Gravitational Bremsstrahlung from Scattering Amplitudes

Julio Parra-Martinez

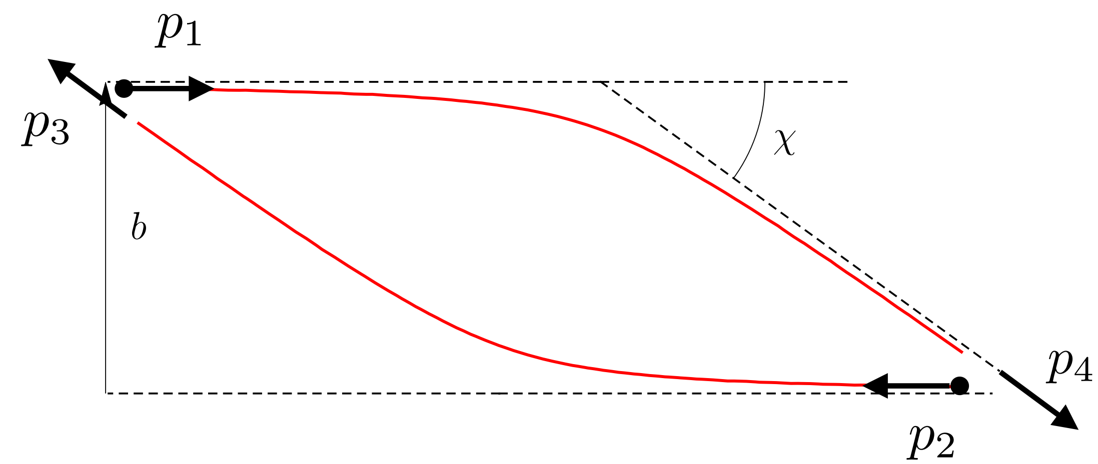
w/ Ruf, Zeng [2005.04236] & w/ Herrmann, Ruf, Zeng [201X.XXXXX]

"Part II of Michael's talk"

@ QCD Meets Gravity 2020, Northwestern University

Multiple approaches

[ACV; BCRSSZ; Porto, Kälin;...]



- From scattering amplitudes to observables for the 2-body problem

Scattering amplitude

$\mathcal{M}(q,p)$

Eikonal Resumation

$\delta(b,p)$

EFT matching

$V(r,p)$

Impetus Formula

$\mathbf{p}^2 = p_\infty^2 + \tilde{\mathcal{M}}(r,p)$

Gauge Invariant Observables

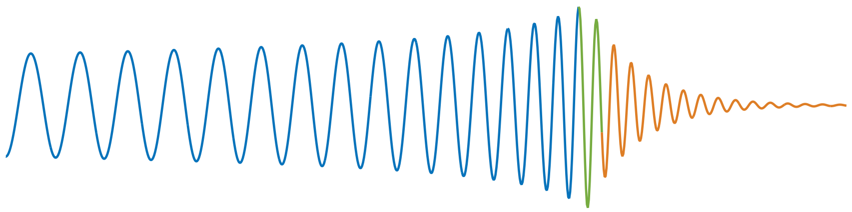
χ $\Delta\Phi$ E^{rad} J^{rad}

see other talks today and tomorrow!

Mostly conservative sector

Next target: radiation

- Key observable: waveform $h_{\mu\nu}(x,t)$



- Today's talk: "inclusive" observables:

    1. are integrated over orbits and celestial sphere
    2. are independent of the phase of GW

    Single scale integrals

- First results for the $\mathcal{N}=8$ scattering angle (including radiation reaction)

  [Di Vecchia, Heissenberg, Russo, Veneziano]

  see talks by Gabriele and Carlo!

- Also in General relativity, using interesting "linear response formula"

  [Bini, Damour; Damour]

$$\chi^{\rm rad} = -\frac{1}{2}\frac{\partial \chi^{\rm cons}}{\partial E}E^{\rm rad} - \frac{1}{2}\frac{\partial \chi^{\rm cons}}{\partial J}J^{\rm rad}$$

KMO'C (in-in) approach

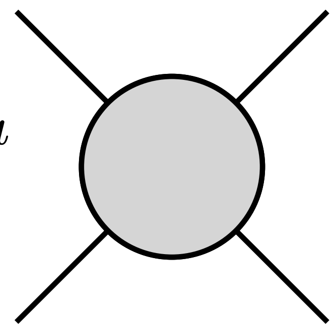
[Kosower, Maybee, O'Connell]

- $\mathcal{M}(q,p)$ → Gauge Invariant Observables

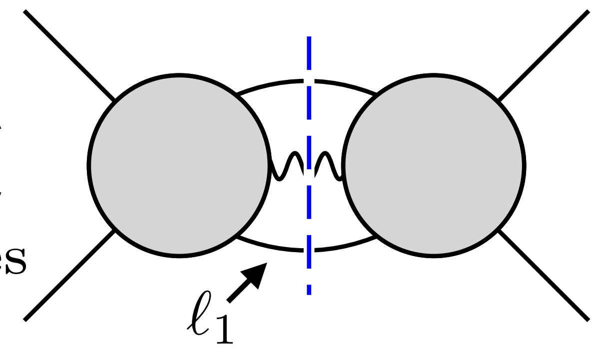
Example 1: Impulse

$$\Delta p_1^\mu = \int d^D q\, \delta(2u_1\cdot q)\delta(2u_2\cdot q)\, e^{ib\cdot q}\, \mathcal{I}^\mu$$

$\mathcal{I}^\mu = q^\mu$



$+ \quad \ell_1^\mu \sum_{\text{states}}$



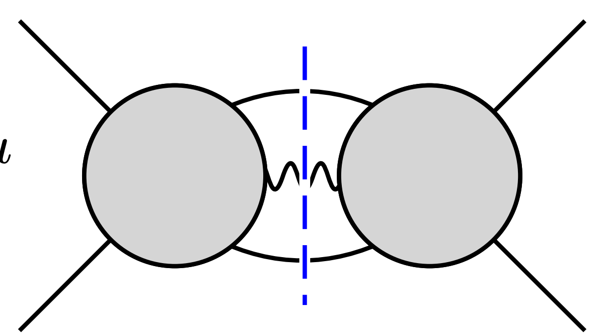
See Ben's talk!

"virtual" "real"

Example 2: Radiated momentum

$$R^\mu = \int d^D q\, \delta(2u_1\cdot q)\delta(2u_2\cdot q)\, e^{ib\cdot q}\, \mathcal{R}^\mu$$

$R^\mu = \sum_{\text{states}} k^\mu$



$\sim \int k^\mu |h_{\alpha\beta}|^2$

Integrals summary

[JPM, Ruf, Zeng]

- Method of regions [Beneke, Smirnov]

$$\frac{m_i^2}{-q^2} \sim \frac{s}{-q^2} \sim J^2 \gg 1$$

hard: $(\omega, \boldsymbol{\ell}) \sim (m, m)$

soft: $(\omega, \boldsymbol{\ell}) \sim (|\boldsymbol{q}|, |\boldsymbol{q}|) \sim J^{-1}\,(m|\boldsymbol{v}|, m|\boldsymbol{v}|)$

potential: $(\omega, \boldsymbol{\ell}) \sim (|\boldsymbol{q}||\boldsymbol{v}|, |\boldsymbol{q}|) \sim J^{-1}\,(m|\boldsymbol{v}|^2, m|\boldsymbol{v}|)$

radiation: $(\omega, \boldsymbol{\ell}) \sim (|\boldsymbol{q}||\boldsymbol{v}|, |\boldsymbol{q}||\boldsymbol{v}|) \sim J^{-1}\,(m|\boldsymbol{v}|^2, m|\boldsymbol{v}|^2)$

- Single variable! canonical form [Henn]

$$\mathrm{d}\vec{I}(y) = \epsilon \textstyle\sum_i A_i \mathrm{dlog}\, \alpha_i(y)\, \vec{I}(y)$$

Boundary conditions via analyticity

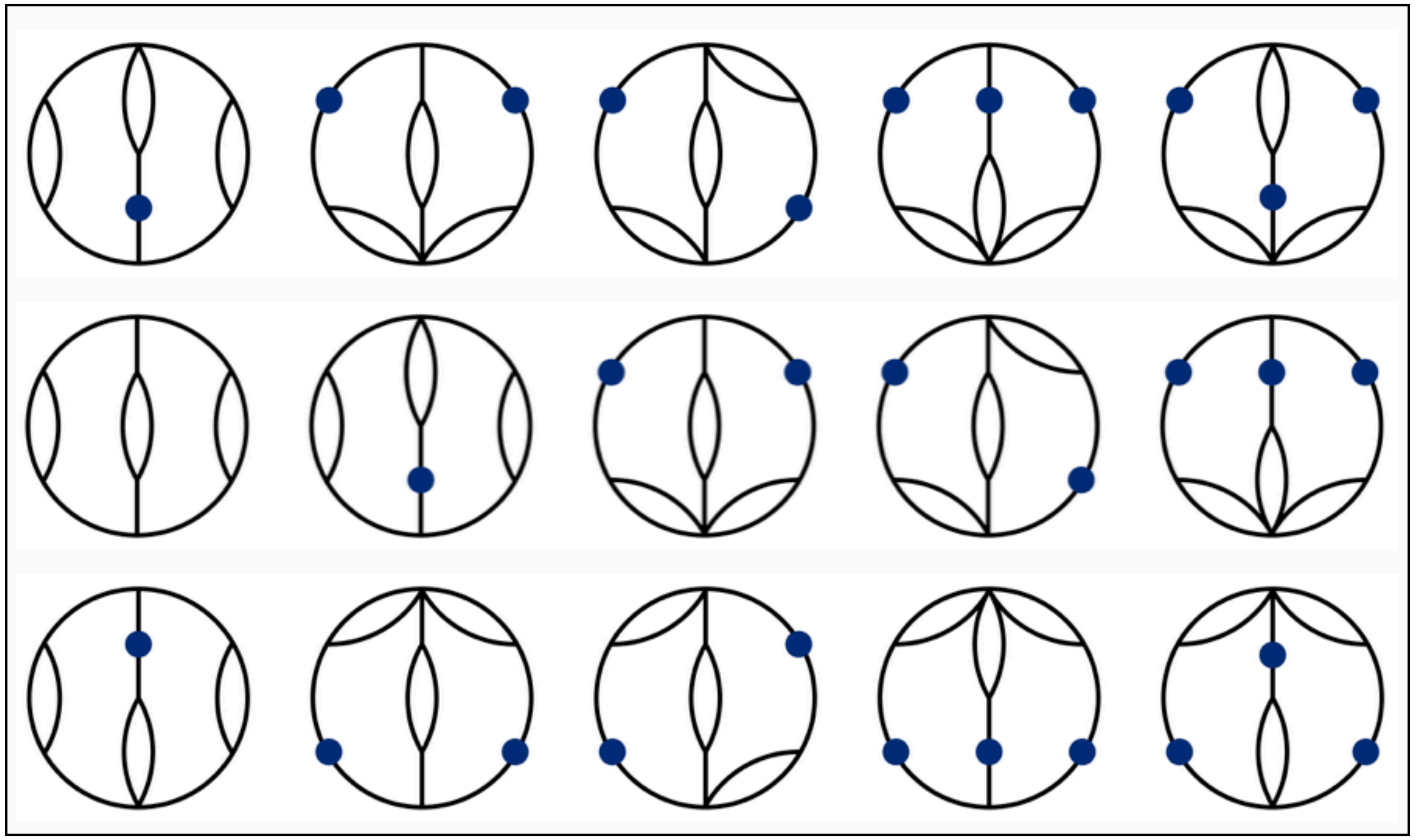
& static limit (potential vs. soft)

- Reverse unitarity for cut integrals [Anastasiou, Melnikov]

$$2\pi i\, \delta(2u_1 \cdot \ell_1) = \frac{1}{2u_1 \cdot \ell_1 - i\epsilon} - \frac{1}{2u_1 \cdot \ell_1 + i\epsilon}$$

See Michael's talk for all the details!

Gravitational Bremsstrahlung in $\mathcal{N} = 8$ supergravity



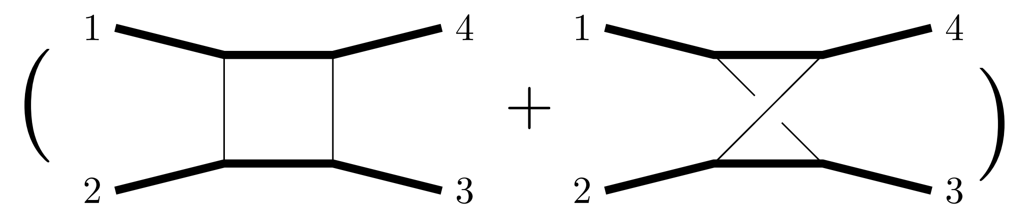
Simplicity in $\mathcal{N}=8$ supergravity

$$\sigma = \frac{p_1 \cdot p_2}{m_1 m_2}$$

$$t = -q^2$$

- One-loop integrand [Brink, Green, Schwarz; Caron-Huot, Zahraee]

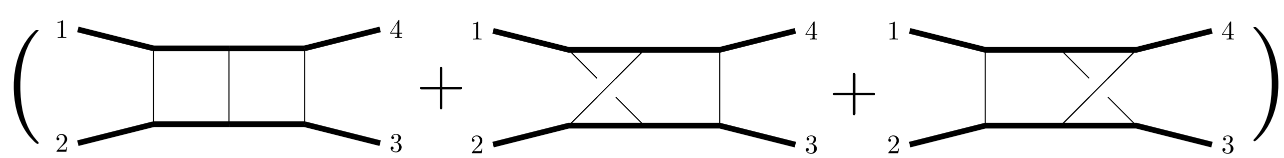
$$\mathcal{M}_4^{(1)} = -i(8\pi G)^2 16 m_1^4 m_2^4 (\sigma - \cos\phi)^4$$



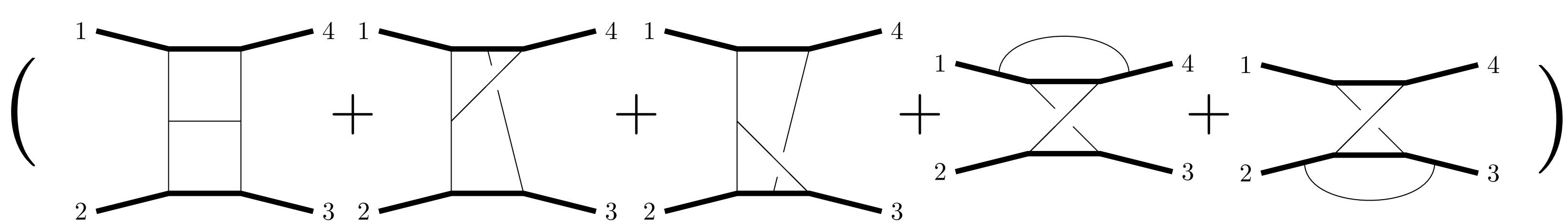
- Two-loop integrand [Bern, Dixon, Perelstein, Rozowski; JPM, Ruf, Zeng]

$$\mathcal{M}_4^{(2)} = -(8\pi G)^3 16 m_1^4 m_2^4 (\sigma - \cos\phi)^4$$

$$\Big[4 m_1^2 m_2^2 (\sigma - \cos\phi)^2$$



$$+ (q^2)^2$$



$$+ (2 \leftrightarrow 3) \Big]$$

Only scalar integrals

- Loop integrand known up to five loops

[Bern, Brink, Carrasco, Chen, Dixon, Edison, Green, Johansson, JPM, Kosower, Perelstein, Roiban, Rozowski, Schwarz, Zeng,…]

Conservative result

[Herrmann, JPM, Ruf, Zeng]

- KMO'C formulas can be used evaluating integrals in potential region.

  Only two-particle cuts necessary

$$\Delta p_1^\mu|_{\text{cons}} = -\frac{16G^3 m_1^2 m_2^2 \sigma^4}{\sigma^2-1}\frac{b^\mu}{b^4}\left(4\,\text{arcsinh}\sqrt{\frac{\sigma-1}{2}} + \frac{\sigma^2 s}{m_1 m_2(\sigma^2-1)^{3/2}}\right)$$

High-energy log

see Carlo's, Gabriele's talks!

- Scattering angle: $\sin\frac{\chi}{2} = \frac{\sqrt{-\Delta p_1^2}}{2p_{\text{cm}}}$ matches result from eikonal/EFT

[JPM, Ruf, Zeng]

- Purely transverse due to "no-triangle" property at one loop more generally by choosing transverse impact parameter

[Caron-Huot, Zahraee]

$$b^\mu_{\text{eik}} = b^\mu + b\cdot\Delta p_1\left(\frac{1}{m_1}\frac{(\gamma u_2 - u_1)^\mu}{\gamma^2-1} - \frac{1}{m_2}\frac{(\gamma u_1 - u_2)^\mu}{\gamma^2-1}\right)$$

Two-loop radiative impulse

[Herrmann, JPM, Ruf, Zeng]

- Evaluating integrals in full soft region

Conservative

$$\Delta p_1^\mu = -\frac{16 G^3 m_1^2 m_2^2 \sigma^4}{\sigma^2-1}\frac{b^\mu}{b^4}\left(-\frac{2\sigma^2}{\sigma^2-1}+\left(4-\frac{4\sigma\left(\sigma^2-2\right)}{\left(\sigma^2-1\right)^{3/2}}\right)\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}+\frac{\sigma^2 s}{m_1 m_2\left(\sigma^2-1\right)^{3/2}}\right)$$
$$-\frac{4\pi G^3 m_1^2 m_2^2 \sigma^4}{(\sigma^2-1)^{3/2}}\frac{\sigma u_2^\mu - u_1^\mu}{|b|^3}\left(-\frac{2\sigma^2}{\sigma^2-1}+\left(4-\frac{4\sigma\left(\sigma^2-2\right)}{\left(\sigma^2-1\right)^{3/2}}\right)\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}+4\log\left(\frac{1}{2}(1+\sigma-\sqrt{\sigma^2-1})\right)\right)$$

- New terms time reversal odd - radiation reaction

$$\log\left(\frac{1}{2}(1+\sigma-\sqrt{\sigma^2-1})\right) = -\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}+\log\sqrt{\frac{\sigma+1}{2}}$$

- Scattering angle with radiation reaction matches eikonal but it also contains more information.

[Di Vecchia, Heissenberg, Russo, Veneziano]

$$\chi \sim \Delta p_1^\perp \qquad\qquad E^{\rm rad} \sim \Delta p_1^u$$

Energy loss

- Radiated momentum $R^\mu = -\Delta p_1 - \Delta p_2$

- Energy loss in rest frame of one of the particles $E^{\rm rad} = u_1 \cdot R = R^0$

$$\frac{E^{\rm rad}}{E_{\rm cm}} = \left(\frac{G m_1 m_2}{J}\right)^3 \frac{m_1^2 m_2^2}{E_{\rm cm}^4} 8\pi\sigma^4(\sigma^2-1)\left(\frac{\sigma^2}{\sigma^2-1} + \frac{2\sigma\,(\sigma^2-2)}{(\sigma^2-1)^{3/2}}\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}} - 2\log\sqrt{\frac{\sigma+1}{2}}\right)$$

- Has expected mass dependence [Kovacs, Thorne; Bini, Damour, Geralico]

- High-energy limit as predicted!

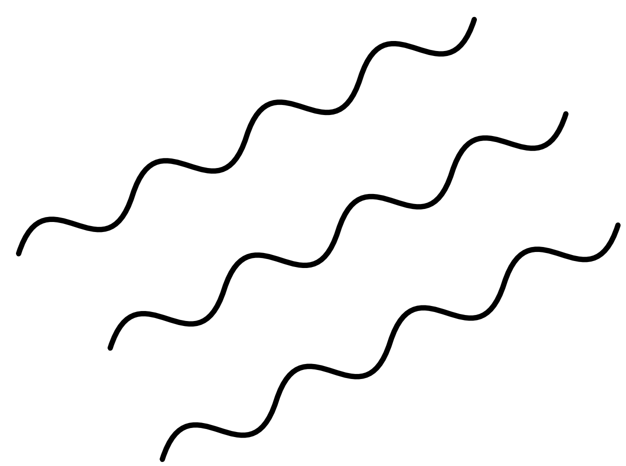
$$E^{\rm rad} \overset{\sigma\to\infty}{\sim} \underbrace{8\pi(2\log 2+1)}_{\sim 60}\frac{G^3 m_1^2 m_2^2}{|b|^3}\sigma^3$$

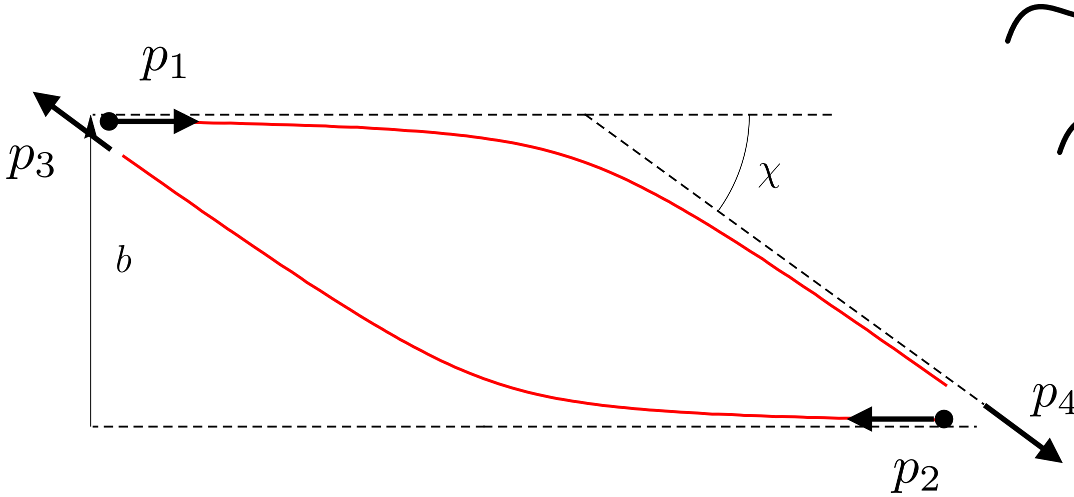
$$\Delta E = (20.0 \pm 0.3)[(m_{\rm A} m_{\rm B})^2/b^3]\gamma^3 \ .$$

[Kovacs, Thorne; Peters]

Modulo coefficient…

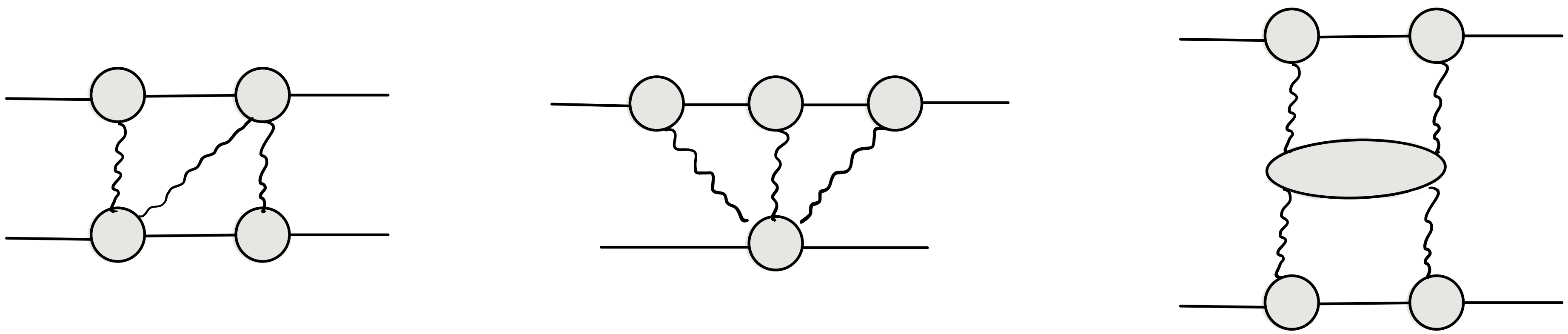
Gravitational Bremsstrahlung in Einstein gravity



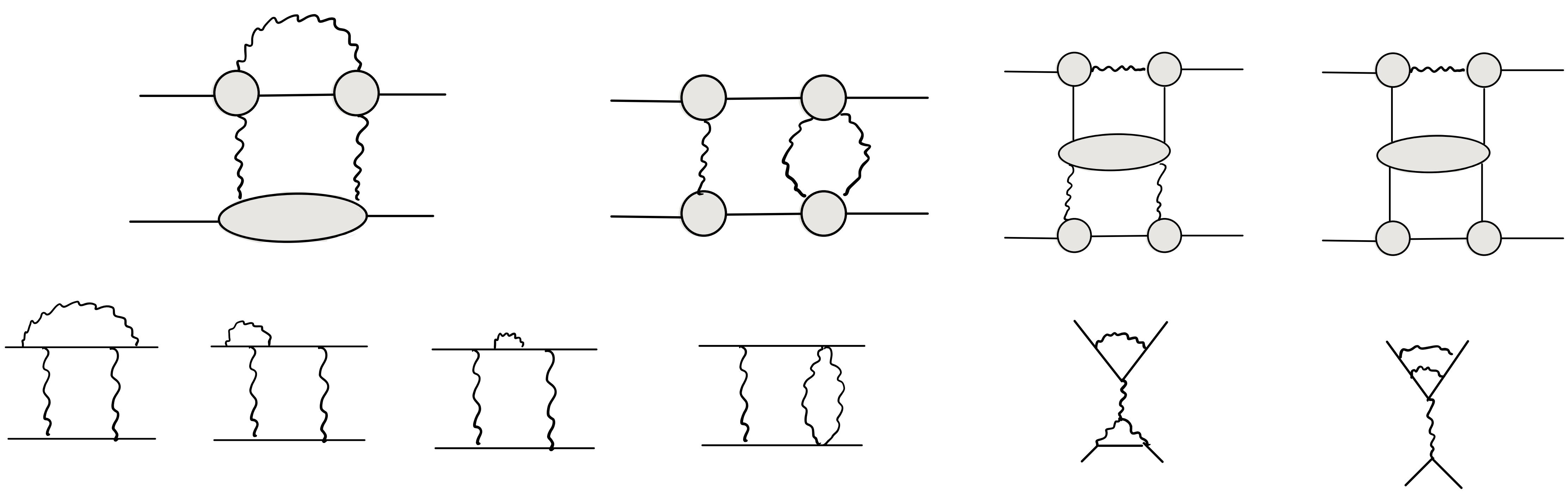


Conservative vs. full soft integrands

- BCRSSZ integrand - potential = “one matter propagator per loop”



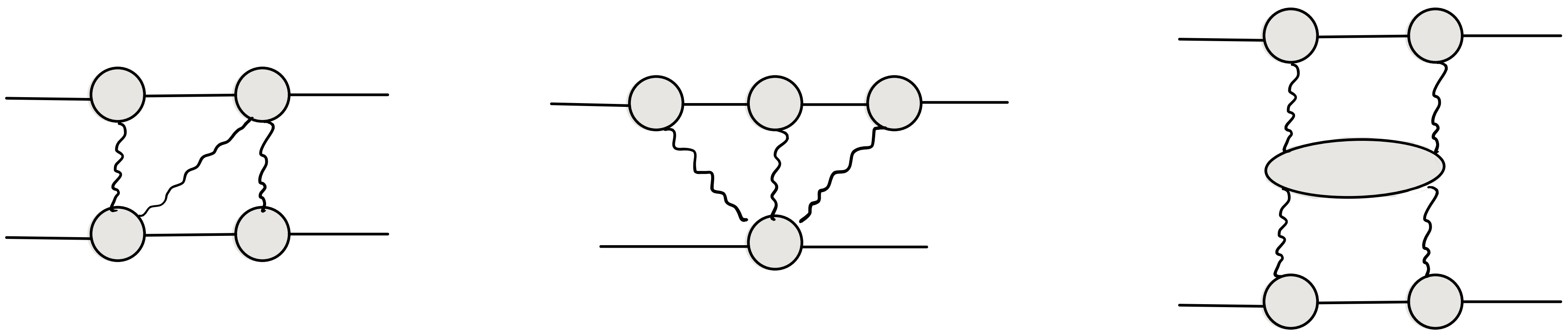
- Not enough for full soft region - need additional unitarity cuts



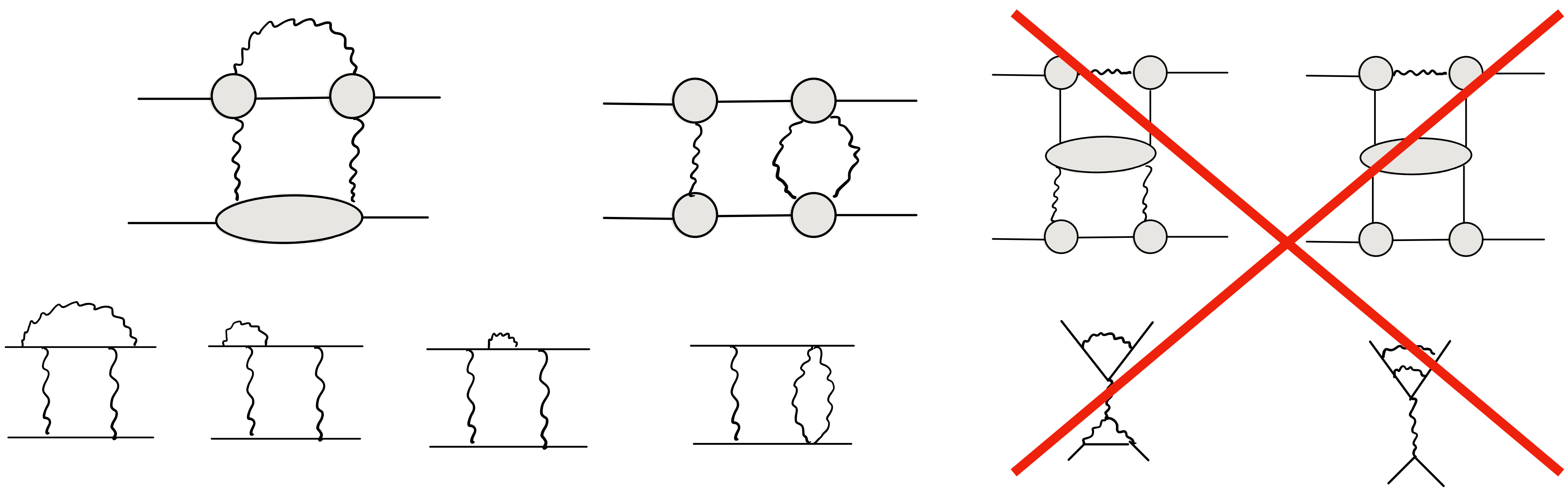
- New rule for soft - “only q-dependence in loop = scaleless”

Conservative vs. full soft integrands

- BCRSSZ integrand - potential = “one matter propagator per loop”



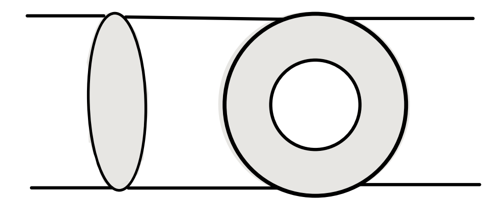
- Not enough for full soft region - need additional unitarity cuts



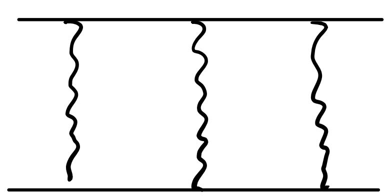
- New rule for soft - “no q-dependence in loop = scaleless”

Conservative vs. full soft integrands

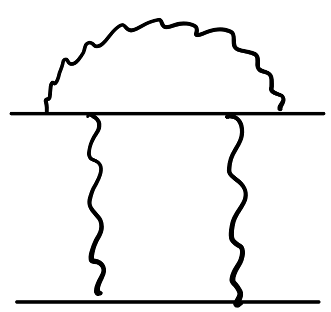
- Some simplifications in the ansatz construction
  - Impose classical cuts (e.g. no matter contacts)
  - Impose iterated two-particle cuts off-shell
  - Use mass/hbar power counting to minimize ansatz



$\propto (s^2 - 2m_1^2 m_2^2)$



$$N \sim m_1^6 m_2^6 (u_1 + q/m_1)^6 (u_2 + q/m_2)^6 \qquad \times \frac{1}{m_1^2} \frac{1}{m_2^2}$$



$$N \sim m_1^4 m_2^8 (u_1 + q/m_1)^8 (u_2 + q/m_2)^4 \qquad \times \frac{1}{m_1} \frac{1}{m_2^3}$$

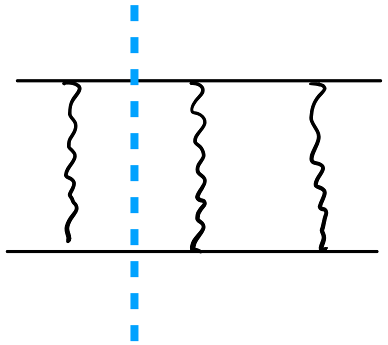
- No new integrals required in Einstein Gravity. $\mathcal{N} = 8$ great toy model

Calculation/checks still underway…
stay tuned!

Conservative impulse

- Conservative impulse - only virtual and two-particle cuts

Trivial subtraction
at classical order



$\propto (s^2 - 2m_1^2 m_2^2)^3$

- Agrees with known $\mathcal{O}(G^3)$ results - non-trivial check [BCRSSZ; Porto Kälin]

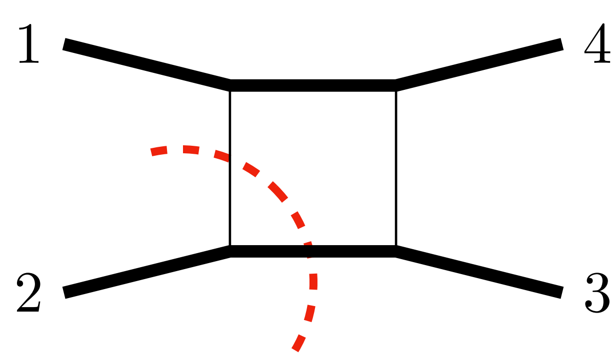
$$\Delta p_1^\mu = \frac{G^3 b_{\text{eik}}^\mu}{|b_{\text{eik}}^2|^2} \left(\frac{16 m_1^2 m_2^2 (4\sigma^4 - 12\sigma^2 - 3) \text{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{(\sigma^2 - 1)} - \frac{4 m_1^2 m_2^2 \sigma (20\sigma^6 - 90\sigma^4 + 120\sigma^2 - 53)}{3(\sigma^2-1)^{5/2}} - \frac{2 m_1 m_2 (m_1^2 + m_2^2)(16\sigma^6 - 32\sigma^4 + 16\sigma^2 - 1)}{3(\sigma^2 - 1)^{5/2}} \right)$$

Conclusions/future work/in progress

- KMO'C formalism encodes "inclusive observables" in a way that requires no more complicated methods than virtual amplitude/ scattering angle.

- First results for the energy loss in $\mathcal{N}=8$, GR around the corner!

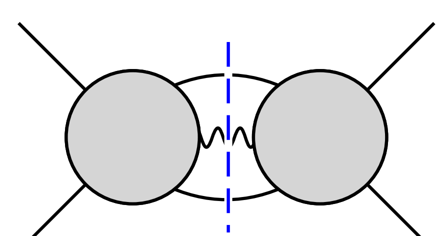
- Can we use KMOC to directly rederive Damour's $J^{\mathrm{rad}} \sim \mathcal{O}(G^2)$? Role of soft gravitons?



For spin see also [Maybee, O'Connell, Vines]

- Experience with collider observables suggests a route for differential/ "less inclusive observables": include "phase-space cuts".

$$\frac{dR^\mu}{d\omega} = \sum_{\text{states}} k^\mu \ \delta(k^0 - \omega)$$



c.f. QCD EE correlation, rapidity distributions

Thank you!