Highly Effective Double Copy Amplitudes

QCD Meets Gravity 2020

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Introduction

double copy invites us to construct & combine our favorite theories using building blocks:

$$\text{SUGRA} = \text{SYM} \otimes \text{SYM}$$

$$\text{Born-Infeld} = \text{YM} \otimes \text{NLSM}$$

something new: composition allows us to generalize to constructing and combining the building blocks themselves:

$$\mathcal{L} = f^{abc} A^{a}_{\mu} \partial^{\mu} \varphi^{b} \varphi^{c}$$

$$\text{NLSM} = J^{a}(\text{simple } \varphi, \text{simple } \varphi)$$

so we may understand the most fundamental pieces of our predictions.

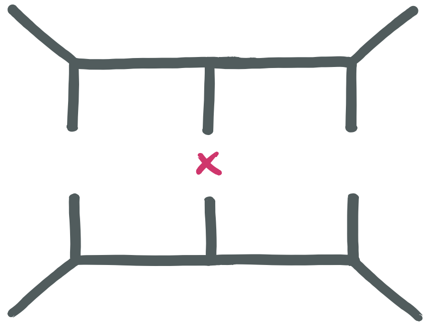
Overview

focus on higher-derivative corrections to gauge and gravity at 4-pt and 5-pt tree level

→ higher derivatives of interest for renormalization and UV predictions

↳ see talks by Caron-Huot, Schlotterer, Shadmi

→ higher multiplicity will make loops accessible via unitarity



only a small number of building blocks needed to construct these corrections

novel color-kinematics dualities emerge at five points

Four Point $\mathcal{N}=4$ Higher Derivative Corrections

construct gauge theory corrections as

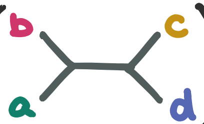
$$\mathcal{A}_4^{\text{SYM+HD}} = \sum_{g\in\{s,t,u\}} \frac{c_g^{\text{HD}}\, n_g^{\text{SYM}}}{d_g}$$

adjoint function of color traces & s_{ij}

obeys SUSY ward

c_g^{HD} built from color & scalar building blocks:

adjoint

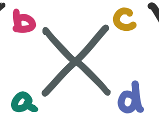
$c^a\Big($  $\Big) = f^{abe}f^{ecd}$

$n_s^{ss} = u-t$

$n_s^{\text{NLSM}} = s(u-t)$

$n^{\text{SYM}}, n^{F^3}, n^{D^2F^4}, \ldots$

perm invt

$c^{\text{PI}}\Big($  $\Big) = d^{abcd}$

$\sigma_2 = s^2+t^2+u^2$

$\sigma_3 = stu$

$st A^{\text{vec}}(1234)$

Composing Solutions

Known solutions (color or kinematics) may be composed into new solutions!

$J^{\times}(j,K)$	$j=$ adjoint	$j=$ PI
$K=$ adjoint	$J^a_s = j^a_t K^a_t - j^a_u K^a_u$ $J^{PI} = j^a_s K^a_s + j^a_t K^a_t + j^a_u K^a_u$	$J^a_s = j^{PI} K^a_s$
$K=$ PI	$J^a_s = K^{PI} j^a_s$	$J^{PI} = j^{PI} K^{PI}$

$$\left.\begin{array}{l} J^a_s(n^{ss}, n^{ss}) = n^{NLSM}_s \\ J^{PI}(n^{ss}, n^{NLSM}) = \sigma_3 \end{array}\right\} \text{structures close quickly}$$

Supersymmetric Corrections

Find just three independent structures for C_g^{HD}:

$$\sigma_2^X \sigma_3^Y c_g \qquad \sigma_2^X \sigma_3^Y J_g^a(c^a, n^{ss}) \qquad \sigma_2^X \sigma_3^Y d^{abcd}\, n_g^{NLSM}$$

an appropriately weighted sum Z_g reproduces the open superstring:

$$\mathcal{A}_4^{OSS} = \sum_g \frac{Z_g\, n_g^{SYM}}{d_g}$$ valid to all orders in α'

and can double copy to $\mathcal{N}=8$ compatible gravity counterterms:

$$\mathcal{M}^{SUGRA}_{+HD} = \sum_g \frac{n_g^{HD}\, n_g^{SYM}}{d_g} = \mathcal{M}^{SUGRA} \times \sum_{X,Y} \sigma_2^X \sigma_3^Y \alpha'^{2X+3Y}$$

Four Point Corrections

just three C_g^{HD} structures encode the open superstring:

$$\sigma_2^X \sigma_3^Y \times \left\{ c_g,\ J_g^a(c^a, n^{ss}),\ d^{abcd}\, n_g^{NLSM} \right\}$$

also find eight independent vector structures; only need four for open bosonic string:

$$\mathcal{A}_4^{OBS} = \sum_g \frac{z_g\, n_g^{YM+(DF)^2}}{d_g}$$

$\leftarrow$ linear combination of

n^{F^3} $\quad n^{(F^3)^2+F^4}$

$n^{D^2F^4}$ $\quad n^{D^4F^4}$

for further details, see 1910.12850

↳ John Joseph M. Carrasco, Laurentiu Rodina, Zanpeng Yin, and SZ

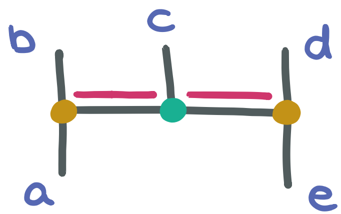
Higher Derivative Corrections at Five Points

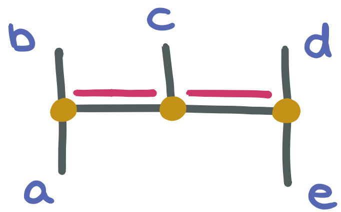
uncover a wealth of algebraic structures at five points

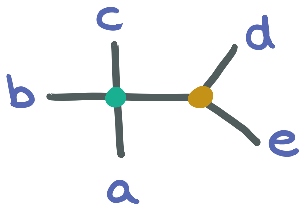
→ many options for composition

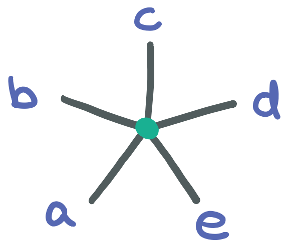
→ we'll find a single scalar kinematic building block generates the factorizing corrections we want

→ new algebraic structures give rise to new color-kinematic dualities in local amplitudes









work in progress with JJMC & LR

Algebraic Structures at Five Points

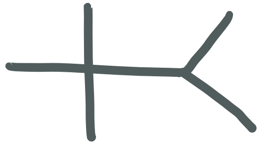
can explore different algebraic structures via the graph topologies at this multiplicity:

→ cubic vertices

↳ adjoint

↳ relaxed adjoint

→ quartic vertices



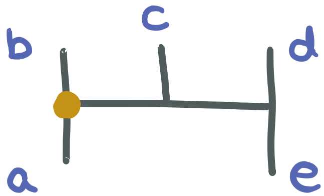
↳ hybrid

→ quintic vertices

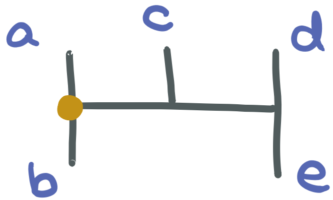
↳ permutation invariant

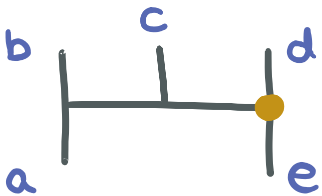
Adjoint at Five Points

adjoint structures obey Jacobi on every internal edge and antisymmetry around every vertex:



$= -$





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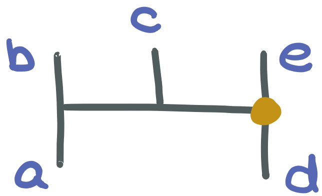

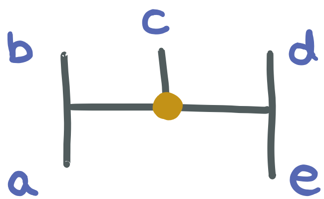

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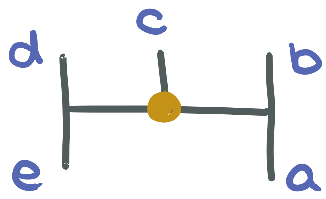
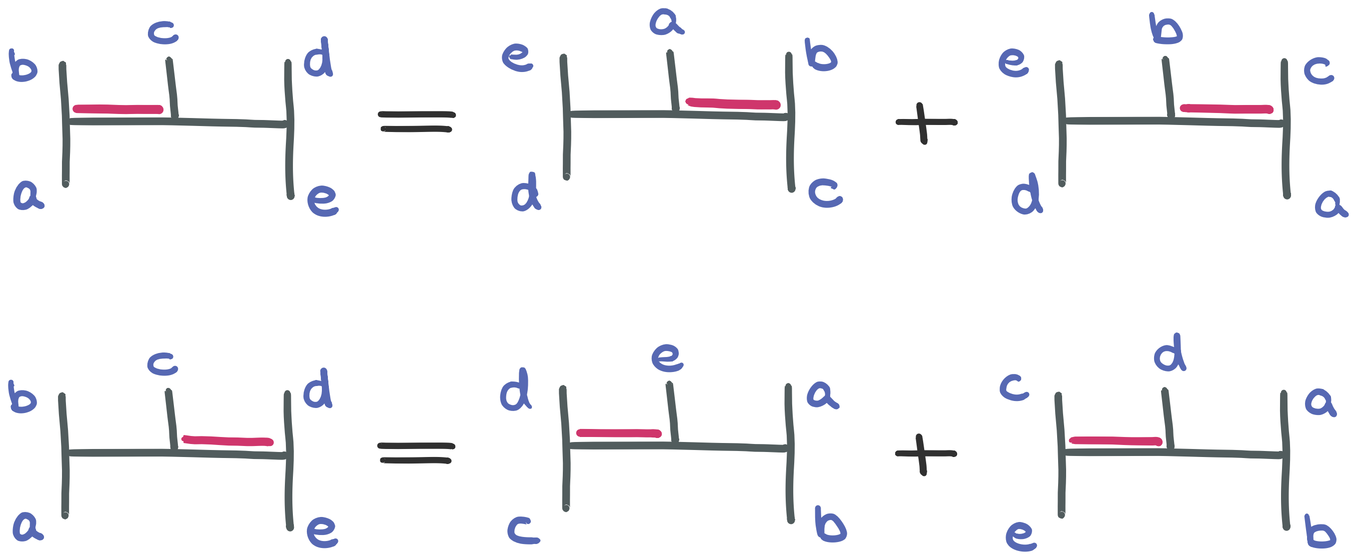
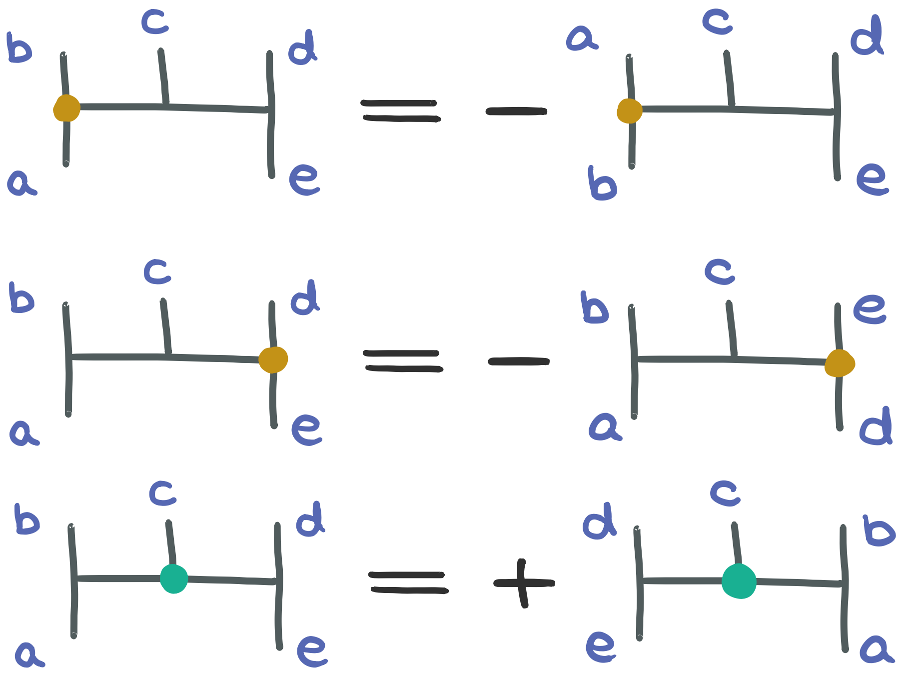


Jacobi at Five Points



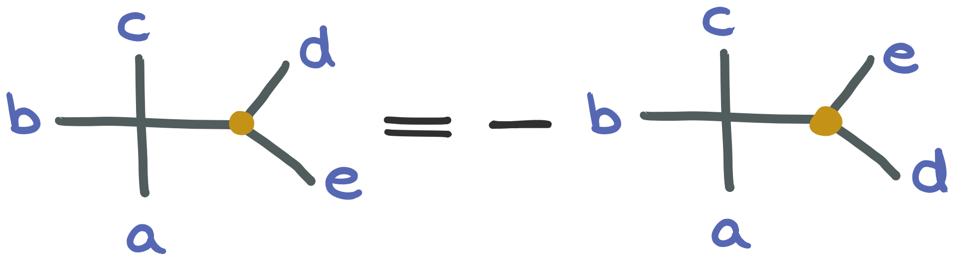
Relaxed Adjoint at Five Points

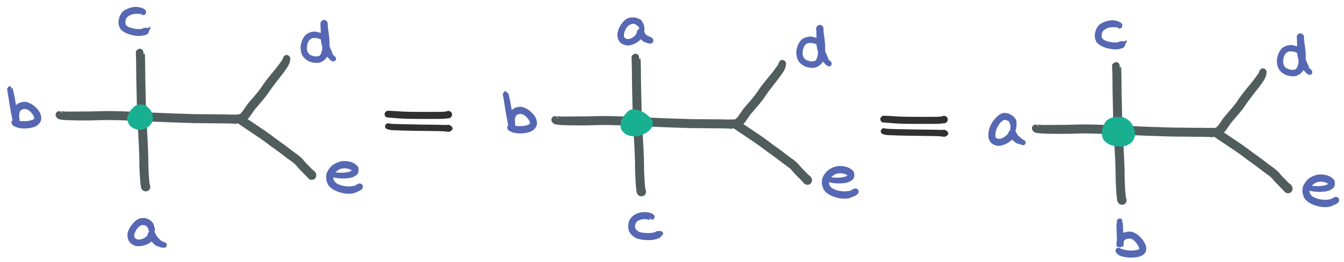
relaxed structures still obey Jacobi on every edge and antisymmetry on outer vertices, but are symmetric about the central vertex:



Hybrid Structures

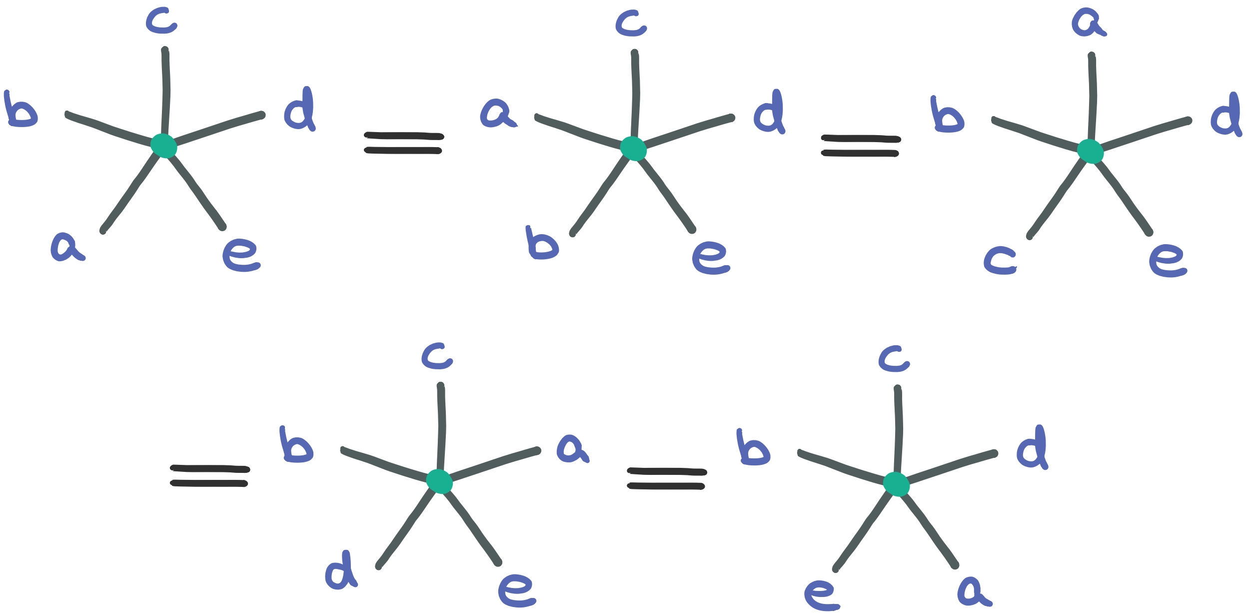
hybrid structures consist of one symmetric quartic vertex and one antisymmetric cubic vertex (as well as obeying four term identities):



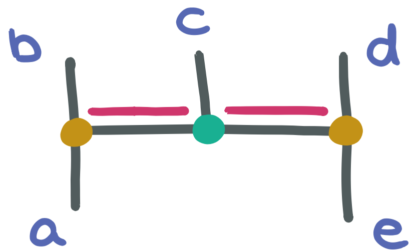


Permutation Invariant Structures

totally symmetric quintic vertices



Relaxed Building Blocks



color structures

$$c_r(12345) = f^3 d^3 f^3(12345)$$
$$+ 3 f^3 d^3 f^3(14253) + \cdots$$

scalar kinematics

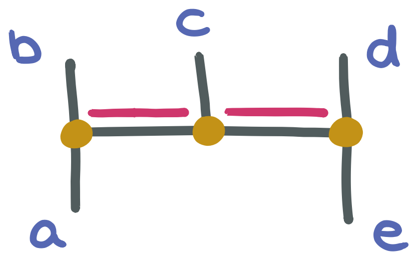
$$r^{(1)}(12345) = s_{12} - 2s_{23}$$
$$- 2s_{34} + s_{45} + 4s_{15}$$

$\hookrightarrow$ repeated composition of linear solution generates ladder of solutions:

$$r^{(2)} = J^r\left(r^{(1)}, r^{(1)}\right)$$
$$\vdots$$
$$r^{(9)} = \sum_{i+j=9} r^{(i)} p^{[j]}$$

closes to products with permutation invariants p!

Adjoint Building Blocks



color structures

$$C_a(12345) = f^3 f^3 f^3(12345)$$

$$C'_a(12345) = 2d^4f^3(12345) + 2d^4f^3(15342)$$
$$-d^4f^3(14253) - 2d^4f^3(14352) + d^4f^3(15243)$$

scalar kinematics

lowest order solution is cubic:

$$a^{(3)} = J^a(r^{(1)}, r^{(2)})$$

$$a^{(4)} = J^a(r^{(1)}, a^{(3)})$$

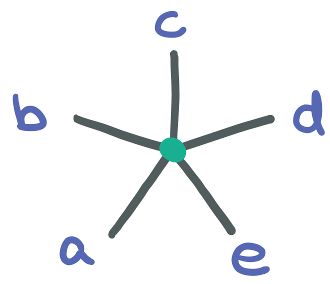
$$\vdots$$

$$a^{(11)} = \sum_{i+j=11} a^{(i)} p^{[j]}$$

Permutation Invariant Building Blocks

color structures

$$c_p(12345) = d^s(12345)$$



scalar kinematics

can compose two relaxed solutions into a permutation invariant:

$$p^{(2)} = J^{PI}(r^{(1)}, r^{(1)}) \equiv \sum_{g \in \Gamma_3}^{15} r^{(1)}(g) r^{(1)}(g)$$

$$p^{(3)} = J^{PI}(r^{(1)}, r^{(2)})$$

$$\vdots$$

$$p^{(10)} = (p^{(2)})^5 + p^{(2)} p^{(8)} + \cdots$$

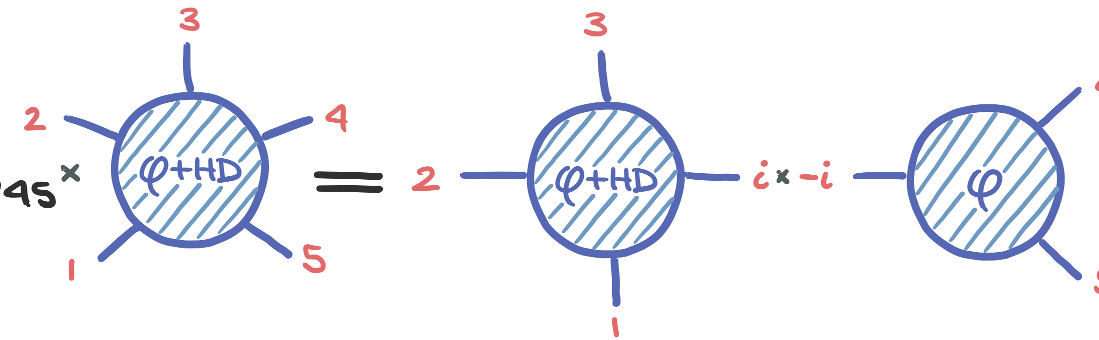
ladder of unique permutation invariants closes!

Factorizing Amplitudes

$$\mathcal{A}_4^{\varphi+HD} = \sum_g \frac{c_g^{HD}\tilde{c}_g}{d_g}$$

consider the equivalent scalar theory:

$$\lim_{s_{45}\to 0} s_{45} \times$$



known 4-pt corrections!

no hd scalar 3pt corrections

so we may fix our five point modified color factors on:

$$\lim_{s_{45}\to 0} s_{45} A^{\varphi+HD}(12345) = A^{\varphi+HD}(123i)A^{\varphi}(-i45)$$

Factorizing Amplitudes

consistently factorizing solutions must satisfy

$$\lim_{s_{45}\to 0} s_{45}\,\underbrace{A^{\varphi+HD}(12345)}_{d^4f^3 \;\&\; f^3f^3f^3} = \underbrace{A^{\varphi+HD}(123i)}_{d^4 \text{ or } f^3f^3}\,\underbrace{A^{\varphi}(-i45)}_{f^{abc}}$$

both found in c_a

compose c_a repeatedly with relaxed scalar kin.

$$N^{(m)} \equiv J^a[\Gamma^{(1)}, N^{(m-1)}]$$

$$N^{(0)} \equiv c_a$$

then fix $N^{(m)}$ on the factorization condition.

Factorizing Amplitudes

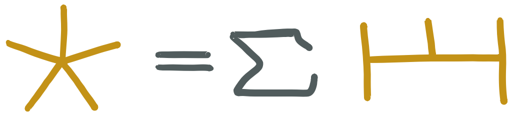
order	d^4f^3	$f^3f^3f^3$
2	1	0
3	0	1
4	1	1
5	1	1
6	1	2
7	1	2
8	2	2
9	1	3

d^4f^3 solutions are truly adjoint – the amplitudes cannot be striated along hybrid algebra

building blocks close under composition to:

$$\sum_{i+j=m} v^{(i)} p^{[j]} \qquad m>7$$

Local Contact Corrections



adjoint color
$\tilde{f}^3\tilde{f}^3\tilde{f}^3$

adjoint function
of s_{ij} and c_x

$$\mathcal{A}^{\varphi+HD} = \sum_{g\in\Gamma_3} \frac{\tilde{c}_a(g)\, c_a^{HD}(g)}{d_g}$$

$x = \{d^5, d^4f^3,$
$f^3f^3f^3, f^3d^3f^3\}$

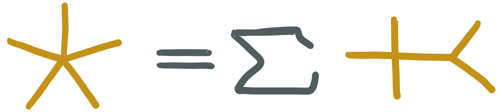
sum over
cubic graphs

propagators
will cancel

what if we striate along the c_x color factors
that survive the double copy with sYM?

Doubly Dual Local Contact Amplitudes

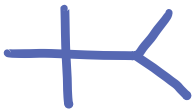
For example, let's look at
$X = d^4f^3$ amplitudes:



$$\mathcal{A}^{\varphi+\mathrm{HD}} = \sum_{g \in \Gamma_{4,3}} d^4f^3(g)\, y_{df}(g)$$

d^4f^3-dual function of s_{ij} and $\tilde{f}^3\tilde{f}^3\tilde{f}^3$

sum over relevant graphs



manifestly local construction

Doubly Dual Local Contact Amplitudes

may construct local amplitudes by demanding doubly color-dual structure:

$$\mathcal{A}^{\varphi+HD} = \sum_{g \in \Gamma_X} c_X(g)\, \psi_X(g)$$

fixed on X-type algebraic relations

then cast into DDM basis:

$$\mathcal{A}^{\varphi+HD} = \sum_{\sigma \in S_3} \tilde{f}^3\tilde{f}^3\tilde{f}^3(1\sigma5)\, A(1\sigma5)$$

fix on $(m-3)!$ to impose adjoint duality

Comparing to String Theory

much like at four points, coefficients of both factorizing and local contact amplitudes may be fixed to the five point open superstring

↳ see e.g. 1106.2645, 1106.2646 Mafra, Schlotterer, Stieberger

1307.3534 Green, Mafra, Schlotterer

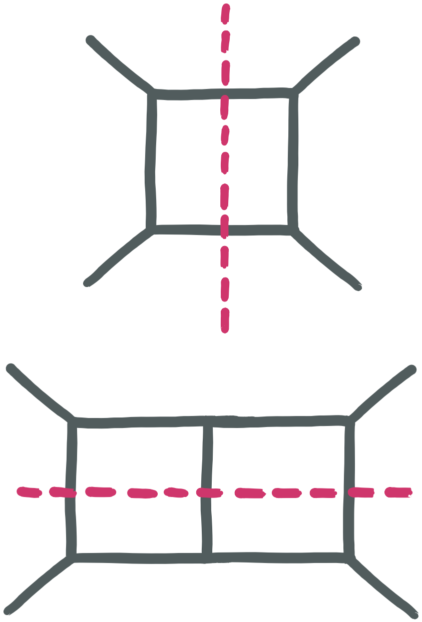
only structures consistent with reflection contribute:

$$A(\overrightarrow{abcde}) = -A(\overleftarrow{edcba})$$

$f^3f^3f^3$	d^4f^3	$f^3d^3f^3$	d^5
✓	✓	✗	✗

Further Work

- → search for continued structure and building blocks at loop level
  - do composition rules exist at (multi)loop level?



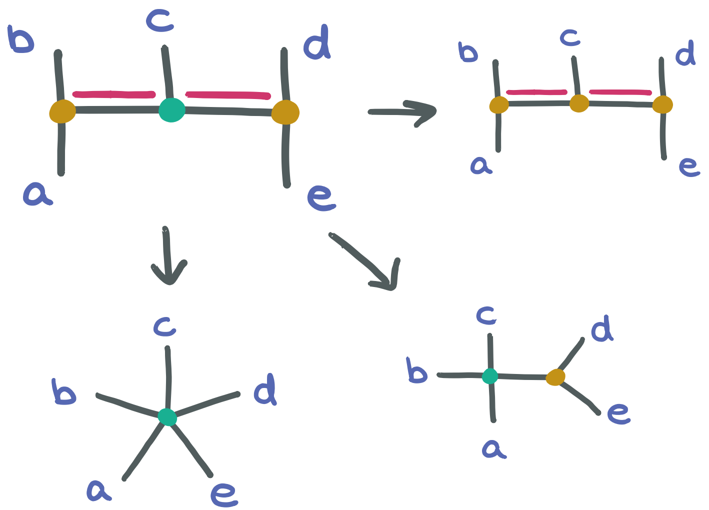
- → S-matrix to operators
  - $d^4 f^3 \to n_n^{vec}$ vector found via ansatz, but what's the corresponding gravity theory?

Summary

only a few building blocks required to write down towers of higher derivative corrections

constructive alternative to ansatze

novel c/k dualities emerge in local corrections:



$$\mathcal{A}^{\varphi+\mathrm{HD}} = \sum_{K} d^4f^3 \underbrace{y(\vec{c}^{\,a}, s_{ij})}_{d^4f^3\ \text{c/k dual}} = \sum_{H} \vec{c}^{\,a} \underbrace{A(d^4f^3, s_{ij})}_{\text{adjoint c/k dual}}$$