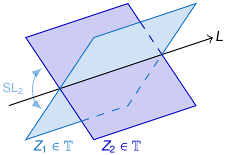
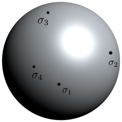
Twistors for Ambitwistor strings

Yvonne Geyer

Chulalongkorn University, Bangkok



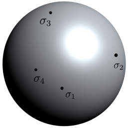


QCD meets Gravity
2020

arXiv:2012.xxxxx with L. Mason, D. Skinner
& work with G. Albonico, L.Mason

Worldsheet representations of FT amplitudes

- RNS ambitwistor string [Mason-Skinner]



$$S = \int_\Sigma P \cdot \overline{D} X - \frac{\tilde{e}}{2} P^2 + S_M$$

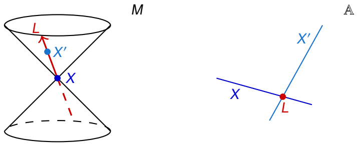
  - $X^\mu \in \Omega^0$, $P_\mu \in \Omega^{(1,0)}$
  - *chiral* worldsheet theory: $\overline{D} = \bar{\partial} + e\partial$
  - worldsheet 'matter': $S_M = S_\psi + S_{\tilde{\psi}}$, $S_\psi = \int_\Sigma \psi \cdot \overline{D} \psi + \chi P \cdot \psi$

Target space: Geometry of $\mathbb{A}$

ambitwistor space $\sim$ space of null geodesics

- Vector representation

$$\mathbb{A} = \left\{(X, P) \in T^*\mathbb{M} \,|\, P^2 = 0\right\} / \{P \cdot \partial_X\}$$



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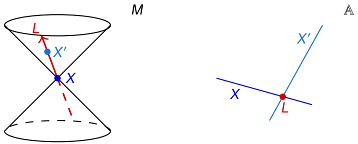

- Twistor representation (4d)

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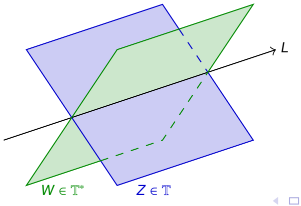

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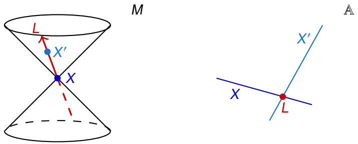

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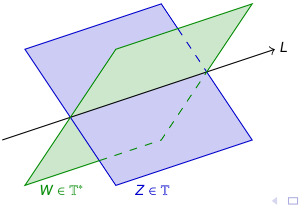

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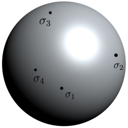

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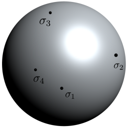

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- **4d twistor/ambitwistor string** [Berkovits-Witten,Skinner]

$$S = \int_\Sigma W \cdot \bar{D} Z - W \cdot \bar{D} Z + a\, Z \cdot W$$

  - $Z \in \Omega^0(K_\Sigma^{1/2} \otimes \mathbb{T})$, $W \in \Omega^0(K_\Sigma^{1/2} \otimes \mathbb{T}^*)$
  - correlators: RSVW / CS-formulae

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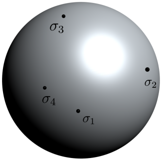

d dimensions

- RNS $\mathbb{A}$-string
- localization: $P^2 = 0$
- [Cachazo-He-Yuan, Mason-Skinner, Berkovits]

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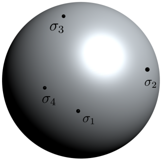

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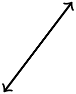

$d = 4$ dimensions

- twistorial $\mathbb{A}$-string
- localization: $\langle\lambda(\sigma_+)\,\kappa_+\rangle = 0$
  $[\tilde{\lambda}(\sigma_-)\,\tilde{\kappa}_-] = 0$
- [Berkovits-Witten, Roiban-Spradlin-Volovich, Skinner]

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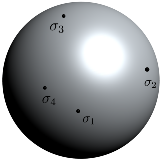

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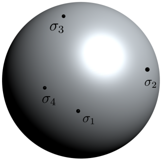

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$d = 5, 6$ dimensions

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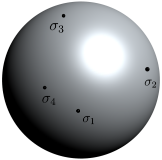

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TODAY

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6d preamble: spinor-helicity

- Vectors: $$k_\mu = \gamma^{AB} k_{AB} , \qquad k_{AB} = k_{[AB]} = \frac{1}{2}\varepsilon_{ABCD}k^{CD}$$

6d preamble: spinor-helicity

$\mu = 0, \ldots, 5$

$\mathrm{Spin}(6, \mathbb{C}) \simeq \mathrm{SL}(4, \mathbb{C})$
$A, B = 0, \ldots, 3$

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little group:
$\mathrm{Spin}(4,\mathbb{C}) \simeq \mathrm{SL}(2,\mathbb{C}) \times \mathrm{SL}(2,\mathbb{C})$
$a = 0, 1, \;\; \dot a = \dot 0, \dot 1$

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Momentum eigenstates:

- $F_A^B = \epsilon_A \epsilon^B$
- EoM: $\epsilon_A k^{AB} = 0 \;\Rightarrow\; \epsilon_A = \epsilon_a \kappa_A^a$
  $\epsilon^A k_{AB} = 0 \;\Rightarrow\; \epsilon^A = \epsilon_{\dot a} \kappa^{A \dot a}$

Twistors and light rays in 6d

- Twistors:
  - pure spinors for the conformal group:

$$\mathbb{T} = \left\{ [Z = (\lambda_A, \mu^B)] \in \mathbb{CP}^7 \,\middle|\, Z \cdot Z = 2\mu^A \lambda_A = 0 \right\}$$

  - incidence relation: $\mu^A = x^{AB} \lambda_B$
    $\Rightarrow [Z] \sim$ totally null self-dual 3-plane (α-plane)

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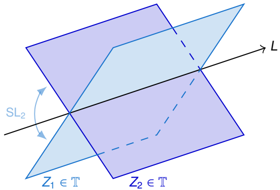
- Ambitwistors:
  - α-planes intersect $\Leftrightarrow$ $Z_1 \cdot Z_2 = 0$
    (along null ray) $\Leftrightarrow$ $Z_1 + uZ_2 \in \mathbb{T}$



  - $\mathbb{PA}$ $\sim$ space of lines in $\mathbb{T}$

$$\mathbb{PA} = \left\{ [Z_a] \in \mathbb{CP}^7 \,\middle|\, Z_a \cdot Z_b = 0 \right\} / \, \mathrm{SL}(2, \mathbb{C})$$

6d ambitwistor string

- Chiral 2d CFT:

ambitwistor string

$$S_{\mathbb{A}} = \frac{1}{2\pi}\int_{\Sigma} \varepsilon_{ab}\, Z^a \cdot \bar{D} Z^b + A_{ab}\, Z^a \cdot Z^b$$

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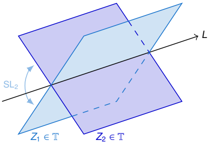
- anomaly-free for S_M with $c = 40$
- target space $\sim \mathbb{PA}$



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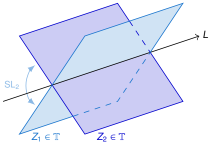

- Spectrum:

bi-adjoint scalar $\oplus$ higher order conformally inv. (gauge $\oplus$ gravity)

Vertex operators

- Penrose transform ($n \geq -1$):

$$H^{0,1}(\mathbb{PA}, \mathcal{O}(n)) \;\cong\; \left\{ \text{lin. fields } \phi^0_{(a_0 \ldots a_n)} \right\} \,/\, \left\{ \phi^0_{(a_0 \ldots a_n)} = \nabla_{(a_0} \xi^0_{a_1 \ldots a_n)} \right\}$$

  - off-shell correspondence

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    $\Rightarrow$ field equations from quantum consistency

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- VOs in twistor variables
  - Spinorial resolution
    - P_μ is null: $P_{AB} = \langle \lambda_A(\sigma)\lambda_B(\sigma)\rangle$
    - Scatt. equs: $k_i \cdot P(\sigma_i) = \det\left(\kappa^a_{iA}, \lambda^a_A(\sigma_i)\right) = 0$

polarized SE

$$u_{ia}\,\lambda^a_A\,(\sigma_i) = v_{ia}\,\kappa^a_{iA}$$

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$\exists \ (u^a_i, v^a_i)$

normal.: $\langle v_i \epsilon_i \rangle = 1$

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Path integral:
$\lambda^a_A(\sigma) = \sum_i \frac{u^a_i \epsilon_{iA}}{\sigma - \sigma_i}$

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polarized SE

$$u_{ia}\,\lambda^a_A(\sigma_i) = v_{ia}\,\kappa^a_{iA}$$

  - Reproduces $P = \sum_i \frac{k_i}{\sigma - \sigma_i}$ due to $\mathrm{Res}_{\sigma_i}\langle \lambda_A \lambda_B\rangle = k_{iAB}$

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  - VO: $V_i = \int d^2u_i\, d^2v_i\, \bar{\delta}(\langle v_i\epsilon_i\rangle - 1)\,\bar{\delta}^4\left(\langle u_i\lambda_A\rangle - \langle v_i\kappa_{iA}\rangle\right) w\, e^{i\langle u_i\mu^A\rangle\epsilon_{iA}}$

Vertex operators

- Penrose transform ($n \geq -1$):

$$H^{0,1}(\mathbb{PA}, \mathcal{O}(n)) \cong \left\{\text{lin. fields } \phi^0_{(a_0 \dots a_n)}\right\} / \left\{\phi^0_{(a_0 \dots a_n)} = \nabla_{(a_0}\xi^0_{a_1 \dots a_n)}\right\}$$

  - off-shell correspondence
  - vertex operator as pull-back: $V_{\text{RNS}} \sim w\, \bar{\delta}(k \cdot P) e^{ik \cdot X}$

    $\Rightarrow$ field equations from quantum consistency

- VOs in twistor variables
  - Spinorial resolution
    - P_μ is null: $P_{AB} = \langle \lambda_A(\sigma) \lambda_B(\sigma) \rangle$
    - Scatt. equs: $k_i \cdot P(\sigma_i) = \det\left(\kappa^a_{iA}, \lambda^a_A(\sigma_i)\right) = 0$

polarized SE

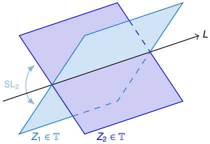
$$u_{ia}\, \lambda^a_A\, (\sigma_i) = v_{ia}\, \kappa^a_{iA}$$

  - VO: $V_i = \int d^2 u_i\, d^2 v_i\, \bar{\delta}(\langle v_i \epsilon_i \rangle - 1)\, \bar{\delta}^4\, (\langle u_i \lambda_A \rangle - \langle v_i \kappa_{iA} \rangle)\, w\, e^{i \langle u_i \mu^A \rangle \epsilon_{iA}}$

Aside: Triality

- Ambitwistors: $[Z = (\lambda_A, \mu^B)]$

$$S_{\mathbb{A}} = \frac{1}{2\pi} \int_\Sigma \varepsilon_{ab}\, Z^a \cdot \overline{D} Z^b + A_{ab}\, Z^a \cdot Z^b$$



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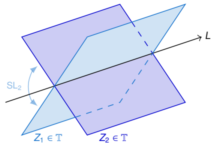

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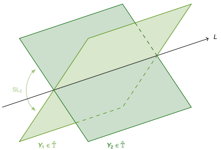

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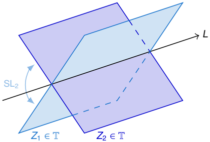

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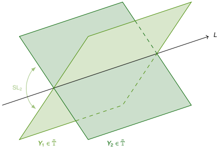


- Embedding description $\mathbb{M}_6 \subset \mathbb{CP}^7$:

$$S_{\mathbb{A}} = \frac{1}{2\pi} \int_\Sigma \varepsilon_{\tilde{a}\tilde{b}}\, X^{\tilde{a}} \cdot \overline{D} X^{\tilde{b}} + A_{\tilde{a}\tilde{b}}\, X^{\tilde{a}} \cdot X^{\tilde{b}}$$

  - $\mathbb{M}_6 = \left\{ [X = (x^{AB}, s, t)] \in \mathbb{CP}^7 \,\middle|\, X \cdot X = x^{AB} x_{AB} + 2st = 0 \right\}$
  - reminiscent of [Adamo-Monteiro-Paulos]
  - reduces to CHY for biadjoint scalar

Question:

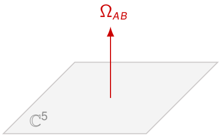
What about Yang-Mills / gravity?

Reduction to 5d

- Reduction:
  - choose non-null vector $\Omega_\mu = \gamma_\mu^{AB}\,\Omega_{AB}$

    momenta: $\Omega \cdot k = \langle \kappa_A \kappa^A \rangle = 0$

    identify hypersurfaces: $\Omega \cdot \partial/\partial x$



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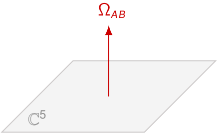


- Ambitwistors:
  - $\mathbb{PA}_5$: $\langle \lambda_A \lambda^A \rangle = 0, \quad \mu^{Aa} \sim \mu^{Aa} + r\Omega^{AB}\lambda_B^a$

    incidence rel.

    $\Omega \cdot \partial_x = \Omega^{AB}\lambda_A^a \partial/\partial\mu^{Ba}$

  - ambitwistor string:

5d $\mathbb{A}$-string

$$S_{\mathbb{A}_5} = S_{\mathbb{A}} + \int_\Sigma a\,\langle \lambda_A \lambda^A \rangle$$

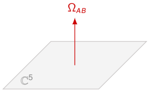
Worldsheet matter

- Super ambitwistors:

  natural supersymmetric extension: $[Z^\alpha = (\lambda_A, \mu^B, \eta^I)] \in \mathbb{CP}^{7|2N}$

$$\mathbb{PA}_{6|N} = \left\{ [Z_a] \in \mathbb{CP}^{7|2N} \,\middle|\, Z_a \cdot Z_b = \mu^A_{(a}\lambda_{A|b)} + \Omega_{IJ}\eta^I_a \eta^J_b = 0 \right\} / \, \mathrm{SL}(2,\mathbb{C})$$

  - reduction to 5d as in bosonic case
  - worldsheet action $\sim$ Green-Schwarz $\mathbb{A}$-string



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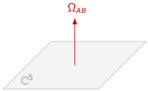

- **Worldsheet matter:**

$$S_{\rho\tilde{\rho}} = \int_\Sigma \tilde{\rho}^A \, \bar{\partial}\rho_A + \chi_a \, \lambda^a_A \rho^A + \tilde{\chi}_a \, \lambda^a_A \tilde{\rho}^A$$

  with fermions $\rho_A, \tilde{\rho}_A \in \Pi\Omega^0(\Sigma, K_\Sigma^{1/2} \otimes \mathbb{S}_A)$

  - vertex operators: $w_{\text{fix}} \sim \delta(\langle u v\rangle)\,\delta(\langle u \tilde{v}\rangle)\, e^{i\langle u\eta^I\rangle q_I}$
    $w_{\text{int}} \sim \left(\epsilon \cdot \langle\lambda\lambda\rangle + F^B_A \rho^A \tilde{\rho}_B\right)\, e^{i\langle u\eta^I\rangle q_I}$

Worldsheet matter and the Double Copy

- Critical models:

  Maximal Supergravity: $S^{\text{sugra}} = S_{5d}^{N=4} + S_{\rho_1} + S_{\rho_2}$

  Maximal super Yang-Mills: $S^{\text{sYM}} = S_{5d}^{N=2} + S_{\rho} + S_{C}$

  Bi-adjoint scalar: $S^{\text{BS}} = S_{5d}^{\text{bos}} + S_{C_1} + S_{C_2}$.

  - anomaly-free after including S_5 from 10d → 5d
  - Double Copy: manifest at level of worldsheet

Correlator

$$\mathscr{M}_n = \left\langle V_1 \, V_2 \ldots V_n \right\rangle$$

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  - EoM: $\bar{\partial}\lambda_A^a = \sum_{i=1}^n u_i^a \epsilon_{iA}\, \bar{\delta}(\sigma - \sigma_i)$
  - Solution: $\lambda_A^a = \sum_{i=1}^n \frac{u_i^a}{\sigma - \sigma_i} \epsilon_{iA}$

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$\Rightarrow$

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$$\prod_{i=1}^n \delta^4\left(\langle u_i \lambda_A(\sigma_i)\rangle - \langle v_i \kappa_{iA}\rangle\right)$$

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$\Rightarrow$ Half-integrands
PT(α)
det$'H$ with $H_{ij} = \sigma^{-1}_{ij}\, \epsilon_{iA} \epsilon^A_j$

Applications in 4d
with G. Albonico, L. Mason

- 4d massive models
  - masses via symmetry reduction:

$$S_{4d}^{m} = S_{\mathbb{A}_5} + \int_{\Sigma} \tilde{a} \left(\tilde{\Omega}^{AB}\langle \lambda_A \lambda_B \rangle - J\right)$$

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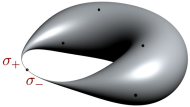

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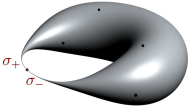

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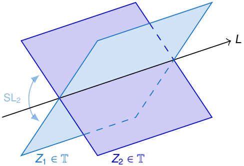
Summary and Outlook



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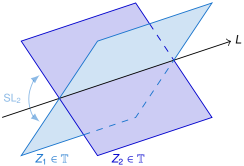

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- Future directions
  - role of $\rho\tilde{\rho}$-system
  - super Yang-Mills / gravity in 6d
  - link to pure spinor string [c.f. Berkovits, Casali, Guillen, Mason]
  - loop amplitudes? [c.f. Wen-Zhang]

Thank you!