QCD meets gravity in neutron stars

and what we can learn with gravitational waves from their binary inspirals

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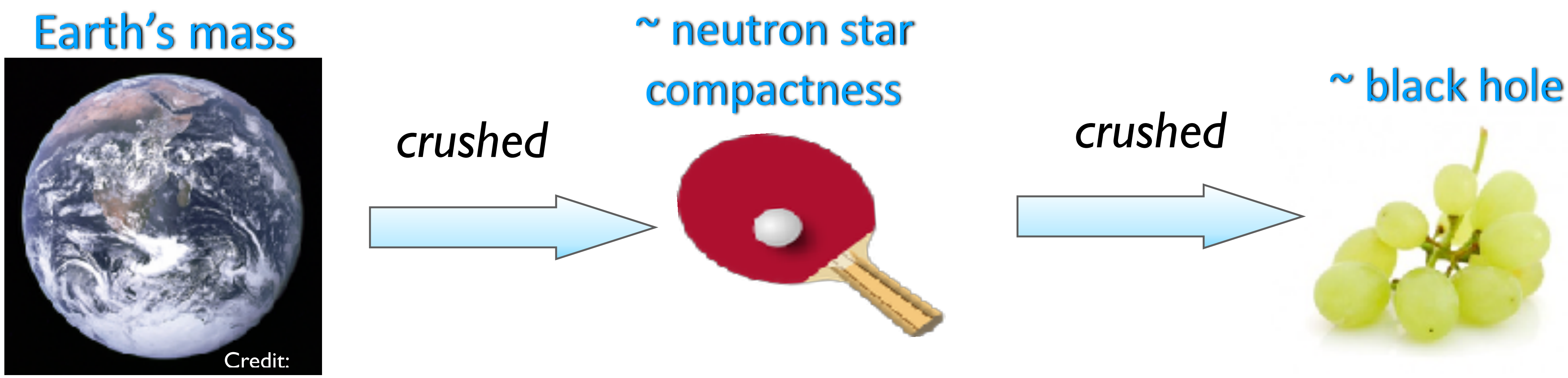
Overview

- Gravitational waves (GWs) now available for probing subatomic physics in unexplored regime with neutron stars in binary systems
- Interpretation of GW signals from binaries relies on theoretical models
- Required: detailed understanding of GW signatures of matter during an inspiral
  - clean, cumulative, currently accessible regime
- Tidal effects are a rich set of matter phenomena during inspiral, two examples:
  - Gravitoelectric tides, fundamental modes
  - Gravitomagnetic tides, inertial modes (slow rotation)
- Summary and outlook

Neutron stars (NSs)

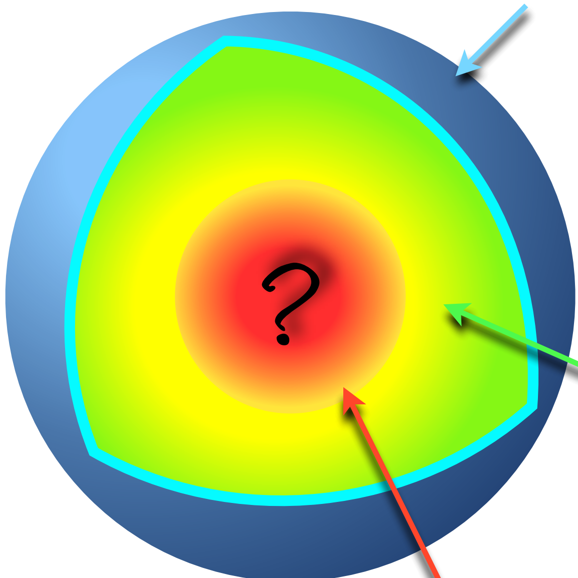
- Gravity compresses matter to up to several times nuclear density



- Thousands observed to date, some masses > 2 Msun
- Quantum pressure (neutron degeneracy) can only support up to ~ 0.7 Msun
- Unique window onto strongly-interacting subatomic matter

Conjectured NS structure

[density of iron ~ **10** g/cm³]



crust ~ *km*

neutron rich nuclei,

free neutrons

~10^6 g/cm³ inverse ***β***-decay

~10^{11} g/cm³ neutron drip

outer core

neutron superfluid, proton superconductor

deep core

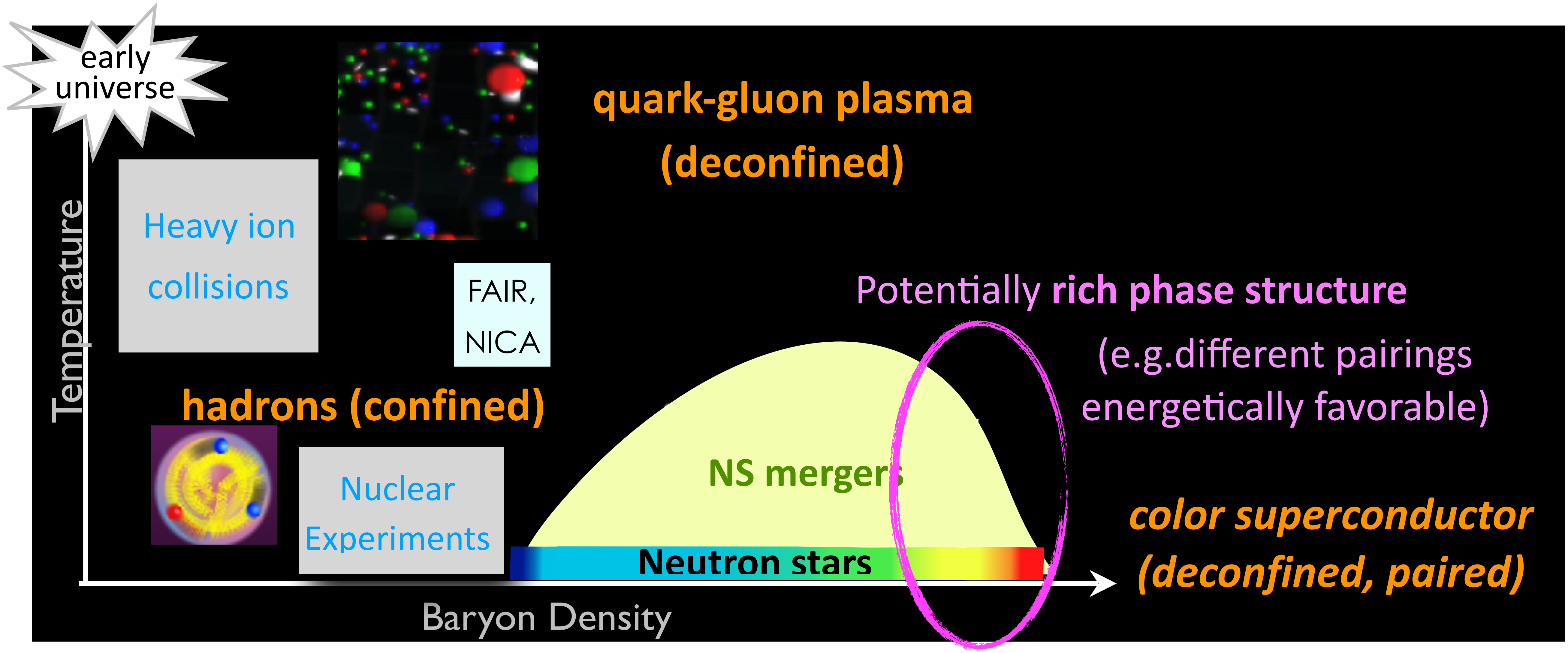
≳2x nuclear density, nucleons overlap -

new degrees of freedom relevant

exotic states of matter? deconfined quarks?

~ few x **10^{14}** g/cm³

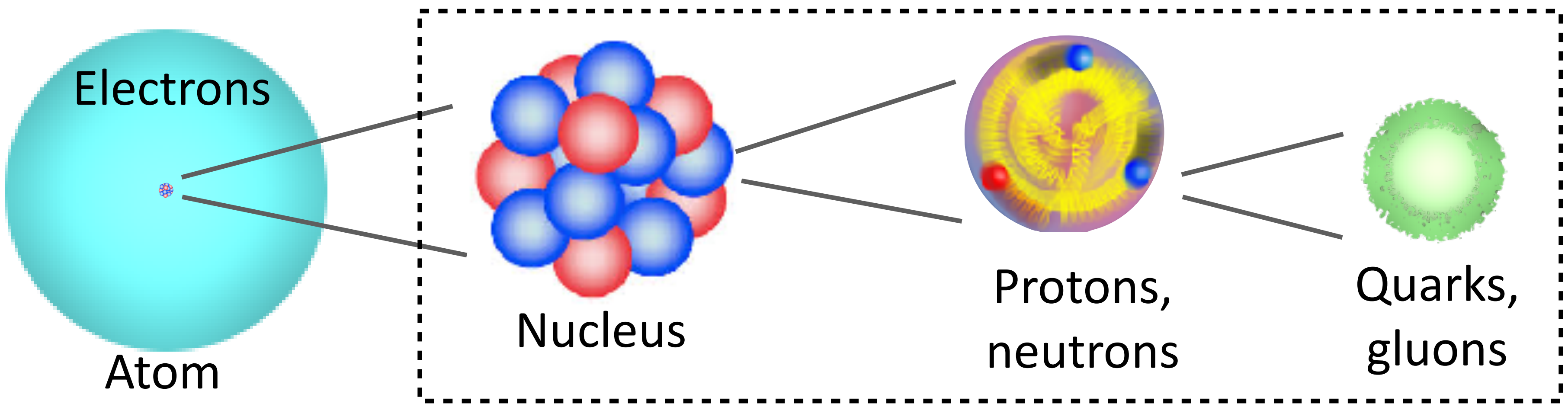
Neutron stars as QCD labs



- discover, characterize new phases of QCD
- Does color-superconducting quark matter exist in the universe?
- Insights into deconfinement
- Deeper understanding of strong interactions

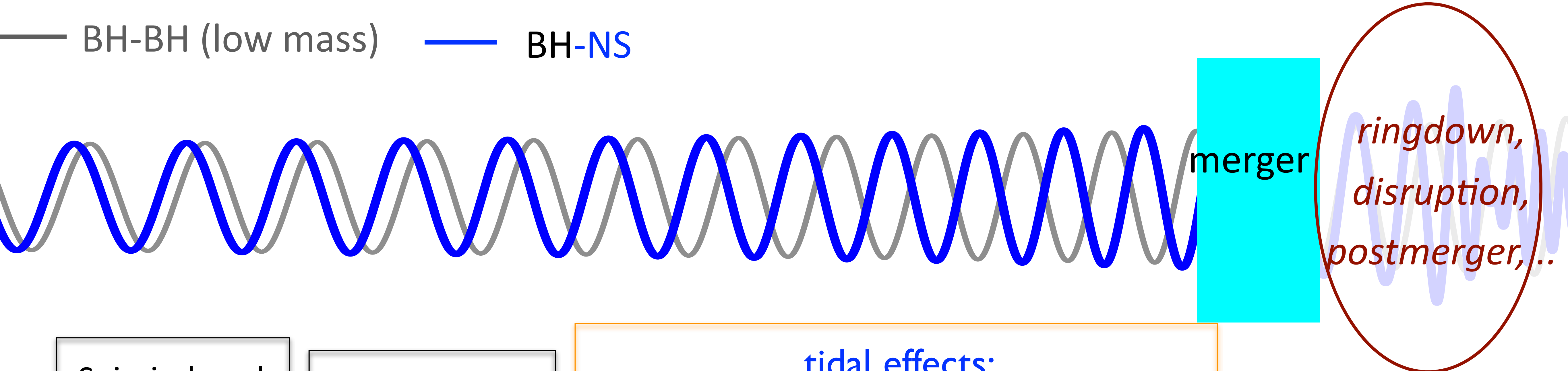
Neutron stars as labs for emergent structural complexity

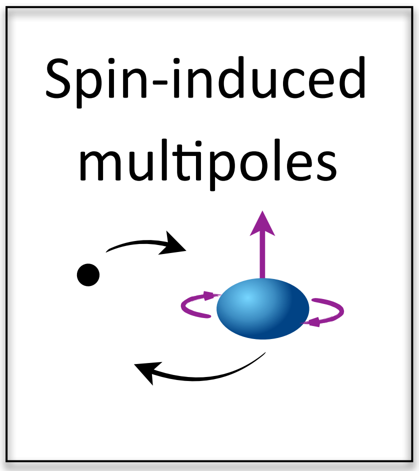
Building blocks of matter

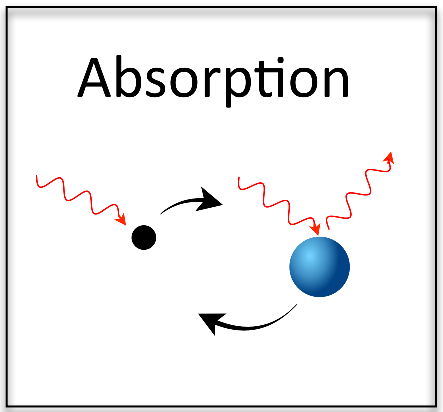


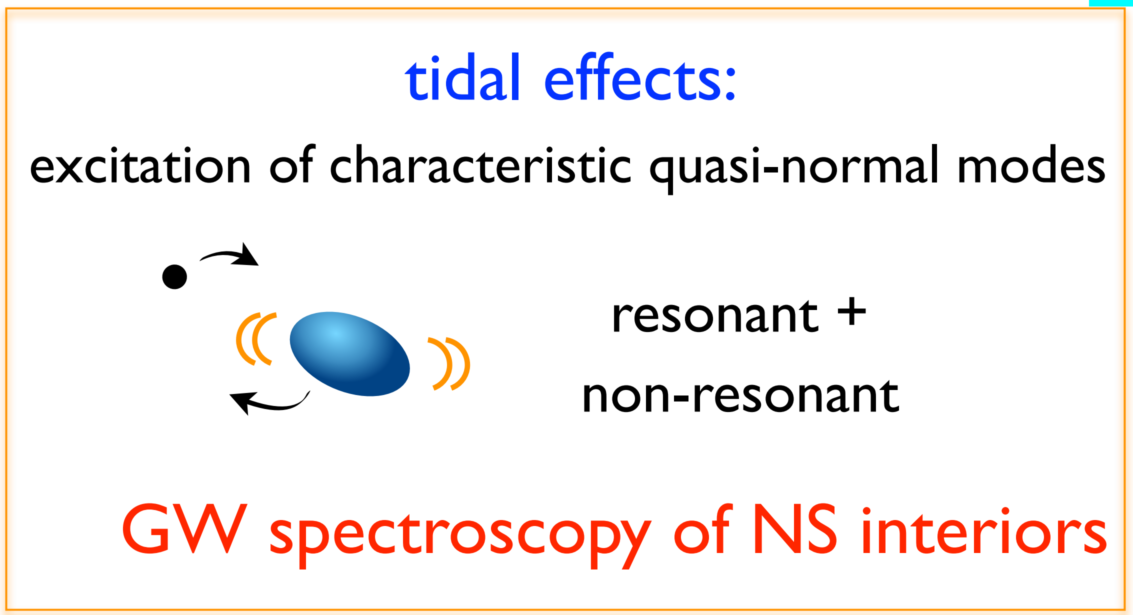
- Collective phenomena, multi-body interactions
- How do nucleons and their fundamental quarks and gluons assemble and interact to create the structure of matter?
- Effects of the excess of neutrons over protons (isospin asymmetry)?

GW signatures of matter









Generic phenomena, apply for any objects that are not classical GR black holes, associated characteristic parameters depend on the object's internal structure

[Flanagan, TH, TH, Damour, Nagar, Poisson, Lai, Steinhoff, Levi, Pani, Vines, Landry, Schmidt, Pratten, Dietrich, Bernuzzi, Blanchet, Faye, Porto, Arun, … many others]

Gravitoelectric tidal effects from quadrupolar fundamental (f -) modes

Based on:

Steinhoff, TH, Buonanno, Taracchini: arXiv:1608.01907

TH+ arXiv:1602.00599

Schmidt, TH: arXiv:1905.00818,

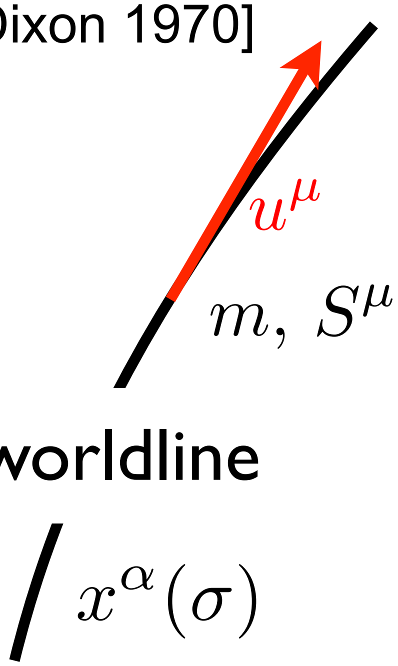
Pratten, Schmidt, TH: arXiv:1905.00817

Flanagan, TH: arXiv:0709.1915

Coupling of NS matter to binary dynamics

World-line skeleton

[Dixon 1970]



u^μ

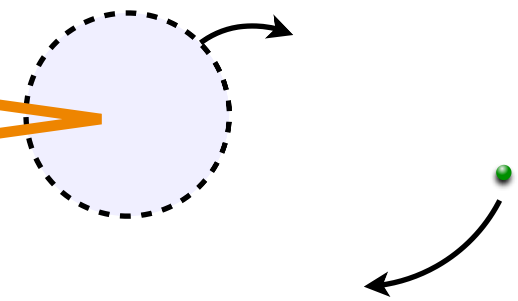
$m,\ S^\mu,\ Q^{\mu\nu}$

worldline

$x^\alpha(\sigma)$

$$\frac{D}{d\sigma} = u^\alpha \nabla_\alpha$$

$$z = \sqrt{-u_\mu u^\mu}$$



dynamics described by an effective action:

$$S = \underbrace{S_{\rm pp}}_{\text{point masses}} + \int d\sigma\ [L_{\rm tidal} + \underbrace{L_{\rm internal}}_{\text{internal dynamics of } Q^{\mu\nu}}]$$

$$L_{\rm tidal} = -\frac{z}{2}\,\mathcal{E}_{\mu\nu}\,Q^{\mu\nu} \qquad \mathcal{E}_{\mu\nu} = C_{\mu\alpha\nu\beta}\frac{u^\alpha u^\beta}{z^2}$$

Weyl tensor (companion)

Induced multipoles behave ≈ as harmonic oscillators:

$$L_{\rm internal} \approx \frac{z}{4\,\lambda\,z^2\,\omega_f^2}\left[\frac{DQ^{\mu\nu}}{d\sigma}\frac{DQ_{\mu\nu}}{d\sigma} - z^2\,\omega_f^2\,Q^{\mu\nu}Q_{\mu\nu}\right]$$

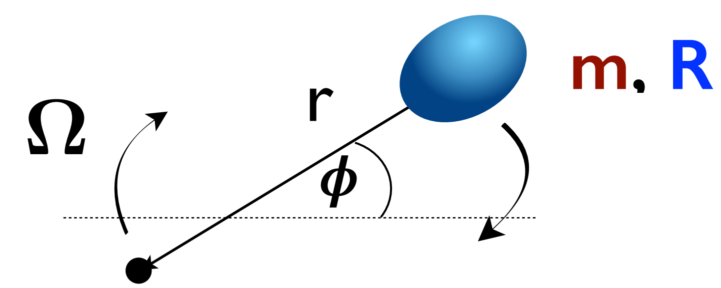
Tidal deformability

fundamental oscillation mode frequency

[Steinhoff, TH +2016, initial work by Goldberger & Rothstein, related work in PN by Vines, Flanagan, …]

Scalings

- f-mode frequency: $\omega_f \sim \sqrt{m/R^3}$ (internal structure - dependent)
- tidal forcing frequency: $\sim \mathbf{2\Omega} \sim \mathbf{2}\sqrt{\boldsymbol{M/r^3}}$



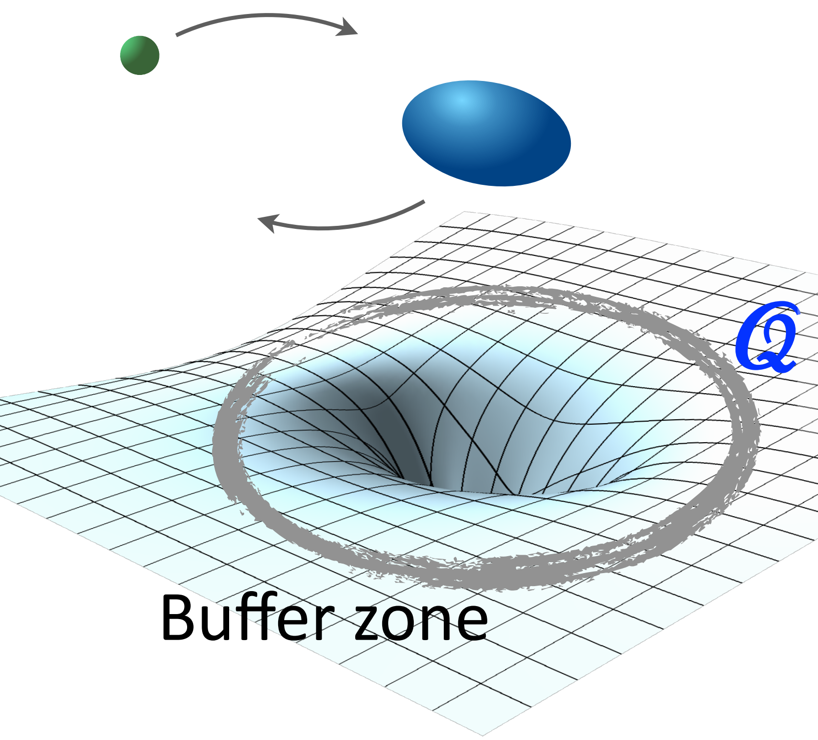
- adiabatic limit $2\Omega \ll \omega_f$: equilibrium solutions $Q_{ij}^{\mathrm{adiabatic}} = -\lambda \mathcal{E}_{ij}$

$$L^{\mathrm{adiabatic}} = L_{\mathrm{pp}} + \frac{z}{4} \lambda \mathcal{E}_{ij} \mathcal{E}^{ij}$$

Tidal invariants

information on NS matter

Information in tidal deformability



Deformation of exterior spacetime away from spherical symmetry

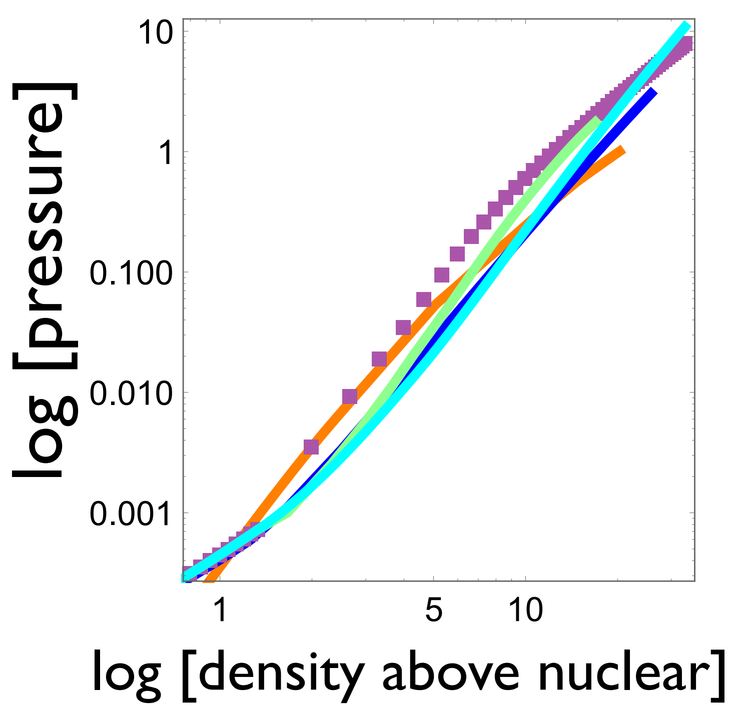
$$\overset{\text{adiabatic}}{\mathcal{Q}_{\rm tidal}} = -\lambda\, \mathcal{E}_{\rm tidal}$$

tidal deformability

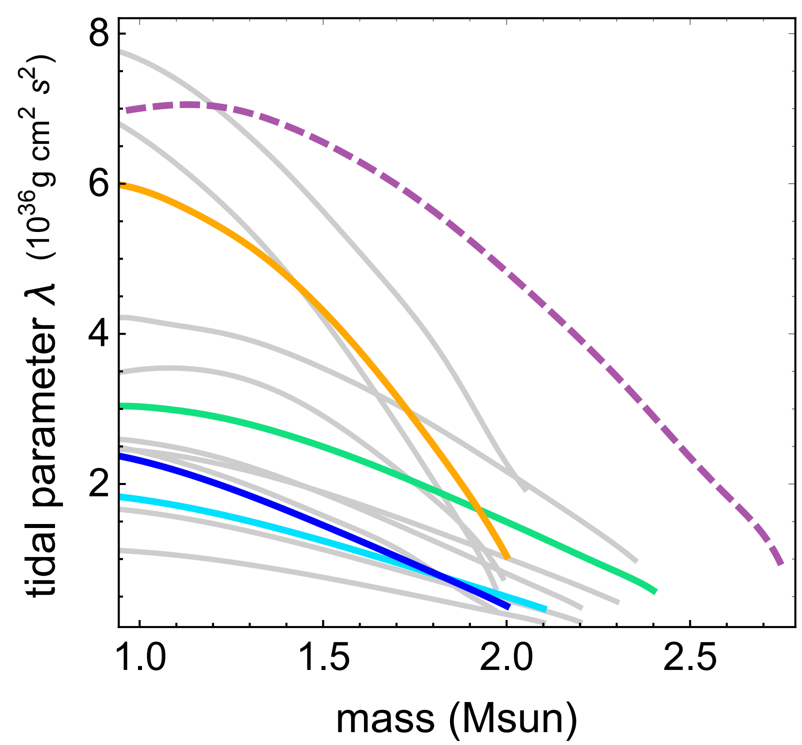
=0 for BHs

[H.S Chia 2010.07300, Casals, LeTiec 2020, Hui+ , Kol & Smolkin , Pani+, …]

equation of state models



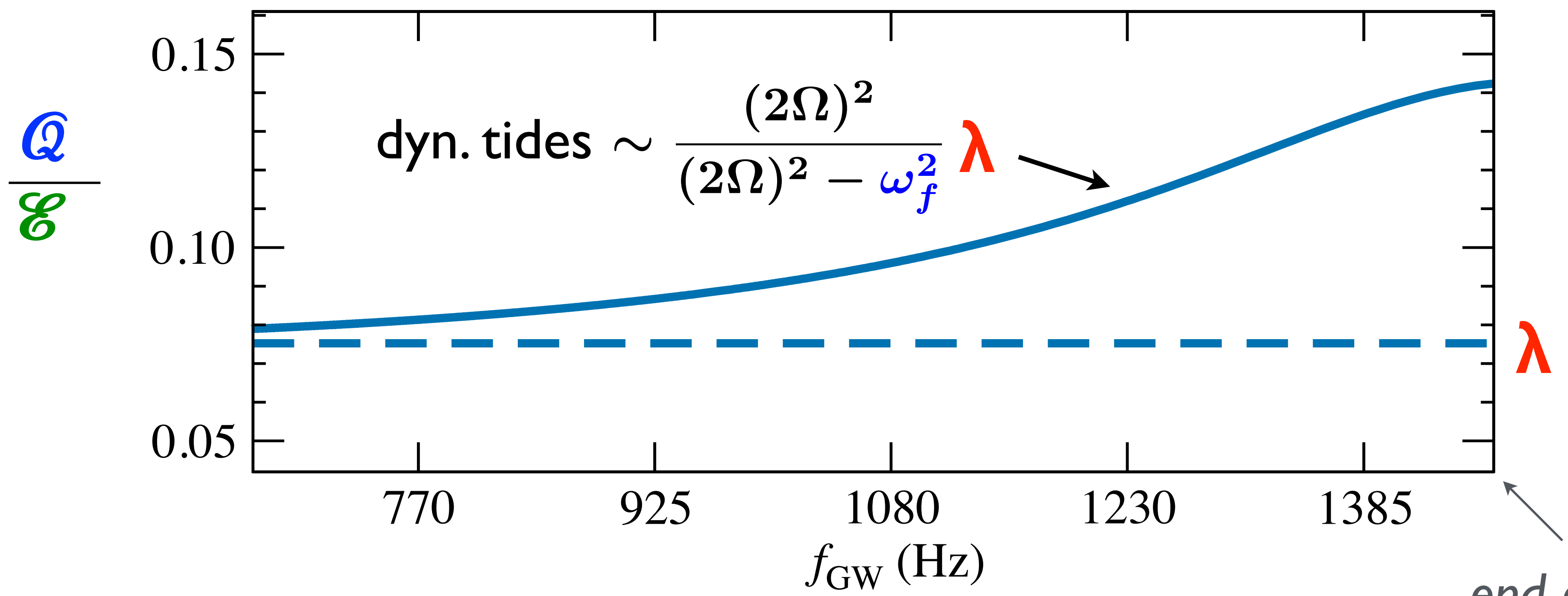
λ vs. mass



computed from the Einstein Field Equations (linearized perturbations to equilibrium NS)

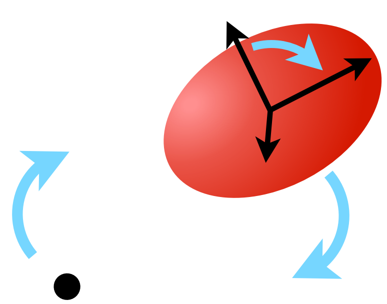
Impact of finite f-mode frequency

Example tidal response during the inspiral (NSBH)



Enhanced tidal response even if resonance is not fully excited

end of inspiral



GR redshift & frame-dragging have opposite effects on response

- Included in the effective one body model SEOBNRv4T
- Can learn interesting information by measuring both λ and ω_f

TH + 2016, Steinhoff, TH 2017, Schmidt, TH 2019, Pratten, Schmidt, TH 2020

Gravitomagnetic tidal effects from quadrupolar inertial modes

Based on:

P. Gupta, J. Steinhoff, TH

arXiv:2011.03508

Inertial modes in slowly rotating NSs

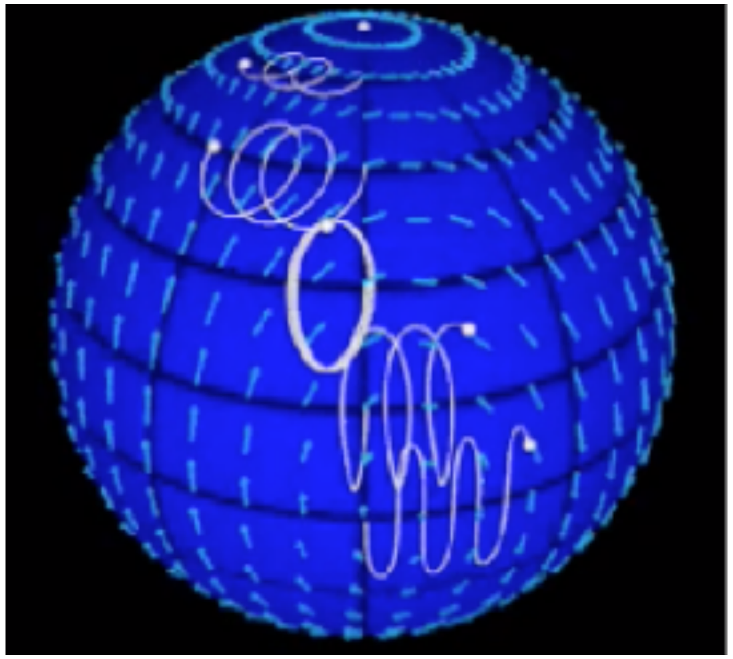
- Restoring force: Coriolis effect

- Mode frequency approximately proportional to *Ω*spin

- Dominant tidal coupling is gravitomagnetic

$$B_{IJ} = *C_{I0J0}$$

Fluid motion in the corotating frame



Agarwal+@UIUC

- Most famous: 'r-modes' = inertial modes with purely magnetic parity (usually ℓ=*m* in a spherical harmonic decomposition). Other *m*-modes have mixed parity contributions.

- Important feature of magnetic modes: shift symmetry

$$Q_{\mathcal{B}}^{IJ} \to Q_{\mathcal{B}}^{IJ} + \mathrm{const}^{IJ}$$

[Friedman & Schutz 1978, Schenck+ 2001]

Relativistic effective action (co-rotating frame)

$$\mathcal{S}^{\mathcal{B}}_{\rm tidal} \approx \int d\tau \left[C^{\mathcal{B}}_{\dot{Q}^2} \dot{Q}^{IJ}_{\mathcal{B}} \dot{Q}^{IJ}_{\mathcal{B}} + C^{\mathcal{B}}_{\Omega Q \dot{Q}} \Omega^{JK} \dot{Q}^{IJ}_{\mathcal{B}} Q^{KI}_{\mathcal{B}} - \frac{1}{2} B_{IJ} \dot{Q}^{IJ}_{\mathcal{B}} + C_{B^2} B_{IJ} B_{IJ} \right]$$

- Unusual features c.f. gravitoelectric case:

$$\mathcal{S}^{\mathcal{E}}_{\rm tidal} \approx \int d\tau \left[\frac{1}{4\lambda\omega_f^2} \left(\dot{Q}^{IJ}_{\mathcal{E}} \dot{Q}^{IJ}_{\mathcal{E}} - \omega_f^2 Q^{IJ}_{\mathcal{E}} Q^{IJ}_{\mathcal{E}} \right) - \frac{1}{2} \mathcal{E}_{IJ} Q^{IJ}_{\mathcal{E}} \right]$$

- Flux quadrupole: $\mathcal{J}^{IJ} \equiv \frac{3}{2} \frac{\partial L}{\partial B_{IJ}} \approx -\frac{3}{4} \dot{Q}^{IJ}_{\mathcal{B}} + 3 C_{B^2} B_{IJ}$

Matter contribution

field contribution

- Relativistic magnetic tidal deformabilities $\boldsymbol{\sigma}$ defined by (adiabatic limit)

$$\mathcal{J}_{IJ} = 2\sigma B_{IJ}$$

Relation of the coefficients to NS parameters

$$\mathcal{S}^{\mathcal{B}}_{\rm tidal} \approx \int d\tau \left[C^{\mathcal{B}}_{\dot{Q}^2} \dot{Q}^{IJ}_{\mathcal{B}} \dot{Q}^{IJ}_{\mathcal{B}} + C^{\mathcal{B}}_{\Omega Q \dot{Q}} \, \Omega^{JK} \dot{Q}^{IJ}_{\mathcal{B}} Q^{KI}_{\mathcal{B}} - \frac{1}{2} B_{IJ} \dot{Q}^{IJ}_{\mathcal{B}} + C_{B^2} B_{IJ} B_{IJ} \right]$$

- Two kinds of relativistic tidal deformabilities: [Landry & Poisson, Pani+, others]
  - $\boldsymbol{\sigma}_{\rm stat}$: static $\dot{Q}^{IJ}_{\mathcal{B}} = 0$
  - $\boldsymbol{\sigma}_{\rm irr}$: irrotational $\Omega = 0$

- Matching determines:

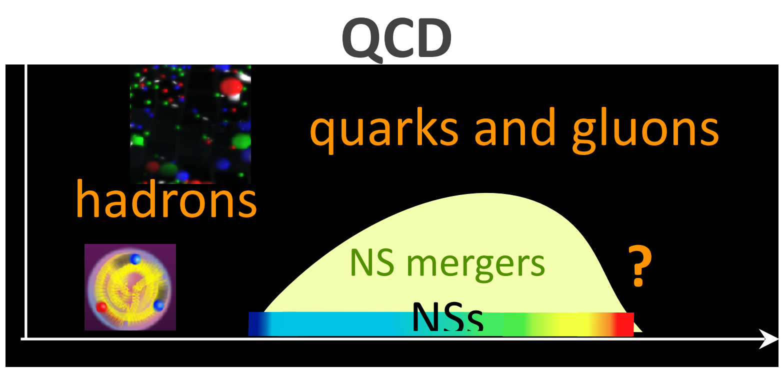
Magnetic mode frequency

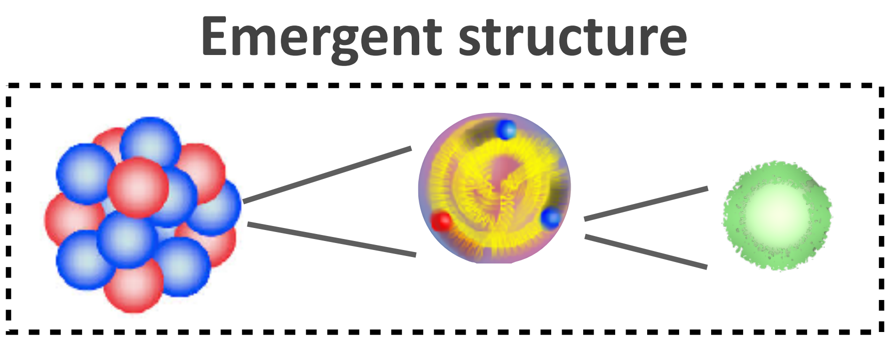
$$C_{B^2} = \frac{2}{3} \sigma_{\rm stat} \qquad C_{\dot{Q}^2} = -\frac{3}{32 \underbrace{(\sigma_{\rm irr} - \sigma_{\rm stat})}_{\text{Matter contribution}}} > 0 \qquad C^{\mathcal{B}}_{\Omega Q \dot{Q}} = -\frac{3\omega^{\mathcal{B}}_{2m}}{16 m \Omega (\sigma_{\rm irr} - \sigma_{\rm stat})} < 0$$

Field contribution

Summary and outlook

Neutron stars can shed light on important questions in subatomic physics





Unique prospects for exploring them with GWs

- Requires accurate theoretical models based on a relativistic description
- Rich phenomena from tidal effects (also have broader applicability to other matter)
- Recent progress, first empirical constraints on gravitoelectric tidal deformability
- Many interesting aspects remain to be explored and understood, plus including more realistic NS physics