Locality Constraints on a Massive Double Copy

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QCD Meets Gravity, 2nd December 2020

Based on arXiv:2004.12948
L Johnson, C Jones, SP

Goals

- Understand factorization, decoupling and locality of a naive massive extension of the BCJ double copy
- Does massive Yang-Mills double copy to massive gravity?

[Johansson, Ochirov, Naculich, Chiodaroli, Gunaydin, Roiban, Bautista, Guevara...]

Massless BCJ Color-Kinematics

[Bern, Carrasco, Johansson]

$$\mathcal{A}_n = \sum_i \frac{c_i n_i}{p_i} \to \mathcal{M}_n = \sum_i \frac{\tilde{n}_i n_i}{p_i}$$

Numerators are not unique because of generalized gauge transformations.
Ex: At 4-point,

$$n_s + n_t + n_u = 0 \text{ and } \mathcal{A}_4 = \frac{c_s n_s}{s} + \frac{c_t n_t}{t} + \frac{c_u n_u}{u}$$

are invariant under

$$n_s \to n_s + s\Delta$$
$$n_t \to n_t + t\Delta$$
$$n_u \to n_u + u\Delta$$

$\Rightarrow$ one function's worth of color-kinematics-satisfying numerators.

Massive BCJ Color-Kinematics

[Johnson, Jones, SP]

$$\mathcal{A}_4 = \frac{c_s n_s}{s+m^2} + \frac{c_t n_t}{t+m^2} + \frac{c_u n_u}{u+m^2}$$

This is invariant under a genralized gauge transformation,

$$\begin{aligned} n_s &\to n_s + (s+m^2)\Delta \\ n_t &\to n_t + (t+m^2)\Delta \\ n_u &\to n_u + (u+m^2)\Delta \end{aligned}$$

BUT kinematic Jacobi identity is not preserved

$$\tilde{n}_s + \tilde{n}_t + \tilde{n}_u = (n_s + n_t + n_u) + (s+t+u+3m^2)\Delta = -m^2\Delta\,.$$

$\Rightarrow$ there is no generalized gauge freedom.

A Corollary

Start with numerators in a massive theory that do not satisfy kinematic Jacobi,

$$n_s + n_t + n_u = \Delta$$

Performing a generalized gauge transformation

$$\begin{aligned} n_s &\to n_s + (s + m^2)\frac{\Delta}{m^2} \\ n_t &\to n_t + (t + m^2)\frac{\Delta}{m^2} \\ n_u &\to n_u + (u + m^2)\frac{\Delta}{m^2} \end{aligned}$$

results in

$$\tilde{n}_s + \tilde{n}_t + \tilde{n}_u = (n_s + n_t + n_u) + (s + t + u + 3m^2)\frac{\Delta}{m^2} = 0\,.$$

$\Rightarrow$ Color-kinematics is trivial for massive theories at 4-point.

Finding Numerators in Massive Theories

- Systematic way of finding the unique set of numerators

$$\begin{bmatrix} \mathcal{A}_4[1234] \\ \mathcal{A}_4[1324] \end{bmatrix} = \begin{bmatrix} \frac{1}{s+m^2} + \frac{1}{u+m^2} & \frac{1}{u+m^2} \\ -\frac{1}{u+m^2} & \frac{1}{t+m^2} - \frac{1}{u+m^2} \end{bmatrix} \begin{bmatrix} n_s \\ n_t \end{bmatrix}$$

$$\begin{bmatrix} n_s \\ n_t \end{bmatrix} = \begin{bmatrix} \frac{(m^2+s)(2m^2+s)}{m^2} & -\frac{(m^2+s)(m^2+t)}{m^2} \\ \frac{(m^2+s)(m^2+t)}{m^2} & -\frac{(m^2+t)(2m^2+t)}{m^2} \end{bmatrix} \begin{bmatrix} \mathcal{A}_4[1234] \\ \mathcal{A}_4[1324] \end{bmatrix}$$

- Matrix corresponds to massive bi-adjoint scalar amplitudes
- Matrix is invertable $\Rightarrow$ we can solve for the numerators uniquely
- No null vectors $\Rightarrow$ No BCJ relations
- Kinematic Jacobi-satisfying numerators exist for any massive theory
- No new poles are generated at 4-point

Massless or Decoupling Limit

Naive $m \to 0$ type massless limits are ill-defined because of loss of dofs.
Simple example: a massive photon

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2 A_\mu A^\mu + g(A_\mu A^\mu)^2$$

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Stückelberg trick:

$$\begin{aligned} A_\mu \to & A_\mu + \partial_\mu \phi \\ \delta\mathcal{L} = & - m A_\mu \partial^\mu \phi A^\mu - \frac{1}{2}\partial_\mu \phi \partial^\mu \phi + 4\,\frac{g}{m} A_\mu A^\mu A_\nu \partial^\nu \phi + \frac{g}{m^4}\left(\partial_\mu \phi \partial^\mu \phi\right)^2 \\ & + \frac{g}{m^2}\left(4\,\left(A_\mu \partial^\mu \phi\right)^2 + 2 A_\mu A^\mu \partial_\nu \phi \partial^\nu \phi\right) + 4\,\frac{g}{m^3} A_\mu \partial^\mu \phi \partial_\nu \phi \partial^\nu \phi. \end{aligned}$$

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Only one non-singular limit:

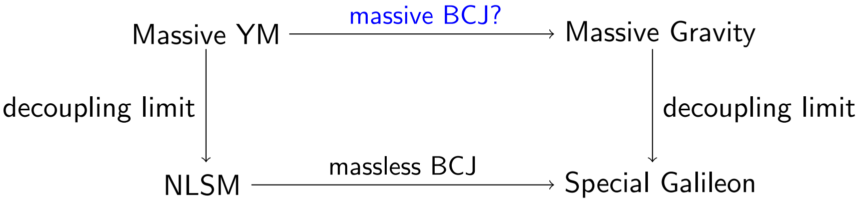
$$g, m \to 0, \quad \Lambda = \left(\frac{m^4}{g} \right)^{1/4}, \, A_\mu, \, \phi \, \text{fixed},$$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{\Lambda^4} (\partial_\mu \phi \partial^\mu \phi)^2$$

Example: Massive Yang-Mills

$$\begin{aligned}\mathcal{L}_{\text{mYM}} =& -\frac{1}{4}\left(\partial_{[\mu}A^a_{\nu]}\right)^2 - \frac{1}{2}m^2 A^a_\mu A^{a\mu} - g f^{abc} A^a_\mu A^b_\nu \partial^\mu A^{c\nu} - \frac{1}{4} g^2 f^{abe} f^{cde} A^a_\mu A^{\mu c} A^b_\nu A^{\nu d} \\ =& \mathcal{L}_{\text{YM}} - \frac{1}{2} m^2 A^a_\mu A^{a\mu}\end{aligned}$$

Goldstone boson equivalence theorem: Longitudinal modes decouple to give NLSM pions in the high energy/massless/decoupling limit.



Can we construct massive gravity from massive YM?

Double-Copying Massive Yang-Mills

Strategy for constructing BCJ form of the amplitude:

$$\text{Feynman rules} \longrightarrow \text{Partial amplitude} \longrightarrow \text{BCJ numerators}$$

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$$A^{\mu} \otimes A^{\nu} = 3 \otimes 3 = 5 \oplus 3 \oplus 1 = h^{\mu\nu} \oplus B^{\mu\nu} \oplus \phi.$$

- Build projection operators
- Calculate amplitudes with different external states
- Use these to check factorization of graviton amplitudes at higher-point

Massive Gravity as a Double Copy: 3-point

At 3-point,

$$\mathcal{A}_3 = 2g\big((\epsilon_2\cdot\epsilon_3)(\epsilon_1\cdot p_2)+(\epsilon_1\cdot\epsilon_3)(\epsilon_2\cdot p_3)+(\epsilon_1\cdot\epsilon_2)(\epsilon_3\cdot p_1)\big).$$

$$\mathcal{M}_3 = \frac{2}{M_p}\big((\epsilon_2\cdot\epsilon_3)(\epsilon_1\cdot p_2)+(\epsilon_1\cdot\epsilon_3)(\epsilon_2\cdot p_3)+(\epsilon_1\cdot\epsilon_2)(\epsilon_3\cdot p_1)\big)^2.$$

where $\epsilon_i^{\mu\nu}=\epsilon_i^{\mu}\epsilon_i^{\nu}$ is replaced by projection operator for different states:

- Dilaton parity is violated so graviton amplitudes have dilaton channels.
- B-parity is preserved.

Massive Gravity as a Double Copy: 4-point

At 4-point,

$$n_s = [(\epsilon_1 \cdot \epsilon_2) p_1^\mu + 2(\epsilon_1 \cdot p_2)\epsilon_2^\mu - (1 \leftrightarrow 2)] \left(g_{\mu\nu} + \frac{(-p_{1\mu} - p_{2\mu})(p_{3\nu} + p_{4\nu})}{m^2} \right)$$
$$\times [(\epsilon_3 \cdot \epsilon_4) {p_3}^\nu + 2(\epsilon_3 \cdot p_4){\epsilon_4}^\nu - (3 \leftrightarrow 4)]$$
$$+ (s + m^2)[(\epsilon_1 \cdot \epsilon_3)(\epsilon_2 \cdot \epsilon_4) - (\epsilon_1 \cdot \epsilon_4)(\epsilon_2 \cdot \epsilon_3)],$$

with $n_t = n_s|_{1\to3\to2\to1}$ and $n_u = n_s|_{1\to2\to3\to1}$.

$$\mathcal{M}_4 = \frac{{n_s}^2}{s + m^2} + \frac{{n_t}^2}{t + m^2} + \frac{{n_u}^2}{u + m^2}$$

- Factorizes into $\mathcal{A}_3(hh\phi)$ and $\mathcal{A}_3(hhh)$ correctly
- Matches well-known dRGT theory of ghost-free massive gravity.

[deRham, Gabadadze, Tolley]

[Momeni, Rumbutis, Tolley]

Pions and Special Galileons

$$\mathcal{A}_4(g_L g_L g_L g_L) \xrightarrow{\text{decouple}} \mathcal{A}_4(\pi\pi\pi\pi)$$

IF massive double copy and decoupling limit commute, we expect

$$\mathcal{M}_4(h_L h_L h_L h_L) \xrightarrow{\text{decouple}} \mathcal{M}_4(\phi\phi\phi\phi)$$

to be the only non-zero contribution. We find

$$\mathcal{M}(0000) = \frac{7}{144} st(s+t) \quad \checkmark$$
$$\mathcal{M}(2^+000) = -\frac{1}{24\sqrt{6}} st(s+t).$$

$\Rightarrow$ massless limit and massive BCJ do not commute.

Double Copy at 5-Point

Again,

$$\text{Feynman rules} \longrightarrow \text{Partial amplitude} \longrightarrow \text{BCJ numerators}$$

- Unwieldy answer for numerators
- Study singularity structure numerically

Pick a set of independent $p_i \cdot p_j$, $\epsilon_i \cdot p_j$ and $\epsilon_i \cdot \epsilon_j$

$\downarrow$

Assign numerical values to all except one, say s_{12}

$\downarrow$

Study behaviour of amplitude around $s_{12} = -m^2$

- Factorizes correctly on physical poles

Note: Using the KLT form, we can show factorization on physical poles for *n*-point amplitudes!

Spurious Poles

BUT we have spurious poles in the denominator:

$$\alpha_1 s_{12}^4 + \alpha_2 s_{12}^3 + \alpha_3 s_{12}^2 + \alpha_4 s_{12} + \alpha_5 \,,$$

where α_i are functions of the mass and other Mandelstam variables.

In special kinematic configurations, the exact locations of the spurious poles can be found and the amplitude evaluated on such a non-physical pole gives a nonzero residue $\Rightarrow$ (massive YM)$^2 \neq$ massive gravity.

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Where did these come from?

$$\mathcal{A}[\alpha] = M \cdot n$$

where M is a matrix of propagators i.e. contains no spurious singularities.

When do Spurious Poles Arise?

We find numerators by

$$n = M^{-1} \cdot \mathcal{A}[\alpha] = \frac{1}{\det M}(\text{matrix of cofactors}) \cdot \mathcal{A}[\alpha]$$

where the only source of spurious poles can be the numerator of det M.

At 5-point, the numerator is proportional to

$$m^8\Big(s_{12}^4 + \left(2s_{13} + 2s_{14} + 2s_{23} + 2s_{24} - 6m^2\right) s_{12}^3 + \big(s_{13}^2 + 2s_{13}s_{14} + 2s_{13}s_{23} + 4s_{13}s_{24} + s_{14}^2 + 4s_{14}s_{23} + 2s_{14}s_{24} + s_{23}^2$$
$$+2s_{23}s_{24} + s_{24}^2 - 6m^2 s_{13} - 6m^2 s_{14} - 6m^2 s_{23} - 6m^2 s_{24} + m^4\big) s_{12}^2 + \big(2s_{13}^2 s_{24} + 2s_{13}s_{14}s_{23} + 2s_{13}s_{14}s_{24} + 2s_{13}s_{23}s_{24}$$
$$+2s_{13}s_{24}^2 + 2s_{14}^2 s_{23} + 2s_{14}s_{23}^2 + 2s_{14}s_{23}s_{24} + 4m^2 s_{13}s_{14} - 4m^2 s_{13}s_{23} - 6m^2 s_{13}s_{24} - 6m^2 s_{14}s_{23} - 4m^2 s_{14}s_{24}$$
$$+4m^2 s_{23}s_{24} - 8m^4 s_{13} - 8m^4 s_{14} - 8m^4 s_{23} - 8m^4 s_{24} + 24m^6\big) s_{12} + s_{13}^2 s_{24}^2 - 2s_{13}s_{14}s_{23}s_{24} + s_{14}^2 s_{23}^2 + 4m^2 s_{13}^2 s_{14}$$
$$+4m^2 s_{13}s_{14}^2 + 4m^2 s_{13}s_{14}s_{23} + 4m^2 s_{13}s_{14}s_{24} + 4m^2 s_{13}s_{23}s_{24} + 4m^2 s_{14}s_{23}s_{24} + 4m^2 s_{23}^2 s_{24} + 4m^2 s_{23}s_{24}^2 - 8m^4 s_{13}^2$$
$$-20m^4 s_{13}s_{14} - 8m^4 s_{13}s_{23} - 8m^4 s_{13}s_{24} - 8m^4 s_{14}^2 - 8m^4 s_{14}s_{23} - 8m^4 s_{14}s_{24} - 8m^4 s_{23}^2 - 20m^4 s_{23}s_{24} - 8m^4 s_{24}^2$$
$$+24m^6 s_{13} + 24m^6 s_{14} + 24m^6 s_{23} + 24m^6 s_{24} - 20m^8\Big).$$

Canceling the Spurious Poles

These spurious singularities could be canceled but they were not in (massive YM)^2 =(massive gravity).

In the massless case,

BCJ relations $\Rightarrow$ Local amplitudes

In the massive case,

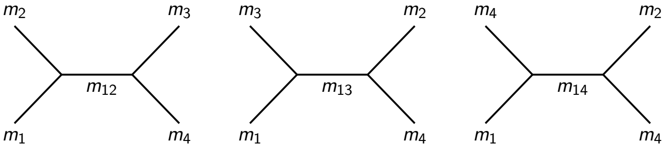
No BCJ relations $\Rightarrow$ Amplitudes with spurious poles

i.e. extra condition needs to be satisfied for locality

How else can we get rid of spurious poles? By considering a more general spectrum.

Spectral Condition

M has rank $(n-3)!$ i.e. the following spectral condition holds:



$$m_{12}^2 + m_{13}^2 + m_{14}^2 = m_1^2 + m_2^2 + m_3^2 + m_4^2.$$

$\Rightarrow$ clearly not satisfied by mYM or any other single mass spectrum.

Satisfied by KK towers, Compton amplitudes, Bhabha scattering.

[Johansson, Ochirov, Chiodaroli, Gunaydin, Roiban, Guevara, Bautista...]

	Spectral Condition	Spurious Pole Cancellation
Local	✓	✓
Commutes with massless limit	✓	×
BCJ Relations	✓	×

Up Next:

- Landscape exploration: What theories satisfy the spectral condition/cancel the spurious singularities?
- 3 Dimensions: Spurious pole-ynomial proportional to 3d Gram determinant... Can massive BCJ work in 3d?
- Higher derivative corrections: Can we add operators/dofs to massive YM to get a healthy double copy?
- Multiple masses: Generalize to case of multiple masses exchanged on each channel

Questions?