New Techniques for Rational-Terms of Two-Loop Amplitudes

Sebastian Pögel

Based on work with David Kosower

QCD meets Gravity VI, December 2nd 2020

SAGEX
Scattering Amplitudes:
from Geometry to Experiment

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Why Two Loops?

- Theory predictions at NNLO: accuracy of order ~3%

Why All-Plus?

- Simple structure (easiest after N=4 SYM)
  - Easiest amplitude with rational terms
- Excellent testing ground for new techniques

Why Rational Terms?

- Not cut-constructible in four dimensions
- Missing piece for all-*n* two-loop all-plus in Yang-Mills

Decomposition of two-loop amplitudes

[Catani, hep-ph/9802439]

$$A_n^{(2)} \simeq \underbrace{\left(A_n^{(0)}\mathcal{I}_n^{(2)}\right)}^{\text{Tree} \times \text{2-Loop}} + \overbrace{\left(A_n^{(1)}\mathcal{I}_n^{(1)}\right)}^{\text{1-Loop} \times \text{1-Loop}} + \overbrace{\mathcal{P}_n^{(2)} + \mathcal{R}_n^{(2)}}^{\text{Finite } F^{(2)}}$$

$A_n^{(0)}$: 0 for All-Plus

$\mathcal{P}_n^{(2)}$: Finite Polylogs

$\mathcal{R}_n^{(2)}$: Rational Terms

Feature of two-loop all-plus

Nested loops

$$A_n^{(2)} \simeq \left(A_n^{(1)} \mathcal{I}_n^{(1)} \right) + \mathcal{P}_n^{(2)} + \mathcal{R}_n^{(2)}$$

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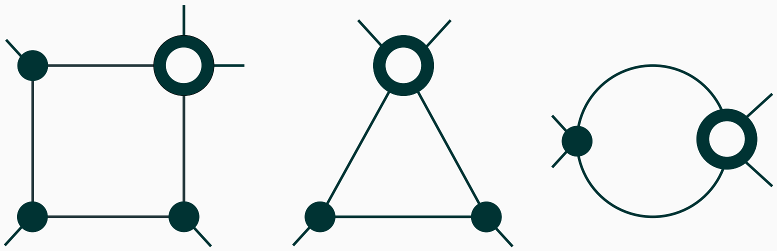

All-n expressions available
[Dunbar, Jehu, Perkins, 1604.06631]

Two-Loop All-Plus

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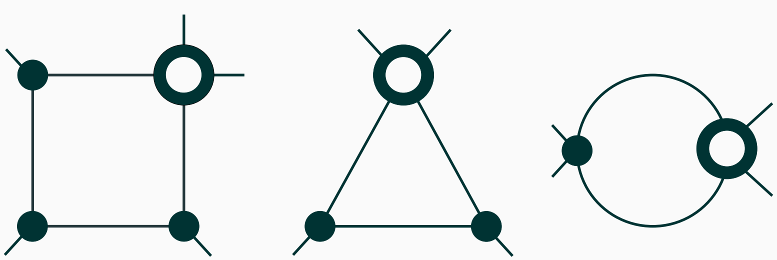


All-n expressions available

[Dunbar, Jehu, Perkins, 1604.06631]

?

4-, 5-, 6- and 7-point known

[Bern, Dixon, Kosower, 0001001]
[Dunbar, Jehu, Perkins, 1604.06631]
[Dunbar, Jehu, Perkins, 1710.10071]

D-dimensional unitarity

Dimensional reconstruction from six dimensions

$$A^{(2)} = A^{2g}_{6\mathrm{D}} + (D_s - 6) A^{sg}_{6\mathrm{D}} + (D_s - 6)^2 A^{2s}_{6\mathrm{D}}$$

Two gluon loops — One scalar, one gluon loop — Two scalar loops

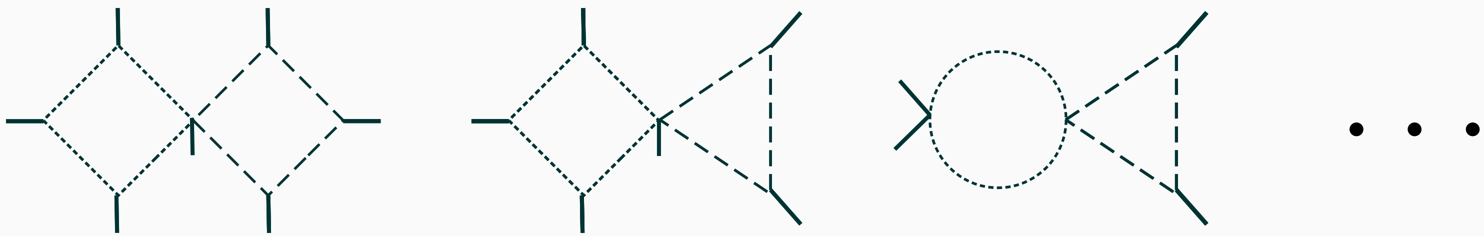
Conjecture for all-plus at leading color:

[Badger, Mogull, Peraro, 1606.02244, 1607.00311]

$$F^{(2)} = \underbrace{(D_s - 2) P^{(2)}}_{\mathcal{P}^{(2)}} + \underbrace{(D_s - 2)^2 R^{(2)}}_{\mathcal{R}^{(2)}} + \mathcal{O}(\epsilon)$$

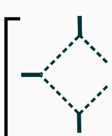
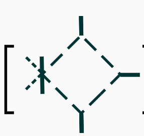
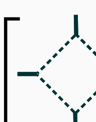
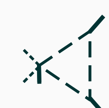
Sufficient to compute rational terms of $A^{2s}_{6\mathrm{D}}$

Loops of different scalar flavors

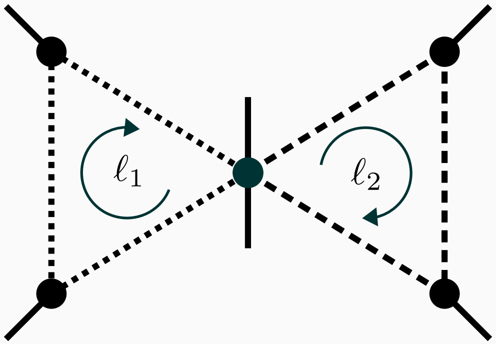


All (one-loop)² topologies

With conjecture:

$\mathcal{R}^{(2)} \propto \mathcal{R}^{(1)} \Big[$  $\times \mathcal{R}^{(1)} \big[$  $\big]\Big] + \mathcal{R}^{(1)} \Big[$  $\times \mathcal{R}^{(1)} \big[$  $\big]\Big] + \dots$

Two-Loop Ingredients



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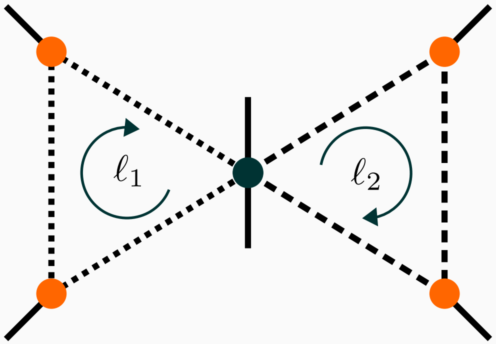

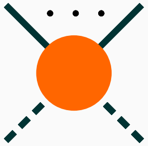

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[Ferrario, Rodrigo, Talavera, hep-th/0602043]

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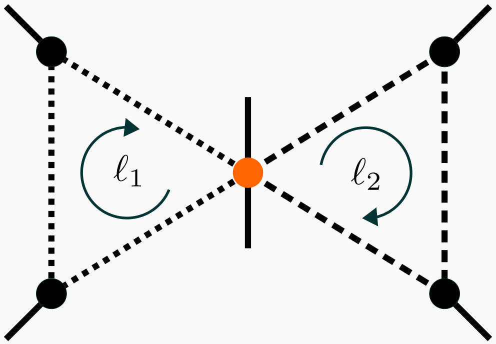

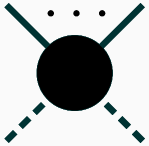

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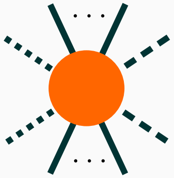

Computable via BCFW

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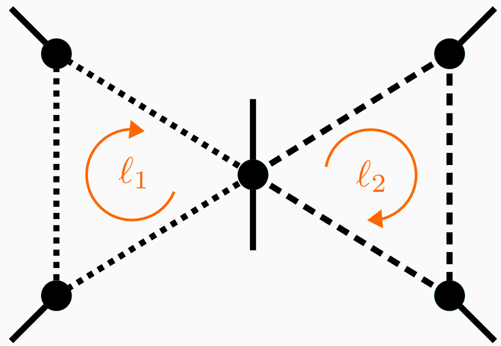

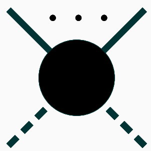

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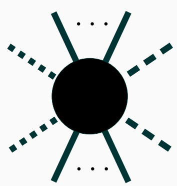

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$\ell_\square, \ell_\triangle, \ell_\circ$ Parametrization following [Badger, 0806.4600]

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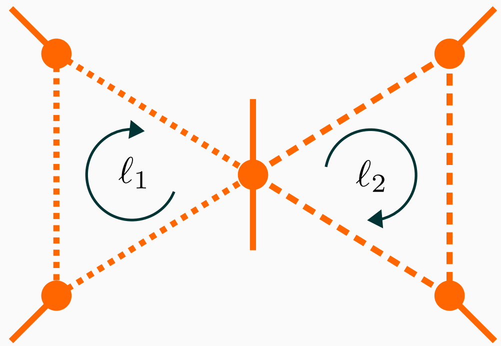

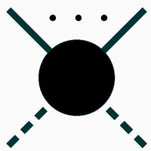

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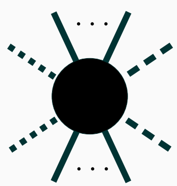

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$\ell_\square, \ell_\triangle, \ell_\circ$

Parametrization following [Badger, 0806.4600]

$C_\square[\mu^4], C_\triangle[\mu^2],$
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Residues via series expansion in Mathematica.
Parameter integrals known
[Forde, 0704.1835, Kilgore, 0711.5015]

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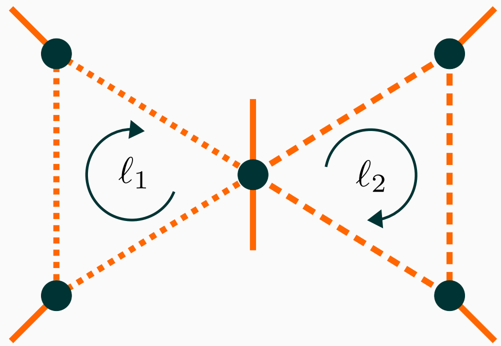

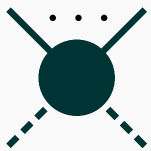

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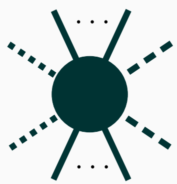

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$I_\Box[\mu^4], I_\triangle[\mu^2], I_\circ[\mu^2]$ — Integrals are well known, e.g.
[Badger, 0806.4600]

Verification

Numeric Checks:

- Computation automated in Mathematica
  - ➔ Rational kinematics possible ⇨ exact results
- Nested construction verified for 4-, 5-, 6-, and 7-gluon amplitudes

Analytic Checks:

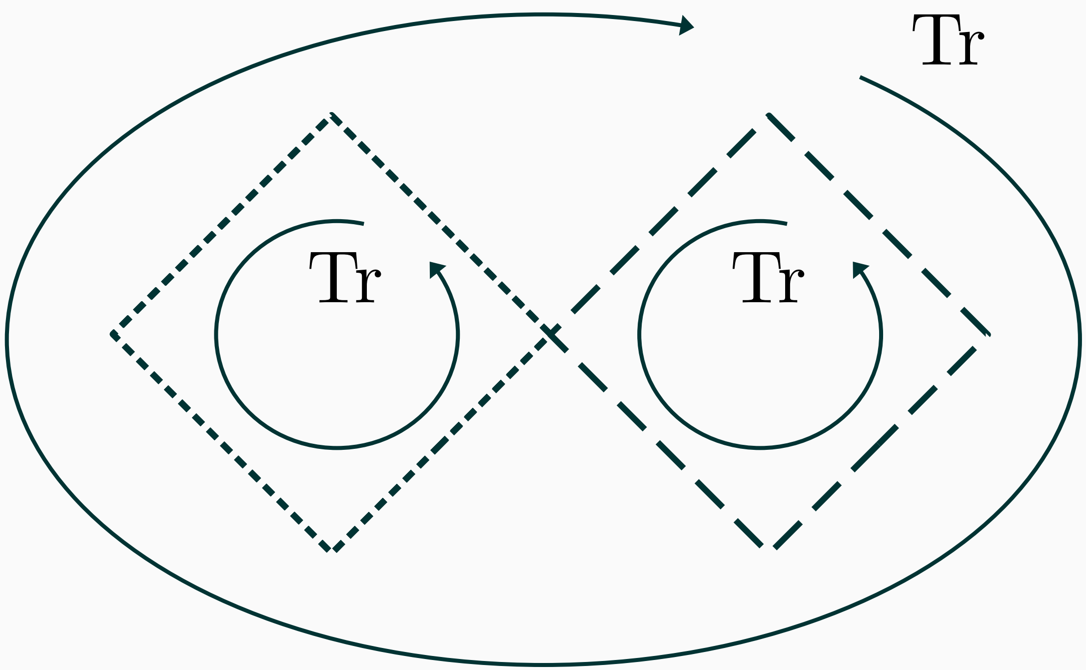
- Parametrized kinematics lead to analytic results
- Parametrized momentum twistors
  - Rederived compact 4-, and 5-gluon expressions

Color decomposition of two-loop amplitudes

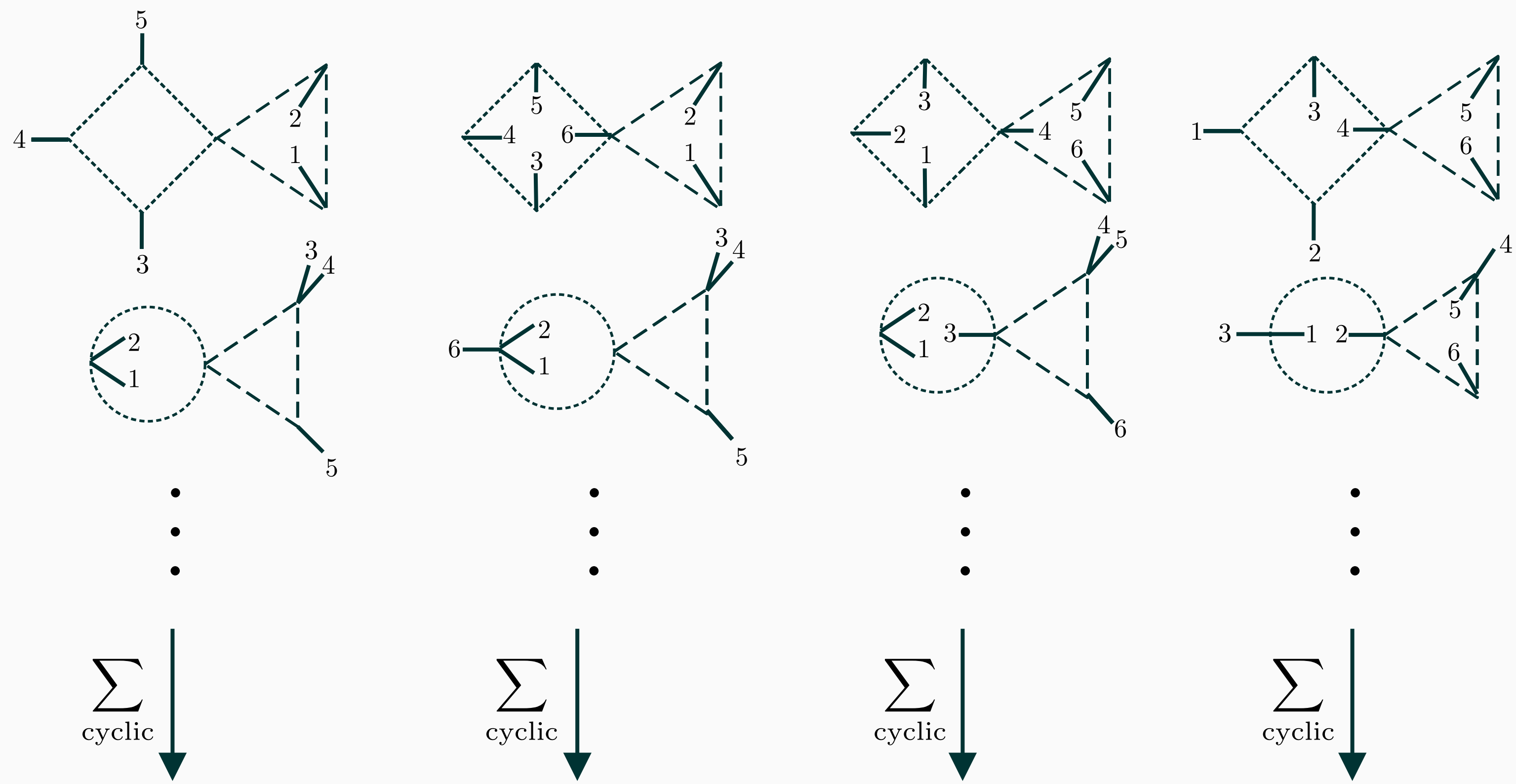
[Dalgleish, Dunbar, Perkins, Strong, 2003.00897]

$$
\begin{aligned}
\mathcal{A}^{(2)} = {} & N_c^2 \mathrm{Tr}[T^{a_1} \ldots T^{a_n}] A^{(2)}_{n:1}(a_1, \ldots, a_n) \\
& + N_c \mathrm{Tr}[T^{a_1} \ldots T^{a_{r-1}}] \mathrm{Tr}[T^{a_r} \ldots T^{a_n}] A^{(2)}_{n:r}(a_1, \ldots, a_{r-1}; a_r, \ldots, a_n) \\
& + \mathrm{Tr}[T^{a_1} \ldots T^{a_s}] \mathrm{Tr}[T^{a_{s+1}} \ldots T^{a_{s+t}}] \mathrm{Tr}[T^{a_{s+t+1}} \ldots T^{a_n}] \\
& \qquad \times A^{(2)}_{n:s,t}(a_1, \ldots, a_s; a_{s+1}, \ldots, a_{s+t}; a_{s+t+1}, \ldots, a_n) \\
& + \mathrm{Tr}[T^{a_1} \ldots T^{a_n}] A^{(2)}_{n:1B}(a_1, \ldots, a_n)
\end{aligned}
$$

Extension of (one-loop)² construction to Non-Planar Cases



$\mathrm{Tr}(12)\mathrm{Tr}(345)$ $\mathrm{Tr}(12)\mathrm{Tr}(3456)$ $\mathrm{Tr}(123)\mathrm{Tr}(456)$ $\mathrm{Tr}(12)\mathrm{Tr}(34)\mathrm{Tr}(56)$



$R^{(2)}_{5:3}$ $R^{(2)}_{6:3}$ $R^{(2)}_{6:4}$ $R^{(2)}_{6:2,2}$

Again (one-loop)² topologies sufficient

Verified numerically against known results

[Dalgleish, Dunbar, Perkins, Strong, 2003.00897]
[Dunbar, Godwin, Perkins, Strong, 1911.06547]

Gravity All-Plus

Graviton All-Plus

For positive helicity gravitons

Nested loop picture appears to work as well

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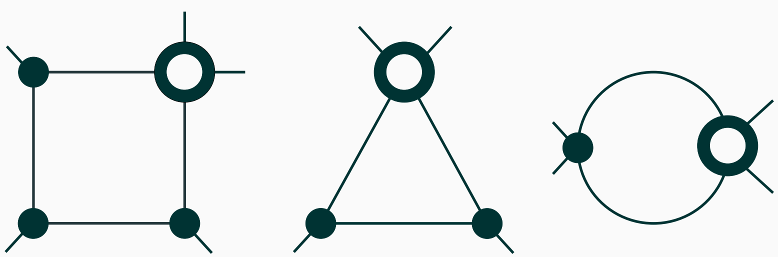

Computed for 4 and 5 gravitons

[Dunbar, Jehu, Perkins, 1701.02934]

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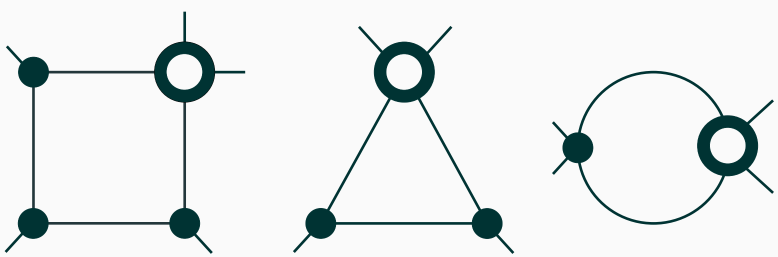

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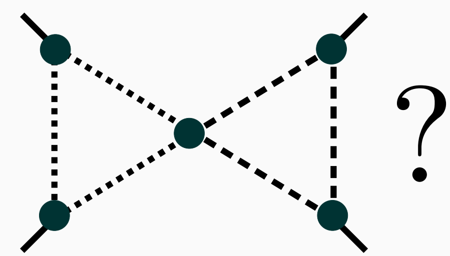

Result from numerical unitarity available for 4 gravitons

[Abreu, Cordero, Ita, Jacquier, Page, Ruf, Sotnikov, 2002.12374]

Two-Loop Gravity Ingredients

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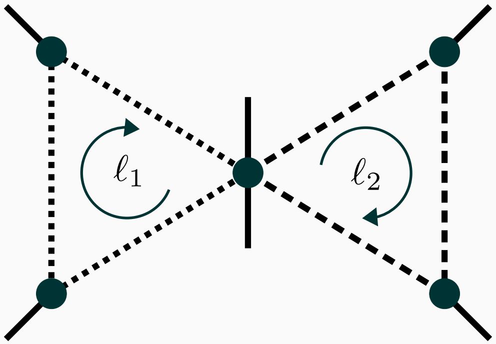

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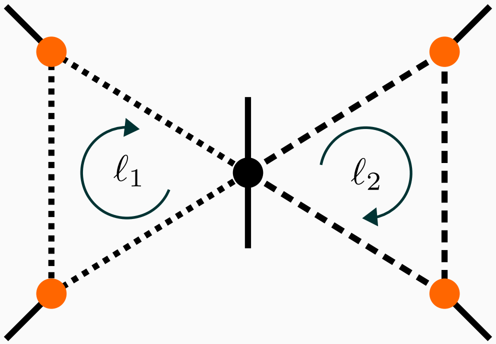

Computable from YM amplitudes via KLT

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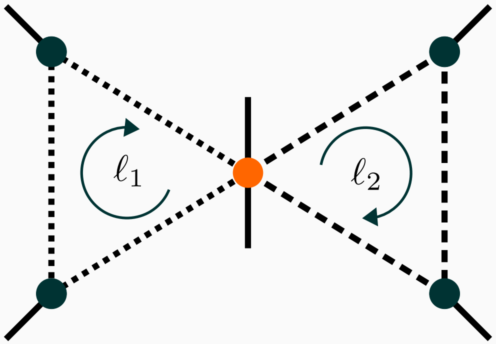

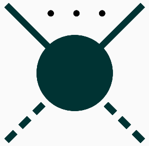

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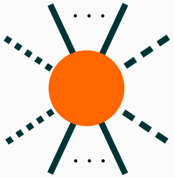

Four-scalar amplitude from YM via KLT?
Adding gravitons: BCFW

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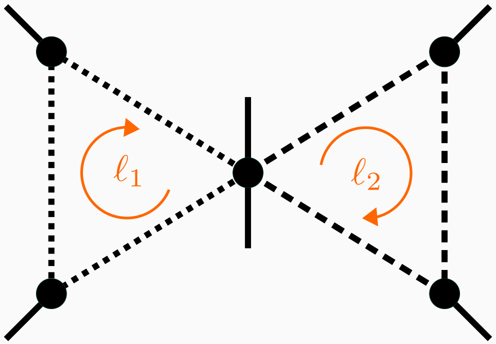

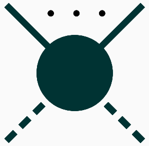

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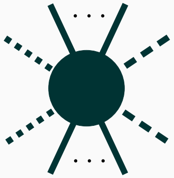

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$\ell_\Box, \ell_\triangle, \ell_\circ$ Parametrization as in YM

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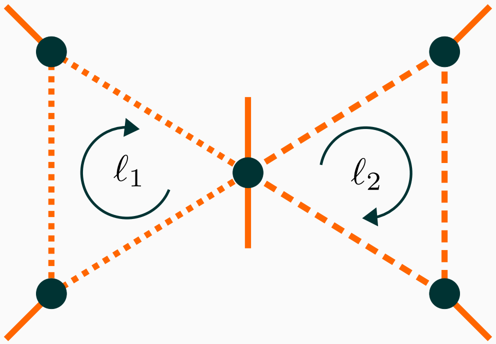

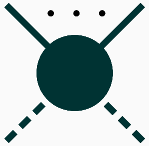

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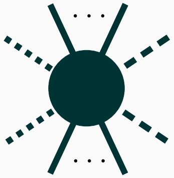

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$\ell_\square, \ell_\triangle, \ell_\circ$

Parametrization as in YM

$C_\square[\mu^8], C_\square[\mu^6], C_\square[\mu^4],$
$C_\triangle[\mu^6], C_\triangle[\mu^5], C_\triangle[\mu^2],$
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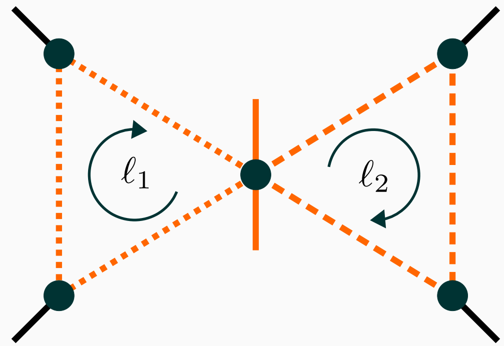
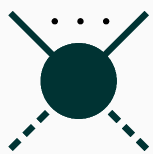
As in YM, but higher orders in parameters
Parameter integrals either known [Kilgore, 0711.5015]
or derived





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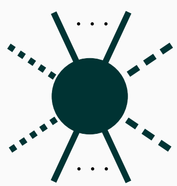

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$I_\triangle[\mu^6], I_\triangle[\mu^4], I_\triangle[\mu^2],$
$I_\circ[\mu^4], I_\circ[\mu^2]$

Integrals computed using dimension shifting identities

Conclusion

Cut-constructible part of two-loop all-plus amplitudes:

- Loops can be nested, to be evaluated one-by-one

Rational terms:

- Nested loop picture also applicable

$$\mathcal{R}^{(2)} \propto \mathcal{R}^{2s(2)}_{6D} = \mathcal{R}^{(1)}\Big[\mathcal{R}^{(1)}\Big]$$

- Holds both for planar and non-planar partial amplitudes

All-Plus Graviton Amplitudes

- Similarity to Yang-Mills in computational approach
  - ➔ Nested loop approach for rationals likely also viable