Anomalous dimensions from S-matrix

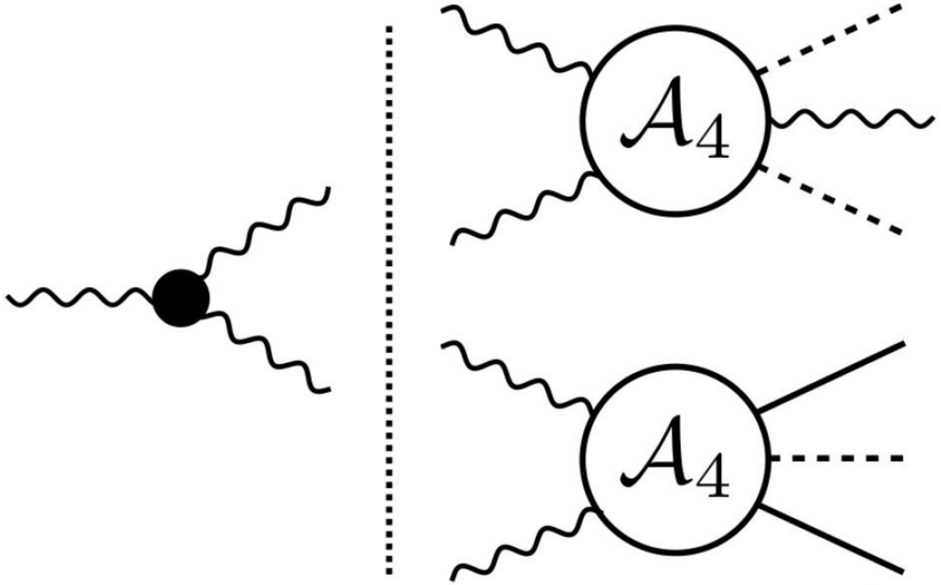
DESY Theory Journal Club 2020

15 October 2020

Papers to discuss:

Elias Miró, Ingoldby, Riembau. arXiv 2005.06983 (JHEP 09 (2020) 163)

Baratella, Fernandez, Pomarol. arXiv 2005.07129 (Nucl. Phys. B (2020) 115155)

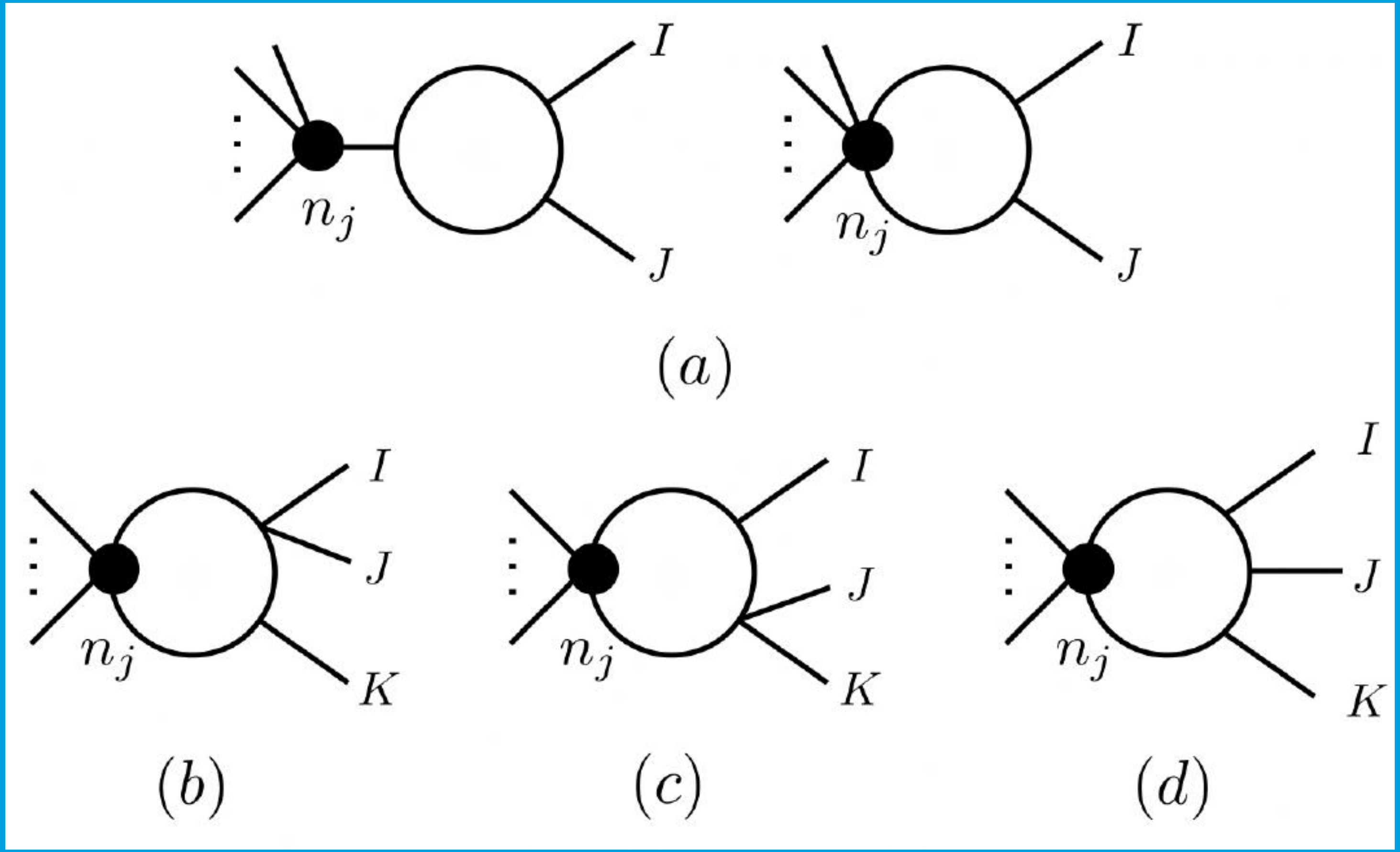


Alejo N. Rossia

HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

Introduction and formalism

Based on: *Elias Miró et al. (EMIR)* 2005.06983



What do they study/compute?

- RG running of SMEFT operators. → Anomalous dimensions

Dominated by

(Peskin, Schroeder)

$$\left[M\frac{\partial}{\partial M}+\beta(\lambda)\frac{\partial}{\partial\lambda}+n\gamma(\lambda)\right]G^{(n)}(\{x_i\};M,\lambda)=0. \tag{12.41}$$

- Can we compute those coefficients more efficiently with S-matrix techniques?
- Does massless spinor helicity formalism help?
- How?

Massless spinor helicity formalism

- **When computing UV anomalous dimensions, all SM can be taken as massless**

On-shell massless particle with spin *s*.

pair of spinors $\{\lambda_a, \tilde{\lambda}_{\dot{a}}\}$ $\quad SL(2,\mathbb{C})$

$(1/2, 0) \qquad (0, 1/2)$

$\in \mathbb{C}^{2\times 1}$

helicity $h = \pm|s|$

Constrains the power of the spinors in an amplitude

Invariants and other relations

$$(\lambda_1)_a(\lambda_2)_b\epsilon^{ab} \equiv \langle 12\rangle \quad \text{and} \quad (\tilde{\lambda}_1)_a(\tilde{\lambda}_2)_b\epsilon^{ab} \equiv [12]\,, \tag{2.2}$$

$$(p_i + p_j)^2 = \langle ij\rangle\,[ji] \equiv s_{ij} \qquad p^\mu = \lambda^a(\sigma^\mu)_{a\dot{a}}\tilde{\lambda}^{\dot{a}}$$

Form factors

- **Form factors:**

$$F_O(\vec{n}) \equiv {}_{\text{out}}\langle \vec{n}|\, O(0)\, |0\rangle \ , \tag{1.1}$$

- **Callan-Symanzik Eq. for FFs:**

$$\left(\mu\partial_\mu + \gamma - \gamma_{\text{IR}} + \cancel{\beta_g\partial_g}\right)\ F_O(\vec{n};\mu) = 0\,,$$

$$\gamma_i \equiv \frac{dC_{\mathcal{O}_i}}{d\ln\mu} = \sum_j \gamma_{ij}\, C_{\mathcal{O}_j}$$

- **Dilatation operator and fundamental relation:**

Dim.Reg.

$$D \equiv \sum_{\text{all particles}} p_i \frac{\partial}{\partial p_i}$$

$$DF_O \approx -\mu\partial_\mu F_O^{(1)} = (\gamma - \gamma_{\text{IR}} + \beta_g\partial_g)^{(1)} F_O^{(0)}$$

$$e^{-i\pi D}\, F_O^*(\vec{n}) = \sum_{\vec{m}} S_{nm}\, F_O^*(\vec{m})\,.$$

(Caron-Huot and Wilhelm, 1607.06448)

Master formula

$$\langle \vec{n}|\, O_j \,|0\rangle^{(0)} \left(\gamma_{ji} - \gamma^{i}_{\mathrm{IR}}\delta_{ij}\right)^{(1)} = -\frac{1}{\pi}\, \langle \vec{n}|\, \mathcal{M} \otimes O_i \,|0\rangle^{(0)}$$

(Caron-Huot and Wilhelm, 1607.06448)

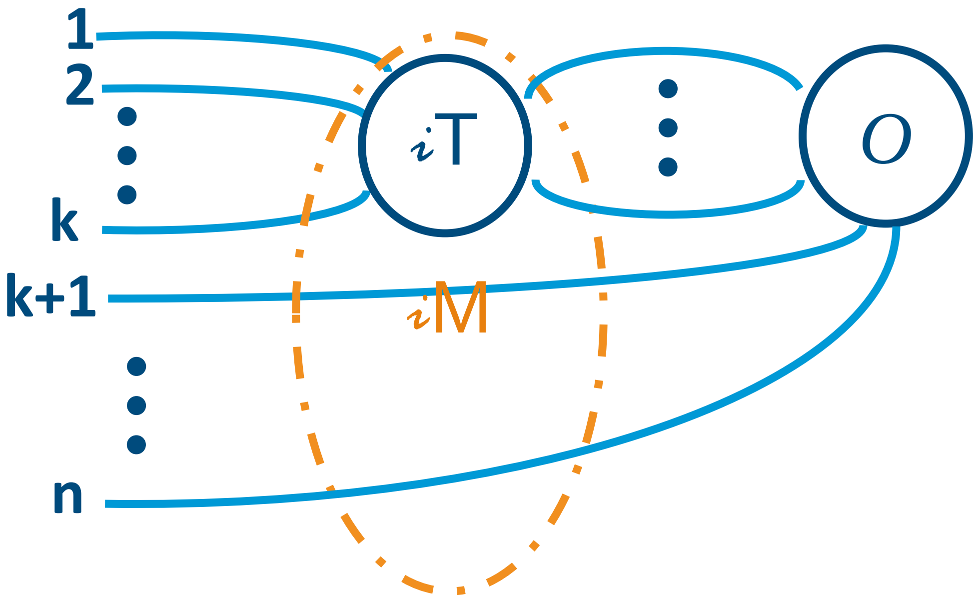
Minimal Form Factor

$$\sum_{\vec{m}} |\vec{m}\rangle_{\mathrm{in}}\ {}_{\mathrm{in}}\langle \vec{m}|$$

$$S = \mathbb{1} + i\mathcal{M}$$

Not fully connected

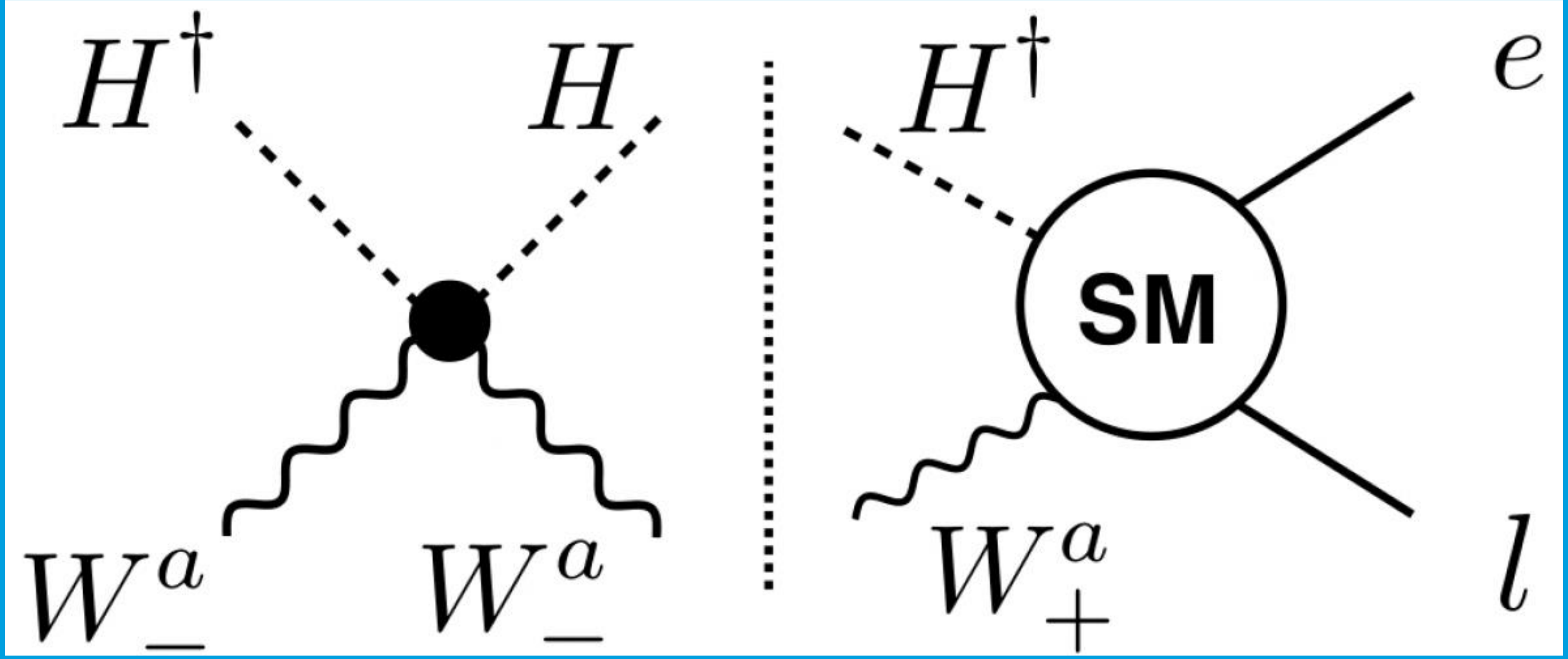
Tree-level SM amp.



Strategy: Fix the operator on the right, compute, identify the blocks of the result with other MFFs, get the anomalous dimensions.

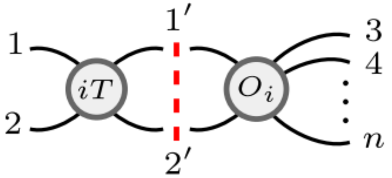
Computing at 1 loop

Based on: *Elias Miró et al.* 2005.06983



Simplified master formula

$$\langle \vec{n}|\, O_j\, |0\rangle^{(0)} \left(\gamma_{ji} - \gamma^{i}_{\mathrm{IR}}\delta_{ij}\right)^{(1)} = -\frac{1}{\pi}\, \langle \vec{n}|\, \mathcal{M} \otimes O_i\, |0\rangle^{(0)}$$



$$= -\frac{1}{2\pi} \int \mathrm{dLI}' \,(2\pi)^4 \delta^{(4)}\left(p_1 + p_2 - p_1' - p_2'\right) M(12;1'2')\, F_{O_i}(1'2'3\ldots n)$$

2→2 scat.: operators with same amount of legs

Rotation trick:

$$p_1 + p_2 = p_1' + p_2' \text{ throughout the whole phase-space integration}$$

$$\begin{pmatrix} \lambda_1' \\ \lambda_2' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta e^{i\phi} \\ \sin\theta e^{-i\phi} & \cos\theta \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix}$$

$$F_{O_j}(12\ldots n)\left(\gamma_{ji} - \gamma^{i}_{\mathrm{IR}}\delta_{ij}\right) = -\frac{1}{16\pi^2}\int_0^{2\pi}\frac{d\phi}{2\pi}\int_0^{\pi/2} 2\sin\theta\cos\theta d\theta\, M(1,2;1'2')\, F_{O_i}(1'2'3\ldots n)$$

Example without IR divergencies

Operators: $O_{\|} = |H|^2 |D^\mu H|^2$ $\qquad$ $O_\perp = |H^\dagger D_\mu H|^2$

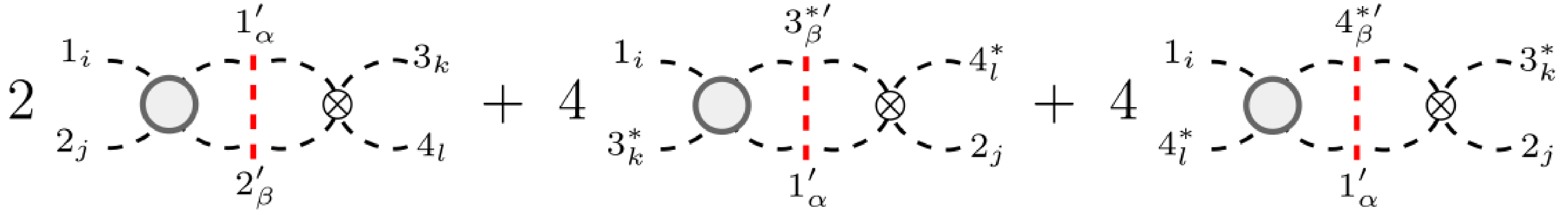
$$F_{\|}(1_i 2_j 3_k^* 4_l^*) = \delta_i^l \delta_j^k \ \langle 14\rangle [14] + \delta_j^l \delta_i^k \ \langle 13\rangle [13]$$

$$F_{\perp}(1_i 2_j 3_k^* 4_l^*) = \delta_i^l \delta_j^k \ \langle 13\rangle [13] + \delta_j^l \delta_i^k \ \langle 14\rangle [14]$$

Matrix element:

$$M(1_i 2_j 3_k^* 4_l^*) = -2\lambda \left(\delta_i^k \delta_j^l + \delta_j^k \delta_i^l\right)$$

Different channels:



For $F_\perp$

$$-\frac{1}{16\pi^2}\int d\Omega_2 \left[-2\lambda(\delta_i^\alpha \delta_j^\beta + \delta_j^\alpha \delta_i^\beta)\right]\left[\delta_\alpha^l \delta_\beta^k \ \langle 1'3\rangle [1'3] + \delta_\beta^l \delta_\alpha^k \ \langle 1'4\rangle [1'4]\right] = \frac{2\lambda}{16\pi^2}\left[\delta_i^l \delta_j^k + \delta_j^l \delta_i^k\right]\left[\langle 13\rangle [13] + \langle 14\rangle [14]\right]$$

Example without IR divergencies

Final result :

$$F_{O_j}(12\ldots n)\,(\gamma_{ji}-\gamma_{\rm IR}^{i}\delta_{ij}) = -\frac{1}{16\pi^2}\int_0^{2\pi}\frac{d\phi}{2\pi}\int_0^{\pi/2} 2\sin\theta\cos\theta d\theta\, M(1,2;1'2')\, F_\perp(1_i 2_j 3_k^* 4_l^*)$$

$$F_{O_j}(12\ldots n)\,(\gamma_{ji}-\gamma_{\rm IR}^{i}\delta_{ij}) = \frac{4\lambda}{16\pi^2}\,[\,3\,F_\perp(1_i 2_j 3_k^* 4_l^*) + (2-N)\,F_\parallel(1_i 2_j 3_k^* 4_l^*)\,]$$

IR divergencies?

Extracted from literature

$$\gamma_{\rm IR}(s_{ij};\mu) = \sum_{i<j} T_i^A\cdot T_j^A\,\gamma_{\rm cusp}\,\log\frac{\mu^2}{-s_{ij}} + \sum_i \gamma_i^{\rm coll}\,, \qquad (2.24)$$

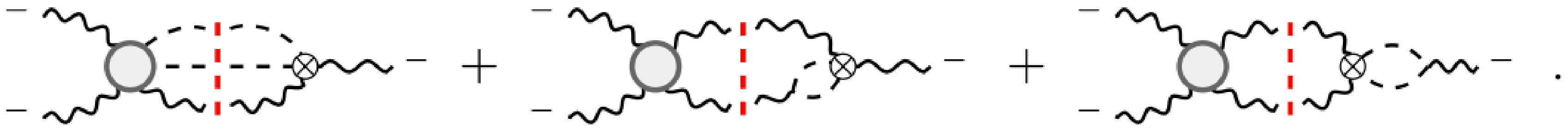
$$\gamma_{ji}F_{O_j} = -\frac{1}{16\pi^2}\int d\Omega_2\left(M(12;1'2')F_{O_i}(1'2'3\ldots n) + \frac{2g^2T_1^AT_2^AF_{O_i}(123\ldots n)}{\sin^2\theta\cos^2\theta}\right) + F_{O_i}\sum_k\gamma_k^{\rm coll}\,,$$

Example: gauge mixing for the operators shown before.

$$M(1_i 2_j 3_k^* 4_l^*) = g'^2Y_H^2\,\delta_i^k\delta_j^l\left(1-2\frac{\langle 12\rangle\langle 34\rangle}{\langle 13\rangle\langle 24\rangle}\right) + g'^2Y_H^2\,\delta_i^l\delta_j^k\left(1-2\frac{\langle 12\rangle\langle 34\rangle}{\langle 23\rangle\langle 41\rangle}\right)$$

“Easy” 2-loop computations

Based on: *Elias Miró et al.* 2005.06983



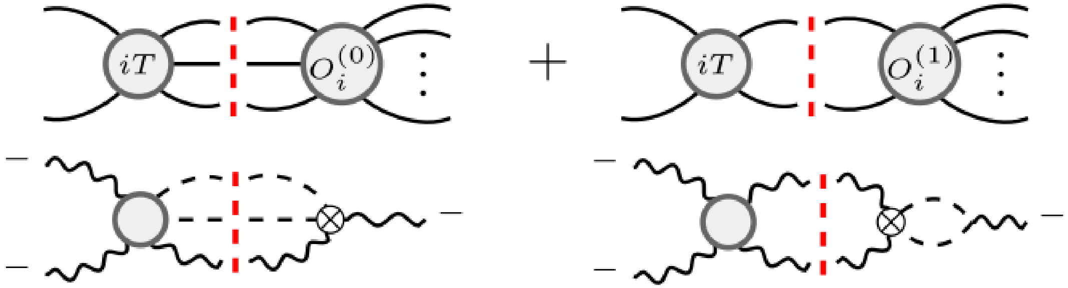
(4.4)

Why “easy”?

- No 1-loop contribution, i.e. the anomalous dimension still corresponds to the leading single logarithm.

$$DF_O \approx -\mu\partial_\mu F_O^{(1)} = (\gamma - \gamma_{\mathrm{IR}} + \beta_g \partial_g)^{(1)} F_O^{(0)}$$

- With the following general form:

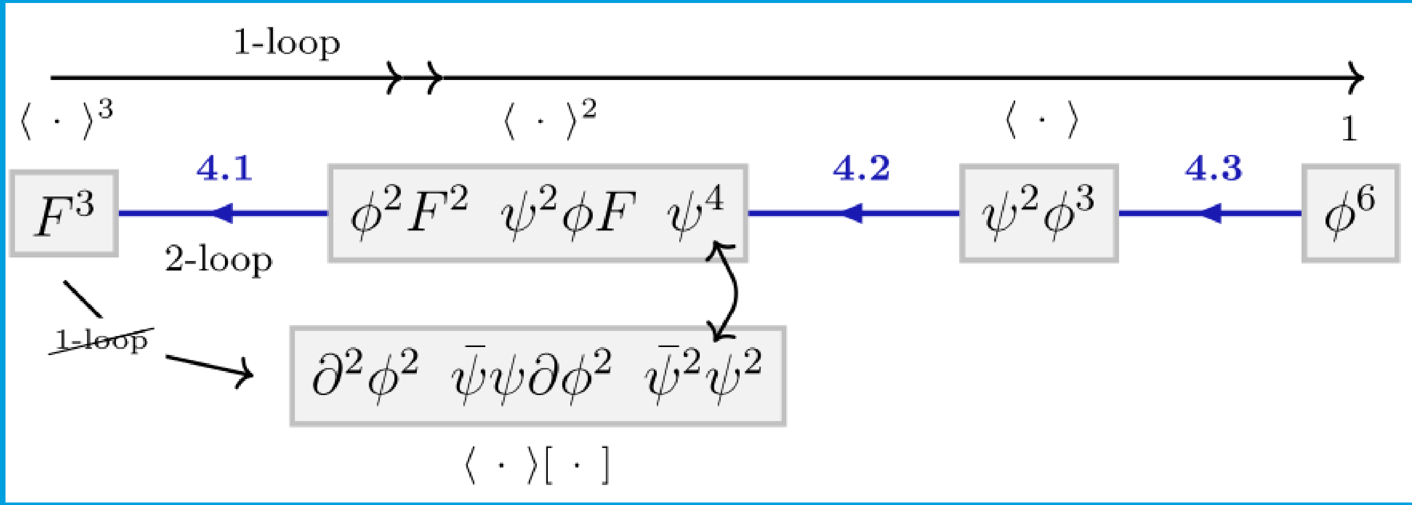


- Where is the extra difficulty?

$$\frac{\langle 12\rangle[12]}{(16\pi^2)^2 3!}\int d\Omega_3\, M(12;1'2'3')\, F_{O_i}(1'2'3'4\ldots n) \qquad \begin{pmatrix}\lambda_1'\\ \lambda_2'\\ \lambda_3'\end{pmatrix} = \begin{pmatrix}\cos\theta_1 & -e^{i\phi}\cos\theta_3\sin\theta_1\\ \cos\theta_2\sin\theta_1 & e^{i\phi}\left(\cos\theta_1\cos\theta_2\cos\theta_3 - e^{i\delta}\sin\theta_2\sin\theta_3\right)\\ \sin\theta_1\sin\theta_2 & e^{i\phi}\left(\cos\theta_1\cos\theta_3\sin\theta_2 + e^{i\delta}\cos\theta_2\sin\theta_3\right)\end{pmatrix}\begin{pmatrix}\lambda_1\\ \lambda_2\end{pmatrix}$$

Difficulty doesn’t escalate so much with loops

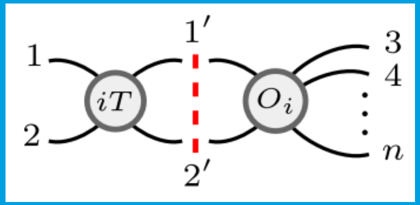
γ Matrix structure



$$\langle \vec{n}|\, O_j \,|0\rangle^{(0)} \left(\gamma_{ji} - \gamma^{i}_{\mathrm{IR}}\delta_{ij}\right)^{(1)} = -\frac{1}{\pi}\, \langle \vec{n}|\, \mathcal{M} \otimes O_i \,|0\rangle^{(0)}$$

At 1 loop:

This operator has equal or more legs than this



Based on: *Elias Miró et al.* 2005.06983

What did the other group do?

Basics of *Baratella et al.* 2005.07129 and comparison with *Elias Miró et al.*

A philosophical difference

Baratella et al. 2005.07129

- On-shell amplitudes based, no operators nor Lagrangian.
- Formula based in amplitudes.

Elias Miró et al. 2005.06983

- Lagrangian and high dimensional operator based.
- Formula based in form factors.

But very similar!

$$\gamma_{ij}\,\mathcal{A}_{\mathcal{O}_i}(1,2,\ldots) = -\frac{1}{4\pi^3}\frac{C_{\mathcal{O}_i}}{C_{\mathcal{O}_j}}\int d\text{LIPS}\sum_{\ell_1,\ell_2}\widehat{\mathcal{A}}_{\mathcal{O}_j}(1,2,\ldots,\ell_1,\ell_2)\times\mathcal{A}_4(-\ell_1,-\ell_2,\ldots)\,.$$

$$\langle\vec{n}|\,O_j\,|0\rangle^{(0)}\left(\gamma_{ji}-\gamma^{i}_{\text{IR}}\delta_{ij}\right)^{(1)} = -\frac{1}{\pi}\,\langle\vec{n}|\,\mathcal{M}\otimes O_i\,|0\rangle^{(0)}$$

- Base of amplitudes at order E^2/Λ^2
- Restricted to cases without IR div.
- Cancellations among triangles, boxes…

- Dim. 6 operators
- Take IR div. from literature
- More straightforward computation.

Looking at those formulas in detail

$$\sum_{i=1}^{n} p_i^\mu = 0$$

Elias Miró et al.
2005.06983

$$\langle \vec{n}| O_j |0\rangle^{(0)} (\gamma_{ji} - \gamma^i_{\text{IR}}\delta_{ij})^{(1)} = -\frac{1}{\pi} \langle \vec{n}| \mathcal{M} \otimes O_i |0\rangle^{(0)}$$

$$\sum_{i=1}^{n} p_i^\mu \neq 0$$

Baratella et al.
2005.07129

$$\gamma_{ij}\, \mathcal{A}_{\mathcal{O}_i}(1,2,\ldots) = -\frac{1}{4\pi^3}\frac{C_{\mathcal{O}_i}}{C_{\mathcal{O}_j}} \int d\text{LIPS} \sum_{\ell_1,\ell_2} \widehat{\mathcal{A}}_{\mathcal{O}_j}(1,2,\ldots,\ell_1,\ell_2) \times \mathcal{A}_4(-\ell_1,-\ell_2,\ldots)\,.$$

$$\sum_{i=1}^{n} p_i^\mu = 0$$

Note: This difference is discussed in 2005.07129, where the FFs are used to prove their formula.

$$\gamma_{ij}\, \mathcal{F}_{\mathcal{O}_i}(1,2,\ldots,n) = -\frac{1}{4\pi^3} \int d\text{LIPS} \sum_{\substack{\text{ext. legs}\\ \text{distrib.}}} \sum_{\ell_1,\ell_2} \widehat{\mathcal{F}}_{\mathcal{O}_j}(\ldots,-\ell_1,-\ell_2) \times \mathcal{A}_4(\ell_2,\ell_1,\ldots)\,, \qquad (18)$$

And what’s the practical difference?

Fortunately, they both compute a dipole renormalization.

Baratella et al.
2005.07129

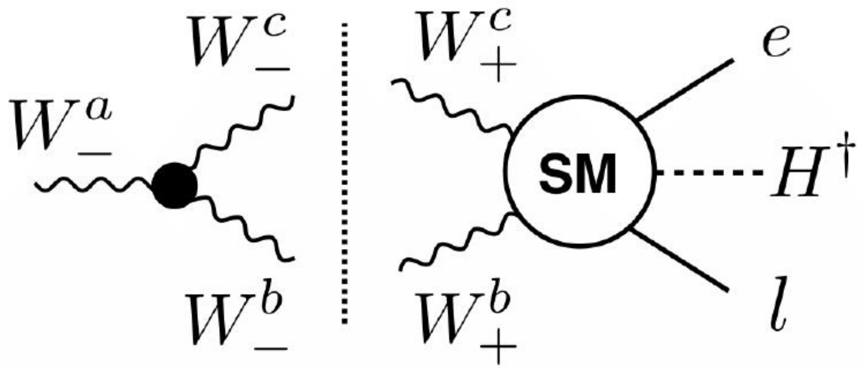
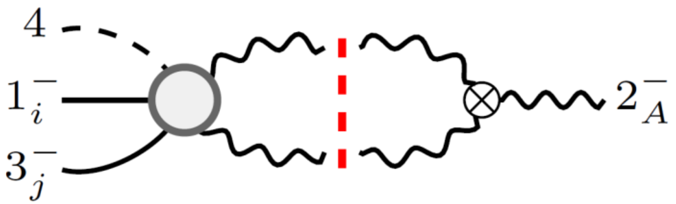


Figure 9: *Potential extra contribution from C_{W^3} to the anomalous dimension of C_{WHle}. This should be considered for the anomalous dimension of the form-factor $\mathcal{F}_{WHle}$, Eq. (18) (where $p_b + p_c + p_a \neq 0$), but not when using Eq. (16).*

Elias Miró et al.
2005.06983



$$- \frac{1}{16\pi^2} \int \mathrm{d}\Omega_2 \, M(1_i^- 3_j^- 4; x_{\mathsf{X}}^- y_{\mathsf{Y}}^-) \, F_{3G}(x_{\mathsf{X}}^- y_{\mathsf{Y}}^- 2_A^-)$$

Did they get the same?

Baratella et al.
2005.07129

$$\gamma_{GHqd} = \frac{3g_3^2 y_d}{16\pi^2} C_{G^3}$$

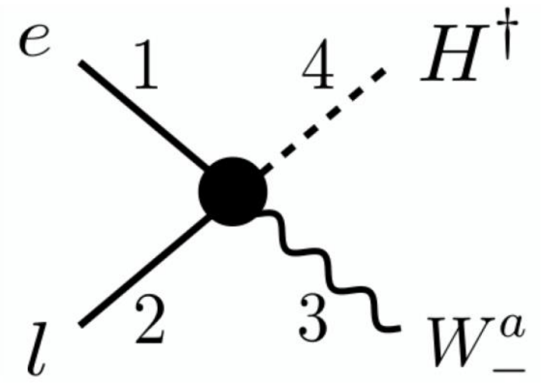
Elias Miró et al.
2005.06983

$$\gamma_{qG\leftarrow 3G} = \frac{g_s^2 y}{16\pi^2}\frac{N_c}{2}$$

Matching between
amplitudes and operators

What did *Baratella et al.* compute?

- **All 1 loop contributions to:** γ_{WHle}



- **For free:** γ_{GHqd}

Final words

- Many more technical details in Appendix slides and in the papers.
- EMIR's method seems easier to generalize and better to attack more loops.
- BFP's approach might be seen as more model independent.
- They both end up doing almost identical computations.
- BFP's method avoids some contributions.
- No clear way to use BFP's approach to compute 2-loop corrections with a non-vanishing 1-loop.
- No proposal to compute IR anomalous dimensions in either paper.
- 2 loop anomalous dimensions matrix for SMEFT seems attainable in the near future.
- More to read: 2005.12917, 1910.05831, 1607.06448, 2005.10261…

Thank you for your attention

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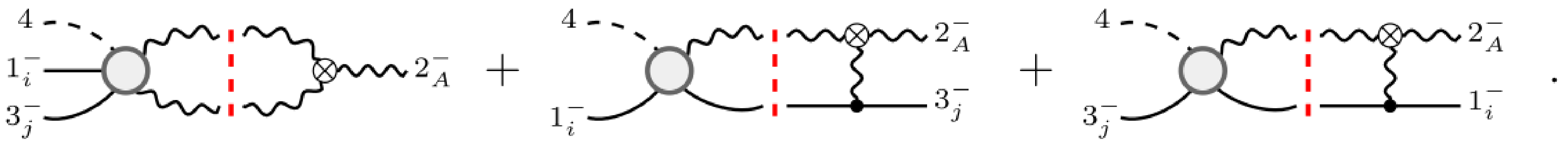
Appendix

Non minimal FFs in EMIR 2005.06983

Appear for mixing between operators with different amount of legs.

Example: dipole renormalization.



(2.36)

The trick of choosing a convenient basis

You have this integral: $$-\frac{1}{16\pi^2}\int \mathrm{d}\Omega_2\, M(1_i^- 3_j^- 4; x_{\mathsf{X}}^- y_{\mathsf{Y}}^-)\, F_{3G}(x_{\mathsf{X}}^- y_{\mathsf{Y}}^- 2_A^-)$$

Rotate the spinors like: $$(|x\rangle, |y\rangle) = R(\theta,\phi).(|a\rangle, |b\rangle)$$

Define a and b such that: $|a\rangle$ and $|b\rangle$ are on-shell and $p_a + p_b = p_1 + p_3 + p_4$

A possible choice: $$|a\rangle = |1\rangle\sqrt{\frac{s_{134}}{s_{13}+s_{14}}}\,, \qquad |b\rangle = (|3\rangle[13] + |4\rangle[14])\frac{1}{\sqrt{s_{13}+s_{14}}}$$

The elephant in the room of *BFP* 2005.07129 (I)

The details behind the master formula of *BFP* 2005.07129

- **At 1 loop, Passarino-Veltman decomposition:**

$$\mathcal{A}_{\text{loop}} = \sum_a C_2^{(a)} I_2^{(a)} + \sum_b C_3^{(b)} I_3^{(b)} + \sum_c C_4^{(c)} I_4^{(c)} + R$$

$$I_2^{(a)} = \frac{1}{16\pi^2}\left(\frac{1}{\epsilon} + \ln\left(\frac{\mu^2}{-P_a^2}\right) + \cdots\right)$$

Contribution to the anomalous dimensions...

…but mixed with IR anomalous dimensions.

Then:

$$\gamma_i \mathcal{A}_{\mathcal{O}_i} = -\frac{C_{\mathcal{O}_i}}{8\pi^2}\sum_a C_2^{(a)} + \cancel{\gamma_{\text{IR}}}\, \mathcal{A}_{\mathcal{O}_i}$$

Restrict themselves to cases without IR part.

How to obtain the bubble coefficients $C_2^{(a)}$? 2-cuts!

"(…) 2-cuts (…) are in one-to-one correspondence with bubble coefficients. Each 2-cut picks up a unique $C_2^{(a)}$."

"(…) in general, 2-cuts can also contain terms coming from triangle and boxes."

The elephant in the room of *BFP* 2005.07129 (II)

Dealing with triangles and boxes in *BFP* 2005.07129

- **They prove that, after summing over all 2-cuts, boxes and triangles cancel out from:**

$$\gamma_{ij}\,\mathcal{A}_{\mathcal{O}_i}(1,2,\ldots,n) = -\frac{1}{4\pi^3}\frac{C_{\mathcal{O}_i}}{C_{\mathcal{O}_j}}\int d\mathrm{LIPS}\sum_{\substack{\text{ext. legs}\\ \text{distrib.}}}\sum_{\ell_1,\ell_2}\widehat{\mathcal{A}}_{\mathcal{O}_j}(\ldots,-\ell_1,-\ell_2)\times\mathcal{A}_4(\ell_2,\ell_1,\ldots)$$

- **Even more, if the amplitudes have the same number of legs, there are no boxes nor triangles.**

- **If you have to deal with boxes and triangles, then it might be useful the alternative formula:**

$$\gamma_{ij}\,\mathcal{A}_{\mathcal{O}_i}(1,2,\ldots,n) = i\frac{C_{\mathcal{O}_i}}{C_{\mathcal{O}_j}}\int\frac{d\mathrm{LIPS}}{8\pi^4}\sum_{\substack{\text{ext. legs}\\ \text{distrib.}}}\sum_{\ell_1,\ell_2}\int_{\mathcal{C}}\frac{dz}{z}\widehat{\mathcal{A}}_{\mathcal{O}_j}(\ldots,-\ell_1(z),-\ell_2(z))\times\mathcal{A}_4(\ell_2(z),\ell_1(z),\ldots)\,. \tag{22}$$