QCD Meets Gravity VI

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Post-Newtonian meets Ultra-Relativistic after all*

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*based on: P. Di Vecchia, C. Heissenberg, R. Russo, GV, 2008.12743 (PLB), & to appear

See also Carlo Heissenberg's talk

CERN

COLLÈGE DE FRANCE 1530

A two years old puzzle

- In 1710.10599, T. Damour suggested that useful input for the EOB could be obtained from the high-energy/UR regime of gravitational scattering discussed, among others, by ACV since the late eighties.
- Made sense: the PM(=loop) expansion could then be anchored to two limiting cases: the much studied small-velocity PN expansion on the one hand, and the UR limit on the other (NB: both limits are classical)
- In 1901.04424/1908.01493 an impressive calculation by BCRSSZ led to the first "complete" 3PM result (GR for two massive scalars). Announced @ QCDMG-IV-12.2018
- Checked to be consistent up to "5PN" (later also to "6PN") order but presented a puzzle.

- The high-energy (or just the massless) limit of the BCRSSZ result exhibited a logarithmic divergence in contrast with a perfectly finite 1990 result by ACV.
- NB: gravity is supposedly free from $m=0$ (collinear) divergences. Quite some controversy came up.
  - Furthermore, both the massless limit of ACV90 (DNRVW1911, BIP-MR2002) and the BCRSSZ log-enhancement @ large s/m^2 (P-MRZ2005) have been claimed/checked to be "universal" and yet looked mutually incompatible.
  - The prevailing attitude till a few months ago was that the limits $q \ll m \ll E$ and $m \ll q \ll E$ are unrelated: PN and UR don't meet/talk!
  - But the solution of the puzzle turned out to be different...that's the main topic of this talk.

Outline

- The gravitational eikonal approach
- Sharpening the puzzle
  - Extending the ACV-90 argument
  - Calculation of the massive UR 3-particle cut
- 3PM massive N=8 SUGRA @ all $p/m=v(1-v^2)^{-1/2}$
  - Recovering smooth UR limit from full soft region through RR (see also Carlo's talk)
  - Recasting in real-analytic, xing-symm. form
- Radiation reaction via soft theorems
- Connection with Damour's 2010.01641.
- Summary

new!

The gravitational eikonal approach (1987…)

- Eikonal resummation in b-space restores unitarity by converting a p.w.unitarity violating amplitude into a large phase: $S(b) \sim e^{2i\delta}$
- $2\delta \sim (\text{Action})/h \gg 1$ => class. limit
- Going back to q-space one recovers GR expectations (or derives new results!) through a saddle point approximation.
- For a review (including string & brane effects) see e.g. my 2015 Les Houches lecture notes or the short account given in my slides at the AEI workshop in August.

Sharpening the puzzle

I. Extending the ACV90 argument

- To put *ACV90* on more rigorous and general grounds (e.g. @ $q < m$) we use general properties of the scattering amplitude (for the HE scattering of two scalars in a generic theory of gravity)
  - Real analyticity: $A^*(s^*,t) = A(s,t)$
  - Xing symm.: $A(s,t) = A(u,t)$, $u=-s-t+2m_1^2+2m_2^2$
  - Asymptotic behavior in order to write suitably subtracted fixed-t disp. relations.
- From those we get informations on the high-energy limit of $\rho = \mathrm{Re}A(s,t)/\mathrm{Im}A(s,t)$ in analogy with what is done in high-energy soft hadronic physics.
- Popular in '70s, now again w/ LHC data, also used recently by Caron-Huot et al. (see his talk?)

Typical results: for $\boxed{\text{Im } A \sim s^n \log^p s}$

ρ = ReA(s,t)/ImA(s,t)

For n even $\boxed{\rho \sim -\frac{2\log s}{(1+p)\pi}}$

while for n odd, p > 0, $\boxed{\rho \sim \frac{\pi p}{2\log s}}$

These model-independent results allow to express Re δ_2 in terms of Im δ_2 & of tree + 1-loop quantities.

$$\boxed{2Re\delta_2 = \frac{\pi p}{2\log s}(2Im\delta_2) - (4-3p)\frac{\delta_0}{s}(2\nabla\delta_0)^2 - (p-1)(2\delta_0)(2Im\delta_1)}$$

non-universal quantum piece aka Δ_1

For p generic (e.g. p=2) we are left with a non-universal piece ~ Imδ_1 Also, since neither Imδ_1 nor δ_0 have a log s, Re δ_2 would not have a finite UR limit. And, indeed, p=2 is the BCRSSZ value...

$$\boxed{2Re\delta_2 = \frac{\pi p}{2\log s}(2Im\delta_2) - (4-3p)\frac{\delta_0}{s}(2\nabla\delta_0)^2 - (p-1)(2\delta_0)(2Im\delta_1)}$$

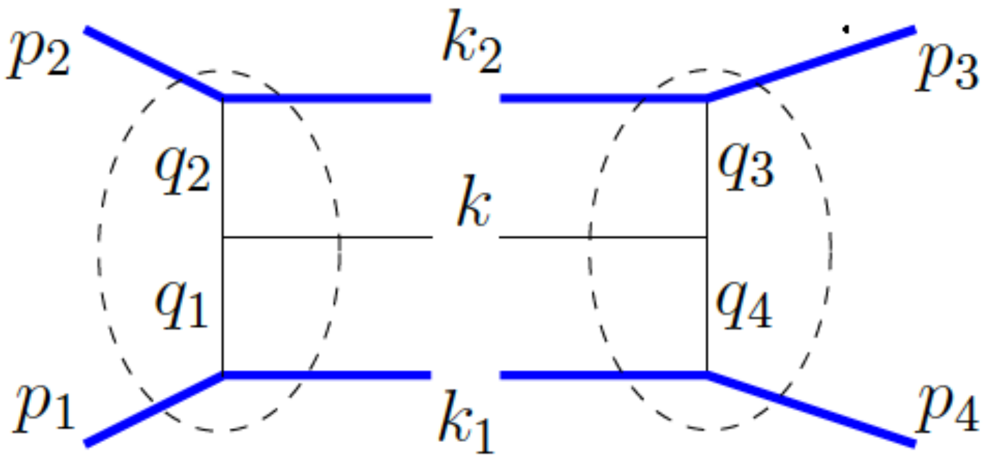
On the contrary, if p=1, Reδ_2 does not depend on the non-universal Imδ_1 and approaches a finite UR limit. The above relation simplifies to:

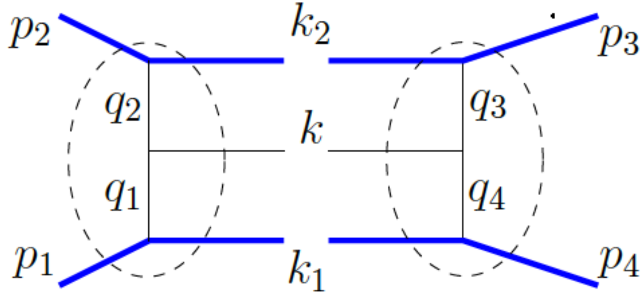
$$\boxed{2Re\delta_2 = \frac{\pi}{2\log s}(2Im\delta_2) - \frac{\delta_0}{s}(2\nabla\delta_0)^2}$$

The logarithmically growing term in Imδ_2 has an IR divergence which, however, cancels against the δ_0 term yielding a finite result for Reδ_2, see below.

II. Revisiting the 3-particle unitarity cut

- Given the general A&X result it is crucial to compute the full 3-p. unitarity cut at high energy (~$\mathrm{Im}\ A_2^{(3pc)}$ which, in b-space, gives $\mathrm{Im}\ \delta_2$).
- Was done in ACV90 for $m=0$. We have redone the whole calculation for a generic q^2/m^2 ratio
- Its kinematics is shown below (NB: called H diagram in both ACV and BCRSSZ, not the same!)





- The calculation amounts to convoluting two tree-level 5-point functions evaluated in the (double)-Regge limit. In momentum space one finds:

$$
Im A_2^{(3p)} = \frac{(16\pi G_N)^3 s^3}{2\pi} \int dy \int \frac{d^{D-2}\vec{q}_1}{(2\pi)^{D-2}} \int \frac{d^{D-2}\vec{q}_2}{(2\pi)^{D-2}} \frac{1}{(\vec{k}^2)^2} \times \left[\frac{\left[(\vec{q}_1\vec{q}_4)(\vec{q}_2\vec{q}_3) + (\vec{q}_1\vec{q}_2)(\vec{q}_3\vec{q}_4) - (\vec{q}_1\vec{q}_3)(\vec{q}_2\vec{q}_4)\right]^2}{\vec{q}_1^{\,2}\vec{q}_2^{\,2}\vec{q}_3^{\,2}\vec{q}_4^{\,2}} + 1 - \frac{(\vec{q}_1\vec{q}_2)^2}{\vec{q}_1^{\,2}\vec{q}_2^{\,2}} - \frac{(\vec{q}_3\vec{q}_4)}{\vec{q}_3^{\,2}\vec{q}_4^{\,2}} \right] \tag{3.8}
$$

It is less IR singular than the "H" diagram of BCRSSZ. As a result, we only get a single log s enhancement (p=1!) from the y integral.

In b-space this gives ($D = 4 - 2\varepsilon$)

$$Im\widetilde{M}_H(s,b) = \frac{(8G_N s)^3 \log s(\pi b^2)^{3\epsilon}\Gamma^3(1-\epsilon)}{16\pi b^2}\left[-\frac{1}{4\epsilon}+\frac{1}{2}+\mathcal{O}(\epsilon)\right]$$

to be identified with $4s\ Im\ \delta_2$. Therefore $p=1$, the IR singular term exactly cancels against the δ_0 piece, and one recovers the finite ACV90 result at $\varepsilon = 0$!

$$\boxed{2Re\delta_2 = \frac{4G^3 s^2}{\hbar b^2}}$$

As a result the clash between ACV and BCRSSZ looked more and more puzzling.

3PM massive N=8 SUGRA

@ all $p/m=v(1-v^2)^{-1/2}$

- We then decided to compute the full amplitude in N=8 SUGRA (with massive external states introduced via KK compactification, $\cos\phi=0$)
- Arbitrary energy, i.e. from deep NR to deep UR.
- We used the method of IBP+DE, but included contributions from the full soft (potential plus radiation) region.
- It is known (Cf. Ruf's talk yesterday) that including only the potential region implies loss of x-ing symmetry and manifest Lorentz invariance, both crucial in the ACV90 argument.
- Keeping the full soft region restores them.

- We found that, at high energy, the sum of planar and non-planar boxes gives a contribution of the same order as the one coming from the H topology.
- The $\log^2 s$ terms in $\mathrm{Im}\, A_2$ cancel in the sum; same for the $\log s$ term in $\mathrm{Re}\, A_2$ (Cf. the A&X argument!)
- The final result confirms asymptotically ACV90 and extends it to arbitrary kinematics.
- It was later confirmed by a different method by HP-MRZ (pr. comm.) and is quite illuminating.
- I will give it later and refer to Carlo's talk for details

Real-analytic, xing-symmetric form of N=8 results

- We have seen importance of analyticity and x-ing symmetry in relating different quantities and in simplifying computations
- Interesting and useful to recast our 3PM result (in N=8 SUGRA) in an explicitly analytic & xing-symmetric form.
- We got the following result (DHRV, to appear)

Defining as usual:

$$\sigma = \frac{s - m_1^2 - m_2^2}{2m_1 m_2} \geq 1$$

$$x = \sigma - \sqrt{\sigma^2 - 1} \leq 1$$

in s-channel physical region

and their crossing counterparts:

$$\bar{\sigma} = \frac{u - m_1^2 - m_2^2}{2m_1 m_2} = -(\sigma - \frac{q^2}{2m_1 m_2})$$

$$\bar{x} = \bar{\sigma} - \sqrt{\bar{\sigma}^2 - 1} = -\frac{1}{x}\left(1 - \frac{q^2/(2m_1 m_2)}{\sqrt{\sigma^2 - 1}}\right) + O(q^4)$$

we found

$$A_2(s,q^2) = \frac{(8\pi G_N)^3}{(4\pi)^4}\left(\frac{4\pi e^{-\gamma_E}}{q^2}\right)^{2\epsilon}(\sigma^4+\bar\sigma^4)\hat A_2(s,q^2)$$

$$\hat A_2(s,q^2) = \frac{\hat A_2^{(2)}(s,q^2)}{\epsilon^2} + \frac{\hat A_2^{(1)}(s,q^2)}{\epsilon}$$

(O(1/ε in b)

(O(1) in b)

where

$$\boxed{\begin{aligned}\hat A_2^{(2)}(s,q^2) = \Bigg[&-\frac{8\pi^2 m_1^4 m_2^4}{q^2}\left(\frac{\sigma^2}{\sigma^2-1}+\frac{\bar\sigma^2}{\bar\sigma^2-1}\right)\\ &-8m_1^3m_2^3\Bigg(\frac{\sigma}{(\sigma^2-1)^2}\log^2(-x)+\frac{\bar\sigma}{(\bar\sigma^2-1)^2}\log^2(-\bar x)\\ &+2\left[\frac{\sigma^2}{(\sigma^2-1)^{3/2}}\log(-x)+\frac{\bar\sigma^2}{(\bar\sigma^2-1)^{3/2}}\log(-\bar x)\right]\Bigg)\Bigg]\end{aligned}}$$

and (here we eliminated barred variables)

real & irrel.

imaginary

log-enh. 1/2 PN

$$
\begin{aligned}
\hat{A}_2^{(1)}(s,q^2) &= 4\pi^2 m_1^2 m_2^2 (m_1^2+m_2^2)\frac{\sigma^2}{(\sigma^2-1)^2} \\
&-32 m_1^3 m_2^3 \frac{\sigma^2}{(\sigma^2-1)^{3/2}}\left(\log(-x)-\log(x)\right) \\
&+4 m_1^3 m_2^3 \frac{\sigma(\sigma^2-2)}{(\sigma^2-1)^2}\left(\log(-x)-\log(x)\right)\left(Li_2(x^2)-Li_2(\frac{1}{x^2})\right) \\
&-\frac{8}{3} m_1^3 m_2^3 \frac{1}{(\sigma^2-1)^{1/2}}\left[\left(\log^3(-x)-\log^3(x)\right)+\pi^2\left(\log(-x)-\log(x)\right)\right] \\
&-8 m_1^3 m_2^3 \frac{\sigma^3}{(\sigma^2-1)^2}\left(\log(-x)-\log(x)\right)\left(\log(-x)+\log(x)\right) \\
&+8 m_1^3 m_2^3 \frac{\sigma^2}{(\sigma^2-1)^{3/2}}\left(\log(-x)-\log(x)\right)\left(\log(1-x^2)+\log(1-\frac{1}{x^2})\right)
\end{aligned}
$$

log-enh. 2 PN

irrel.

1/2 PN

Connecting Re δ_2 and Im δ_2 for RR

- Two contributions stick out since their real parts correspond to half-integer PN (sum is 1.5PN)

$$+4m_1^3m_2^3\frac{\sigma(\sigma^2-2)}{(\sigma^2-1)^2}\left(\log(-x)-\log(x)\right)\left(Li_2(x^2)-Li_2(\frac{1}{x^2})\right)$$

$$+8m_1^3m_2^3\frac{\sigma^2}{(\sigma^2-1)^{3/2}}\left(\log(-x)-\log(x)\right)\left(\log(1-x^2)+\log(1-\frac{1}{x^2})\right)$$

- In both, Re δ_2 and Im δ_2 appear in the combination

$$i\frac{(1-x)^{-2\epsilon}+(x-1)^{-2\epsilon}}{(-2\epsilon)}=i\left[\frac{1}{(-\epsilon)}+2\log(1-x)\right]+\pi+\mathcal{O}(\epsilon)$$

IR sing. in Im δ_2

RR contribution to Re δ_2

Connecting soft theorems to RR?

- The above results imply a new intriguing connection between soft theorems (i.e. the ZFL of $dE^{rad}/d\omega$) and the RR contribution to the 3PM eikonal phase.

$$\boxed{Re\ 2\delta_2^{RR} = -\pi\epsilon\ Im\ 2\delta_2(\epsilon \to 0) = \frac{\pi}{4\hbar}\frac{dE^{rad}}{d\omega}(\omega \to 0)}$$

ZFL (Smarr 1977)

for instance we found:

$$\frac{dE^{rad}}{d\omega}(\omega \ll v/b \to 0) = \frac{32G^3}{5\pi}\frac{m_1^2 m_2^2}{b^2}$$

in agreement with Kovacs & Thorne 1978

3PM eikonal in N=8
(from AEI workshop, see CH's talk)

P-MRZ

$$\mathrm{Re}(\delta_2) = \frac{2G^3(2m_1 m_2 \sigma)^2}{b^2}$$

$$\left[\frac{\sigma^4}{(\sigma^2-1)^2} - \cosh^{-1}(\sigma)\left(\frac{\sigma^2}{\sigma^2-1} - \frac{\sigma^3\left(\sigma^2-2\right)}{(\sigma^2-1)^{5/2}}\right)\right]$$

New

3PM sc. angle N=8
(from AEI workshop, see CH's talk)

$$\chi_{3\mathrm{PM}} = \frac{2G^3(2m_1m_2\sigma)^3}{J^3}\left(S + \frac{(2m_1m_2\sigma)}{s}(B+A+C)\right)$$

$$S = -\frac{1}{3}\frac{\sigma^3}{(\sigma^2-1)^{3/2}}$$

"Schwarzshild."

$$B = -\cosh^{-1}(\sigma)$$

P-MRZ (2PN)

cancel @ large σ

$$A = \frac{\sigma^2}{\sigma^2-1}$$

ACV-limit

$$C = \cosh^{-1}(\sigma)\frac{\sigma(\sigma^2-2)}{(\sigma^2-1)^{3/2}}$$

1.5PN

Connection with Damour 2010.01641?

- In 2010.01641 Damour has derived the RR part of the scattering angle in GR using a smart shortcut.
- Used a previous result with Bini (1210.2834) relating RR to loss of energy and angular momentum
- He argued that, at 3PM, only latter enters
- He then computed J^{rad} to $O(G^2)$ and got the 3PM RR correction to the BCRSSZ deflection angle recovering smoothness and the ACV90 UR limit.
- Our method provides instead the RR correction to the eikonal phase Re δ_2 directly from the ZFL of the energy-loss-spectrum at $O(G^3)$.
- We understood mathematically why the two calculations lead to the same result

- We have used the DHRV connection to re-derive Damour's result in GR. Our method computes directly Re δ_2 instead of the scattering angle. Result:

1PN

2PN(BCRSSZ)

$$2Re\delta_2 = \frac{2G^3 m_1 m_2 s}{\hbar b^2(\sigma^2-1)^{3/2}}(12\sigma^4 - 10\sigma^2 + 1)$$
$$-\frac{4G^3 m_1^2 m_2^2}{\hbar b^2(\sigma^2-1)^{1/2}}\left(\frac{\sigma(14\sigma^2+25)}{3} + (4\sigma^4 - 12\sigma^2 - 3)\frac{cosh^{-1}(\sigma)}{(\sigma^2-1)^{1/2}}\right)$$
$$+\frac{2G^3 m_1^2 m_2^2 (2\sigma^2-1)^2}{\hbar b^2(\sigma^2-1)^2}\left(\frac{8-5\sigma^2}{3} + \sigma(2\sigma^2-3)\frac{cosh^{-1}(\sigma)}{(\sigma^2-1)^{1/2}}\right)$$

2.5PN

UR-limit: log s terms becomes subleading &

$$(48 - 56/3 - 40/3)\sigma^2 = 16\sigma^2$$ => ACV90!

- Another check would be to use TD's approach for N=8 SUGRA and obtain the DHRV result for the RR contribution to the scattering angle;
- Instead, we checked our (equivalent) connection between the soft limit of radiated energy and the RR.
- In N=8 the total radiated energy comes from the graviton, the dilaton, 2 vectors, and 2 scalars.
- They add up to reproduce the correct RR term in Re δ_2:

2.5PN 1.5PN

$$\frac{1}{3}\left[\overbrace{(8-5\sigma^2+3\sigma(2\sigma^2-3)F(\sigma))}^{\text{grav.}}+\overbrace{(\sigma^2+2-3\sigma F(\sigma))}^{\text{dil.}}+\overbrace{8(\sigma^2-1)}^{\text{vect.}}+\overbrace{2(\sigma^2-1)}^{\text{scal.}}\right]$$

$$\boxed{=2(\sigma^2+\sigma(\sigma^2-2)F(\sigma))}\,;\;\; F(\sigma)=\frac{\cosh^{-1}(\sigma)}{\sqrt{\sigma^2-1}}$$

Summarizing

- We have seen how the inclusion of the full soft region allows for the eikonal phase and the (physical) scattering angle, including RR terms, to be continuous functions of $p/m=v(1-v^2)^{-1/2}$ so that Newtonian and UR classical limits get smoothly connected (meet!)
- We have uncovered (via analyticity) an intriguing relation between the RR terms and the $O(G^3)$ ZFL of the $dE^{rad}/d\omega$ spectrum controlled by soft theorems.
- This looks mathematically equivalent to Damour's recent connection between RR and radiated angular momentum @ $O(G^2)$, but the precise physical reason for the equivalence remains to be understood.

From Amplitudes (Dublin, July 2019)

- Eventually, one would like to extend these results to arbitrary masses and kinematics and to combine them with recent ones on the conservative gravitational potential at 3PM level, leading hopefully to a full understanding of gravitational scattering and radiation at that level.

Thank you!