Bootstrapping Cosmological Correlations

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QCD Meets Gravity VI

Based on work with

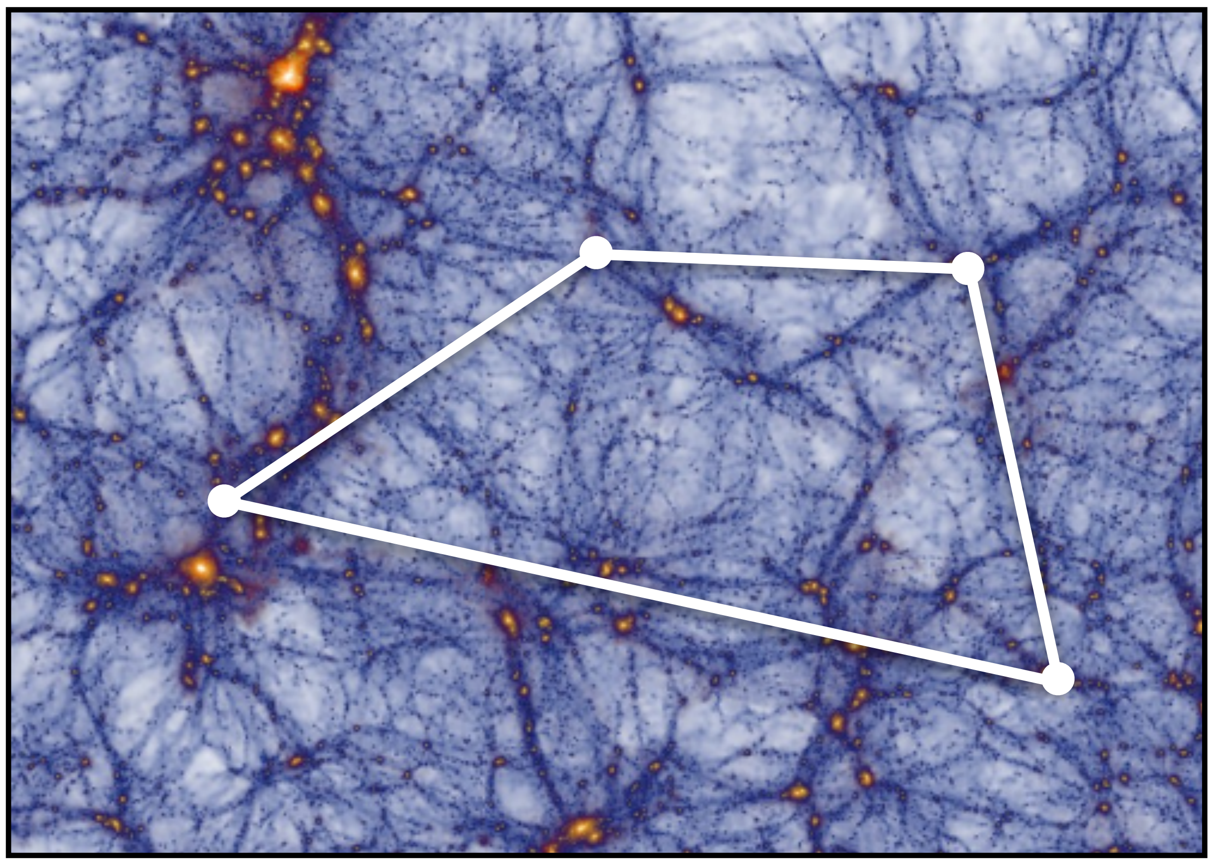
Nima Arkani-Hamed, Wei Ming Chen, Aaron Hillman, Hayden Lee, Manuel Loparco, Guilherme Pimentel, Carlos Duaso Pueyo and Austin Joyce

Interesting related work by

Paolo Benincasa, Charlotte Sleight, Massimo Taronna, Enrico Pajer, Aaron Hillman, Hiroshi Isono, Toshifumi Noumi, Daniel Green, Rafael Porto, David Stefanyszyn, Jakub Supel, Sebastian Cespedes, Anne-Christine Davis, Scott Melville, Harry Goodhew, Sadra Jazayeri, David Meltzer, Allic Sivaramakrishnan, Lorenz Eberhardt, Shota Komatsu, Sebastian Mizera, Paul McFadden, Kostas Skenderis, Adam Bzowski, Soner Albayrak, Savan Kharel, Tanguy Grall, Claudio Coriano, Matteo Maglio, Yi Wang, Gary Shiu, Arthur Lipstein, Joseph Farrow, Raman Sundrum, Soubhik Kumar, Antonio Riotto, Alex Kehagias, Garrett Goon, Kurt Hinterbichler, Mark Trodden, Sandip Trivedi, Suvrat Raju, Juan Maldacena, …

Cosmological Correlations

Spatial correlations are the fundamental observables in cosmology:

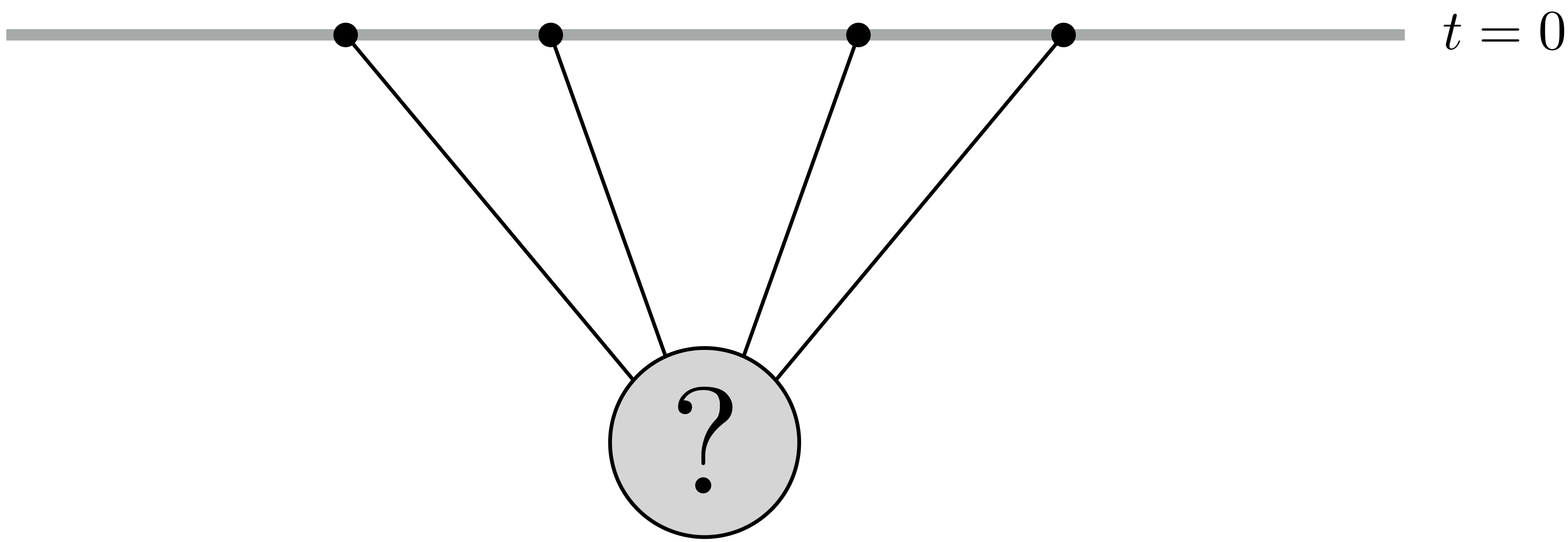


$$= \langle \delta\rho(\vec{x}_1)\delta\rho(\vec{x}_2)\cdots\delta\rho(\vec{x}_N)\rangle$$

These correlations encode the history of the universe.

Back to the Future

All correlations can be traced back to the beginning of the hot Big Bang. If inflation is correct, then this is the boundary of an approximate dS spacetime:



Can we bootstrap the allowed boundary correlators?

- Conceptual advantage: focus directly on observables.
- Practical advantage: simplify calculations.

Outline

1. Singularities of correlators
    - Cosmological wavefunction
    - Total energy singularity
    - Partial energy singularities

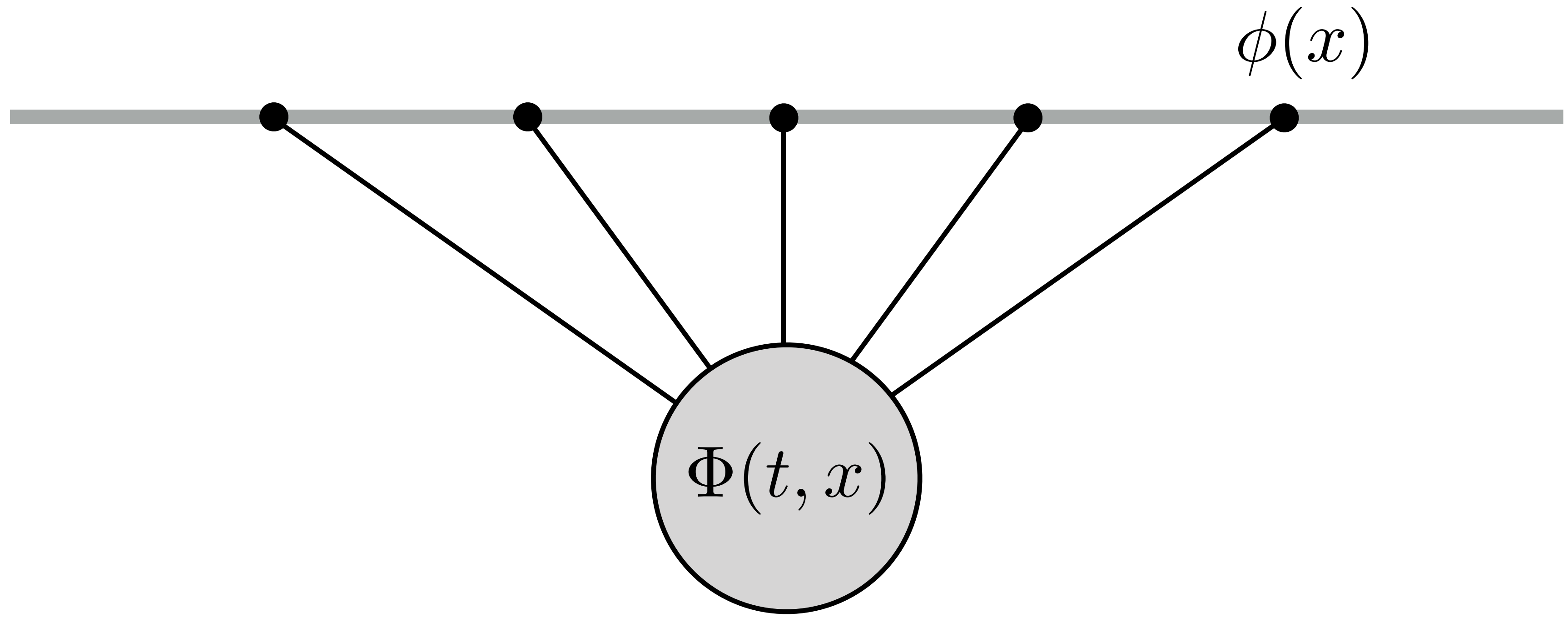
2. Bootstrapping correlators
    - Constraints from local evolution
    - Constraints from bulk unitarity
    - Weight-shifting of scalar seeds
    - From flat space to de Sitter

3. Applications
    - Exchange of massive particles
    - Spinning correlators

4. Open problems

Cosmological Wavefunction

Fluctuations on the boundary can be described by a wavefunctional:



$$\Psi[\phi] \equiv \langle \phi | 0 \rangle$$

Boundary correlators are given by

$$\langle \phi(x_1) \ldots \phi(x_N) \rangle = \int \mathcal{D}\phi \; \phi(x_1) \ldots \phi(x_N) \, |\Psi[\phi]|^2$$

Cosmological Wavefunction

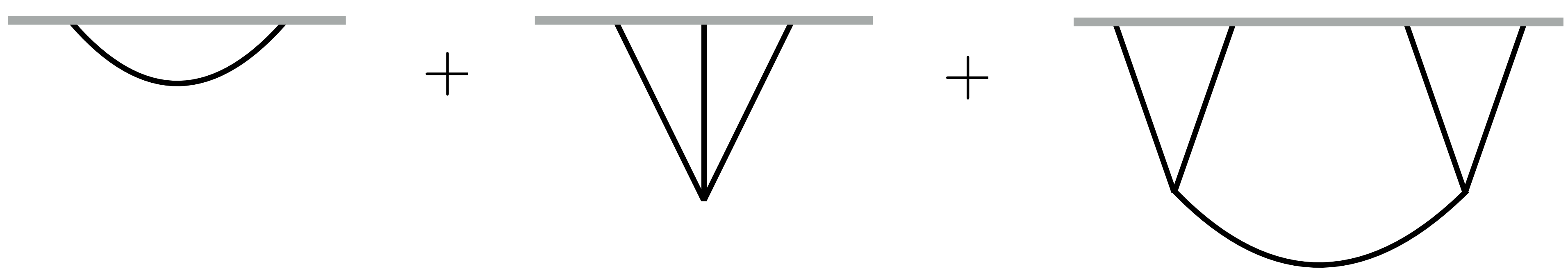
In perturbation theory, the wavefunction is

$$\Psi[\phi] = \exp\left(-\sum_N \int \mathrm{d}^3x_1 \ldots \mathrm{d}^3x_N \, \underline{\Psi_N(x_1, \ldots, x_N)} \, \phi(x_1) \ldots \phi(x_N) \right)$$

wavefunction coefficient

The wavefunction coefficients can be computed in a Feynman-Witten diagram expansion:

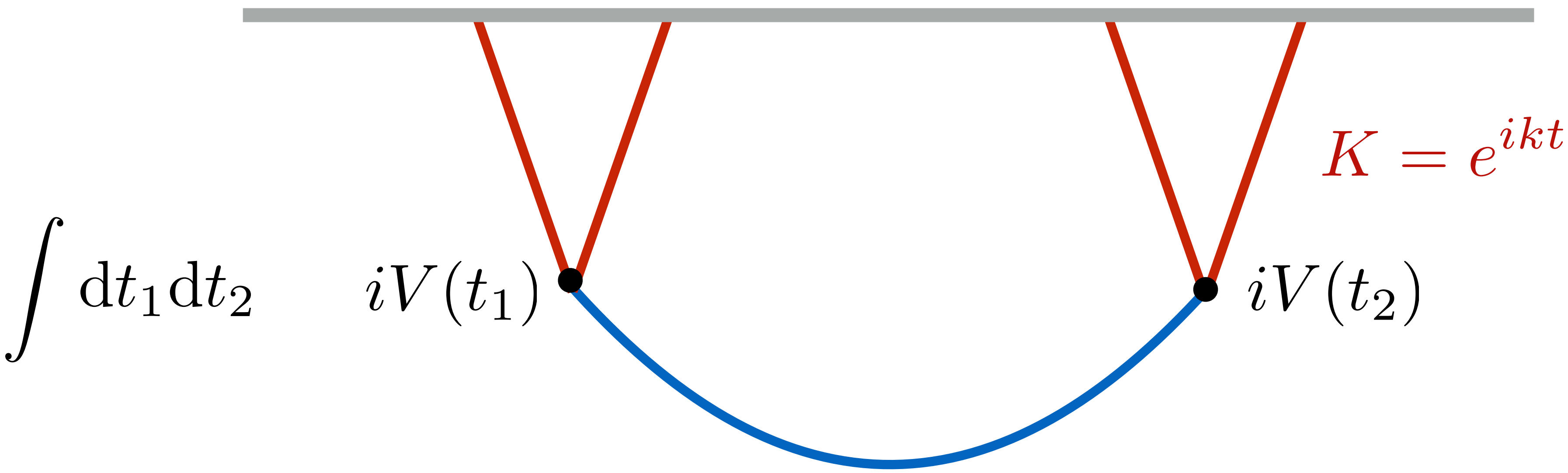
$\Psi_N =$



$+ \quad \cdots$

Feynman Rules

The Feynman rules are



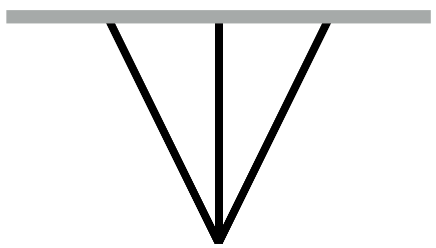
$$G(t_1, t_2) = G_F(t_1, t_2) - \frac{1}{2k_I} e^{ik_I(t_1+t_2)} \qquad \text{(flat space)}$$

$$\lim_{t \to 0} \Phi(t, x) = \phi(x)$$

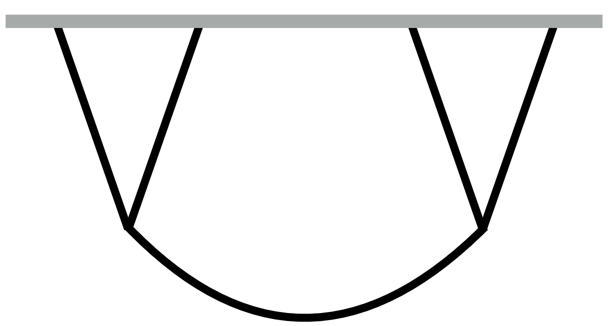
In inflation, the propagators will satisfy de Sitter symmetry, but the vertex factors may include symmetry breaking.

Massless Scalars in Flat Space

For massless scalars in flat space, we find



$$= i\int_{-\infty}^{0} \mathrm{d}t \; e^{iEt} = \frac{1}{E} \qquad E \equiv k_1 + k_2 + k_3$$



$$= -\int_{-\infty}^{0} \mathrm{d}t_1 \mathrm{d}t_2 \, e^{ik_{12}t_1} \, G_{k_I}(t_1, t_2) \, e^{ik_{34}t_2} \qquad k_{34} \equiv k_3 + k_4$$

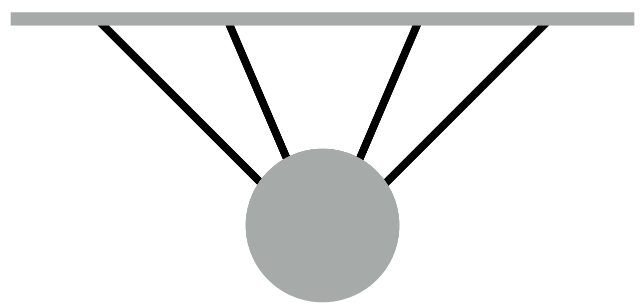
$$= \frac{1}{\underbrace{(k_{12} + k_I)}_{E_L} \underbrace{(k_{34} + k_I)}_{E_R} \underbrace{(k_{12} + k_{34})}_{E}}$$

Correlators are singular when energy is conserved.

Total Energy Singularity

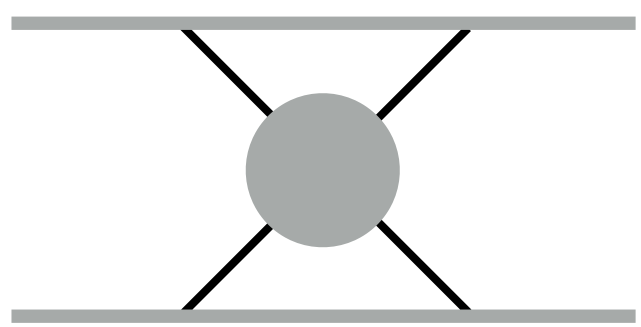
Every correlator has a singularity at vanishing total energy:

$$\Psi_N =$$



$$\sim \int_{-\infty}^{0} \mathrm{d}t\, e^{iEt}\, f(t) \;=\; \frac{A_N}{E^p} + \cdots$$

$$A_N =$$



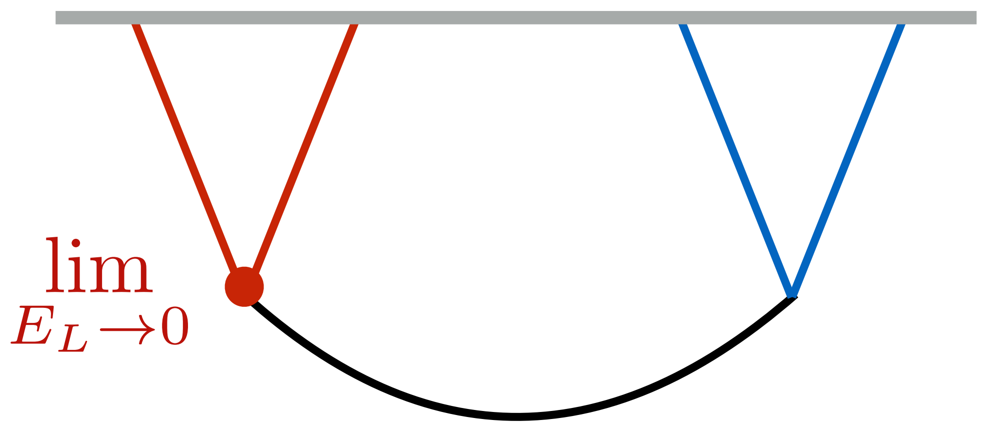
$$\sim \int_{-\infty}^{\infty} \mathrm{d}t\, e^{iEt} \cdots \;=\; M_N \delta(E)$$

The residue of the total energy singularity is the corresponding amplitude.

Raju [2012]
Maldacena and Pimentel [2011]

Partial Energy Singularities

Exchange diagrams lead to additional singularities:



$$= \frac{A_L \times \tilde{\Psi}_R}{E_L^q} + \cdots$$

Bulk-to-bulk propagator:

$$G_{k_I}(t_L, t_R) \xrightarrow{t_L \to -\infty} \frac{e^{ik_I t_L}}{2k_I}\left(e^{-ik_I t_R} - e^{+ik_I t_R}\right)$$

Shifted correlator:

$$\tilde{\Psi}_R \equiv \frac{1}{2k_I}\Big(\Psi(k_{34} - k_I) - \Psi(k_{34} + k_I)\Big)$$

Bootstrapping Correlators

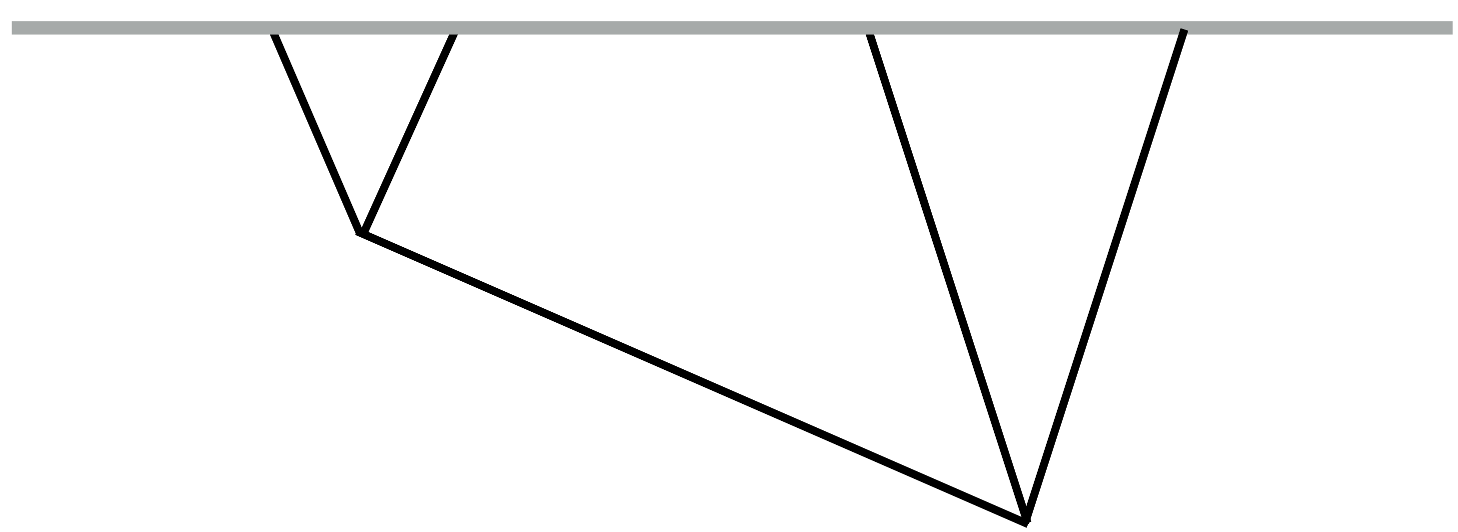
Use these energy singularities as an input:

- Three-point correlators can be constructed from the amplitudes.
- Symmetries are encoded in the residues of the energy singularities.

Pajer, Stefanyszyn and Supel [2020]
Pajer [2020]

In simple examples, the correlators are completely fixed by their singularities.

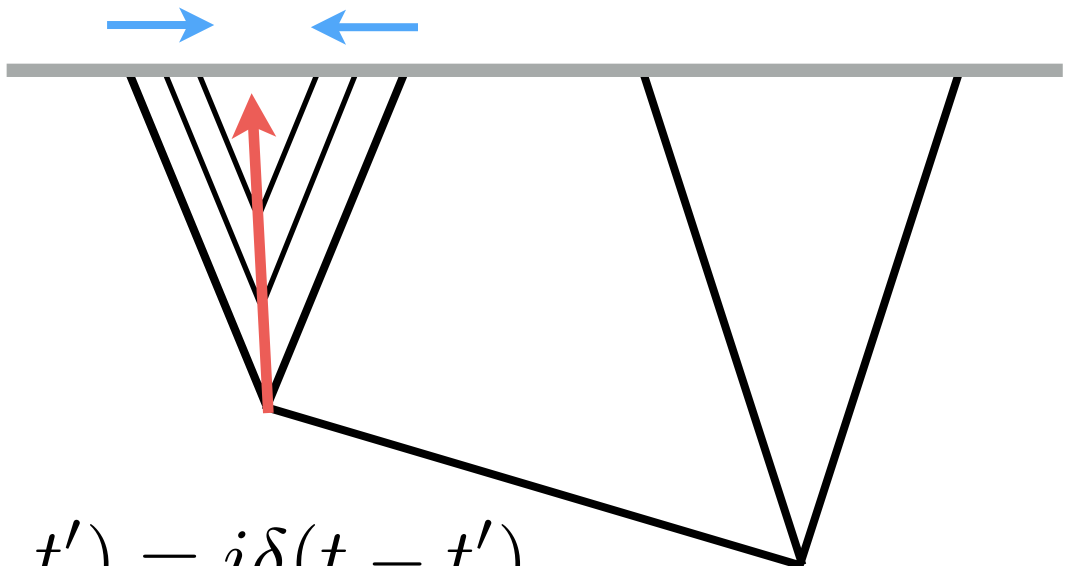
In general, we need a way to connect the singular limits.

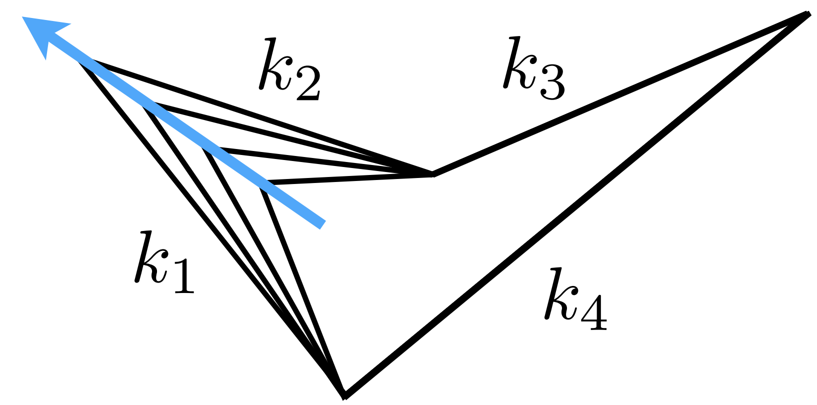


Arkani-Hamed, DB, Lee and Pimentel [2018]

Time Without Time

Time evolution in the bulk = momentum scaling on the boundary:





$$(\Box - M^2)G(t,t') = i\delta(t-t')$$

$$\left[(p^2-1)\partial_p^2 + 2p\,\partial_p - M^2\right]\Psi_4 = C_4 \qquad p \equiv k_{12}/k_I$$

Unique differential operator that kills partial energy singularities.

contact interaction: can include symmetry breaking

The solutions to these differential equations can be systematically classified.

Arkani-Hamed, DB, Lee and Pimentel [2018]

Seed Correlators

The differential equation is simplest for conformally coupled scalars exchanging a scalar.

- More complicated correlators can be generated by shifting the weights (masses and spins) of the external and internal fields:

seed solution

$$\Psi_4 = \mathcal{D}\, \Psi_4^{cc}$$

weight-shifting operators

- Symmetry-breaking interactions can be captured by transmuting de Sitter-invariant seeds:

$$\dot{\phi}^2\sigma \quad \longrightarrow \quad \Psi_4 = \partial_p^2 \partial_q^2\, \Psi_4^{\rm dS}$$

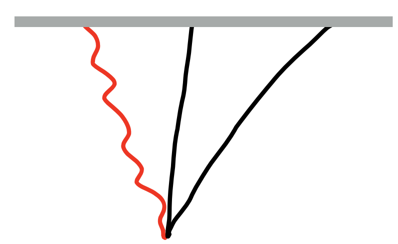
$$(\partial_i\phi)^2\sigma \quad \longrightarrow \quad \Psi_4 = (\vec{k}_1\cdot\vec{k}_2)(\vec{k}_3\cdot\vec{k}_4)\, \Psi_4^{\rm dS}$$

Spinning Correlators

At tree level, the correlators of massless particles with spin are rational functions. The results are highly constrained by their singularities.

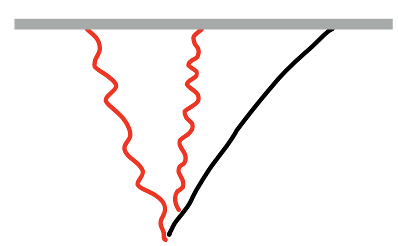
- Three-point correlators can be constructed directly from amplitudes.
- In flat space, we have

$$\langle J_S^- \phi\phi \rangle \;=$$



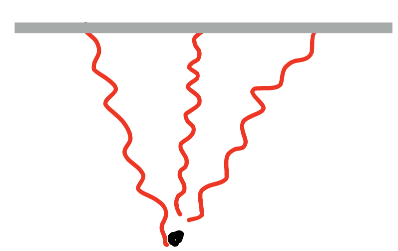
$$= \left(\frac{\langle 12\rangle\langle 13\rangle}{\langle 23\rangle}\right)^S \frac{1}{E}$$

$$\langle J_S^- J_S^- \phi \rangle \;=$$



$$= \langle 12\rangle^{2S} \frac{1}{E}$$

$$\langle J_S^+ J_S^- J_S^- \rangle \;=$$



$$= \left(\frac{\langle 23\rangle^3}{\langle 12\rangle\langle 13\rangle}\right)^S \frac{1}{E}$$

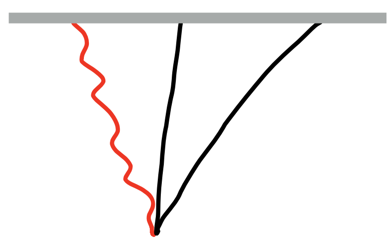
DB, Duaso Pueyo, Joyce, Lee and Pimentel [2020]

Spinning Correlators

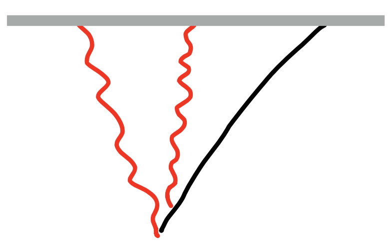
Using the relation between the mode functions

$$\boxed{\phi^{\mathrm{dS}} = (1 - ikt)e^{ikt} = (1 - k\partial_k)\phi^{\mathrm{flat}}}$$

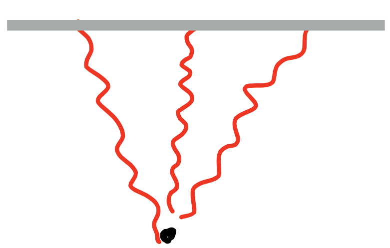
these results can be uplifted to de Sitter space:



$$= \left(\frac{\langle 12\rangle\langle 13\rangle}{\langle 23\rangle}\right)^{S} \prod_{j=1}^{S-1} \left[\overbrace{(2j-1) - k_1\partial_{k_1}}^{\text{transmutation}}\right] \frac{1}{E}$$



$$= \langle 12\rangle^{2S} \prod_{j=1}^{S-1} \left[(2j-1) - k_1\partial_{k_1}\right]\left[(2j-1) - k_2\partial_{k_2}\right] \frac{1}{E}$$



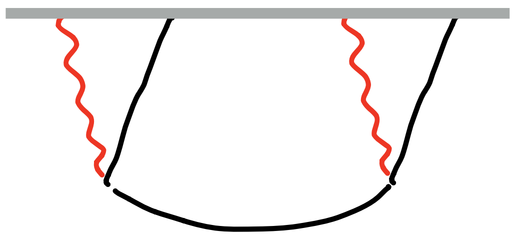
$$= \left(\frac{\langle 23\rangle^{3}}{\langle 12\rangle\langle 13\rangle}\right)^{S} \prod_{i=1}^{3}\prod_{j=1}^{S-1} \left[(2j-1) - k_i\partial_{k_i}\right] \left(\int_E^{\infty} \mathrm{d}E\right)^{2S-2} \frac{1}{E}$$

DB, Duaso Pueyo, Joyce, Lee and Pimentel [2020]

Spinning Correlators

Four-point correlators are constrained by consistent factorization.
For example, the s-channel of the gravitational Compton correlator is

fixed by partial energy singularities



$$= (\vec{\xi}_1 \cdot \vec{k}_2)^2 (\vec{\xi}_3 \cdot \vec{k}_4)^2 \left[\frac{1}{E_L^2 E_R^2} \left(\frac{2k_I k_1 k_3}{E^2} + \frac{2k_1 k_3 + E_L k_3 + E_R k_1}{E} \right) \frac{1}{E_L E_R} \left(\frac{2k_1 k_3}{E^3} + \frac{k_{13}}{E^2} + \frac{1}{E} \right) \right]$$

fixed by total energy singularity

unfixed

Fixing the subleading poles requires additional input.

DB, Duaso Pueyo, Joyce, Lee and Pimentel [2020]

Constraints from Unitarity

Unitarity in the bulk = factorization on the boundary:

Bulk-to-bulk propagator:

$$\mathrm{Re}[G_k(t_1,t_2)] = -\frac{1}{2k}\left(e^{ikt_1} - e^{-ikt_1}\right)\left(e^{ikt_2} - e^{-ikt_2}\right) \qquad \text{(flat space)}$$

↓

Cutting rule:

$$\Psi_4(k_i) + \Psi_4^*(-k_i) \;=\; 2k_I^{2\Delta_I-3}\,\tilde{\Psi}_3^L \otimes \tilde{\Psi}_3^R$$

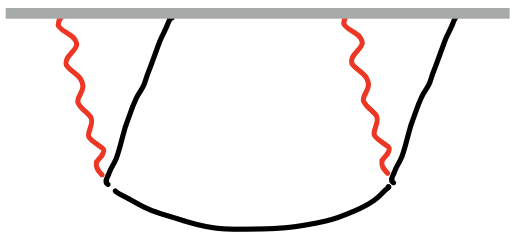
Melzer and Sivaramakrishnan [2020]
Goodhew, Jazayeri and Pajer [2020]
Cespedes, Davis and Melville [2020]

The unitarity cut requires consistent factorization away from the energy poles.

This fixes the subleading singularities up to contact terms.

From Flat Space to de Sitter

The correlator can also be obtained by uplifting the flat space result to de Sitter:



transmutation

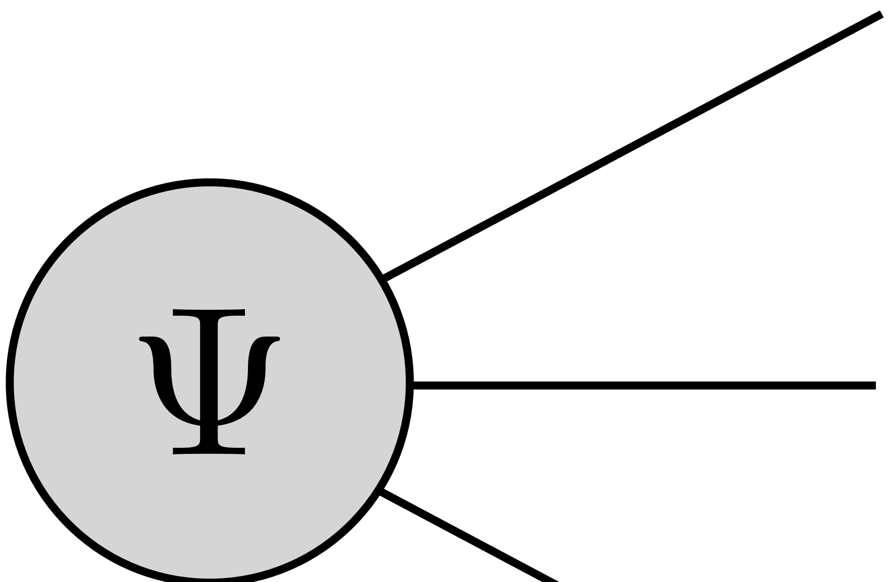
$$= (\vec{\xi}_1 \cdot \vec{k}_2)^2 (\vec{\xi}_3 \cdot \vec{k}_4)^2 \overline{\left[1 - k_1 \partial_{k_1}\right]\left[1 - k_3 \partial_{k_3}\right]} \, \underline{\frac{1}{E E_L E_R}}$$

seed

$$= (\vec{\xi}_1 \cdot \vec{k}_2)^2 (\vec{\xi}_3 \cdot \vec{k}_4)^2 \left[\frac{1}{E_L^2 E_R^2} \left(\frac{2 k_I k_1 k_3}{E^2} + \frac{2 k_1 k_3 + E_L k_3 + E_R k_1}{E} \right) \frac{1}{E_L E_R} \left(\frac{2 k_1 k_3}{E^3} + \frac{k_{13}}{E^2} + \frac{1}{E} \right) \right]$$

The physics is controlled by the leading-order poles. The subleading poles are linked to them by symmetry.

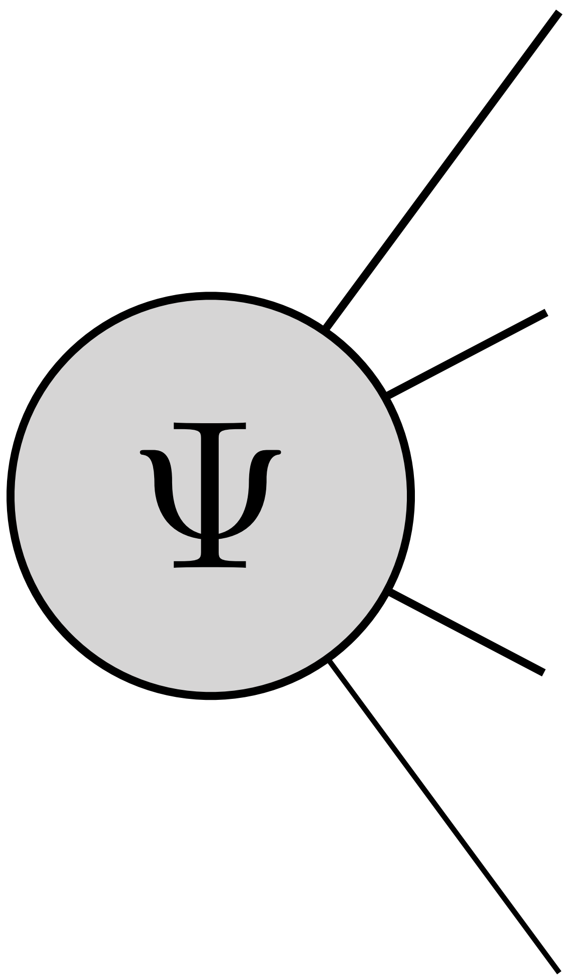
Summary



- **Singularities** control much of the structure of cosmological correlators.
- Simple building blocks are connected by **unitarity** and **local evolution**.
- **Simple seeds** can be transmuted into more complex correlators.

We now have a large amount of **theoretical data** to analyze.
Are there hidden structures to be discovered?

Open Problems



- The bootstrap is still channel by channel. Can we go **beyond Feynman diagrams**?
- Most explicit results are for dS-invariant correlators. Can we classify **boost-breaking correlators**?
- Can we go **beyond perturbation theory**? What are the nonperturbative bootstrap constraints?
- What are the extra constraints imposed by **ultraviolet completion**?

Simons Symposium

Amplitudes Meet Cosmology

May 16 - 22, 2021

Organizers: N. Arkani-Hamed, D. Baumann, JJ Carrasco and D.Green

Thank you for your attention!