Towards the Complete 5PN (for Generic Compact Binaries) with Spins

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QCD meets Gravity VI
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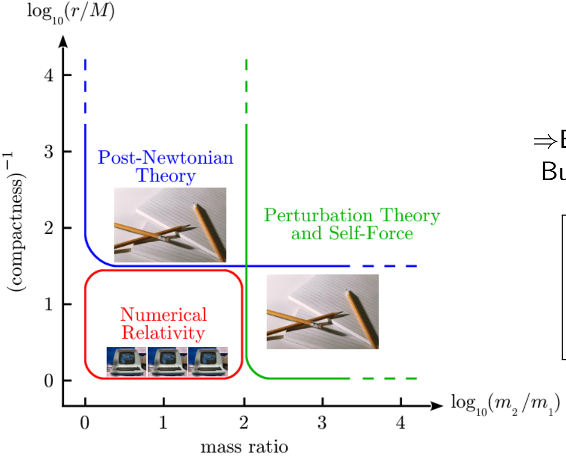
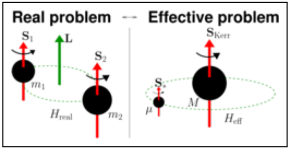
Theory of GW templates



⇒Effective One-Body (EOB)
Buonanno & Damour 1999



- Real world: High demand on accurate theoretical waveform templates
- Modeling of waveforms requires PN parameters of up to 6th PN order! Up to 10% possible discrepancy w simulations at 5PN [nPN$\equiv v^{2n}$]

State of the Art in PN Gravity

Complete state of the art of PN theory for compact binary dynamics

l \ n	$(N^0)LO$	$N^{(1)}LO$	N^2LO	N^3LO	N^4LO	N^5LO
S^0	1	0	3	0	25	0
S^1	2	7	32	**174**		
S^2	2	2	**18**	**52**		
S^3	4	**24**				
S^4	3	**5**				

- Each entry at the PN order $n + l + \text{Parity}(l)/2$
- A measure for loop computational scale: number of (highest) n-loop graphs that enter at N^nLO of up to the lth multipole moment S^l.
- Gray area corresponds to gravitational Compton scattering with $s \geq 3/2$ as classical S^l corresponds to quantum $s = l/2$

State of the Art in PN Gravity Theory

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- 2019: Complete 4PN (w/o spin) corroborated via EFT [Foffa & Sturani]
  - Novel approach for higher-PN corrections [Bini, Damour, Geralico]
  - Generic 4.5PN (with spin) approached via EFT [ML & al. - this talk]
- 2020: Good year for PN gravity!
  - 5PN w/o spin completed [Bini,Damour,Geralico... x2]
  - Generic 4.5 & 5PN (w spin) continued via EFT [this talk]
  - 4.5 & 5PN (w spin) approached à la Bini & al. [Antonelli & al.]

Spin as extra particle DOF

Effective action of spinning particle

- $u^\mu \equiv dy^\mu/d\sigma$, $\Omega^{\mu\nu} \equiv e^\mu_A \frac{De^{A\nu}}{D\sigma} \Rightarrow L_{\text{pp}}\left[\bar{g}_{\mu\nu}, u_\mu, \Omega^{\mu\nu}\right]$
- $S_{\mu\nu} \equiv -2\frac{\partial L}{\partial \Omega^{\mu\nu}}$ spin as further particle DOF – classical source

$$\Rightarrow S_{\text{pp}}(\sigma) = \int d\sigma \left[-p_\mu u^\mu - \frac{1}{2} S_{\mu\nu}\Omega^{\mu\nu} + L_{\text{NMC}}\left[\bar{g}_{\mu\nu}\left(y^\mu\right), u^\mu, S_{\mu\nu}\right]\right]$$

- This form implicitly assumes initial "covariant gauge":

$$e^\mu_{[0]} = \frac{p^\mu}{\sqrt{p^2}}, \quad S_{\mu\nu}p^\nu = 0$$

[Tulczyjew 1959]

- Linear momentum $p_\mu \equiv -\frac{\partial L}{\partial u^\mu} = m\frac{u^\mu}{\sqrt{u^2}} + \mathcal{O}(RS^2)$

For EFT of spin – gauge of both rotational DOFs should be fixed at level of one-particle action

Leading non-minimal couplings to all orders in spin

[**ML** & Steinhoff, JHEP 2014, JHEP 2015]

Spin-induced Wilson coefficients of linear-in-curvature to all orders in spin:

$$L_{\text{NMC}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!} \frac{C_{ES^{2n}}}{m^{2n-1}} D_{\mu_{2n}} \cdots D_{\mu_3} \frac{E_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \cdots S^{\mu_{2n-1}} S^{\mu_{2n}}$$
$$+ \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{C_{BS^{2n+1}}}{m^{2n}} D_{\mu_{2n+1}} \cdots D_{\mu_3} \frac{B_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \cdots S^{\mu_{2n-1}} S^{\mu_{2n}} S^{\mu_{2n+1}}$$

- Love numbers of Kerr - very recent progress [e.g. Casals & Le Tiec; Chia 2020]

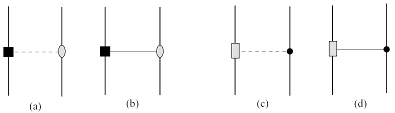
Leading - linear in curvature - spin couplings up to 5PN order

- $L_{ES^2} = -\frac{C_{ES^2}}{2m} \frac{E_{\mu\nu}}{\sqrt{u^2}} S^\mu S^\nu$, Quadrupole @2PN
- $L_{BS^3} = -\frac{C_{BS^3}}{6m^2} \frac{D_\lambda B_{\mu\nu}}{\sqrt{u^2}} S^\mu S^\nu S^\lambda$, Octupole @3.5PN
- $L_{ES^4} = \frac{C_{ES^4}}{24m^3} \frac{D_\lambda D_\kappa E_{\mu\nu}}{\sqrt{u^2}} S^\mu S^\nu S^\lambda S^\kappa$, Hexadecapole @4PN

LO cubic & quartic in spin

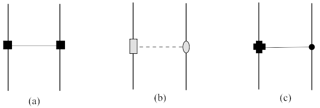
[**ML** & Steinhoff, JHEP 2014]

Feynman diagrams of LO **cubic** in spin sector



- On left pair – quadrupole-dipole, on right – octupole-monopole

Feynman diagrams of LO **quartic** in spin sector

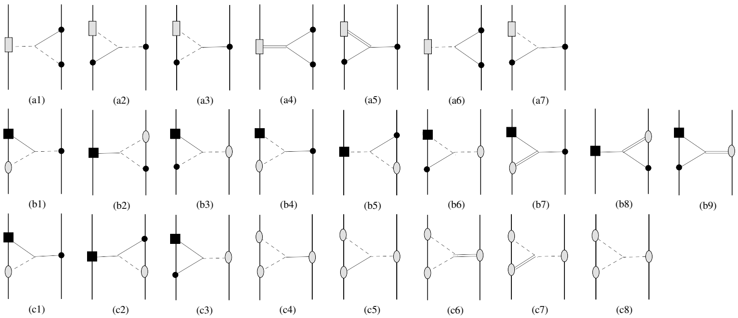


- On left and right – quadrupole-quadrupole and hexadecapole-monopole
- In middle – octupole-dipole

NLO Cubic-in-Spin sector

ML, Mougiakakos, Vieira; JHEP 2020

Feynman graphs of one-loop



- Graphs include all relevant interactions among the spin-induced quadrupole, octupole, and the mass and spin
- There are nonlinearities originating strictly from minimal coupling
- Derivation of NLO Feynman rules becomes quite intricate and subtle

NLO Cubic-in-Spin sector

ML, Mougiakakos, Vieira; JHEP 2020

New Feature: Extra one- and two-graviton exchange

(a1) (a2) (b1) (b2)

- $L_S = -\frac{1}{2}\hat{S}_{ab}\hat{\Omega}^{ab}_{\text{flat}} - \frac{1}{2}\hat{S}_{ab}\omega_\mu^{ab}u^\mu - \frac{\hat{S}_{ab}p^b}{p^2}\frac{Dp^a}{D\lambda} \Rightarrow L_{S^3}$
- $p_\mu = -\frac{\partial L}{\partial u^\mu} = \frac{m}{u}u_\mu + \Delta p_\mu(RS^2),\ \bar{p}_\kappa \equiv \frac{m}{u}u_\kappa$
  $\Delta p_\kappa[S] \equiv p_\kappa - \bar{p}_\kappa \simeq \frac{C_{ES^2}}{2m}S^\mu S^\nu\left(\frac{2}{u}R_{\mu\alpha\nu\kappa}u^\alpha - \frac{1}{u^3}R_{\mu\alpha\nu\beta}u^\alpha u^\beta u_\kappa\right)$
- $\hat{\Lambda}^a_{[0]} = \delta^a_0, \quad \hat{S}^{ab}(p_b + p\delta_{0b}) = 0$ – canonical gauge
- New type of worldline-graviton couplings to "composite" octupole expressed in terms of "elementary" spin multipoles

Extending Non-Minimal Couplings with Spin

Extending effective action beyond linear in curvature

[**ML**, Mcleod, von Hippel; **ML** & Teng JHEP 2020]

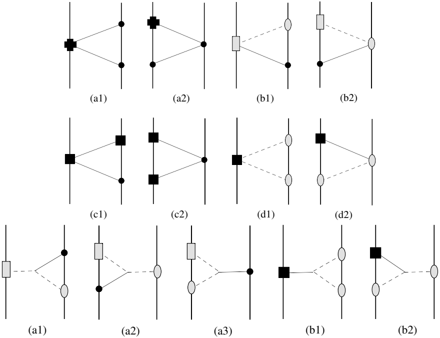
$$
\begin{aligned}
L_{\mathrm{NMC(R^2)}} &= C_{E^2}\frac{E_{\alpha\beta}E^{\alpha\beta}}{\sqrt{u^2}^{\,3}} + C_{B^2}\frac{B_{\alpha\beta}B^{\alpha\beta}}{\sqrt{u^2}^{\,3}} + \dots \\
&+ C_{E^2S^2}S^{\mu}S^{\nu}\frac{E_{\mu\alpha}E_{\nu}^{\alpha}}{\sqrt{u^2}^{\,3}} + C_{B^2S^2}S^{\mu}S^{\nu}\frac{B_{\mu\alpha}B_{\nu}^{\alpha}}{\sqrt{u^2}^{\,3}} \\
&+ C_{E^2S^4}S^{\mu}S^{\nu}S^{\kappa}S^{\rho}\frac{E_{\mu\nu}E_{\kappa\rho}}{\sqrt{u^2}^{\,3}} + C_{B^2S^4}S^{\mu}S^{\nu}S^{\kappa}S^{\rho}\frac{B_{\mu\nu}B_{\kappa\rho}}{\sqrt{u^2}^{\,3}} \\
&+ C_{\nabla EBS}S^{\mu}\frac{D_{\mu}E_{\alpha\beta}B^{\alpha\beta}}{\sqrt{u^2}^{\,3}} + C_{E\nabla BS}S^{\mu}\frac{E_{\alpha\beta}D_{\mu}B^{\alpha\beta}}{\sqrt{u^2}^{\,3}} \\
&+ C_{\nabla EBS^3}S^{\mu}S^{\nu}S^{\kappa}\frac{D_{\kappa}E_{\mu\alpha}B_{\nu}^{\alpha}}{\sqrt{u^2}^{\,3}} + C_{E\nabla BS^3}S^{\mu}S^{\nu}S^{\kappa}\frac{E_{\mu\alpha}D_{\kappa}B_{\nu}^{\alpha}}{\sqrt{u^2}^{\,3}} \\
&+ C_{(\nabla E)^2S^2}S^{\mu}S^{\nu}\frac{D_{\mu}E_{\alpha\beta}D_{\nu}E^{\alpha\beta}}{\sqrt{u^2}^{\,3}} + C_{(\nabla B)^2S^2}S^{\mu}S^{\nu}\frac{D_{\mu}B_{\alpha\beta}D_{\nu}B^{\alpha\beta}}{\sqrt{u^2}^{\,3}} \\
&+ C_{(\nabla E)^2S^4}S^{\mu}S^{\nu}S^{\kappa}S^{\rho}\frac{D_{\kappa}E_{\mu\alpha}D_{\rho}E_{\nu}^{\alpha}}{\sqrt{u^2}^{\,3}} + C_{(\nabla B)^2S^4}S^{\mu}S^{\nu}S^{\kappa}S^{\rho}\frac{D_{\kappa}B_{\mu\alpha}D_{\rho}B_{\nu}^{\alpha}}{\sqrt{u^2}^{\,3}} + \dots,
\end{aligned}
$$

- Non-spinning case in 1st line: [Bini, Damour, Faye; PRD 2012]

NLO Quartic-in-Spin Sector

ML & Teng; JHEP 2020

Non-linear "elementary" Feynman graphs

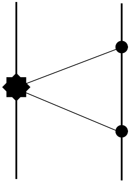


- Though the "elementary" graphs in this sector are fewer and simpler, the situation gets more complicated in terms of new contributions
- More options of worldline-graviton couplings to "composite" hexadecapole in terms of "elementary" spin multipoles, incl. product of Wilson coefficients
- Relevant operators quadratic in the curvature

NLO Quartic-in-Spin Sector

ML & Teng; JHEP 2020

Feynman graph of Quadratic-in-curvature worldline coupling



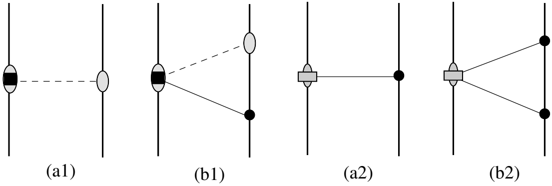
$$L_{S^4(R^2)} = \frac{C_{E^2S^4}}{24m^3} S^\mu S^\nu S^\kappa S^\rho \frac{E_{\mu\nu} E_{\kappa\rho}}{\sqrt{u^2}^3} + \frac{C_{B^2S^4}}{24m^3} S^\mu S^\nu S^\kappa S^\rho \frac{B_{\mu\nu} B_{\kappa\rho}}{\sqrt{u^2}^3}$$

- Turns out only electric operator enters at this order.
- New (unstudied) Wilson coefficient.

NLO Quartic-in-Spin sector

ML & Teng; JHEP 2020

"Composite" worldline couplings

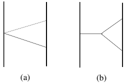


- $L_S = -\frac{1}{2}\hat{S}_{ab}\hat{\Omega}^{ab}_{\text{flat}} - \frac{1}{2}\hat{S}_{ab}\omega_{\mu}^{ab}u^{\mu} - \frac{\hat{S}_{ab}p^b}{p^2}\frac{Dp^a}{D\lambda} \Rightarrow L_{S^3}, L_{S^4}$
- $p_{\mu} = -\frac{\partial L}{\partial u^{\mu}} = \frac{m}{u}u_{\mu} + \Delta p_{\mu}(RS^2),\ \bar{p}_{\kappa} \equiv \frac{m}{u}u_{\kappa}$
- New type of worldline-graviton couplings to "composite" hexadecapole expressed in terms of "elementary" spin multipoles

Graph Topologies for NNNLO linear, quadratic-in-spin

[**ML**, Mcleod, von Hippel 2020]

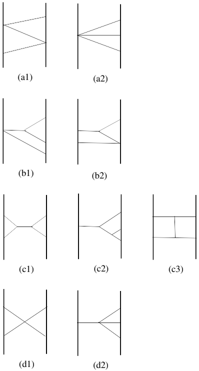
Single topology at $O(G)$:
One-graviton exchange.



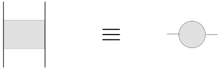
Topologies at $O(G^2)$:
(a) Two-graviton exchange;
(b) Cubic self-interaction
$\equiv$ One-loop topology.

$$\int_{\vec{p}_1} \frac{e^{i\vec{p}_1\cdot(\vec{x}_1-\vec{x}_2)}}{\vec{p}_1^{\,2}} \int_{\vec{p}_2} \frac{e^{i\vec{p}_2\cdot(\vec{x}_1-\vec{x}_2)}}{\vec{p}_2^{\,2}},$$

$$p_1 + p_2 \rightarrow p, \quad p_2 \rightarrow k_1, \quad \rightarrow \int_{\vec{p}} e^{i\vec{p}\cdot(\vec{x}_1-\vec{x}_2)} \int_{\vec{k}_1} \frac{1}{\vec{k}_1^2(\vec{p}-\vec{k}_1)^2}$$



Topologies at $O(G^3)$



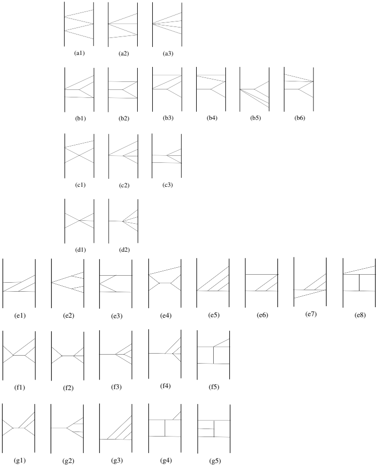
[Kol & Shir 2013]
Standard multi-loop

n-loop master integrals & IBPs – `EFTofPNG` code

A topology at G^{n+1} is rank r, when r basic n-loop integral types form its n-loop integral.

Graph Topologies for NNNLO linear, quadratic-in-spin

[**ML**, Mcleod, von Hippel 2020]

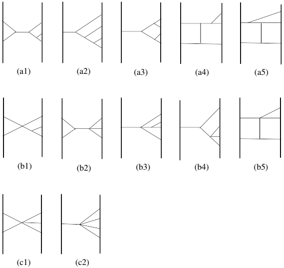


Topologies at $O(G^4)$

At G^n the loop order n_L

$$n_L \equiv 2n - \sum_{i=1}^{n+1} m_i$$

with m_i gravitons on insertion i



3-loop topologies

EFT of gravitating spinning objects

ML, Rept. Prog. Phys. 2020

- Real-world scalability
  $\Rightarrow$ Continuous development of public `EFTofPNG` code
- Self-contained framework
  $\Rightarrow$ Direct derivation of useful & physical quantities
  $\Rightarrow$ Self-consistency checks
  $\Rightarrow$ In progress w **Roger Morales**
- Complete precision frontier with spins at 4.5 & 5PN
  $\Rightarrow$ Within reach, underway
- Lessons from classical nonlinear higher-spin interactions