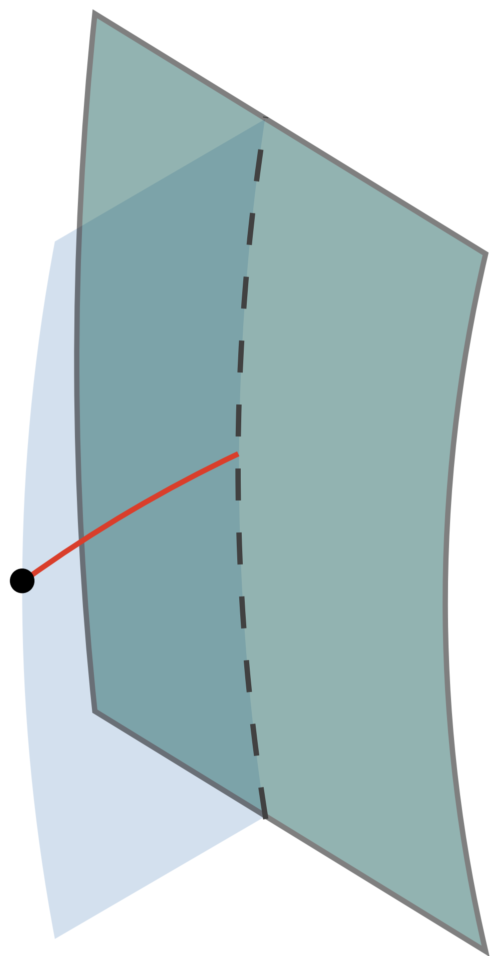
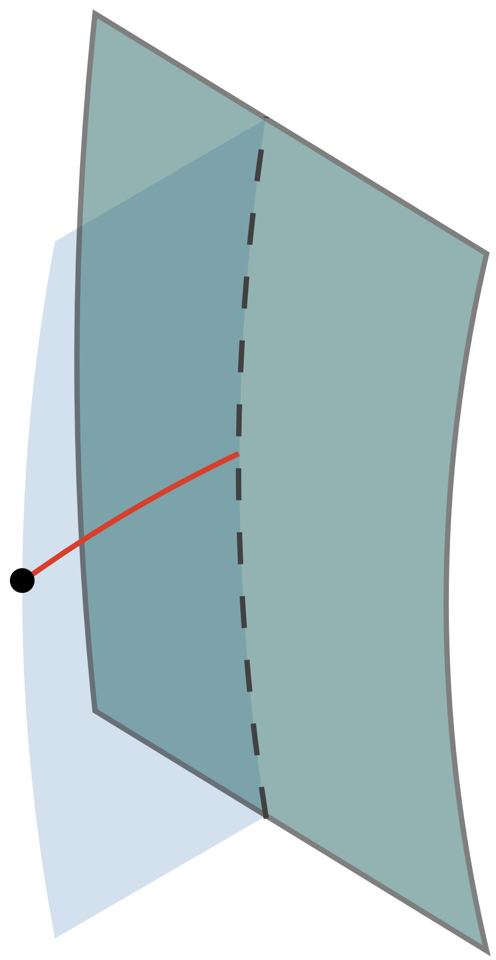


QCD meets gravity VI, Dec 2020

Another dimension of Kerr

Dónal O'Connell
University of Edinburgh



QCD meets gravity VI, Dec 2020

or Newman-Janis & the dynamics of Kerr

with Alfredo Guevara, Ben Maybee, Alexander Ochirov and Justin Vines

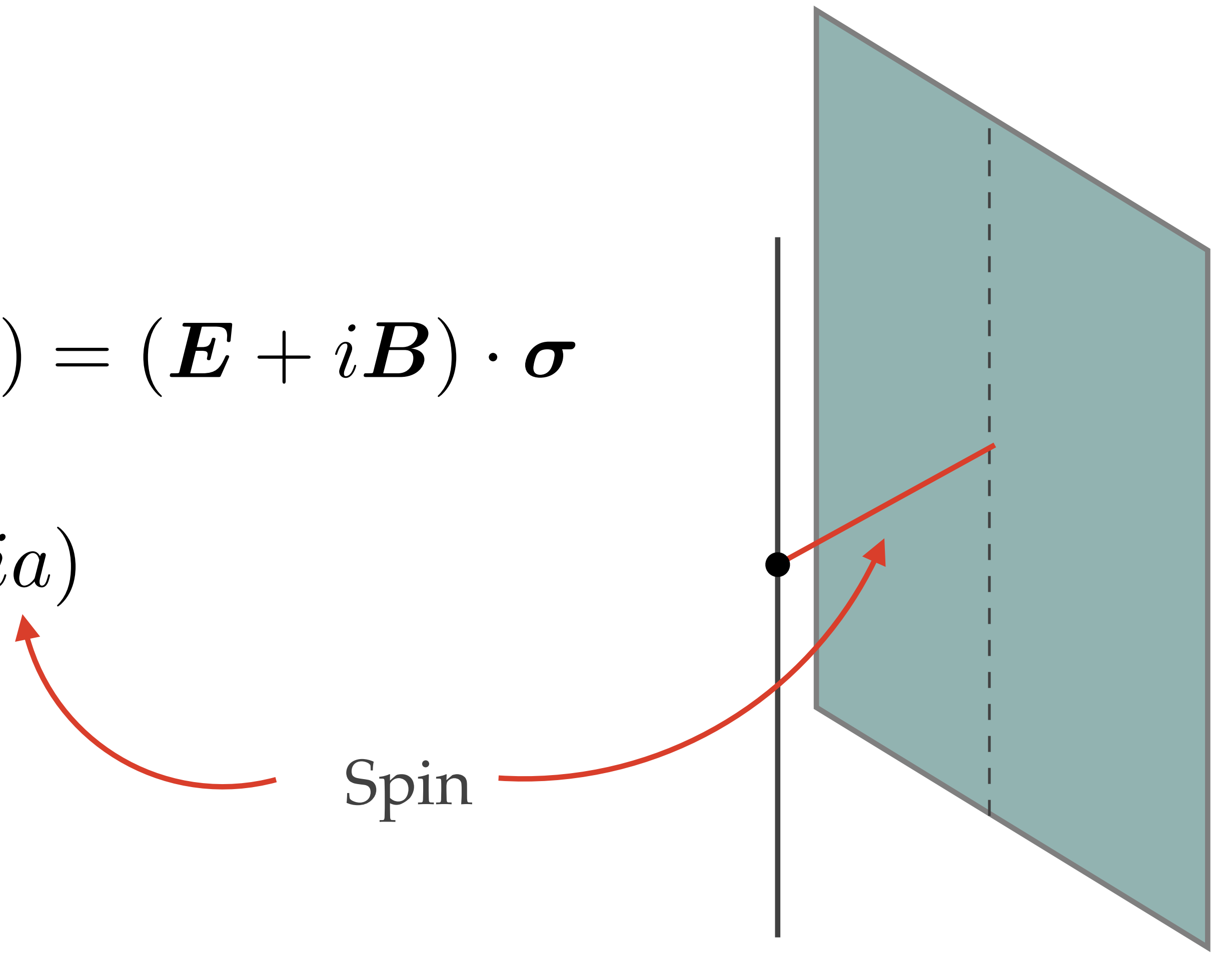
Newman-Janis

Surprising simplicity:

- Exact Kerr
- $\sqrt{\text{Kerr}}$ (single copy)

$$F_{\mu\nu}(x) \leftrightarrow \phi(x) = \sigma^{\mu\nu} F_{\mu\nu}(x) = (\boldsymbol{E} + i\boldsymbol{B}) \cdot \boldsymbol{\sigma}$$

$$\phi^{\sqrt{\text{Kerr}}}(x) = \phi^{\text{Coulomb}}(x + ia)$$



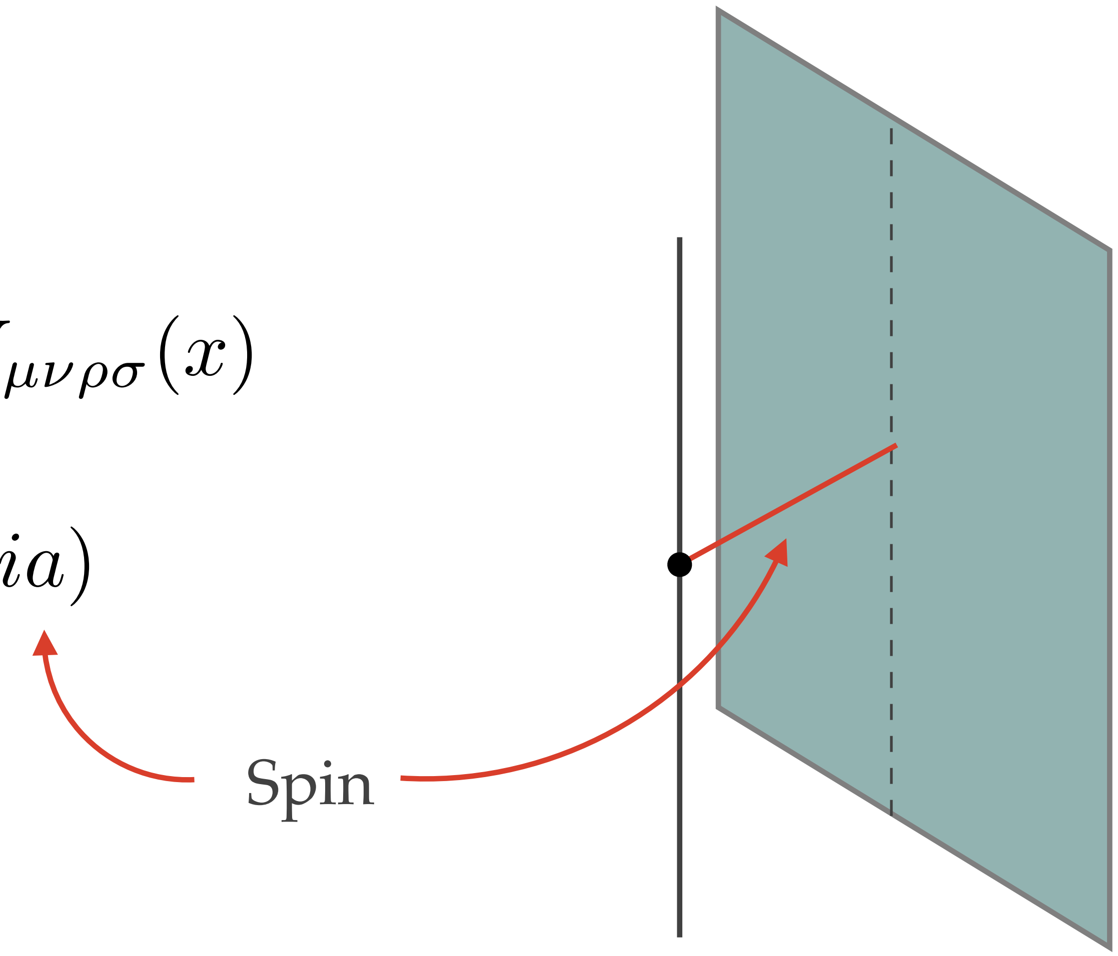
Newman-Janis

Surprising simplicity:

- Exact Kerr
- $\sqrt{\text{Kerr}}$ (single copy)

$$W_{\mu\nu\rho\sigma}(x) \leftrightarrow \Psi(x) = \sigma^{\mu\nu}\sigma^{\rho\sigma}W_{\mu\nu\rho\sigma}(x)$$

$$\Psi^{\text{Kerr}}(x) = \Psi^{\text{Schwarzschild}}(x + ia)$$

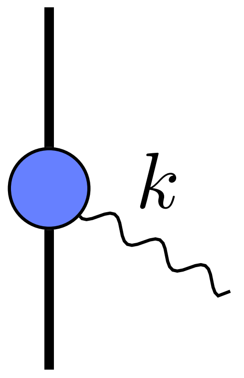


Newman-Janis

Arises naturally from large-spin limit of minimally coupled massive amplitudes

$$\mathcal{A}_3^S = \mathcal{A}_3^{\text{Coulomb}} \langle \mathbf{12} \rangle^{2S}$$

$$\langle \mathbf{12} \rangle^{2S} \simeq \left(1 + \frac{1}{2S} \frac{s \cdot k}{m} \right)^{2S} \to \exp(a \cdot k)$$



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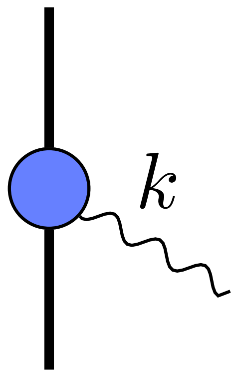
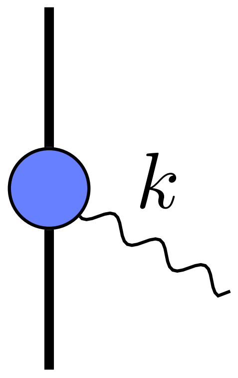
a is a length!



Newman-Janis

Arises naturally from large-spin limit of minimally coupled massive amplitudes

$$\mathcal{A}_3^{\sqrt{\text{Kerr}}} = \mathcal{A}_3^{\text{Coulomb}} e^{k\cdot a}$$
$$\mathcal{M}_3^{\text{Kerr}} = \mathcal{M}_3^{\text{Schw}} e^{k\cdot a}$$



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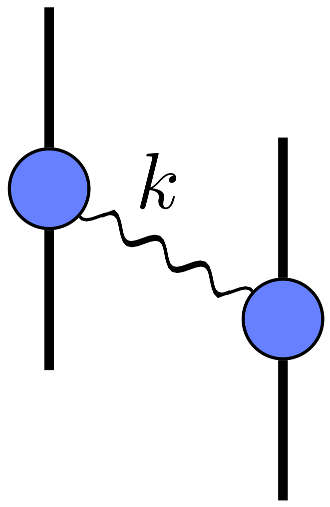

But amplitudes describe interactions, not just solutions?

Newman-Janis

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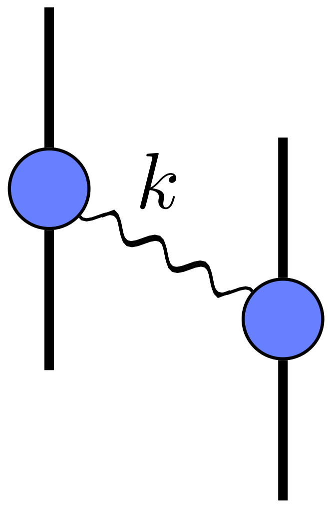

But amplitudes describe interactions, not just solutions?

Newman-Janis for classical dynamics?

$\sqrt{\text{Kerr}}$ Effective Action

We know the amplitude…

$$\mathcal{A}_3^{\sqrt{\text{Kerr}}} \longrightarrow S_{\text{effective}}$$

$\sqrt{\text{Kerr}}$ Effective Action

We know the amplitude…

$$\mathcal{A}_3^{\sqrt{\text{Kerr}}} \longrightarrow S_{\text{effective}}$$

$$\searrow \quad \swarrow$$

$$\phi(x) = \text{Re} \int \mathrm{d}\Phi(k)\, \delta(k \cdot u)\, |k\rangle|k\rangle\, e^{-ik\cdot x} \mathcal{A}_3$$

Monteiro, Peinador Veiga, Sergola
& DOC

$\sqrt{\text{Kerr}}$ Effective Action

We know the amplitude…

$$\mathcal{A}_3^{\sqrt{\text{Kerr}}} \longrightarrow S_{\text{effective}}$$

$$\searrow \qquad \swarrow$$

$$\phi(x) = \text{Re} \int \boxed{\mathrm{d}\Phi(k)\, \delta(k \cdot u)}\, |k\rangle |k\rangle\, e^{-ik\cdot x} \mathcal{A}_3$$

Zero-energy on-shell phase space: exists in (2,2) signature

Monteiro, Peinador Veiga, Sergola & DOC

Match the EFT in (2,2) signature

$\sqrt{\text{Kerr}}$ Effective Action

Most general parity-even action with one photon

Porto, Rothstein, Levi, Steinhoff, …
Hanson & Regge

$$S = S_{\rm pp} + Q \sum_{n=1}^{\infty} \int \mathrm{d}\tau \left(\alpha_n (a \cdot \partial)^{2(n-1)} F^*_{\mu\nu} u^\mu a^\nu + \beta_n (a \cdot \partial)^{2n-1} F_{\mu\nu} u^\mu a^\nu \right)$$

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Spinning, charged, massive point particle

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Odd number of a's
Need F^*

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Even number of a's
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Odd number of a's
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Even number of a's
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Spinning, charged, massive point particle

Systematically ignoring higher dimension operators!

$\sqrt{\text{Kerr}}$ Effective Action

Most general parity-even action with one photon

$$S_{\text{EFT}} = Q \sum_{n=1}^{\infty} \int \mathrm{d}\tau \, u^\mu a^\nu \left(\alpha_n (a \cdot \partial)^{2(n-1)} F^*_{\mu\nu} + \beta_n (a \cdot \partial)^{2n-1} F_{\mu\nu} \right)$$

Continue to (2,2): most natural to do so in direction of spin

$$\phi^{\sqrt{\text{Kerr}}}(x, y, z) = \phi^{\text{Coulomb}}(x, y, z + ia) \qquad z \to -iz$$

$$\to \phi^{\text{Coulomb}}(x, y, z - a)$$

$\sqrt{\text{Kerr}}$ Effective Action

Most general parity-even action with one photon

$$S_{\text{EFT}} = Q \sum_{n=1}^{\infty} \int \mathrm{d}\tau \, u^\mu a^\nu \left(\alpha_n (a \cdot \partial)^{2(n-1)} F^*_{\mu\nu} + \beta_n (a \cdot \partial)^{2n-1} F_{\mu\nu} \right)$$

Adjust to absorb some signs

Continue to (2,2): most natural to do so in direction of spin

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In (2,2): real self-dual / anti-self dual parts

$$F^{\pm} = F \pm F^{*}$$

$\sqrt{\text{Kerr}}$ Effective Action

Most general parity-even action with one photon

$$S_{\text{EFT}} = Q \sum_{n=0}^{\infty} \int \mathrm{d}\tau \, u^{\mu} a^{\nu} \left(\alpha_n (a \cdot \partial)^n F^{+}_{\mu\nu} + \beta_n (a \cdot \partial)^n F^{-}_{\mu\nu} \right)$$

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$$F^{\pm} = F \pm F^{*}$$

$\sqrt{\text{Kerr}}$ Effective Action

Most general parity-even action with one photon

$$S_{\mathrm{EFT}} = Q \sum_{n=0}^{\infty} \int \mathrm{d}\tau \, u^{\mu} a^{\nu} \left(\alpha_n (a \cdot \partial)^n F^{+}_{\mu\nu} + \chi.c. \right)$$

"Chiral conjugate"

In (2,2): real self-dual / anti-self dual parts

$$F^{\pm} = F \pm F^{*}$$

$\sqrt{\text{Kerr}}$ Effective Action

Match to $\mathcal{A}_3^{\text{Coulomb}} e^{ik\cdot a}$:

$$S_{\text{EFT}} = -Q \int \mathrm{d}\tau \, u^\mu a^\nu \left(\sum_{n=0}^{\infty} \frac{(a\cdot\partial)^n}{(n+1)!} F^+_{\mu\nu} + \chi.c. \right)$$

$$= -Q \int \mathrm{d}\tau \, u^\mu a^\nu \left(\frac{e^{a\cdot\partial} - 1}{a\cdot\partial} F^+_{\mu\nu}(x) + \chi.c. \right)$$

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Translation operator: NJ?

$$= -Q \int \mathrm{d}\tau\, u^\mu a^\nu \left(\frac{e^{a\cdot\partial} - 1}{a\cdot\partial} F^+_{\mu\nu}(x) + \chi.c. \right)$$

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Inverse derivative: integral?

Another dimension of Kerr

Action is

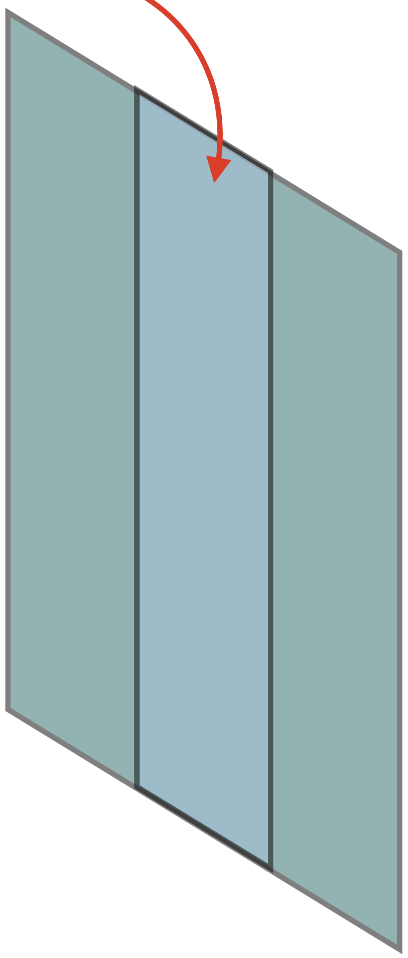
$$S_{\text{int}} = -Q\int \mathrm{d}\tau\, u\cdot A - Q\int \mathrm{d}\tau \int_0^1 \mathrm{d}\lambda\, u^\mu a^\nu\, F^+_{\mu\nu}(x+\lambda a) + \chi.c..$$

Another dimension of Kerr

Action is

$$S_{\text{int}} = -Q\int \mathrm{d}\tau\, u\cdot A - Q\int \mathrm{d}\tau \int_0^1 \mathrm{d}\lambda\, u^{\mu} a^{\nu}\, F^{+}_{\mu\nu}(x+\lambda a) + \chi.c.\,.$$

Two-dimensional world-sheet Σ



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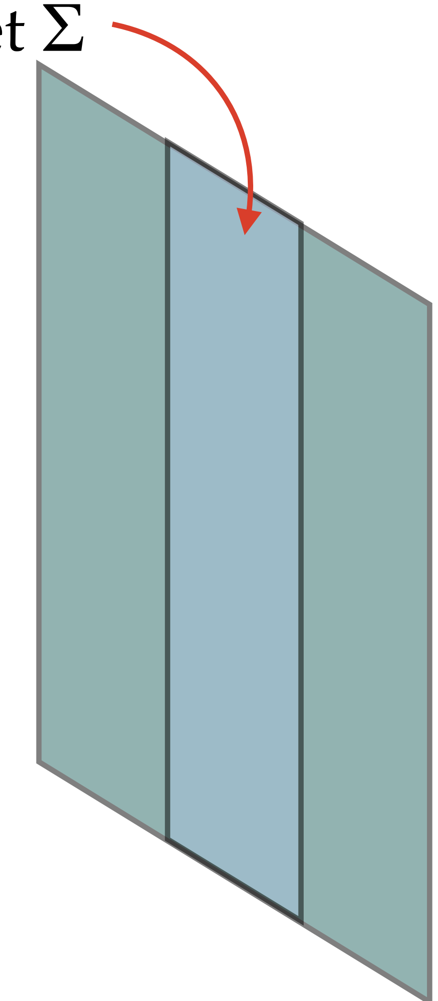
Two-dimensional world-sheet Σ



Worldsheet has boundaries:

1. "Near" boundary at $\lambda = 0$
2. "Far" boundary at $\lambda = 1$

$\sqrt{\text{Kerr}}$ Effective Action

Structure familiar from brane world EFTs:

$$S_{\rm int} = -Q\int_{\partial\Sigma_n} A_\mu \mathrm{d}x^\mu - \frac{Q}{2}\int_\Sigma F^+_{\mu\nu}(z)\,\mathrm{d}z^\mu \wedge \mathrm{d}z^\nu + \chi.c. + \cdots$$

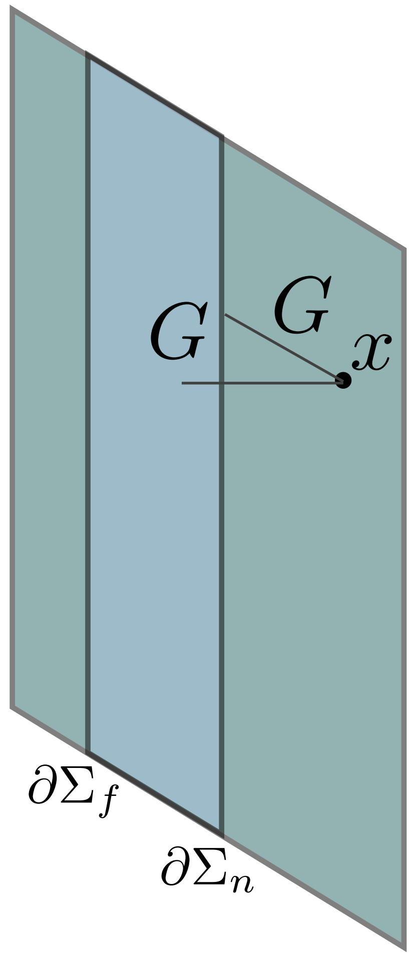
Operator integrated over "near" boundary

Operator integrated over "bulk"

$\sqrt{\text{Kerr}}$ Effective Action

Recover NJ shift: compute field sourced by worldsheet

$$\phi(x) = Q\,\sigma_{\mu\nu}u^{\mu}\partial^{\nu}\int_{\partial\Sigma_n}\mathrm{d}\tau\, G(x-u\tau) + Q\,\sigma_{\mu\nu}u^{\mu}\partial^{\nu}\int_{\Sigma}\mathrm{d}\tau\mathrm{d}\lambda\,\frac{\mathrm{d}}{\mathrm{d}\lambda}G(x-u\tau-a\lambda)$$



$\sqrt{\text{Kerr}}$ Effective Action

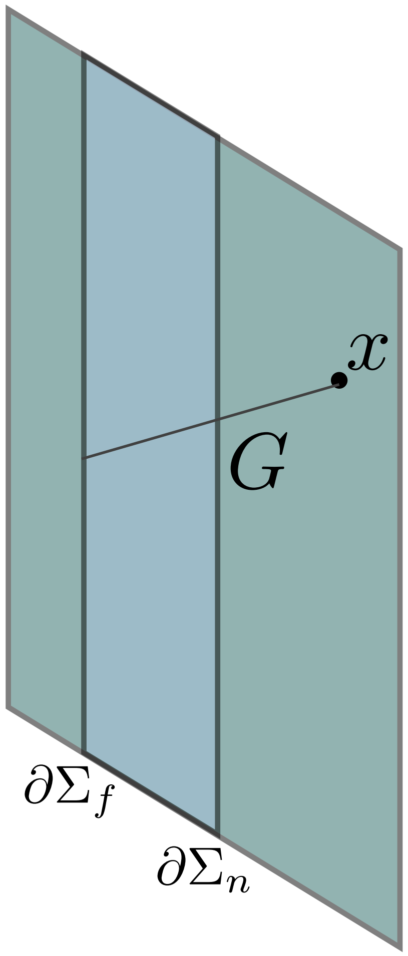
Recover NJ shift: compute field sourced by worldsheet

$$\phi(x) = Q\,\sigma_{\mu\nu}u^\mu\partial^\nu \int_{\partial\Sigma_n} \mathrm{d}\tau\, G(x-u\tau)$$

$$+\, Q\,\sigma_{\mu\nu}u^\mu\partial^\nu \int_{\Sigma} \mathrm{d}\tau\mathrm{d}\lambda\, \frac{\mathrm{d}}{\mathrm{d}\lambda} G(x-u\tau-a\lambda)$$

Integrate by parts

$$= Q\,\sigma_{\mu\nu}u^\mu\partial^\nu \int_{\partial\Sigma_f} \mathrm{d}\tau\, G(x-u\tau-a)$$



$\sqrt{\text{Kerr}}$ Effective Action

Analytic continuation back to Minkowski

$$S_{\text{int}} = -Q \int_{\partial\Sigma_n} \mathrm{d}\tau \, u \cdot A - Q \, \mathrm{Re} \int_{\Sigma} \mathrm{d}\tau \mathrm{d}\lambda \, i F^{+}_{\mu\nu}(r + i\lambda a) \, u^{\mu} a^{\nu} + \cdots$$

$\sqrt{\text{Kerr}}$ Effective Action

Analytic continuation back to Minkowski

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$$F^{+}_{\mu\nu} = F_{\mu\nu} + iF^{*}_{\mu\nu}$$

$\sqrt{\text{Kerr}}$ Effective Action

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neglected

$$F^{+}_{\mu\nu} = F_{\mu\nu} + i F^{*}_{\mu\nu}$$

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Analytic continuation back to Minkowski

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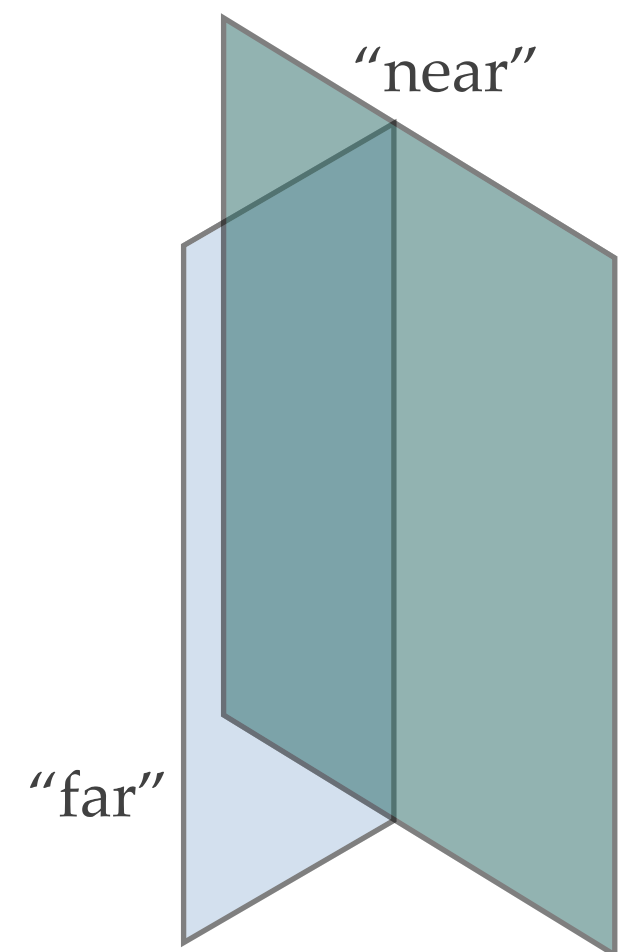
$$F^{+}_{\mu\nu} = F_{\mu\nu} + i F^{*}_{\mu\nu}$$

Worldsheet embedded in complexified spacetime

$$\phi^{\sqrt{\text{Kerr}}}(x) = \phi^{\text{Coulomb}}(x + ia)$$

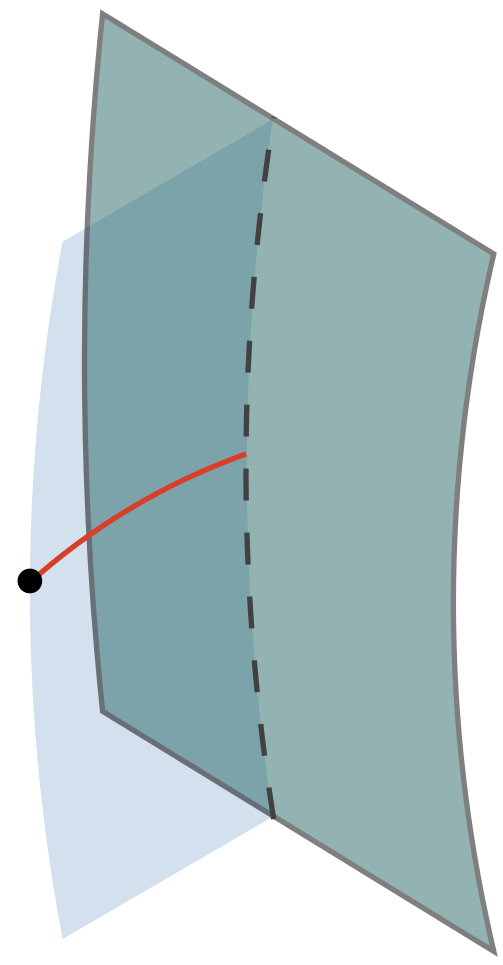
→ Ben's talk



Kerr Effective Action

First, embed $\sqrt{\text{Kerr}}$ in curved space

$$S_{\text{int}} = -Q\int_{\partial\Sigma_n} A_\mu \mathrm{d}x^\mu - \frac{Q}{2}\,\mathrm{Re}\int_\Sigma F^+_{\mu\nu}(z)\,\mathrm{d}z^\mu\wedge\mathrm{d}z^\nu$$



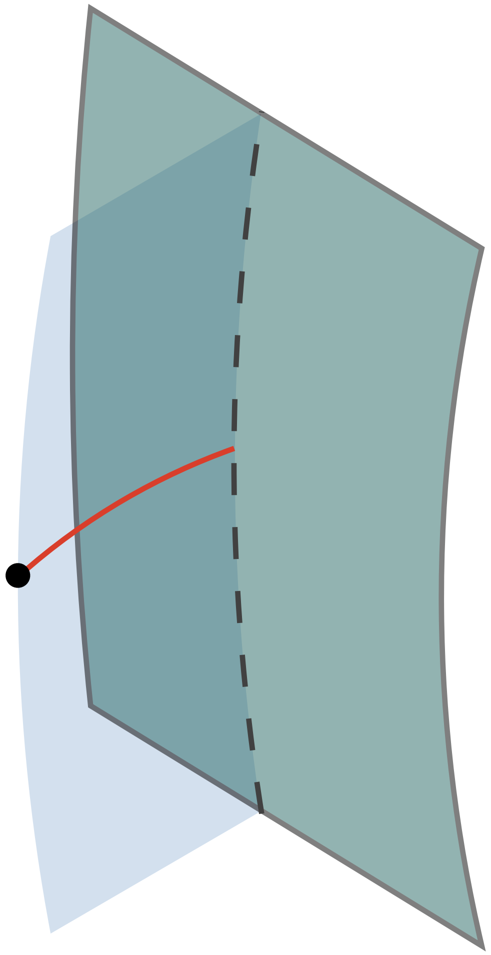
$$z^\mu(\tau,\lambda) = r^\mu(\tau) + i\lambda a^\mu(\tau) - \frac{1}{2}\Gamma^\mu_{\nu\rho}a^\mu a^\nu + \cdots$$

Geodesic in direction a^μ

Kerr Effective Action

Second, double copy:

$$S_{\text{EFT}} = -\,\text{Im}\int_{\Sigma} \mathrm{d}\tau \mathrm{d}\lambda \, c^a F^{a+}_{\mu\nu}(z)\, u^\mu a^\nu + \cdots$$



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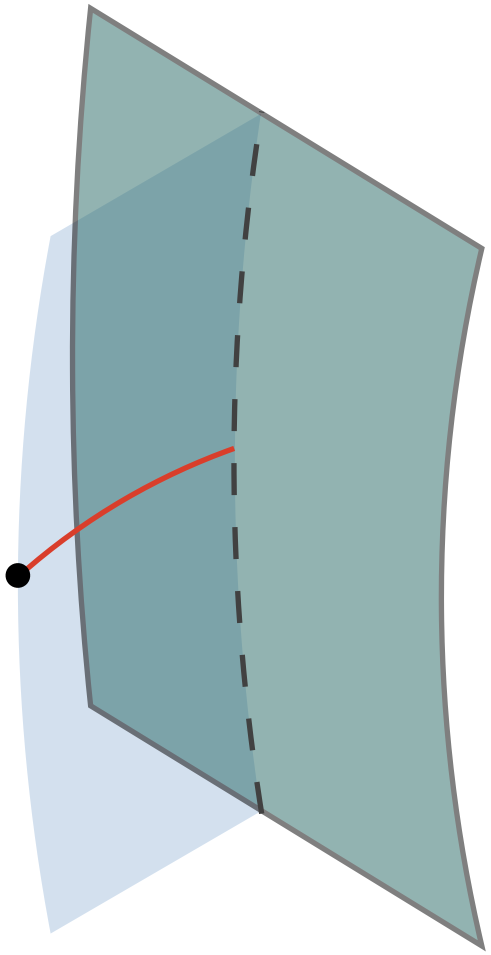

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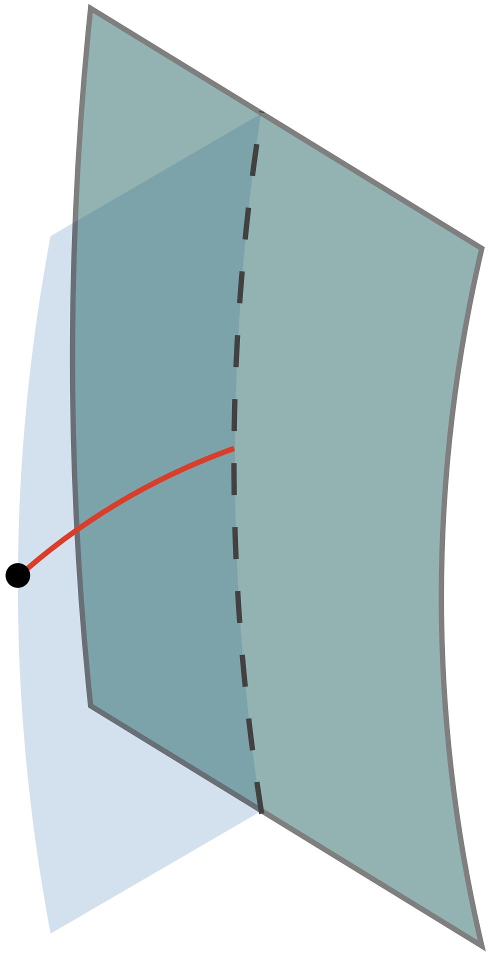
$$\downarrow$$

$$S_{\text{EFT}} = -\operatorname{Im}\int_\Sigma \mathrm{d}\tau\mathrm{d}\lambda\, u^\mu(z)\omega^+_{\mu\rho\sigma}(z)\, p^\rho a^\sigma + \cdots$$



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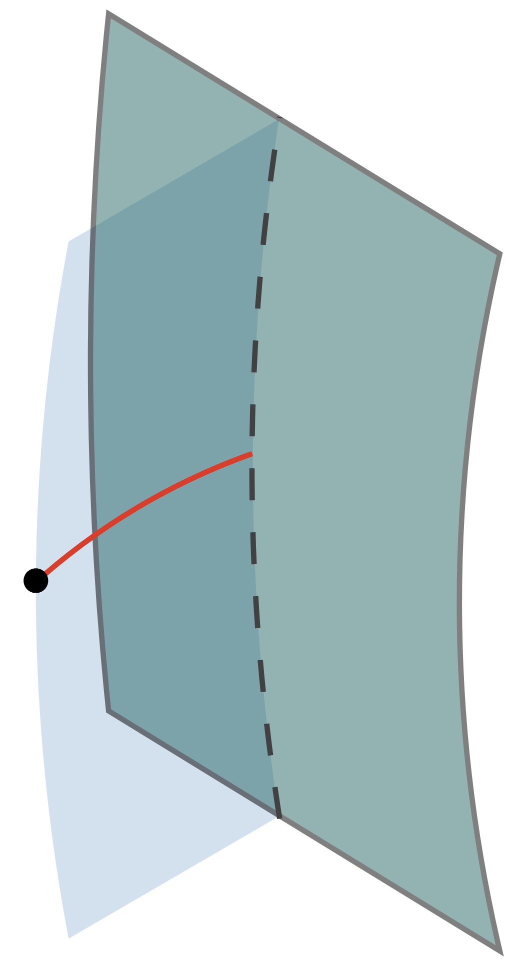
↓

$$S_{\mathrm{EFT}} = -\,\mathrm{Im}\int_\Sigma \mathrm{d}\tau\mathrm{d}\lambda\, u^\mu(z)\omega^{+}_{\mu\rho\sigma}(z)\, p^\rho a^\sigma + \cdots$$

$$= -\frac{1}{2}\int \mathrm{d}\tau\,\left(S_{\mu\nu}\Omega^{\mu\nu} + R_{\mu\alpha\nu\beta}a^\mu u^\alpha a^\nu u^\beta + \cdots\right)$$

Spin kinetic terms!

All Kerr leading interactions



Porto, Rothstein, Levi & Steinhoff

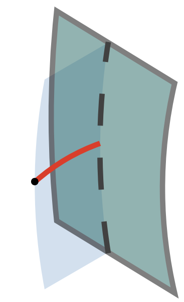
Conclusions

1. Newman-Janis trick?

2. Kerr seems to have the structure of a complex worldsheet?
   - Only three-point information: what about Compton?
   - Use worldsheet symmetries to restrict higher-dimension operators?

3. Newman-Janis for interactions: spinor-helicity. See Ben’s talk

Conclusions



1. Newman-Janis ~~trick~~?

   insight!

2. Kerr seems to have the structure of a complex worldsheet?
   - Only three-point information: what about Compton?
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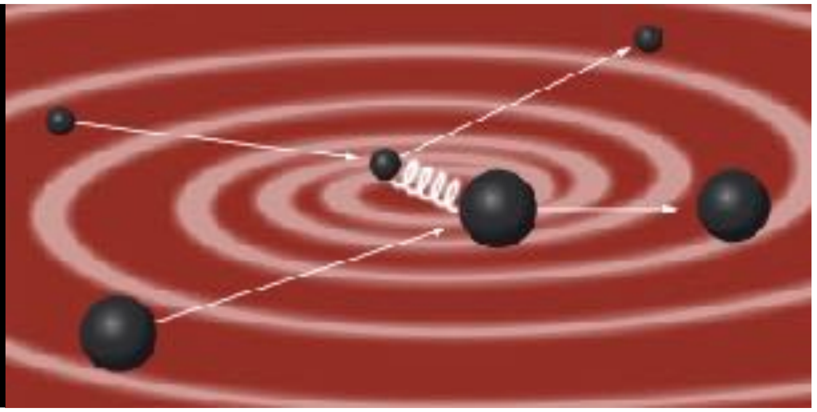

KITP program *"High-Precision Gravitational Waves"*

April 4 - Jun 10, 2022

Application deadline: *Mar 1, 2021*

https://www.kitp.ucsb.edu/activities/gwaves22

Coordinators: A. Buonanno, D. Kosower, I. Rothstein, A. Zimmerman & DOC
Scientific Advisors: L. Barack, Z. Bern, F. Pretorius