

Towards on-shell electroweak EFT(s)

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YS, Yaniv Weiss 1809.09644

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develop an on-shell approach to EFT (=EFfective Theory*) extensions of the standard model

2 main themes:

- toolbox for EFT computations, analyses
- an on-shell understanding of EWSB/Higgs mechanism: how does $SU(2) \times U(1)$ emerge? relations between different masses?

bottom-up (bootstrap) approach: can explore general EFT extensions of standard model: weakly/strongly coupled

* credit: Baratella Fernandez von Harling Pomarol

Outline

- ▶ Motivation for going on-shell
- ▶ Massive spinors
- ▶ Warm-up example
- ▶ Spinor bases
- ▶ The electroweak EFT:
 - ▶ Smooth high energy behavior and perturbative unitarity
 - ▶ Relearning elements of EWSB

Our current (massive) toolbox

general 3- and 4-point amplitudes of massive/massless particles of various spins:

[Durieux Kitahara YS Weiss](#); [Durieux Machado](#); [Durieux Kitahara Machado YS Weiss](#)

electroweak: spins 0, 1/2, 1

- ▶ detailed gluing prescription (massive spin 1/2, 1)
- ▶ bases for all massive 3-points
- ▶ bases for all massive 4-points (any dimension): generic amplitudes, contact-terms
- ▶ matching to broken-phase SMEFT 3-point (+) couplings

beyond electroweak:

- ▶ bases for all massive 3-point $\text{spin} \leq 3$
- ▶ bases for all 4-point massless contact terms $\text{spin} \leq 2$; higher points for spins 0, 1/2, 1, 2, $\text{dim} \leq 8$

See also: [Christensen Field](#); [Christensen Field Moore Pinto](#)
[Herderschee Koren Trott](#); [Aoude Machado](#); [Bachu Yellespur..](#)

On-shell Effective theories: motivation

There has been a lot of activity on EFT extensions of SM recently, both theory & experiment: developing global analyses of observables, different types of EFTs: SMEFT, HEFT..

a bit sad reflecting our ignorance about the origin of EWSB
& the attendant Naturalness problem:

also good: ignorance acknowledged, can hope to learn something new..

→ **bottom-up EFTs**

a well-defined framework for quantifying this ignorance
= parametrizing deviations from SM

On-shell Effective theories: motivation

input:

- ▶ SM fields [+ possibly: some BSM fields]
- ▶ SM symmetry (global, gauge)
- ▶ + Lorentz, locality

→ most general Lagrangian

$$\mathcal{L}_{\text{EFT}} = \sum_i \frac{c_i}{\Lambda^{n_i}} \mathcal{O}_i$$

On-shell Effective theories: motivation

→ **bottom-up** (bootstrap) on-shell approach is very natural:

input:

- ▶ SM particles [+ possibly: some BSM particles]
- ▶ symmetry (global, gauge)
- ▶ + Lorentz, locality

→ most general amplitudes:

starting with 3-point amplitudes & gluing these into higher-point/higher-order amplitudes

gauge symmetry emerges from the requirement of consistent interactions of spin-1 particles (Bose statistics, factorization..)

On-shell Effective theories: motivation

1st step of EFT construction: identify basis of operators \mathcal{O}_i (complete, independent)

modulo field redefinitions, EOMs, gauge redundancies

mathematically: polynomials of operators subject to a set of constraints

beautifully solved using Hilbert Series [Jenkins Manohar](#); [Lehman Martin](#); [Henning Lu](#) [Melia Murayama](#)

On-shell Effective theories: motivation

on-shell construction: only deal with physical quantities

→ this 1st step is mostly circumvented (field redefinitions, gauge redundancies)

the remainder simplifies considerably:

polynomials of **operators** subject to a set of constraints (EOM, momentum conservation)

maps to:

polynomials in the Mandelstam invariants (just numbers..) subject to a set of kinematical constraints, e.g., $s + t + u = \sum m^2$

used in Henning Lu Melia Murayama

by-product: counting & classification of EFT operators

On-shell Effective theories: motivation

- SM applications:

$$\mathcal{L}_{\text{EFT}} = \sum_i \frac{c_i}{\Lambda^{n_i}} \mathcal{O}_i$$

typically: start with full $SU(2) \times U(1)$ (SMEFT)

EWSB: \mathcal{O}_i modify masses, SM couplings

→ redefine input parameters

on-shell construction: relate physical observables (largely), directly measurable

future work

- [computation: avoid Feynman diagrams, particularly complicated with EFT vertices]

on-shell EFTs for the SM

How on-shell do you want to be? or: can you be half on-shell?

yes: work in unbroken electroweak symmetry phase

impose full $SU(2) \times U(1)$ massless particles only

- ▶ good approximation at high-energies
- ▶ all you care about for running, anomalous dimensions
- ▶ mostly used, many powerful results: classification of operators, selection rules, operator mixings, anomalous dimensions, positivity..

Chuang Shen; Azatov Contino Machado Riva; Bern Parra-Martinez Sawyer; Ma Shu Xiao; Baratella Fernandez von Harling Pomarol; Gu Wang Zhang; Falkowski; ..

on-shell EFTs for the SM

focus here: fully on-shell:

work with SM particles: massive W , Z , h ..

want to exploit full power of on-shell approach in relating purely physical observables

also motivated by aforementioned ignorance: back to basics: work with what we know is there

(MASSIVE) EFT AMPLITUDE BASICS

EFT AMPLITUDE BASICS

- Little Group selection rules + spin-statistics determine all n -point Contact Terms (CT) (≥ 3)

3-points: generically complex momenta

- Remaining parts of amplitudes determined by factorization/generalized unitarity

→ bootstrap higher point amplitudes by gluing together lower-point amplitudes + adding CTs

so: derive 3-point amplitudes

get 4-point amplitudes by gluing two 3-points + adding 4-pt CT . . .

CTs \leftrightarrow EFT operators

unknown coefficients of CTs \leftrightarrow unknown Wilson coefficients

0th order task: find CTs

Writing massive amplitudes: LG covariant massive formalism

Arkani-Hamed Huang Huang; Conde Marzolla

- decompose massive momentum in terms of two massless momenta:

$$p = p^{I=1} + p^{I=2}$$

LG [SU(2)] rotates:

$$|\mathbf{p}^I\rangle \rightarrow W_J^I |\mathbf{p}^J\rangle, \quad \text{and} \quad [\mathbf{p}_I| \rightarrow (W^{-1})_I^J [\mathbf{p}_J|.$$

$$I, J = 1, 2$$

- external leg of spin s :

spin s rep can be obtained as a symmetric combination of spin-1/2 reps

$$\mathcal{M}^{I_1 \dots I_{2s}} = |\mathbf{p}\rangle_{\alpha_1}^{I_1} \dots |\mathbf{p}\rangle_{\alpha_{2s}}^{I_{2s}} M^{\{\alpha_1 \dots \alpha_{2s}\}} = |\mathbf{p}\rangle_{\dot{\alpha}_1}^{I_1} \dots |\mathbf{p}\rangle_{\dot{\alpha}_{2s}}^{I_{2s}} \tilde{M}^{\{\dot{\alpha}_1 \dots \dot{\alpha}_{2s}\}}$$

LG indices symmetrized

bold notation: symmetrization over LG indices, e.g. $|\mathbf{p}\rangle [\mathbf{p}| = |p^{\{I}\rangle} [p^{J}]|$

will use boldface for massive momenta too to distinguish them from massless ones

→ amplitudes are expressed in terms of spinor products $\langle ij \rangle, [ij]$
(possibly with momentum insertions)

High-energy limit: massive \rightarrow massless (unbolding)

$$\mathbf{p} = p^1 + p^2$$

choosing the direction of p^1 = choosing spin polarization axis

convenient to recover helicity states in high-energy limit:

take p^1 to coincide with massless limit of \mathbf{p}

then e.g., for a vector of **positive** polarization $I = J = 1$:

$$|\mathbf{p}] |\mathbf{p}] \rightarrow |p] |p]$$

(all others vanish, e.g. $|\mathbf{p}] |\mathbf{p}\rangle \rightarrow 0$)

for a vector of **zero** polarization $(I, J) = (1, 2)$:

$$|\mathbf{p}] |\mathbf{p}\rangle \rightarrow |p] |p\rangle$$

\rightarrow get massless amplitudes simply by **unbolding**

Let's put this to work

Example: vector + 3-gluon amplitude

Z' = SM singlet

YS Weiss 2018

easy to parameterize amplitude up to high dim: here all + helicities and $d = 12$

$$\begin{aligned} \mathcal{M}(Z'(\mathbf{p}_4; g^{a-}(p_1); g^{b-}(p_2); g^{c+}(p_3))) \\ = d^{abc} \langle 12 \rangle^2 \times \left[[34]^2 A + [13][23] \langle 14 \rangle \langle 24 \rangle B + [34] \left([31] \langle 14 \rangle - [32] \langle 24 \rangle \right) C \right] \\ + f^{abc} \langle 12 \rangle^2 \times \left[[34]^2 D + [13][23] \langle 14 \rangle \langle 24 \rangle E + [34] \left([31] \langle 14 \rangle - [32] \langle 24 \rangle \right) F \right] \end{aligned}$$

LG \rightarrow [\dots] \dots [\dots]

A, B, \dots polynomials in invariants; subject to $s_{12} + s_{13} + s_{23} = m_4^2$

Example: vector + 3-gluon amplitude

$$\begin{aligned} A &= \frac{d_8}{\Lambda^4} + \frac{d_{10}^{(1)}}{\Lambda^6} s_{12} + \frac{d_{12}^{(1)} s_{12}^2 + d_{12}^{(2)} s_{13} s_{23}}{\Lambda^8} & B &= \frac{m^2 s_{12} d_{12}^{(6)}}{\Lambda^8} \\ C &= (s_{13} - s_{23}) \frac{m d_{12}^{(5)}}{\Lambda^8} & D &= (s_{23} - s_{13}) \left(\frac{d_{10}^{(3)}}{\Lambda^6} + \frac{d_{12}^{(4)}}{\Lambda^8} s_{12} \right) \\ E &= 0 & F &= \frac{m d_{10}^{(2)}}{\Lambda^6} + \frac{m d_{12}^{(3)}}{\Lambda^8} s_{12} \end{aligned}$$

Example: vector + 3-gluon amplitude

unbolding \rightarrow massless amplitudes:

$$\begin{aligned} & \mathcal{M}(Z'; g^{a-}(p_1); g^{b-}(p_2); g^{c+}(p_3)) \\ &= d^{abc} \langle 12 \rangle^2 \times \left[[34]^2 A + [13][23] \langle 14 \rangle \langle 24 \rangle B + [34] \left([31] \langle 14 \rangle - [32] \langle 24 \rangle \right) C \right] \\ &+ f^{abc} \langle 12 \rangle^2 \times \left[[34]^2 D + [13][23] \langle 14 \rangle \langle 24 \rangle E + [34] \left([31] \langle 14 \rangle - [32] \langle 24 \rangle \right) F \right] \end{aligned}$$

POSITIVE

NEGATIVE

ZERO

TOWARDS ELECTROWEAK EFTs: BASES

Durieux, Kitahara, YS, Weiss

Durieux, Kitahara, Machado, YS, Weiss

Need bases for $n = 3, n \geq 4$ amplitudes with $s = 0, 1/2, 1$

in terms of massive spinor structures*

* spinor structures = no prefactors containing Lorentz invariants like \tilde{s}_{ij}
only: $[ij], \langle ijk \rangle, [ijk] ..$

Bases: 3-points

Bases for all 3-point massive amplitudes were presented in Arkani-Hamed Huang Huang but they are over-complete in some cases

We have derived (independent) bases for all 3-points with $s_i \leq 3$

Bases: 4-points and higher

Different types of bases of interest:

- Spinor-structure basis: minimal set of spinor structures needed to span a generic amplitude

number of elements: $\prod_i (2s_i + 1)$

[Schomerus Sobko Isachenkov; Kravchuk Simmons-Duffin; Henning Lu Melia Murayama](#)

Here interested in writing the amplitudes in terms of massive spinors variables.

Equivalently, can write bases in terms of usual polarizations

[Bonifacio Hinterbichler](#)

Bases: 4-points and higher

denote a spinor structure by $S^{\{I\}}$
 $\{I\}$ collectively denotes LG indices

convenient to construct inner products

$$(S_1, S_2) \equiv \sum_{\{I\}} S_1^{\{I\}*} S_2^{\{I\}}$$

given by functions of the invariants

massless \leftrightarrow massive

Construct the spinor structure basis by starting with the massless basis and bolding **super easy**

- classify spinor structures according to “helicity categories” = helicities of high-energy limits = unbolded versions

- massless limit: spinor-structures of different helicities: inner-product=0

→ $\prod_i (2s_i + 1)$ independent structures

→ bolding them gives the massive basis

works because: bolding = covariantizing wrt massive little group

note: adding extra scalars doesn't change basis: so have a simple prescription for finding bases of $\mathcal{M}(s_1, s_2, s_3, s_4, 0, 0, \dots, 0)$

Bases for EFT amplitudes

To span contact terms only, want a manifestly local form: no negative powers of invariants

Typically, a larger set than in spinor-structure basis

- ▶ Stripped contact term (SCT) basis: set of independent spinor structures (cannot be written as linear combinations of other structures with prefactors that non-negative powers of the invariants)
- ▶ Contact term basis = (SCT) x polynomials in the invariants

Can still rely on massless limit but not as directly

4-point SCT basis

spins	n_{SCT}	n_s	hel. cat.	spinor structures	n_{para}	$\min\{d_{\text{op}}\}$
<i>ssss</i>	1	1	(0000)	constant	1	4
<i>vsss</i>	4 → 3	3	(0000) (+000)	$[121], [131]$ $[1231] \rightarrow [1231] - (1231)$	1 $\beta \rightarrow 1$	5 7
<i>ffss</i>	4	4	(+ + 00) (+ - 00)	$[12]$ $[132]$	2 2	5 6
<i>vvss</i>	10 → 9	9	(0000) (+000) (+ + 00) (+ - 00)	$12, [131][232]$ $[12][132]$ $[12]^2$ $[132]^2 \rightarrow [132]^2 - (132)^2$	1 4 2 $\beta \rightarrow 1$	4,6 6 6 8
<i>ffvs</i>	14 → 12	12	(+ + 00) (+ - 00) (+ + 00) (+ - 00) (+ - 00)	$[12]([313], [323])$ $[13][23]$ $[13][23]$ $[12][3123] \rightarrow \phi$ $[13][312]$	2 2 2 $\beta \rightarrow 0$ 4	6 5 6 8 7
<i>ffff</i>	18	16	(+ + + +) (+ + - -) (+ + - -)	$[12][34], [13][24]$ $[12](34)$ $[12][324]$	2 6 8	6 6 7
<i>vvvs</i>	35 → 29	27	(0000) (+000) (+ + 00) (+ - 00) (+ + 00) (+ - 00)	$[12][343](12), [13][242](13), [23][141](23)$ $[12][13](23)$ $[12]^2 \{ [313], [323] \}$ $[13][132](23)$ $[12][13][23]$ $[12]^2(3123) \rightarrow \phi$	1 6 6 6 2 $\beta \rightarrow 0$	5 5 7 7 7 9
<i>veff</i>	46 → 38	36	(00 + +) (00 - -) (0 - + +) (0 + - -) (0 + - -) (+ + + -) (+ + + +) (- + + +) (+ + - -) (+ - - +)	$(12) \times \{ [12][34], [13][24] \}$ $(14)[231][23], (24)[132][13]$ $(12)[34](241) \rightarrow (12)[34]([241]/m_1 - (142)/m_2)$ $(132) \times \{ [12][34], [13][24] \}$ $(14)[12][23]$ $[12]^2(314)$ $[12] \times \{ [12][34], [13][24] \}$ $(1231)[23][24] \rightarrow \phi$ $[12]^2(34)$ $[14][132](23) \rightarrow [14][132](23) - [24][231](13)$	2 2 $\beta \rightarrow 2$ 4 8 4 2 $\beta \rightarrow 0$ 2 $\beta \rightarrow 2$	5 6 7 7 6 8 7 9 7 8
<i>vvvv</i>	116 → 85	81	(0000) (+000) (+ + 00) (+ - 00) (+ + 00) (+ - 00) (+ + + +) (+ + + +) (+ + + +) (+ + - -)	$\{ [12][34], [13][24] \} \times \{ (12)(34), (13)(24) \}$ $\{ [12][34], [13][24] \} \times [142](34) \rightarrow \dots$ $\{ [12][34], [13][24] \} \times [12](34)$ $[13][14](23)(24)$ $\{ [12][34], [13][24] \} \times [23][134]$ $[12]^2(34)(324) \rightarrow [12]^2(34)([324]/m_4 - (423)/m_3) \rightarrow \dots$ $[12]^2[34]^2, [12][13][24][34], [13]^2[24]^2$ $[12][13][23](4124) \rightarrow \phi$ $[12]^2(34)^2$	1 $\beta \rightarrow 6$ 12 12 8 $\beta \rightarrow 2$ 2 2 $\beta \rightarrow 0$ 6	4 6 6 6 8 8 8 8 10 8

ELECTROWEAK EFTs: some examples

The WWZ coupling

Three nearly-degenerate spin-1 particles: $M_Z^2 - M_W^2 \ll M_W^2$

What do we learn from the WWZ coupling?

Let's first neglect the mass difference

The most general 3-point amplitude (for three degenerate spin-1):

The WWW coupling

$$\begin{aligned} & \mathcal{M}(\mathbf{1}_W^a, \mathbf{2}_W^b, \mathbf{3}_W^c) \\ & \frac{g}{m_W^2} \left(\langle \mathbf{12} \rangle \langle \mathbf{13} \rangle [\mathbf{23}] + \langle \mathbf{12} \rangle [\mathbf{13}] \langle \mathbf{23} \rangle + [\mathbf{12}] \langle \mathbf{13} \rangle \langle \mathbf{23} \rangle \right. \\ & \quad \left. + \langle \mathbf{12} \rangle [\mathbf{13}] [\mathbf{23}] + [\mathbf{12}] \langle \mathbf{13} \rangle [\mathbf{23}] + [\mathbf{12}] [\mathbf{13}] \langle \mathbf{23} \rangle \right) \\ & \quad + \frac{c_6^L}{\Lambda^2} \langle \mathbf{12} \rangle \langle \mathbf{13} \rangle \langle \mathbf{23} \rangle + \frac{c_6^R}{\Lambda^2} [\mathbf{12}] [\mathbf{13}] [\mathbf{23}] \end{aligned}$$

completely antisymmetric \rightarrow completely antisymmetric prefactor

The WWW coupling

$$\mathcal{M}(\mathbf{1}_W^a, \mathbf{2}_W^b, \mathbf{3}_W^c) = \epsilon^{abc} \left\{ \begin{aligned} & \frac{g}{m_W^2} \left(\langle \mathbf{12} \rangle \langle \mathbf{13} \rangle [\mathbf{23}] + \langle \mathbf{12} \rangle [\mathbf{13}] \langle \mathbf{23} \rangle + [\mathbf{12}] \langle \mathbf{13} \rangle \langle \mathbf{23} \rangle \right. \\ & \quad \left. + \langle \mathbf{12} \rangle [\mathbf{13}] [\mathbf{23}] + [\mathbf{12}] \langle \mathbf{13} \rangle [\mathbf{23}] + [\mathbf{12}] [\mathbf{13}] \langle \mathbf{23} \rangle \right) \\ & \quad \left. + \frac{c_6^L}{\Lambda^2} \langle \mathbf{12} \rangle \langle \mathbf{13} \rangle \langle \mathbf{23} \rangle + \frac{c_6^R}{\Lambda^2} [\mathbf{12}] [\mathbf{13}] [\mathbf{23}] \right\} \end{aligned} \right.$$

SU(2) structure emerges! with n degenerate particles: $\epsilon^{abc} \rightarrow$ completely AS tensor

to see full Lie group structure need to consider proper factorization of 4-point amplitude \rightarrow Jacobi identity (as in massless case)

- complete (anti)symmetry of massive 3-vector amplitude is more transparent than massless case, e.g., $[\mathbf{12}]^3 / [\mathbf{13}][\mathbf{23}]$ of $++-$
- masses smooth the singular massless behavior

The WWW coupling

EM charge conservation follows: since the WWW spinor structure is fully antisymmetric \rightarrow vanishes for identical W 's:
 W^+W^+ amplitude vanishes

can see full $SU(2) \times U(1)$ emerge in high-energy limit:

WWZ and $WW\gamma$:

define combination B of Z and γ for which $WWB = 0$

couplings of $3V, Vff, VSS$ related by requirement of consistent factorization.

Renormalizable vs Non-renormalizable (WWW)

identify NR 3-point couplings via “high-energy” behavior (complex momenta)
 $\mathcal{O}(E)$ is good $\mathcal{O}(E^2)$ or higher is bad

$$\begin{aligned} & \frac{g}{m_W^2} \left(\langle \mathbf{12} \rangle \langle \mathbf{13} \rangle [\mathbf{23}] + \langle \mathbf{12} \rangle [\mathbf{13}] \langle \mathbf{23} \rangle + [\mathbf{12}] \langle \mathbf{13} \rangle \langle \mathbf{23} \rangle \right. \\ & \qquad \qquad \qquad \left. + \langle \mathbf{12} \rangle [\mathbf{13}] [\mathbf{23}] + [\mathbf{12}] \langle \mathbf{13} \rangle [\mathbf{23}] + [\mathbf{12}] [\mathbf{13}] \langle \mathbf{23} \rangle \right) \\ & + \frac{c_6^L}{\Lambda^2} \langle \mathbf{12} \rangle \langle \mathbf{13} \rangle \langle \mathbf{23} \rangle + \frac{c_6^R}{\Lambda^2} [\mathbf{12}] [\mathbf{13}] [\mathbf{23}] \end{aligned}$$

at HE:

$\langle \mathbf{12} \rangle \langle \mathbf{13} \rangle [\mathbf{23}] \sim m_W^2 E \rightarrow$ renormalizable
and has smooth limit with $1/m_W^2$ normalization

$[\mathbf{12}] [\mathbf{13}] [\mathbf{23}] \sim E^3 \rightarrow$ requires cutoff $\rightarrow 1/\bar{\Lambda}^2$ normalization

Renormalizable vs Non-renormalizable

identify NR 3-point couplings via “high-energy” behavior (complex momenta)
 $\mathcal{O}(E)$ is good $\mathcal{O}(E^2)$ or higher is bad

3-point version of perturbative unitarity:

Llewellyn Smith; Joglekar; Cornwall Levin Tiktopoulos; Lee Quigg Thacker; Chanowitz Gaillard '70s

$$\mathcal{M}_4 \sim \mathcal{M}_3 \frac{1}{E^2} \mathcal{M}_3$$

→ 3-point amplitude can grow like E , but E^2 requires cutoff

equivalently: want smooth behavior as $m \rightarrow 0$: E/m is bad, $E/\bar{\Lambda}$ is ok

implicitly assume a hierarchy between m and $\bar{\Lambda}$

note: 3-point amplitudes are exact expression: kinematics is fixed
($p_i \cdot p_j$ determined by masses) → in matching to field theory computation:
coefficients sum all loop orders and v/Λ expansion

fermion-fermion- Z coupling

$$\mathcal{M}(\mathbf{1}_{\psi^c}, \mathbf{2}_{\psi}, \mathbf{3}_Z) = \frac{g_R}{m_Z} \langle \mathbf{13} \rangle [\mathbf{23}] + \frac{g_L}{m_Z} [\mathbf{13}] \langle \mathbf{23} \rangle + \frac{c_R}{\Lambda} [\mathbf{13}] [\mathbf{23}] + \frac{c_L}{\Lambda} \langle \mathbf{13} \rangle \langle \mathbf{23} \rangle$$

high energy : $\mathcal{O}(m_Z E)$

$\mathcal{O}(E^2)$

Relearn some QFT (Higgsing)

or really the interactions of massive fermions and vectors

requiring a smooth high-energy limit of separate helicity amplitudes:

- ▶ fermions with a vector-like coupling to the Z do not couple to its longitudinal component
- ▶ the (chiral) coupling of a fermion to the longitudinal Z is proportional to the fermion mass
- ▶ the coupling of a massive fermion to a massless vector is vector-like
- ▶ The mass of a fermion with chiral couplings to a vector must tend to 0 at least as fast as the vector mass [see now](#)
- ▶ ...

Relearn some QFT (Higgsing)

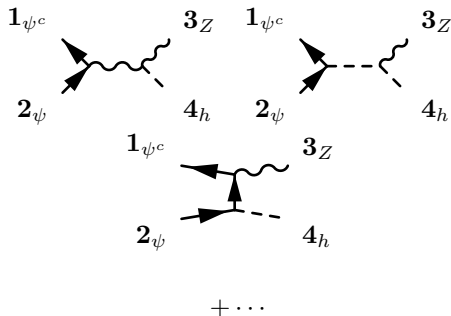
$$\mathcal{M}(\mathbf{1}_{\psi^c}^-, \mathbf{2}_{\psi}^-, \mathbf{3}_Z^+) \sim (g_L - g_R) \frac{m_\psi}{m_Z} \langle 12 \rangle$$

so:

- ▶ either: $g_L - g_R = 0$ **vectorlike** fermion
- ▶ or: m_ψ goes to 0 with m_Z **chiral fermion mass has same origin as vector mass** and goes to zero at least as fast refine when talk about 4-point amplitude

Relearn some QFT (Higgsing): fermion-fermion- Z -higgs amplitude

factorizable part featuring 3-point couplings:



+ new contact terms

fermion-fermion- Z -higgs amplitude

$$\begin{aligned}\mathcal{M}^{\text{contact}}(\mathbf{1}_{\psi^c}, \mathbf{2}_{\psi}, \mathbf{3}_Z, \mathbf{4}_h) &= \frac{C_6^{(1)}}{\bar{\Lambda}^2} [\mathbf{13}][\mathbf{23}] + \frac{C_6^{(2)}}{\bar{\Lambda}^2} [\mathbf{13}]\langle\mathbf{23}\rangle \\ &+ \frac{C_7^{(1)}}{\bar{\Lambda}^3} [\mathbf{312}]\langle\mathbf{13}\rangle + \frac{C_7^{(2)}}{\bar{\Lambda}^3} [\mathbf{321}]\langle\mathbf{23}\rangle \\ &+ \text{angle} \leftrightarrow \text{square}\end{aligned}$$

most general form

each coefficient is a series, e.g.,

$$C_6^{(1)} = c_6^{(1)} + c_8^{(1,1)} \frac{\tilde{s}_{12}}{\bar{\Lambda}^2} + c_8^{(1,2)} \frac{\tilde{s}_{13}}{\bar{\Lambda}^2} + \dots \quad [\tilde{s}_{ij} \equiv 2p_i \cdot p_j]$$

this expression captures all orders in $v/\bar{\Lambda}$

will come back to this but for now look at high-energy behavior

high-energy behavior

require good high-energy behavior: perturbative unitarity: $\mathcal{O}(E)$ growth accompanied by $1/\bar{\Lambda}$ 4-point: genuine energy growth

→ coefficient of $\mathcal{O}(E)$ term vanishes

→ relates fermion-fermion- Z coupling (gauge); fermion-fermion-higgs coupling (Yukawa); ZZh coupling:

$$(g_L - g_R) \left(g_{ZZh} \frac{m_\psi}{m_Z} - y \right) = 0 + \mathcal{O}(m/\bar{\Lambda})$$

- ▶ for a vectorlike fermion $g_L = g_R$: no constraint on mass
- ▶ for a chiral fermion $g_L \neq g_R$:

$$m_\psi = 2 \frac{y}{g_{ZZh}} m_Z$$

fermion mass from Higgs mechanism!

see also Maltoni Mantani Mimasu '19

Conclusions and outlook

- on-shell approach is natural for constructing bottom-up “EFT” extensions of low-energy SM
 - = all possible couplings of SM particles consistent with Lorentz, global symmetries, locality and unitarity
- compact analytic expressions for amplitudes: all orders in v/Λ expansion, straightforward to go to higher orders in derivative expansion (non-trivial part is the spinor structure)
- $SU(2)\times U(1)$ structure & Higgs mechanism beautifully emerge from Lorentz + unitarity/good high energy limit for v hierarchically smaller than Λ
- truly bottom-up: can explore extensions beyond SMEFT, extra Higgses, new particles

Conclusions and outlook

- systematic derivation of $n > 4$ SCT bases
- massive recursions
- include loops—running
- global analysis in terms of on-shell quantities

Ochirov; Franken Schwinn

The Christmas-Tree Table (3-points for $\text{spin} \leq 3$)

n_1	n_2	n_3	$n^{3\text{pt}}$	n_{tot}	spinor structures
0	0	0	1		constant
0	0	1	1		$[3(1-2)3]$
0	0	2	1		$[3(1-2)3]^2$
0	0	3	1		$[3(1-2)3]^3$
0	1/2	1/2	2		$([23], [23])$
0	1/2	3/2	2		$[3(1-2)3] \otimes ([23], [23])$
0	1/2	5/2	2		$[3(1-2)3]^2 \otimes ([23], [23])$
0	1	1	3		$([23]^2, [23][23], [23]^2)$
0	1	2	3		$[3(1-2)3] \otimes ([23]^2, [23][23], [23]^2)$
0	1	3	3		$[3(1-2)3]^2 \otimes ([23]^2, [23][23], [23]^2)$
0	3/2	3/2	4		$([23]^3, [23][23]^2, [23]^2[23], [23]^3)$
0	3/2	5/2	4		$[3(1-2)3] \otimes ([23]^3, [23][23]^2, [23]^2[23], [23]^3)$
0	2	2	5		$([23]^4, [23][23]^3, [23]^2[23]^2, [23]^3[23], [23]^4)$
0	2	3	5		$[3(1-2)3] \otimes ([23]^4, [23][23]^3, [23]^2[23]^2, [23]^3[23], [23]^4)$
0	5/2	5/2	6		$([23]^5, [23][23]^4, [23]^2[23]^3, [23]^3[23]^2, [23]^4[23], [23]^5)$
0	3	3	7		$([23]^5, [23][23]^4, [23]^2[23]^3, [23]^3[23]^2, [23]^4[23]^2, [23]^5[23], [23]^5)$
1/2	1/2	1	4		$([23], [23]) \otimes ([13], [13])$
1/2	1/2	2	4		$[3(1-2)3] \otimes ([23], [23]) \otimes ([13], [13])$
1/2	1/2	3	4		$[3(1-2)3]^2 \otimes ([23], [23]) \otimes ([13], [13])$
1/2	1	3/2	6		$([23]^2, [23][23], [23]^2) \otimes ([13], [13])$
1/2	1	5/2	6		$[3(1-2)3] \otimes ([23]^2, [23][23], [23]^2) \otimes ([13], [13])$
1/2	3/2	2	8		$([23]^3, [23][23]^2, [23]^2[23], [23]^3) \otimes ([13], [13])$
1/2	3/2	3	8		$[3(1-2)3] \otimes ([23]^3, [23][23]^2, [23]^2[23], [23]^3) \otimes ([13], [13])$
1/2	2	5/2	10		$([23]^4, [23][23]^3, [23]^2[23]^2, [23]^3[23], [23]^4) \otimes ([13], [13])$
1/2	5/2	3	12		$([23]^5, [23][23]^4, [23]^2[23]^3, [23]^3[23]^2, [23]^4[23], [23]^5) \otimes ([13], [13])$
1	1	1	7	1	$([12], [12]) \otimes ([23], [23]) \otimes ([13], [13])$
1	1	2	9		$([23]^2, [23][23], [23]^2) \otimes ([13]^2, [13][13], [13]^2)$
1	1	3	9		$[3(1-2)3] \otimes ([23]^2, [23][23], [23]^2) \otimes ([13]^2, [13][13], [13]^2)$
1	3/2	3/2	10	2	$([12], [12]) \otimes ([23]^2, [23][23], [23]^2) \otimes ([13], [13])$
1	3/2	5/2	12		$([23]^3, [23][23]^2, [23]^2[23], [23]^3) \otimes ([13]^2, [13][13], [13]^2)$
1	2	2	13	3	$([12], [12]) \otimes ([23]^3, [23][23]^2, [23]^2[23], [23]^3) \otimes ([13], [13])$
1	2	3	15		$([23]^4, [23][23]^3, [23]^2[23]^2, [23]^3[23], [23]^4) \otimes ([13]^2, [13][13], [13]^2)$
1	5/2	5/2	16	4	$([12], [12]) \otimes ([23]^4, [23][23]^3, [23]^2[23]^2, [23]^3[23], [23]^4) \otimes ([13], [13])$
1	3	3	19	5	$([12], [12]) \otimes ([23]^5, [23][23]^4, [23]^2[23]^3, [23]^3[23]^2, [23]^4[23], [23]^5) \otimes ([13], [13])$
3/2	3/2	2	14	4	$([12], [12]) \otimes ([23]^2, [23][23], [23]^2) \otimes ([13]^2, [13][13], [13]^2)$
3/2	3/2	3	16		$([23]^3, [23][23]^2, [23]^2[23], [23]^3) \otimes ([13]^2, [13][13], [13]^2)$
3/2	2	5/2	18	6	$([12], [12]) \otimes ([23]^3, [23][23]^2, [23]^2[23], [23]^3) \otimes ([13]^2, [13][13], [13]^2)$
3/2	5/2	3	22	8	$([12], [12]) \otimes ([23]^4, [23][23]^3, [23]^2[23]^2, [23]^3[23], [23]^4) \otimes ([13]^2, [13][13], [13]^2)$
2	2	2	19	8	$([12]^2, [12][12], [12]^2) \otimes ([23]^2, [23][23], [23]^2) \otimes ([13]^2, [13][13], [13]^2)$
2	2	3	23	9	$([12], [12]) \otimes ([23]^3, [23][23]^2, [23]^2[23], [23]^3) \otimes ([13]^2, [13][13], [13]^2)$
2	5/2	5/2	24	12	$([12]^2, [12][12], [12]^2) \otimes ([23]^3, [23][23]^2, [23]^2[23], [23]^3) \otimes ([13]^2, [13][13], [13]^2)$
2	3	3	29	16	$([12]^2, [12][12], [12]^2) \otimes ([23]^4, [23][23]^3, [23]^2[23]^2, [23]^3[23], [23]^4) \otimes ([13]^2, [13][13], [13]^2)$
5/2	5/2	3	30	18	$([12]^2, [12][12], [12]^2) \otimes ([23]^5, [23][23]^4, [23]^2[23]^3, [23]^3[23]^2, [23]^4[23], [23]^5) \otimes ([13]^2, [13][13], [13]^2)$
3	3	3	37	27	$([12]^3, [12][12]^2, [12]^2[12], [12]^3) \otimes ([23]^3, [23][23]^2, [23]^2[23], [23]^3) \otimes ([13]^2, [13][13]^2, [13]^2[13], [13]^3)$

THANK YOU

Writing amplitudes: massive

$$\varepsilon_{\alpha\dot{\alpha}}^{\{IJ\}} = \sqrt{2} \frac{|\mathbf{p}\rangle [\mathbf{p}|}{m} \quad I=J=1: +1, \quad I=J=2: -1, \quad I \neq J: 0$$

Example: $h + 3g$ amplitude

YS Weiss 2018

1st example: SM singlet + 3 gluons effectively massless

h is spin-0 SM singlet

all + helicities:

LG \rightarrow $[12][23][13]$ up to function of invariants

1,2,3 label gluon momenta

Contact Terms: no poles, any polynomial of s_{12}, s_{23}, s_{13} subject to

$$s_{12} + s_{23} + s_{13} = m_h^2$$

$$\begin{aligned} \mathcal{M}(h; g^{a+}(p_1)g^{b+}(p_2)g^{c+}(p_3)) &= \frac{[12][13][23]}{\Lambda} \left[\right. \\ & f^{abc} \left(\begin{aligned} &+ \frac{c_7}{\Lambda^2} + \frac{c_{11}}{\Lambda^6} (s_{12}s_{23} + s_{13}s_{23} + s_{12}s_{13}) + \frac{c_{13}}{\Lambda^8} s_{12}s_{13}s_{23} \\ &+ d^{abc} \frac{c'_{13}}{\Lambda^8} (s_{12} - s_{13})(s_{12} - s_{23})(s_{13} - s_{23}) \end{aligned} \right) \\ & \left. + \dots \right] \end{aligned}$$

Example: $h + 3g$ amplitude

$$\mathcal{M}(h; g^{a+}(p_1)g^{b+}(p_2)g^{c+}(p_3)) = \frac{[12][13][23]}{\Lambda} \left[f^{abc} \left(+ \frac{c_7}{\Lambda^2} + \frac{c_{11}}{\Lambda^6} (s_{12}s_{23} + s_{13}s_{23} + s_{12}s_{13}) + \frac{c_{13}}{\Lambda^8} s_{12}s_{13}s_{23} + d^{abc} \frac{c'_{13}}{\Lambda^8} (s_{12} - s_{13})(s_{12} - s_{23})(s_{13} - s_{23}) \right) + \dots \right]$$

here shown for $\dim \leq 13$ but trivial to extend: polynomials in s_{ij} 's (symmetric or antisymmetric)

Example: $h + 3g$ amplitude

adding in factorizable part (from gluing ggg and hgg):

$$\begin{aligned} \mathcal{M}(h; g^{a+}(p_1)g^{b+}(p_2)g^{c+}(p_3)) &= \frac{[12][13][23]}{\Lambda} \left[\right. \\ & f^{abc} \left(-i \frac{m^4 g_s c_5^{hgg}}{s_{12}s_{13}s_{23}} + \frac{c_7}{\Lambda^2} + \frac{c_{11}}{\Lambda^6} (s_{12}s_{23} + s_{13}s_{23} + s_{12}s_{13}) + \frac{c_{13}}{\Lambda^8} s_{12}s_{13}s_{23} \right) \\ & \quad \left. + d^{abc} \frac{c'_{13}}{\Lambda^8} (s_{12} - s_{13})(s_{12} - s_{23})(s_{13} - s_{23}) \right] \\ & + \dots \end{aligned}$$

dim	operators	operators
	$\mathcal{M}(+++)$	$\mathcal{M}(++-)$
5	—	—
7	$h G_{\text{SD}}^3 [1, f^{abc}]$	—
9	—	$\mathcal{D}^2 G_{\text{SD}}^2 G_{\text{ASD}} h [1, f^{abc}]$
11	$\mathcal{D}^4 G_{\text{SD}}^3 h [1, f^{abc}]$	$\mathcal{D}^4 G_{\text{SD}}^2 G_{\text{ASD}} h [1, f^{abc}; 1, d^{abc}]$
13	$\mathcal{D}^6 G_{\text{SD}}^3 h [1, f^{abc}; 1, d^{abc}]$	$\mathcal{D}^6 G_{\text{SD}}^2 G_{\text{ASD}} h [2, f^{abc}; 1, d^{abc}]$

check with [Mathematica notebook of Henning Lu Melia Murayama](#)

ON-SHELL EFTs: BASES

Durieux Kitahara Machado YS Weiss

recall fermion-fermion- Z -higgs amplitude:

$$\begin{aligned}\mathcal{M}^{\text{contact}}(\mathbf{1}_{\psi^c}, \mathbf{2}_{\psi}, \mathbf{3}_Z, \mathbf{4}_h) &= \frac{C_6^{(1)}}{\Lambda^2} [\mathbf{13}][\mathbf{23}] + \frac{C_6^{(2)}}{\Lambda^2} [\mathbf{13}]\langle\mathbf{23}\rangle \\ &+ \frac{C_7^{(1)}}{\Lambda^3} [\mathbf{312}]\langle\mathbf{13}\rangle + \frac{C_7^{(2)}}{\Lambda^3} [\mathbf{321}]\langle\mathbf{23}\rangle \\ &+ \text{angle} \leftrightarrow \text{square}\end{aligned}$$

in red = basic independent spinor-structures—stripped of any \tilde{s}_{ij}

then full contact term basis given by

$$[\mathbf{13}][\mathbf{23}], \quad \tilde{s}_{12} [\mathbf{13}][\mathbf{23}], \quad \tilde{s}_{12}^2 [\mathbf{13}][\mathbf{23}], \quad \dots$$

recall fermion-fermion- Z -higgs amplitude:

$$\begin{aligned} \mathcal{M}^{\text{contact}}(\mathbf{1}_{\psi^c}, \mathbf{2}_{\psi}, \mathbf{3}_Z, \mathbf{4}_h) &= \frac{C_6^{(1)}}{\Lambda^2} [\mathbf{13}][\mathbf{23}] + \frac{C_6^{(2)}}{\Lambda^2} [\mathbf{13}]\langle\mathbf{23}\rangle \\ &+ \frac{C_7^{(1)}}{\Lambda^3} [\mathbf{312}]\langle\mathbf{13}\rangle + \frac{C_7^{(2)}}{\Lambda^3} [\mathbf{321}]\langle\mathbf{23}\rangle \\ &+ \text{angle} \leftrightarrow \text{square} \end{aligned}$$

in red = basic independent spinor-structures—stripped of any \tilde{s}_{ij}

then full contact term basis given by

$$[\mathbf{13}][\mathbf{23}], \quad \tilde{s}_{12} [\mathbf{13}][\mathbf{23}], \quad \tilde{s}_{12}^2 [\mathbf{13}][\mathbf{23}], \quad \dots$$

used the spinor-structure identity:

$$\begin{aligned} [\mathbf{12}]\langle\mathbf{3123}\rangle &= [\mathbf{12}] \left[\tilde{s}_{23} \langle\mathbf{313}\rangle - \tilde{s}_{13} \langle\mathbf{323}\rangle \right] / m_3 \\ &\quad - \tilde{s}_{12} [\mathbf{13}][\mathbf{23}] - m_1 [\mathbf{321}]\langle\mathbf{23}\rangle - m_2 [\mathbf{312}]\langle\mathbf{13}\rangle \end{aligned}$$

bases of $n \geq 4$ -amplitudes

3 types of bases: spanning an amplitude with external legs of spins s_1, s_2, \dots

* spinor structures = no prefactors containing Lorentz invariants like \tilde{s}_{ij}
only: $[ij], \langle ijk \rangle, [ijk] \dots$

- ▶ **contact terms basis**: spanning the manifestly local (pole-free) amplitude
- ▶ **stripped contact term (SCT) basis**: contact term basis with prefactors of invariants stripped off
- ▶ **spinor structure basis**: minimal set of spinor structures spanning the generic amplitude

bases

simple example: massless 4-fermion + + + + amplitude:

spinor-structure basis: $[14][23]$

spans generic amplitude:

$$[13][24] = -\frac{s_{13}}{s_{14}} [14][23] + \text{Schouten } [12][34] = [13][24] - [14][23]$$

stripped-contact term basis: $[14][23], [13][24]$

spans the contact-term (EFT) amplitude): manifestly local

contact-term basis:

$$\text{dim} - 6 : [14][23], [13][24]$$

$$\text{dim} - 8 : s_{13} [14][23], s_{14} [14][23], s_{13} [13][24] \quad \text{but no } s_{14} [13][24]$$

bases: what does redundancy mean

spinor-structure basis: a spinor-structure is redundant if it's a linear combination of other structures, with coefficients = rational functions of the invariants

stripped contact-term basis: a spinor-structure is redundant if it's a linear combination of other structures, with coefficients = polynomials of the invariants (no negative powers)

once you have the SCT basis, getting the CT basis is straightforward: just multiply by invariants and watch out for redundancies (but these can be seen at the massless level)

CONSTRUCTING THE BASES: STUFF WE'LL NEED

bases

but we are mainly interested in **EFT AMPLITUDES:**
(STRIPPED) CONTACT TERM BASIS

(stripped) contact term bases

not as simple as the bolding we saw before but still: can largely rely on massless case:

- if a spinor-structure is redundant in massless case: it is also redundant in massive case so can consider unbolded versions
- + a final stage of reductions:

recall: don't allow negative powers of invariants in spinor relations

but: allow negative powers of masses when longitudinal vectors are involved or higher spins

e.g. many a slide ago..

used the spinor-structure identity:

$$[\mathbf{12}]\langle\mathbf{3123}\rangle = [\mathbf{12}] \left[\tilde{s}_{23} \langle\mathbf{313}\rangle - \tilde{s}_{13} \langle\mathbf{323}\rangle \right] / m_3 \\ - \tilde{s}_{12} [\mathbf{13}][\mathbf{23}] - m_1 \langle\mathbf{321}\rangle [\mathbf{23}] - m_2 \langle\mathbf{312}\rangle [\mathbf{13}]$$

inverse mass in $\langle\mathbf{3}\cdots\mathbf{3}\rangle$: longitudinal vector

(stripped) contact term bases

so: $|i\rangle [i] \rightarrow |i\rangle [i]/m_i$

different ways to see this:

- massive polarization is $|i\rangle [i]/m_i$: so with $1/m_i$ correctly identify dimension of operator
- these are the Goldstone amplitudes \rightarrow scale as p_i/m_i

massless limit: $p_i = |i\rangle [i]$

covariantizing wrt massive little-group: $\rightarrow |i\rangle [i]$

RESULTS:

- ▶ Stripped Contact Term bases for all 4-point amplitudes of massive spin 0, 1/2, 1 (all dimensions)
- ▶ Stripped Contact Term bases for all 4-point amplitudes of massless spin 0, 1/2, 1 (all dimensions)
complementing results of Durieux, Machado '19 for $\text{dim} \leq 8$
- ▶ + all 3-point amplitudes for spin ≤ 3
complementing results of Durieux, Kitahara, YS, Weiss '18 for spin ≤ 1