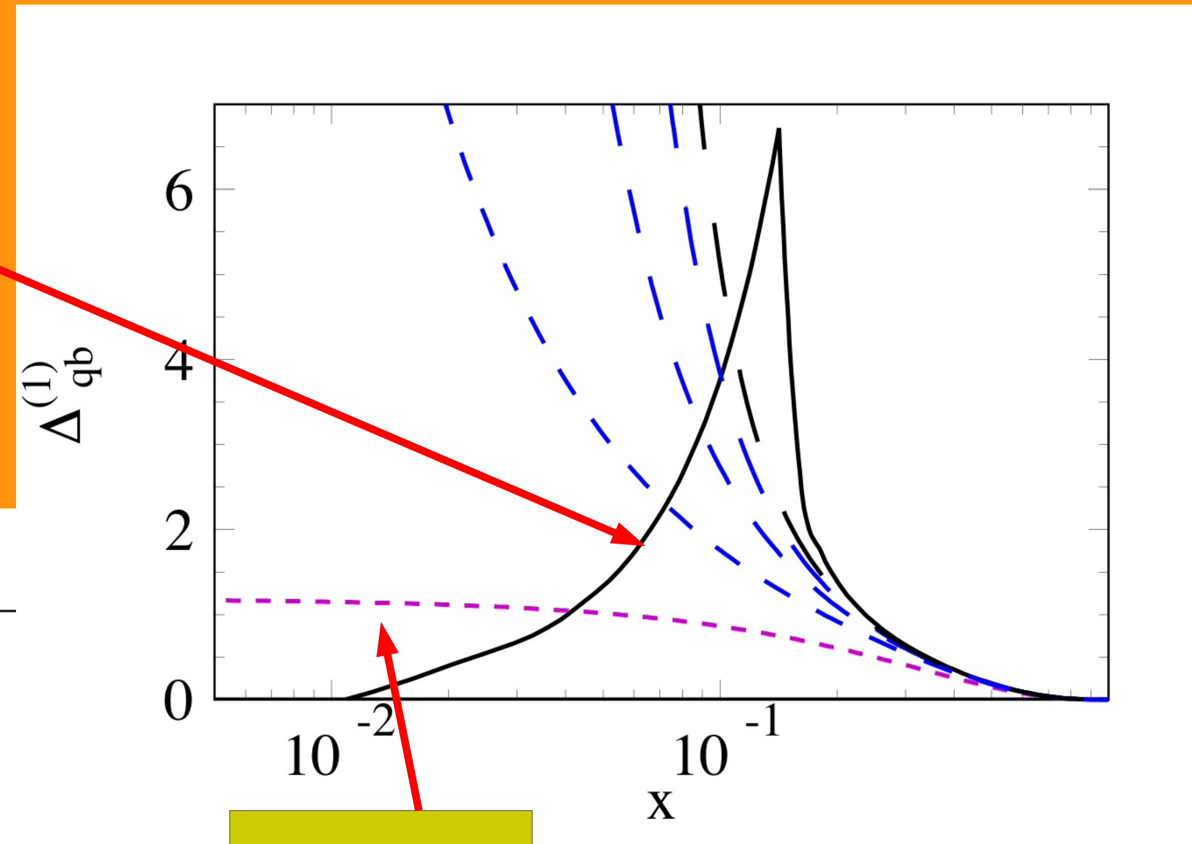
Dots: interpolation

Exact (HIGLU)

$x = \frac{m_h^2}{s}$

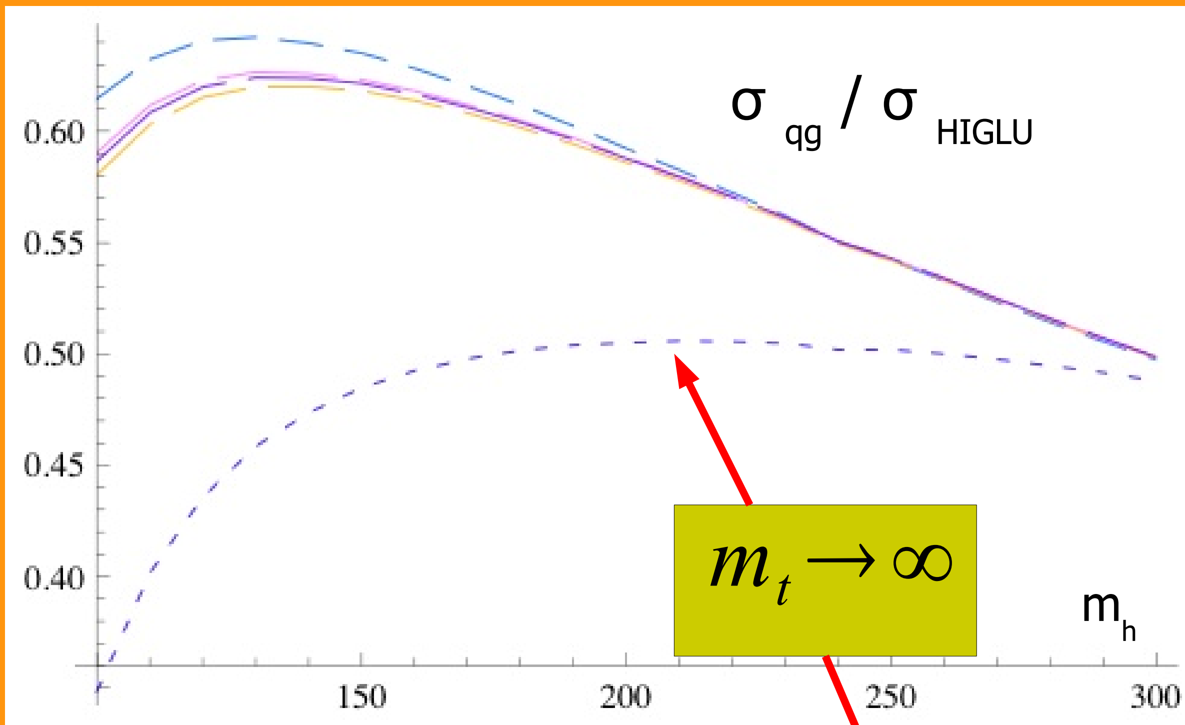
NLO top mass effects, qg and qq channels



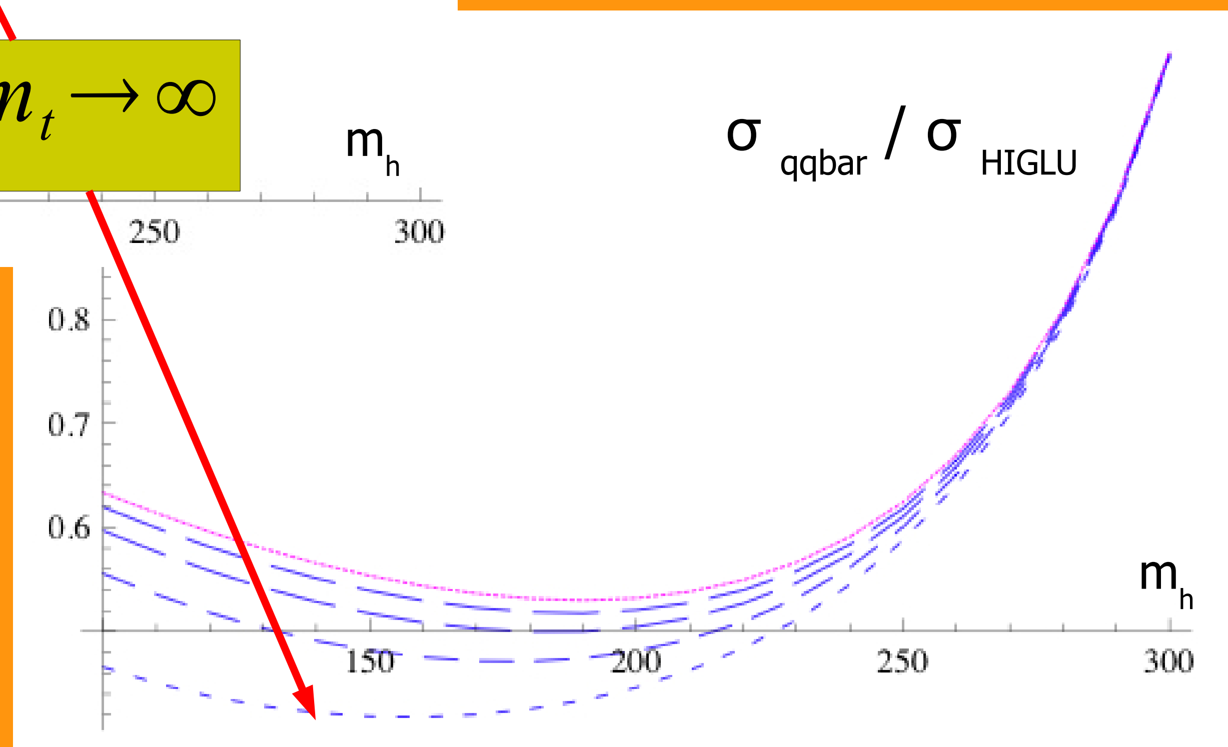


NNLO x→0 asymptotics appeared recently: [Harlander, Mantler, Marzani, Ozeren '09]

NLO qg and qqbar: hadronic study



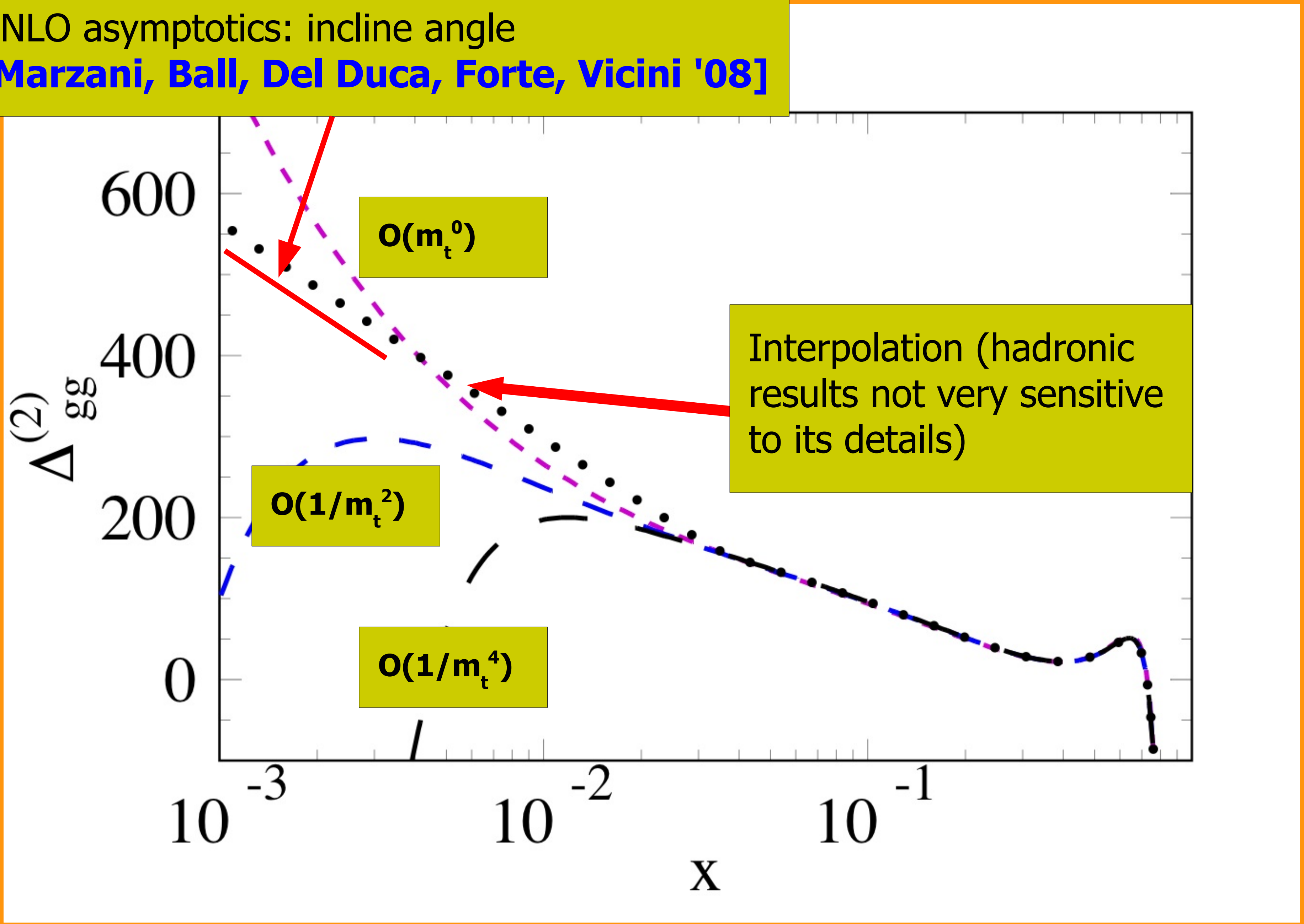
Poor-man's recipe:
use $1/m_t$ expansion below threshold, and heavy top limit above



Not particularly bad: O(50%) difference for subleading terms

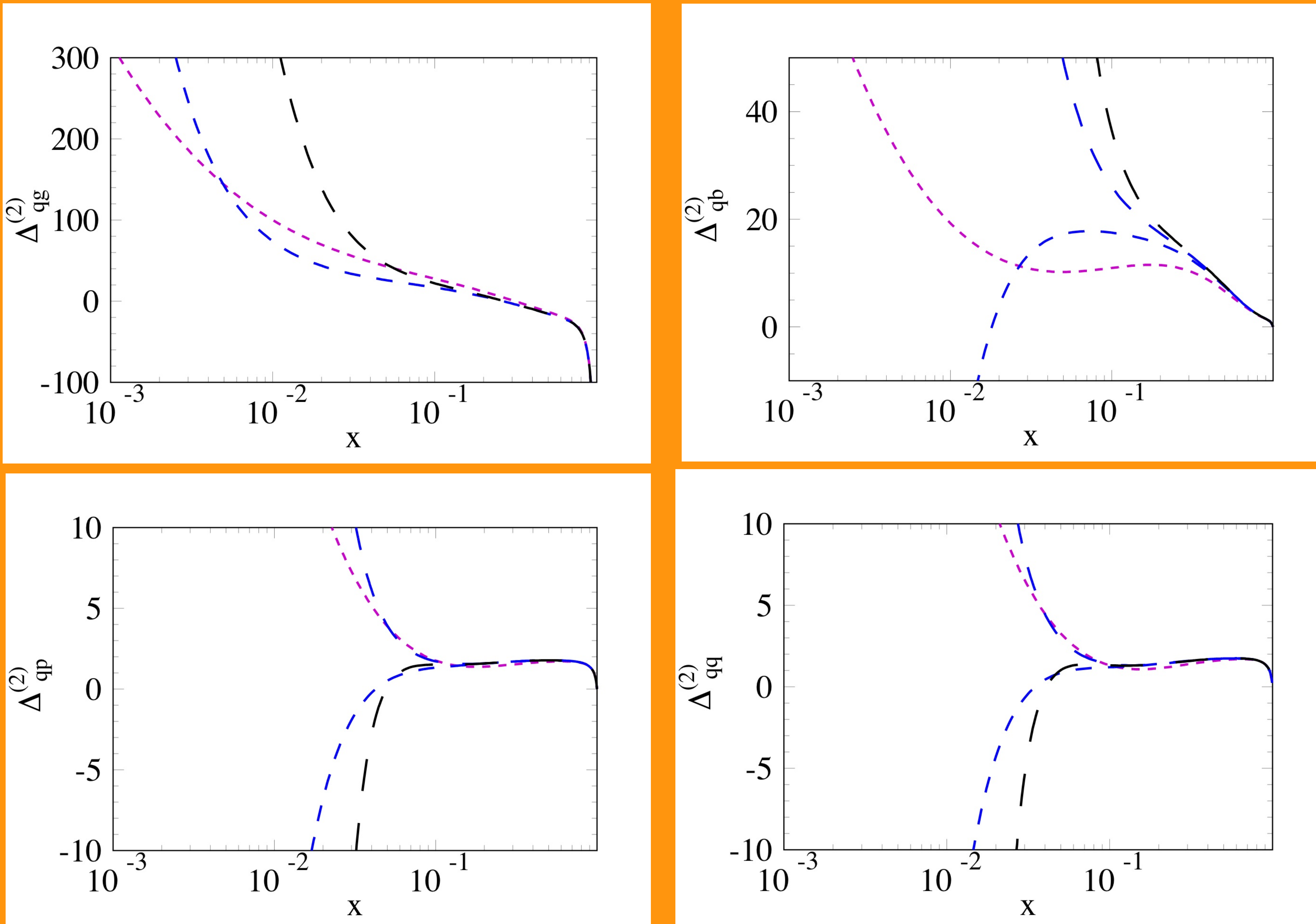
NNLO top mass effects, gg channel

NNLO asymptotics: incline angle
[Marzani, Ball, Del Duca, Forte, Vicini '08]

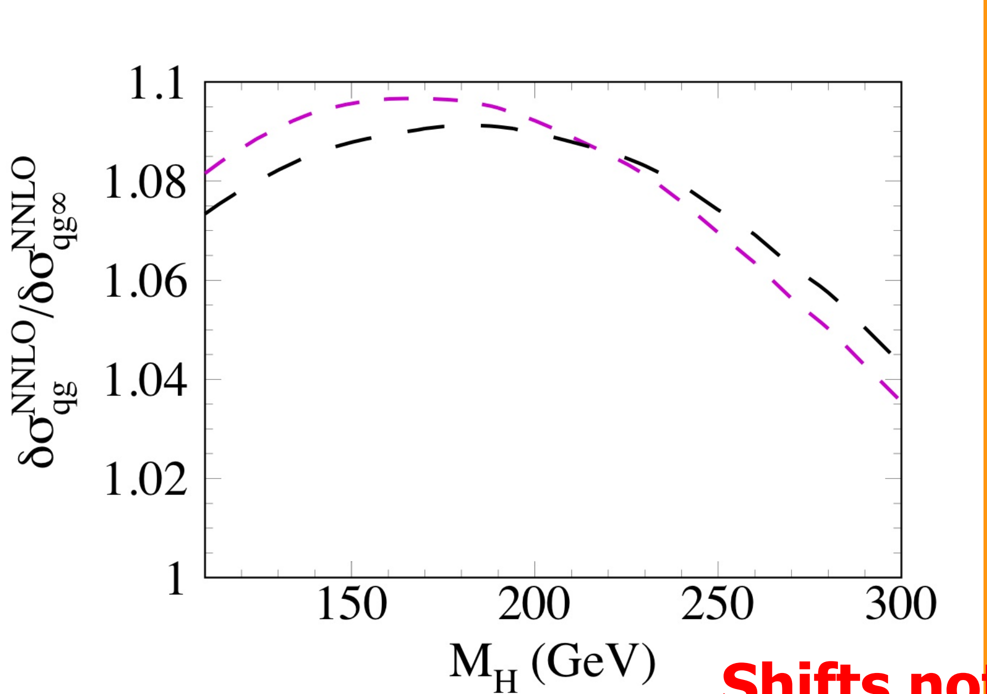


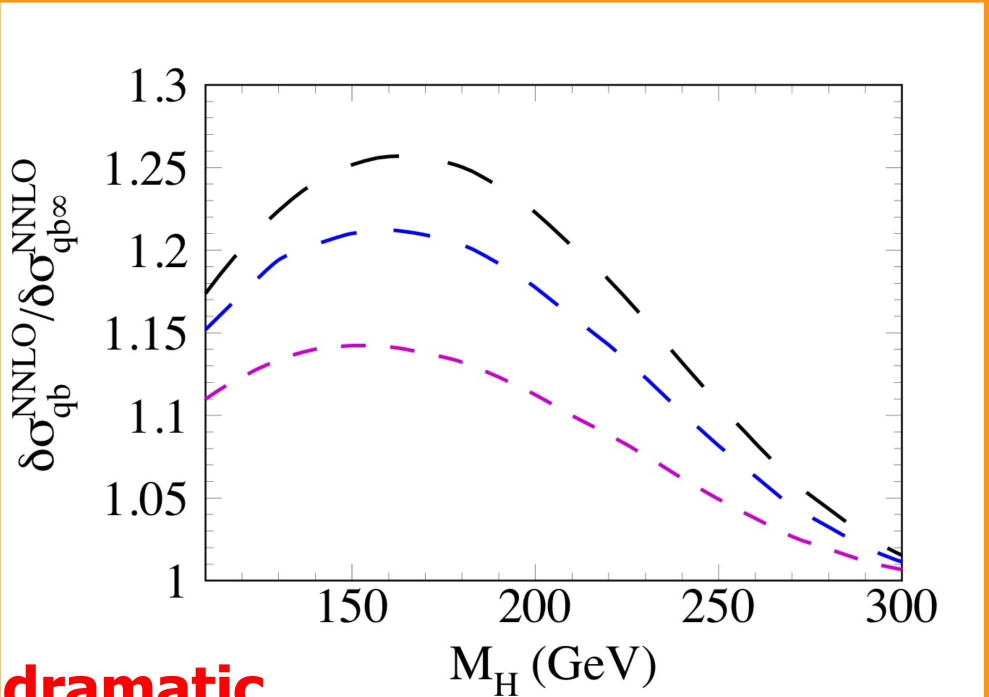
Interpolation (hadronic results not very sensitive to its details)

NNLO top mass effects, quark channels

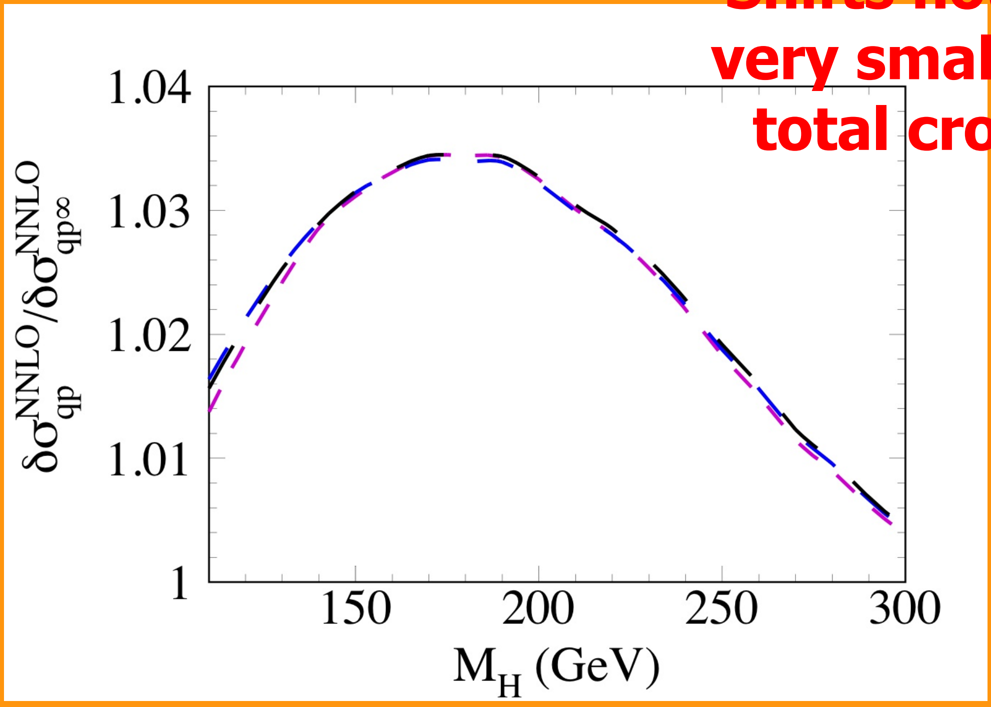


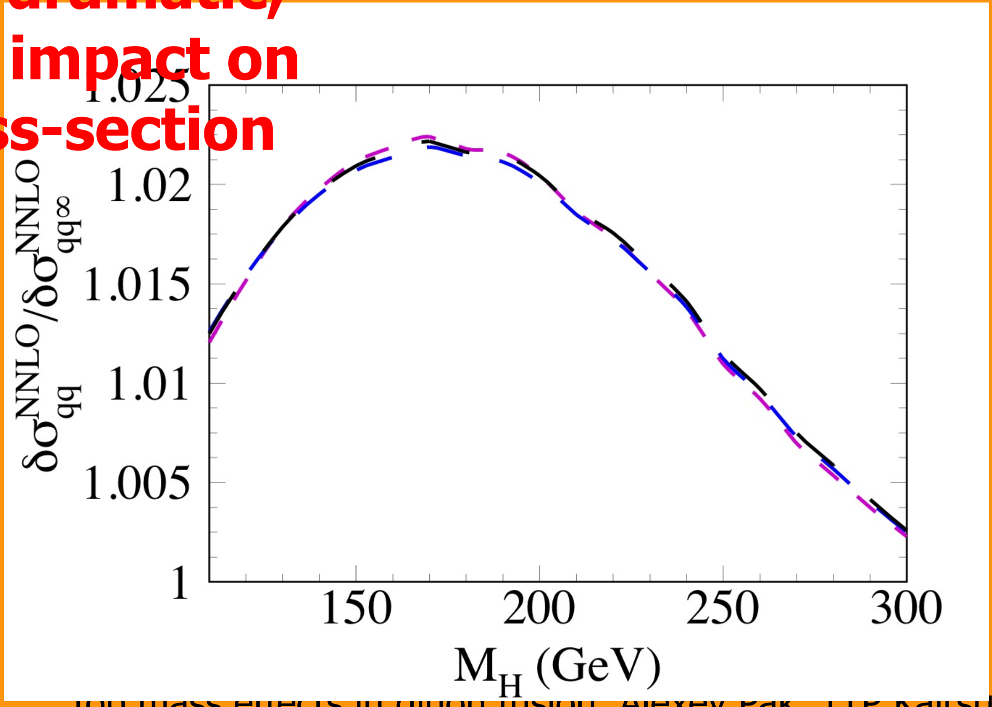
NNLO hadronic results, quark channels



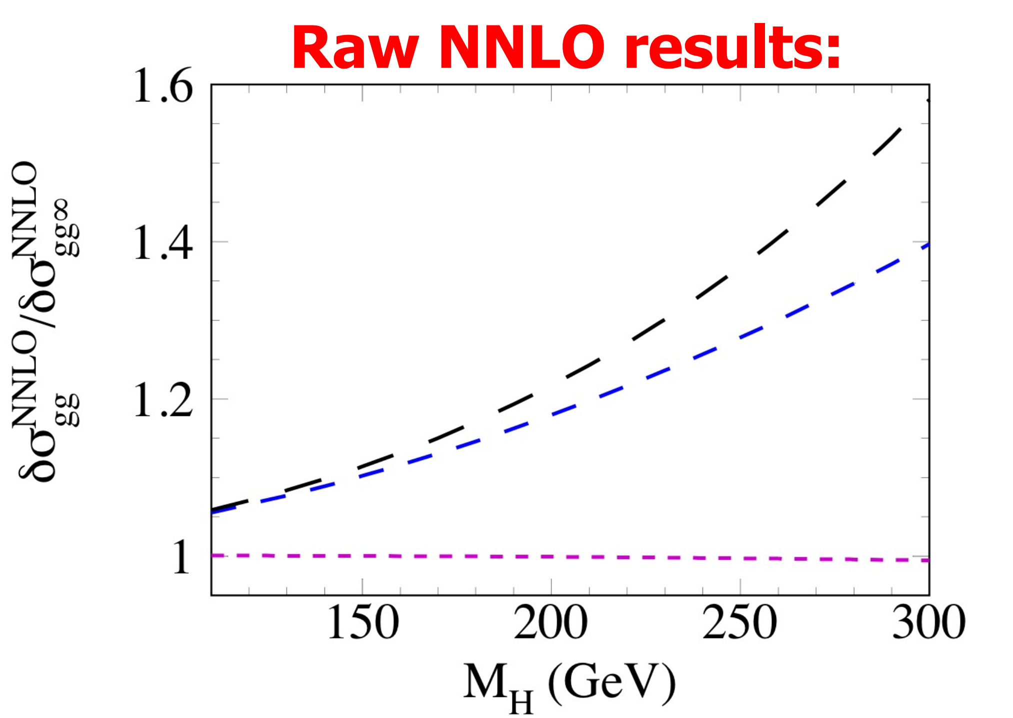


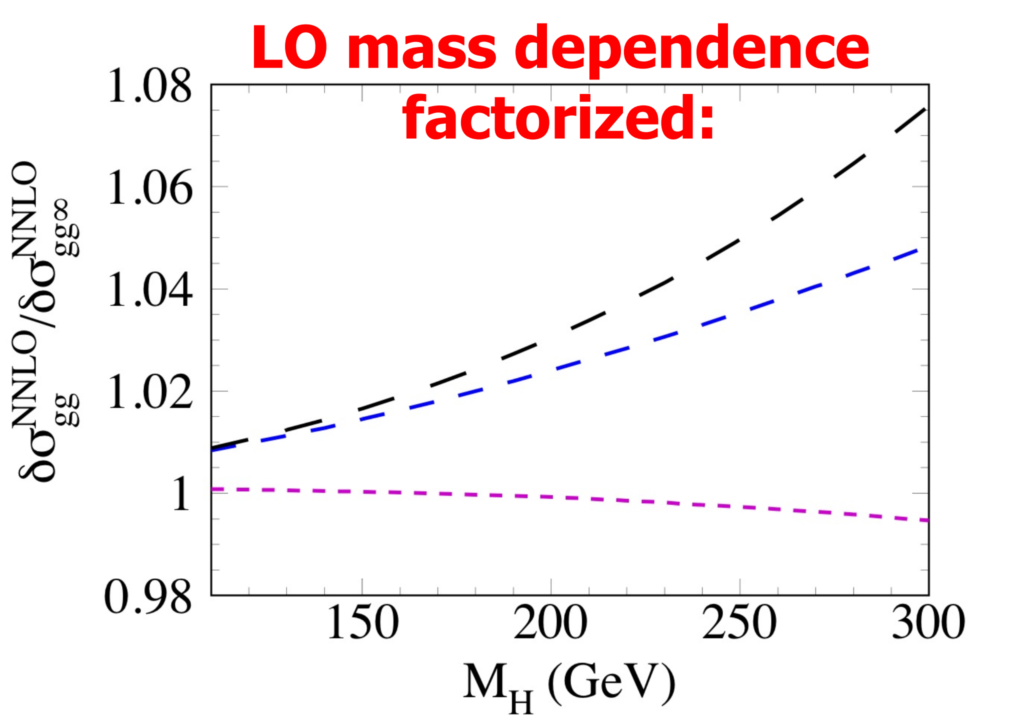
Shifts not dramatic, very small impact on total cross-section





NNLO hadronic results, gg channel

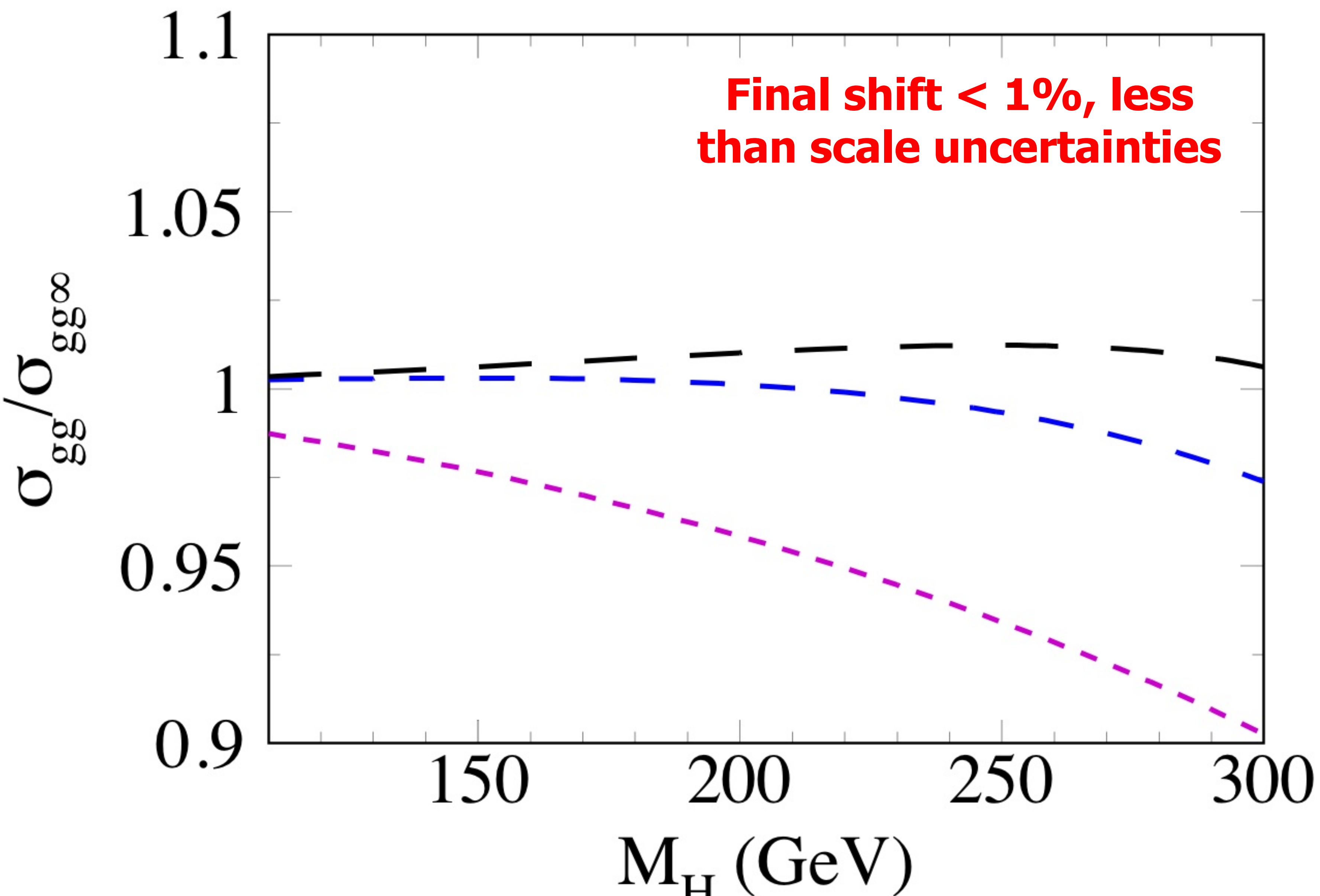




Common recipe:

$$\sigma^{NNLO}_{factorized} = \sigma^{LO}_{exact}(m_t)\left(\frac{\sigma^{NNLO}}{\sigma^{LO}}\right)_{O(1/m_t^n)}$$

NNLO hadronic results, total cross-section



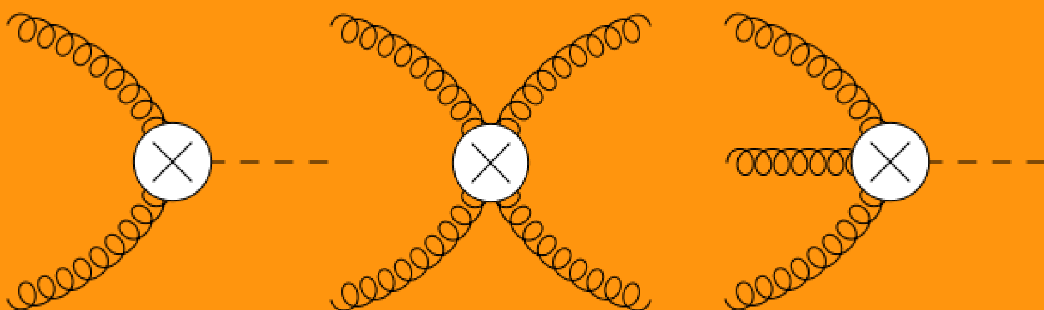
Summary

- Top mass corrections (expanded in $1/m_t$) to Higgs production have been found exactly in x, results by Harlander and Ozeren confirmed
- Shift of hadronic results smaller than scale uncertainties (a non-trivial result!)
- **Verdict: heavy top approximation is justified**

Thank you for your attention!

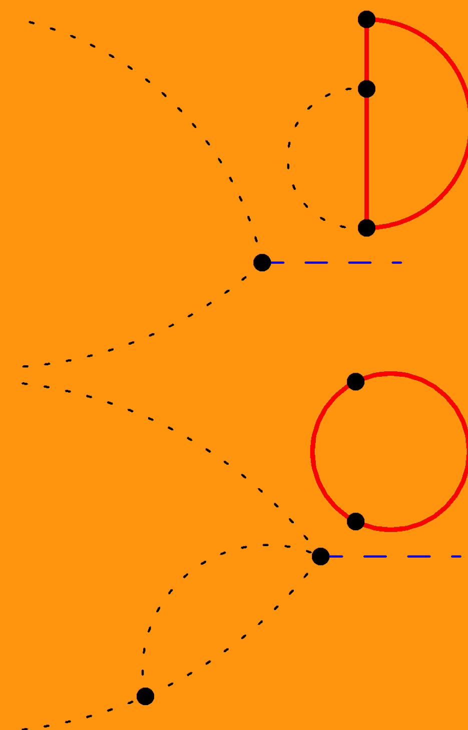
Effective theory vs asymptotic expansion

EFT:
- simpler leading order
- $O(1/m_t)$ corrections: complex power counting, new operators and Feynman rules



Requires Wilson coefficients, renormalizations,...

AE:
- many integrals
- algebraic complexity
- same topologies at every order in $1/m_t$



Can be efficiently automated: same program for any order of AE

Calculation techniques

Asymptotic expansion generates many integrals with various powers of denominator/numerator factors

Integration-by-parts identities:

$$I(a_1, \ldots, a_n; D) = \int \frac{d^D k_1 d^D k_2 \ldots}{D_1^{a_1} D_2^{a_2} \ldots D_n^{a_n}}$$

Consider integral of full D-dimensional divergence:

$$0 = \int d^D k_1 d^D k_2 \ldots \left(\frac{d}{d k_i^\mu} k_j^\mu \right) \frac{1}{D_1^{a_1} D_2^{a_2} \ldots D_n^{a_n}}$$
$$= c_1 I(\ldots, a_l + 1, \ldots) + c_2 I(\ldots, a_m - 1, \ldots) + \ldots$$

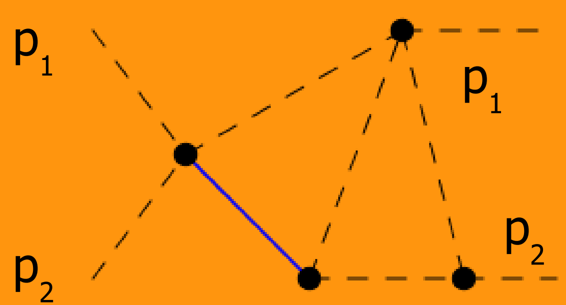
Linear relations between integrals!
Reduce 100000's of integrals in the problem to ~10 **master integrals** (very CPU-intensive step)

Generation and solution of the linear system can be automated (requires fast computer algebra)

Calculation techniques

Master integrals: published but required cross-checks (many typos).
Method of differential equations (DE):

$$U(x,D) =$$



$$x = \frac{m_h^2}{s}, \quad s = (p_1 + p_2)^2$$

Use IBP's to obtain: $\frac{d}{dx} U(x,D) = A(x,D) \cdot U(x,D) + (\text{simpler integrals})$

Solve with Euler method order-by-order in ε, use soft expansion (x = 1 limit) to fix integration constants

Results in terms of Harmonic Polylogarithms - a [relatively] new class of special functions, especially convenient for automated computations. Functions valid for all values of x

$$\int_0^y x^a (1-x)^b (1+x)^c H(1,0,-1,\ldots,x) = y^d (1-y)^e (1+y)^f H(\ldots,y) + \ldots$$

Restoring plus-distributions from HPLs

- DE solution to O(ε^n): $$U = x^n (1-x)^m (1+x)^k H(\ldots, x) + \ldots$$
- Can easily be divergent at x=1, need to restore delta- and plus-pieces
- Soft expansion to O(ε^{n+1}), leading term only: $$C(\epsilon)(1-x)^{k-a\epsilon}$$
- Ansatz:

$$U \to U + C(\epsilon)(1-x)^{k+1}\left[(1-x)^{-1-a\epsilon} - (1-x)^{-1-a\epsilon}\right]$$

Expand in distributions:

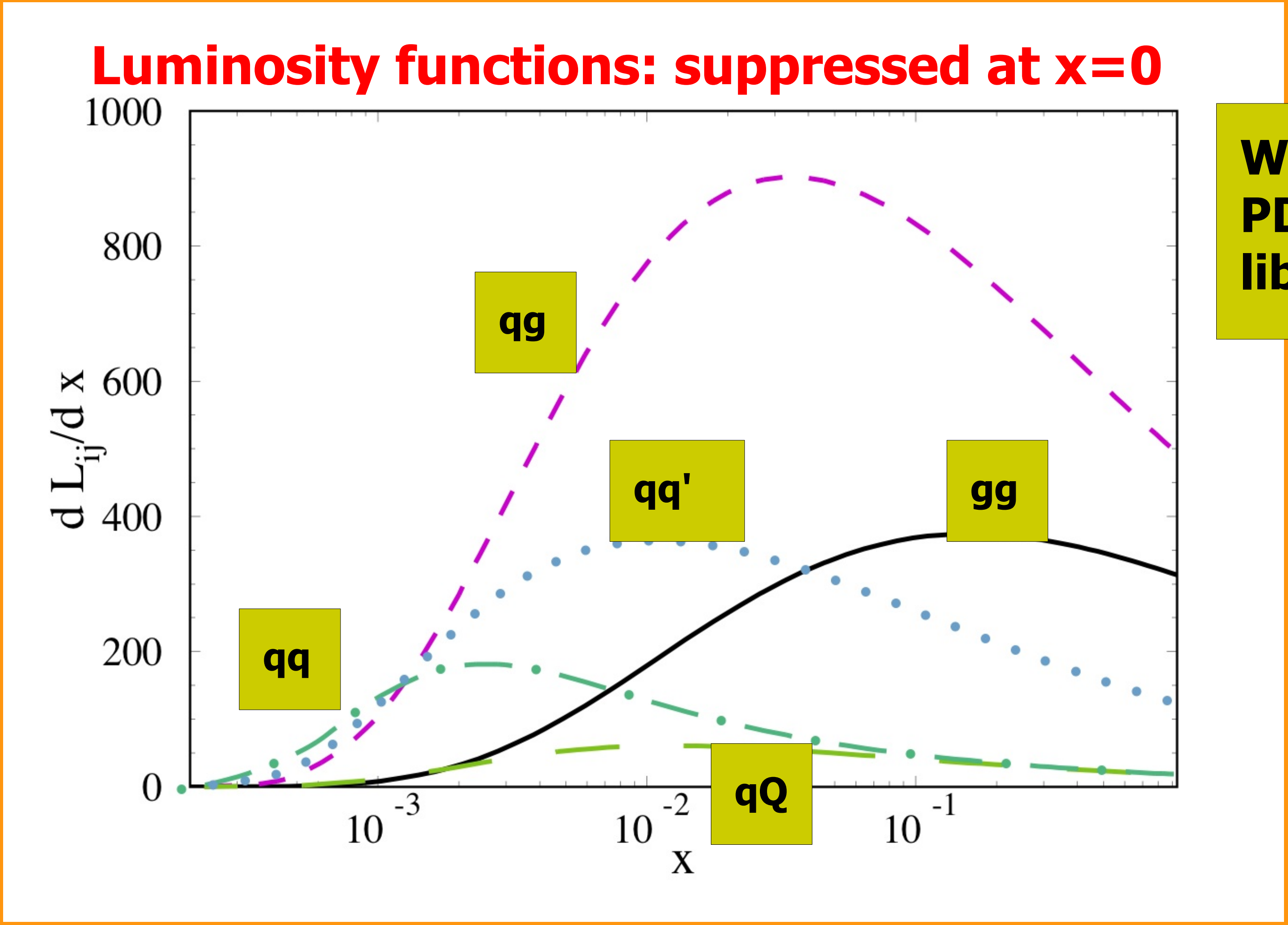
$$\frac{1}{y^{1+a\epsilon}} = \frac{\delta(y)}{a\epsilon} + \left[\frac{1}{y}\right]_+ + \ldots$$

Expand "naively":

$$\frac{1}{y^{1+a\epsilon}} = \frac{1}{y} - \frac{a\ln y}{y} + \ldots$$

- cancels singularities in HPLs

From partonic to hadronic cross-sections



We use MSTW2008 PDFs from LHAPDF library

$$\sigma_{pp\to H+X}=\sum_{ij=gg,\dots}\int_{m_h^2/s}^{1} dx\left[\frac{d\,L_{ij}}{dx}\right](x)\,\sigma_{ij\to H+X}(x)$$