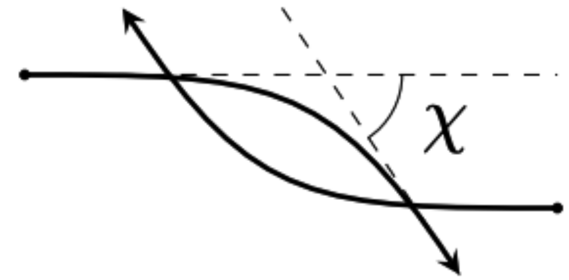
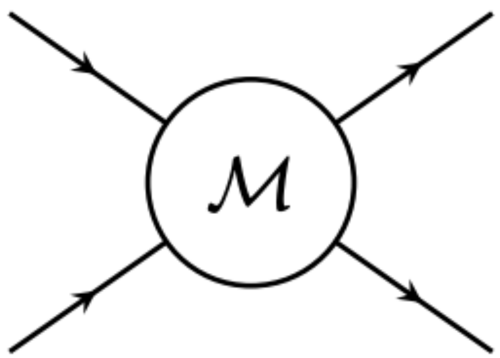
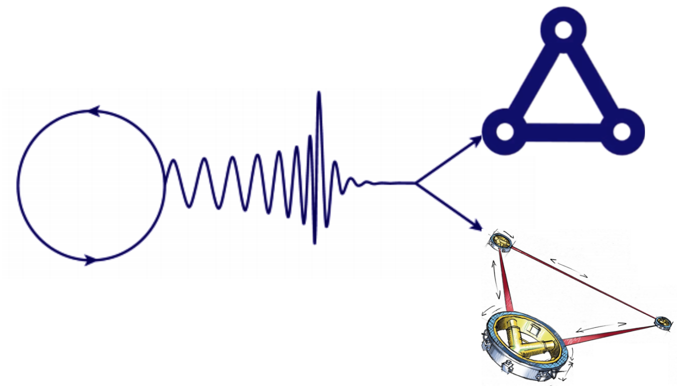
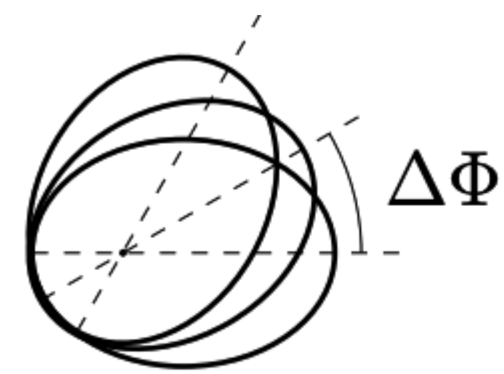
Boundary2Bound & PM EFT: Status and New results









Rafael A. Porto

based on work in collaboration with
Gregor Kälin, Zhengwen Liu and Zixin Yang

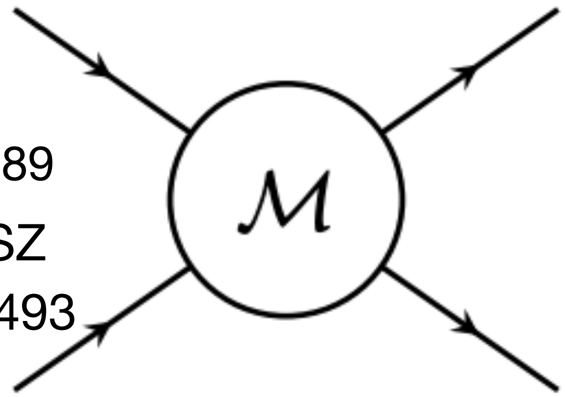
DESY.

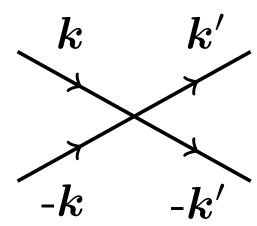
erc European Research Council
Established by the European Commission

Precision Gravity: From the LHC to LISA

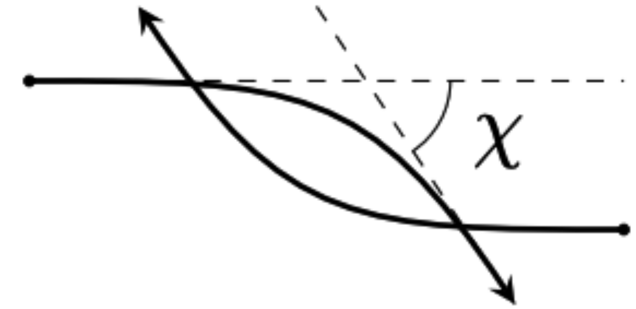
How do we map Observables?

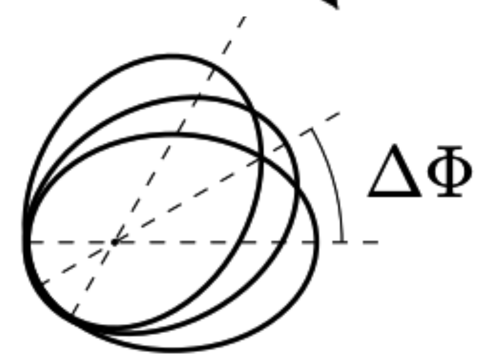
CRS
1808.02489
BCRSSZ
1908.01493





$= -iV(\boldsymbol{k}, \boldsymbol{k}')$





The gravitational interaction is UNIVERSAL!

BUT: Do we need the Hamiltonian?

	0PN	1PN	2PN	3PN	4PN	5PN	6PN	7PN
1PM	$(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + v^{14} + \cdots)\, G$							
2PM	$(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots)\, G^2$							
3PM	$(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + \cdots)\, G^3$							
4PM	$(1 + v^2 + v^4 + v^6 + v^8 + \cdots)\, G^4$							
5PM	$(1 + v^2 + v^4 + v^6 + \cdots)\, G^5$							

Conservative 3PM Hamiltonian

ZB, Cheung, Roiban, Shen, Solon, Zeng (2019)

The O(G^3) 3PM Hamiltonian: $H(\boldsymbol{p}, \boldsymbol{r}) = \sqrt{\boldsymbol{p}^2 + m_1^2} + \sqrt{\boldsymbol{p}^2 + m_2^2} + V(\boldsymbol{p}, \boldsymbol{r})$

$$V(\boldsymbol{p}, \boldsymbol{r}) = \sum_{i=1}^{3} c_i(\boldsymbol{p}^2) \left(\frac{G}{|\boldsymbol{r}|}\right)^i ,$$

Newton in here

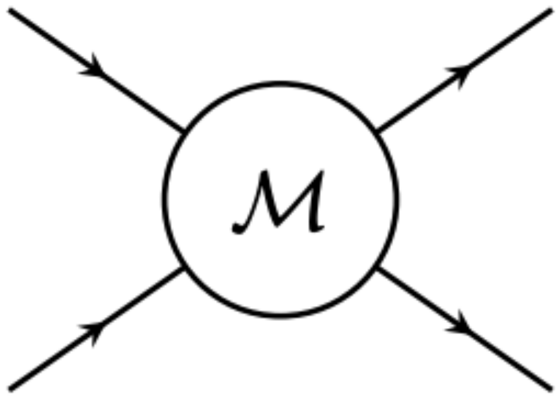
$$c_1 = \frac{\nu^2 m^2}{\gamma^2 \xi}(1 - 2\sigma^2), \qquad c_2 = \frac{\nu^2 m^3}{\gamma^2 \xi}\left[\frac{3}{4}(1 - 5\sigma^2) - \frac{4\nu\sigma(1 - 2\sigma^2)}{\gamma\xi} - \frac{\nu^2(1-\xi)(1 - 2\sigma^2)^2}{2\gamma^3\xi^2}\right],$$

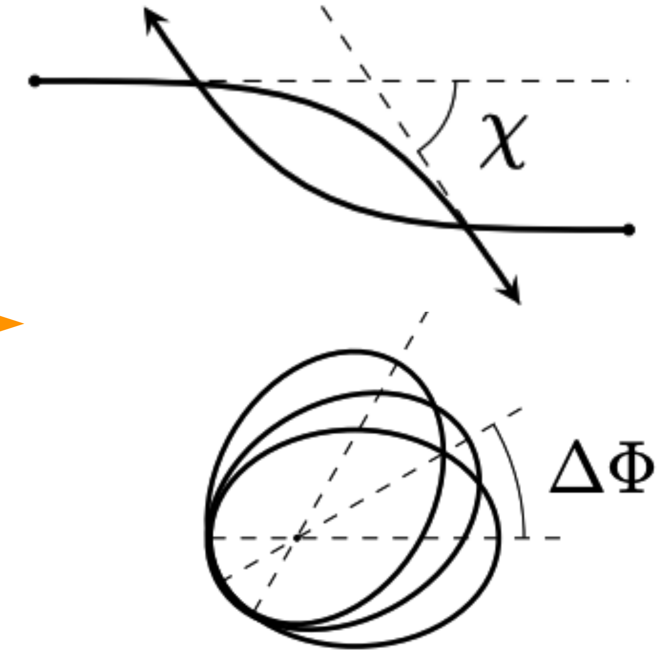
$$c_3 = \frac{\nu^2 m^4}{\gamma^2 \xi}\left[\frac{1}{12}(3 - 6\nu + 206\nu\sigma - 54\sigma^2 + 108\nu\sigma^2 + 4\nu\sigma^3) - \frac{4\nu(3 + 12\sigma^2 - 4\sigma^4)\,\mathrm{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2 - 1}} - \frac{3\nu\gamma(1 - 2\sigma^2)(1 - 5\sigma^2)}{2(1+\gamma)(1+\sigma)} - \frac{3\nu\sigma(7 - 20\sigma^2)}{2\gamma\xi} - \frac{\nu^2(3 + 8\gamma - 3\xi - 15\sigma^2 - 80\gamma\sigma^2 + 15\xi\sigma^2)(1 - 2\sigma^2)}{4\gamma^3\xi^2} + \frac{2\nu^3(3 - 4\xi)\sigma(1 - 2\sigma^2)^2}{\gamma^4\xi^3} + \frac{\nu^4(1 - 2\xi)(1 - 2\sigma^2)^3}{2\gamma^6\xi^4}\right],$$

$$m = m_A + m_B, \qquad \mu = m_A m_B/m, \qquad \nu = \mu/m, \qquad \gamma = E/m,$$
$$\xi = E_1 E_2/E^2, \qquad E = E_1 + E_2, \qquad \sigma = p_1 \cdot p_2/m_1 m_2,$$

Kalin RAP
1910.03008

ON-SHELL SPIRIT: gauge-invariant information!





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$$- \frac{3\nu\gamma\left(1-2\sigma^2\right)\left(1-5\sigma^2\right)}{2(1+\gamma)(1+\sigma)} - \frac{3\nu\sigma\left(7-20\sigma^2\right)}{2\gamma\xi} - \frac{\nu^2\left(3+8\gamma-3\xi-15\sigma^2-80\gamma\sigma^2+15\xi\sigma^2\right)\left(1-2\sigma^2\right)}{4\gamma^3\xi^2}$$
$$\left. + \frac{2\nu^3(3-4\xi)\sigma\left(1-2\sigma^2\right)^2}{\gamma^4\xi^3} + \frac{\nu^4(1-2\xi)\left(1-2\sigma^2\right)^3}{2\gamma^6\xi^4}\right],$$

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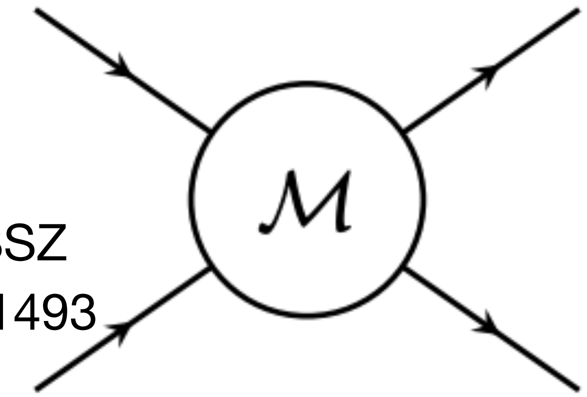
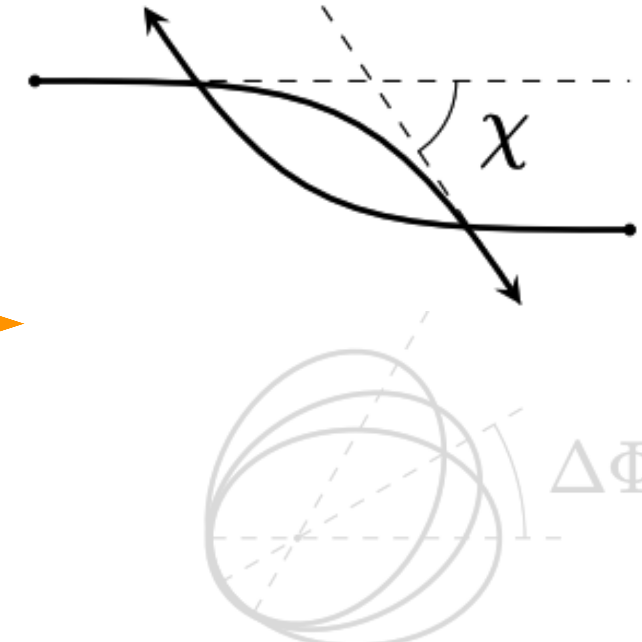
ON-SHELL SPIRIT:
gauge-invariant information!



BCRSSZ
1908.01493



observed to be the same to 3PM order

$$\mathbf{p}^2(r,E) = p_\infty^2(E) + \sum_i^\infty P_i(E)\left(\frac{G}{r}\right)^i$$

$$\widetilde{\mathcal{M}}(r,E) = \sum_{n=1}^\infty \widetilde{\mathcal{M}}_n(E)\left(\frac{G}{r}\right)^n$$

Scattering amplitude

$$\widetilde{\mathcal{M}}(r,E) \equiv \frac{1}{2E}\int \frac{d^3\mathbf{q}}{(2\pi)^3}\mathcal{M}(\mathbf{q},\mathbf{p}^2 = p_\infty^2(E))e^{-i\mathbf{q}\cdot\mathbf{r}}$$

The most exciting phrase
to hear in science,
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Kalin **RAP**
1910.03008

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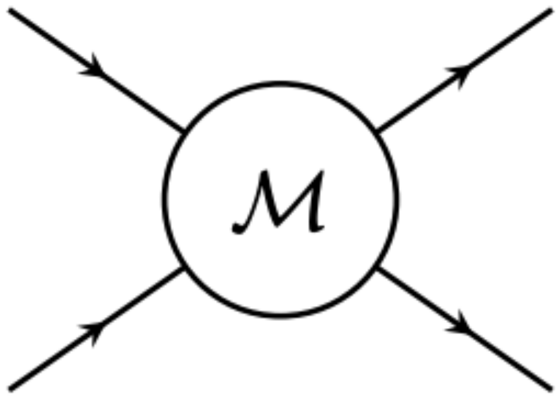

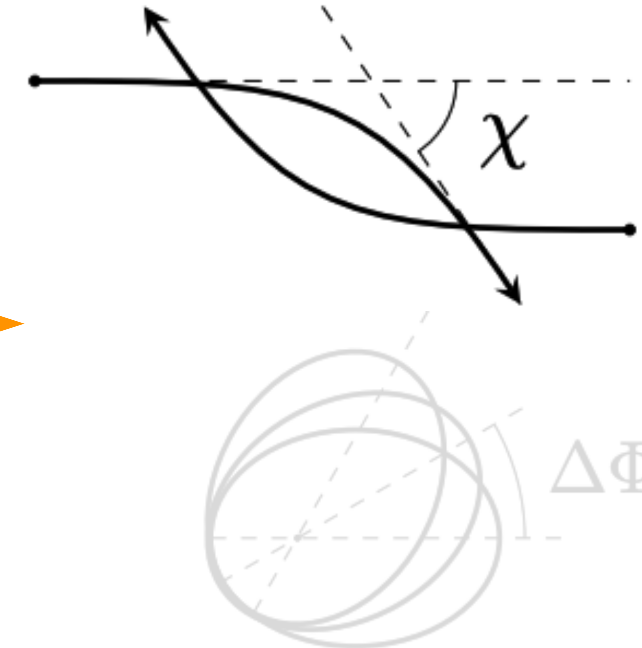

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'Impetus formula' *

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Direct algebraic relationship (Firsov)

$$\bar{p}^2(r,E) = \exp\left[\frac{2}{\pi}\int_{r|\bar{p}(r,E)|}^\infty \frac{\chi_b(\tilde{b},E)\,d\tilde{b}}{\sqrt{\tilde{b}^2 - r^2\bar{p}^2(r,E)}}\right]$$

$$\chi_b^{(n)} = \frac{\sqrt{\pi}}{2}\Gamma\left(\frac{n+1}{2}\right)\sum_{\sigma\in\mathcal{P}(n)}\frac{1}{\Gamma\left(1+\frac{n}{2}-\Sigma^\ell\right)}\prod_\ell \frac{f_{\sigma_\ell}^{\sigma^\ell}}{\sigma^\ell!},$$

*** IR-finite part ('potentials' only)**

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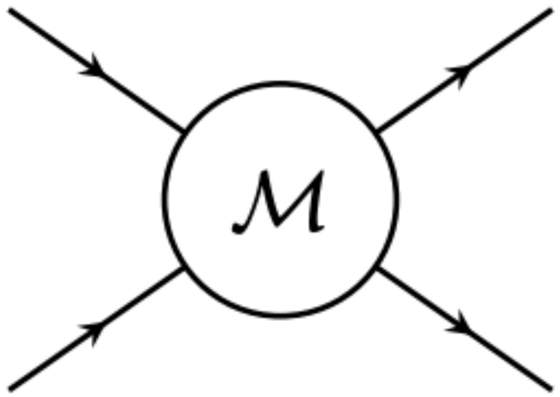
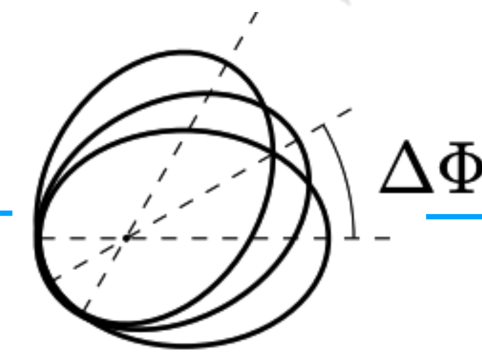
Boundary to Bound Map



Through 'impetus'

$$\boldsymbol{p}^2(r,E) = p_\infty^2(E) + \widetilde{\mathcal{M}}(r,E)$$



$$\mathcal{E} = \frac{E-M}{\mu} < 0$$

Recycle an old idea from Sommerfeld

$$\mathcal{S}_r \equiv \frac{1}{2\pi}\oint p_r dr$$

$$\mathcal{S}_r(J,\mathcal{E}) = \frac{1}{\pi}\int_{r_-}^{r_+}\sqrt{p_\infty^2(\mathcal{E}) + \widetilde{\mathcal{M}}(r,\mathcal{E}) - J^2/r^2}\,\mathrm{d}r$$

Analytic continuation

$$\delta\mathcal{S}_r(J,\mathcal{E},m_a) = -\left(1+\frac{\Delta\Phi}{2\pi}\right)\delta J + \frac{\mu}{\Omega_r}\delta\mathcal{E} - \sum_a \frac{1}{\Omega_r}\left(\langle z_a\rangle - \frac{\partial E(\mathcal{E},m_a)}{\partial m_a}\right)\delta m_a$$

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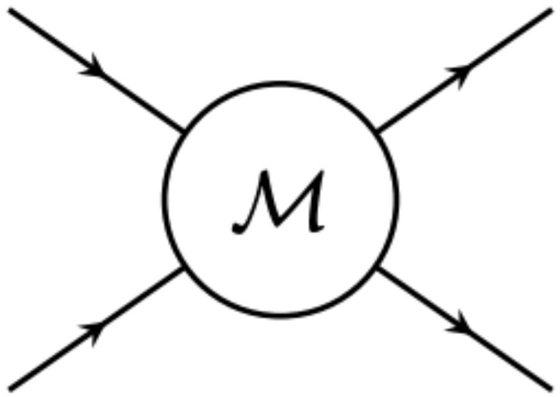

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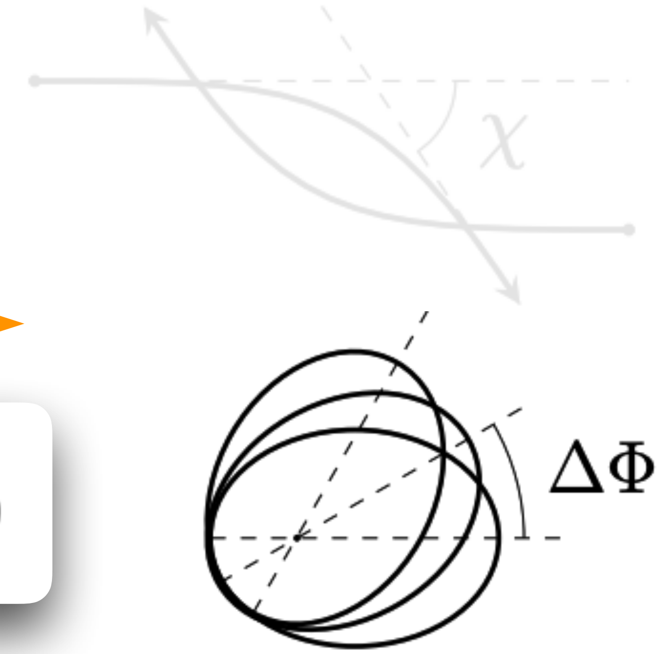

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$$\frac{\Delta\Phi}{2\pi} = \frac{\widetilde{\mathcal{M}}_2 G^2}{2J^2} + \frac{3(\widetilde{\mathcal{M}}_2^2 + 2\widetilde{\mathcal{M}}_1\widetilde{\mathcal{M}}_3 + 2p_\infty^2\widetilde{\mathcal{M}}_4)G^4}{8J^4} + \mathcal{O}(G^6)\,,$$

← **Generalizes to all orders the one-loop result in**
Caron-Huot Zahraee
1810.04694

direct connection from amplitude!

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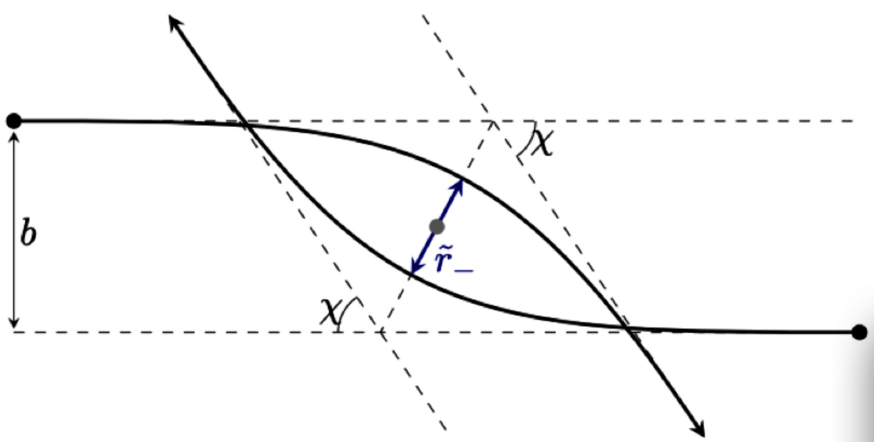
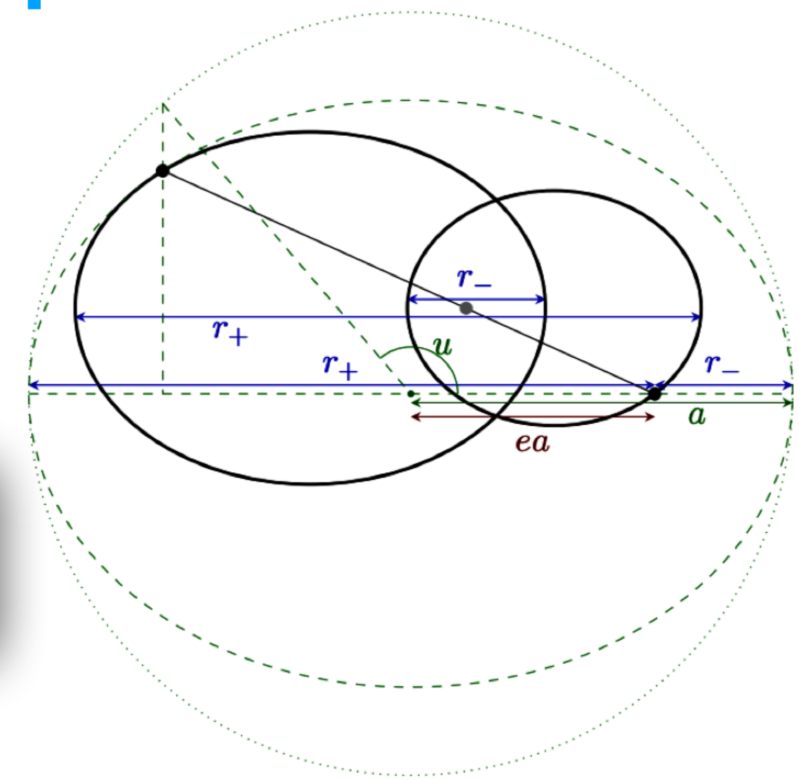
1911.09130

Boundary to Bound Map



Scattering Angle & Periastron Adv.

$$\chi_b^{(n)} = \frac{\sqrt{\pi}}{2}\Gamma\left(\frac{n+1}{2}\right)\sum_{\sigma\in\mathcal{P}(n)}\frac{1}{\Gamma\left(1+\frac{n}{2}-\Sigma^\ell\right)}\prod_\ell \frac{f_{\sigma_\ell}^{\sigma^\ell}}{\sigma^\ell!},$$



Scattering angle

$$\frac{1}{\pi}\int_{\tilde{r}_-(J,\mathcal{E})}^{\infty}\frac{J}{r^2\sqrt{\boldsymbol{p}^2(\mathcal{E},r)-J^2/r^2}}\mathrm{d}r\,;$$

Periastron Advanced

$$\frac{1}{\pi}\int_{r_-(J,\mathcal{E})}^{r_+(J,\mathcal{E})}\frac{J}{r^2\sqrt{\boldsymbol{p}^2(\mathcal{E},r)-J^2/r^2}}\mathrm{d}r$$

$$4\chi_j^{(4)} = \frac{3\pi\hat{p}_\infty^4}{4}\left(f_2^2+2f_1f_3+2f_4\right) = \frac{3\pi}{4M^4\mu^4}\left(\widetilde{\mathcal{M}}_2^2+2\widetilde{\mathcal{M}}_1\widetilde{\mathcal{M}}_3+2p_\infty^2\widetilde{\mathcal{M}}_4\right)$$

$$\frac{\Delta\Phi}{2\pi} = \frac{\widetilde{\mathcal{M}}_2 G^2}{2J^2}+\frac{3(\widetilde{\mathcal{M}}_2^2+2\widetilde{\mathcal{M}}_1\widetilde{\mathcal{M}}_3+2p_\infty^2\widetilde{\mathcal{M}}_4)G^4}{8J^4}+\mathcal{O}(G^6)\,,$$

$$1/j = GM\mu/J$$

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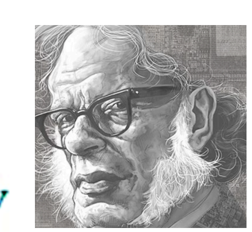

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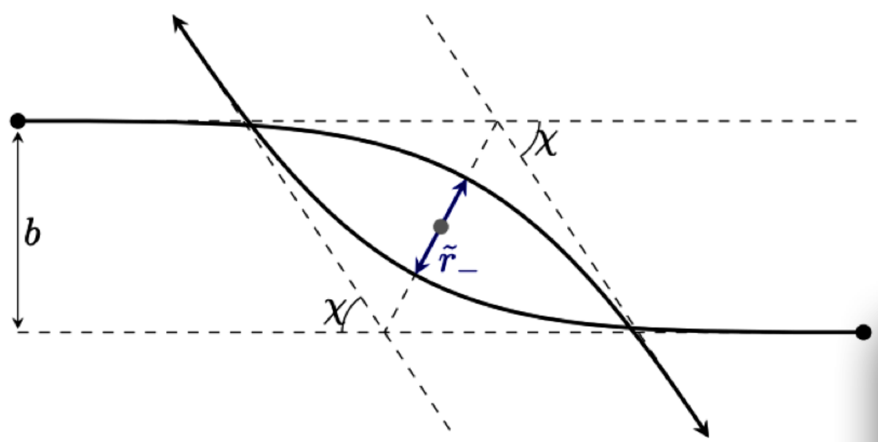
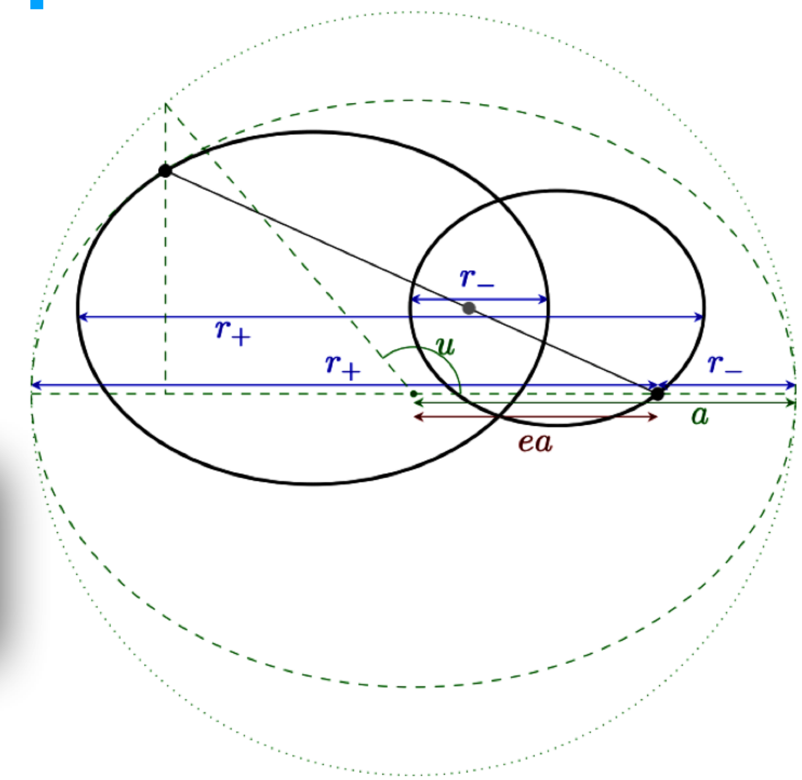


Scattering Angle
TO Periastron Adv.

$$r_-(J,\mathcal{E}) = \tilde{r}_-(J,\mathcal{E}) \qquad J>0,\ \mathcal{E}<0\,.$$
$$r_+(J,\mathcal{E}) = \tilde{r}_-(-J,\mathcal{E}) \qquad J>0,\ \mathcal{E}<0\,,$$



Scattering angle

$$\frac{1}{\pi}\int_{\tilde{r}_-(J,\mathcal{E})}^{\infty} \frac{J}{r^2\sqrt{\boldsymbol{p}^2(\mathcal{E},r)-J^2/r^2}}\mathrm{d}r\,;$$

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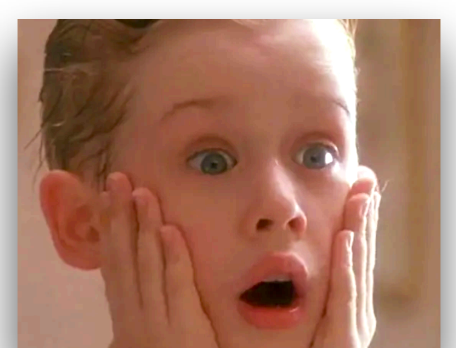

Remarkably!

$$\Delta\Phi(J,\mathcal{E}) = \chi(J,\mathcal{E}) + \chi(-J,\mathcal{E})\,, \qquad \mathcal{E}<0\,,$$

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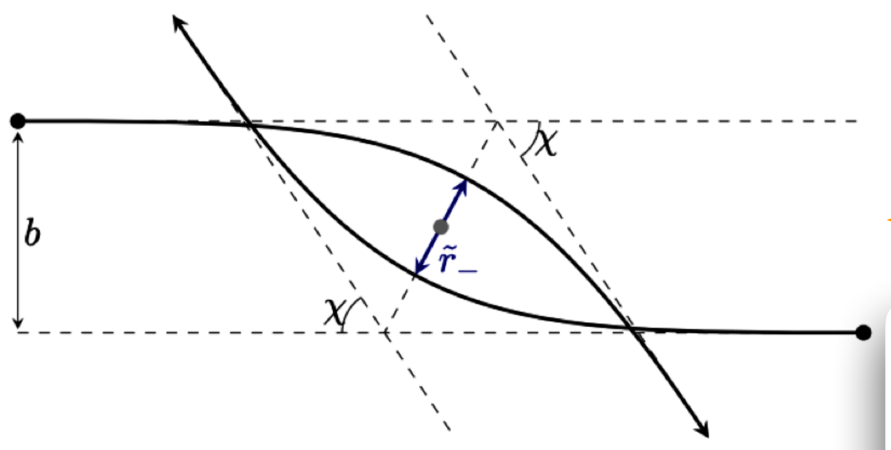
1911.09130

Boundary to Bound Map



$1/j = GM\mu/J$

Through the scattering angle

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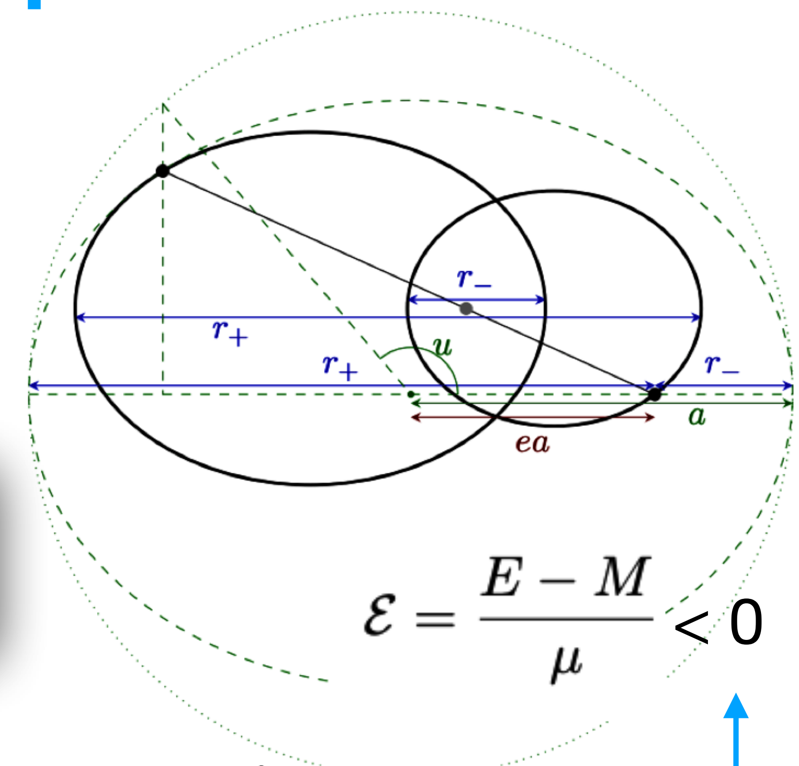

$$\mathcal{E} = \frac{E-M}{\mu} < 0$$

$$1 + \frac{\Delta\Phi}{2\pi} = -\frac{\partial i_r}{\partial j} \longrightarrow \frac{\mathcal{S}_r}{GM\mu} = -\left(j + \frac{2}{\pi}\sum_n \chi_j^{(2n)}(\mathcal{E}) \int \frac{dj}{j^{2n}}\right) + \alpha(\mathcal{E})\,,$$

fixed by the large-j limit

Analytic continuation

CENTRAL OBJECT for the bound problem:

(local & no-spin)

$$i_r(j,\mathcal{E}) \equiv \frac{\mathcal{S}_r}{GM\mu} = \mathrm{sg}(\hat{p}_\infty)\chi_j^{(1)}(\mathcal{E}) - j\left(1 + \frac{2}{\pi}\sum_{n=1} \frac{\chi_j^{(2n)}(\mathcal{E})}{(1-2n)j^{2n}}\right)$$

$$\delta\mathcal{S}_r(J,\mathcal{E},m_a) = -\left(1 + \frac{\Delta\Phi}{2\pi}\right)\delta J + \frac{\mu}{\Omega_r}\delta\mathcal{E} - \sum_a \frac{1}{\Omega_r}\left(\langle z_a\rangle - \frac{\partial E(\mathcal{E},m_a)}{\partial m_a}\right)\delta m_a$$

→ ***ALL the observables!***

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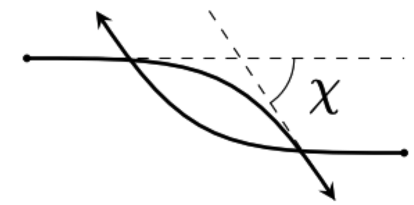
2006.01184

PM EFT for scattering

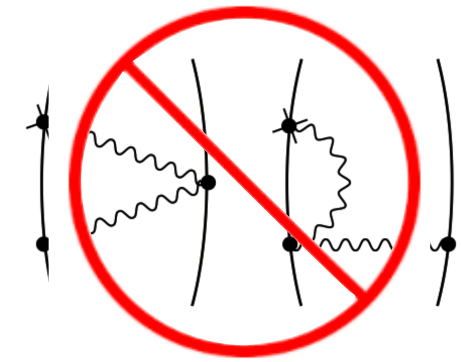


$$e^{iS_{\text{eff}}[x_a]} = \int \mathcal{D}h_{\mu\nu}\, e^{iS_{\text{EH}}[h]+iS_{\text{GF}}[h]+iS_{\text{pp}}[x_a,h]}\,,$$

external source

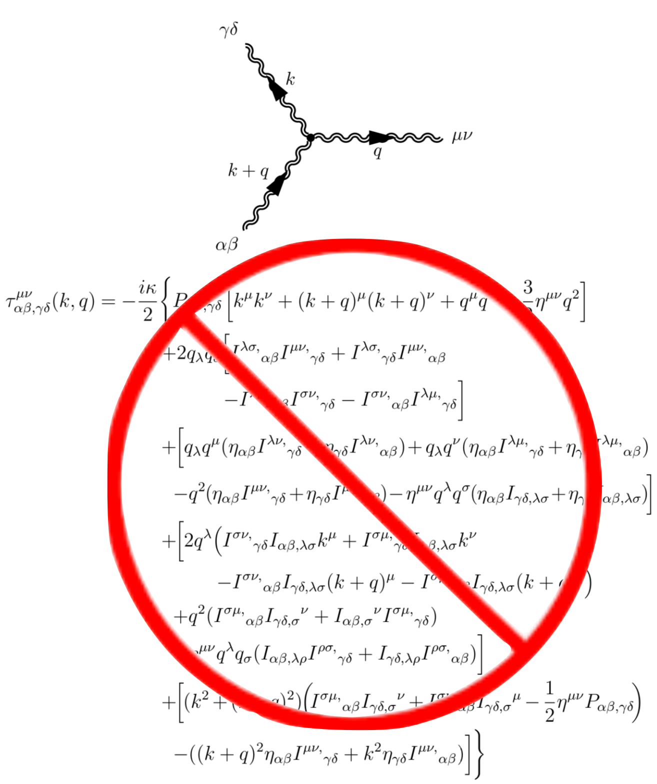
Caveat: No spin nor finite-size yet

$$S_{\text{pp}} = -\sum_a \frac{m_a}{2}\int \mathrm{d}\tau_a\, g_{\mu\nu}(x_a(\tau_a))v_a^\mu(\tau_a)v_a^\nu(\tau_a)\,.$$

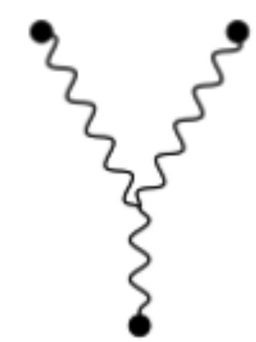


Simplified Feynman rules through GF and total derivatives (but no field redef.)

many terms reduced to:

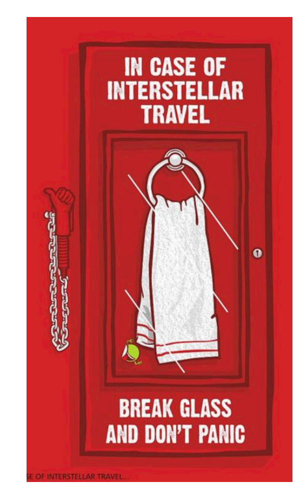


$$M_{\text{Pl}}\mathcal{L}_{hhh} = -\frac{1}{2}h^{\mu\nu}\partial_\mu h^{\rho\sigma}\partial_\nu h_{\rho\sigma} + \frac{1}{2}h^{\mu\nu}\partial_\rho h\partial^\rho h_{\mu\nu} - \frac{1}{8}h\partial_\rho h\partial^\rho h + h^{\mu\nu}\partial_\nu h_{\rho\sigma}\partial^\sigma h_\mu{}^\rho - h^{\mu\nu}\partial_\sigma h_{\nu\rho}\partial^\sigma h_\mu{}^\rho + \frac{1}{4}h\partial_\sigma h_{\nu\rho}\partial^\sigma h^{\nu\rho}\,.$$



Lots of redundancy in GR — No need to panic!

(See also Gregor's talk)



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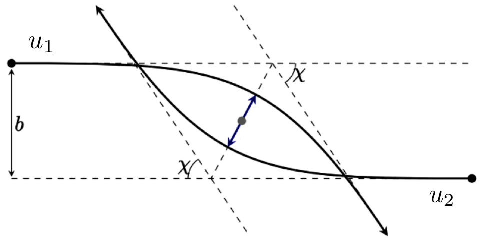
PM EFT for scattering



$$e^{iS_{\rm eff}[x_a]} = \int \mathcal{D}h_{\mu\nu}\, e^{iS_{\rm EH}[h]+iS_{\rm GF}[h]+iS_{\rm pp}[x_a,h]}\,,$$

Post-Minkowskian solution to the equation of motion (Euler-Lagrange eqs.)

$$v_a^\mu(\tau_1) = u_a^\mu + \sum_n \delta^{(n)} v_a^\mu(\tau_a)\,,$$
$$x_a^\mu(\tau_1) = b_a^\mu + u_a^\mu \tau_a + \sum_n \delta^{(n)} x_a^\mu(\tau_a)\,,$$

The true classical motion

Compute total impulse from the action…

$$\Delta p_a^\mu = -\eta^{\mu\nu} \int_{-\infty}^{+\infty} \mathrm{d}\tau_a \frac{\partial \mathcal{L}_{\rm eff}}{\partial x_a^\nu}(x_a(\tau_a))$$

… and deflection angle in the centre-of-mass

$$2\sin\left(\frac{\chi}{2}\right) = \chi - \frac{1}{24}\chi^3 + \mathcal{O}(\chi^5) = \frac{|\Delta \boldsymbol{p}_{1\rm cm}|}{p_\infty} = \frac{\sqrt{-\Delta p_1^2}}{p_\infty}\,,$$

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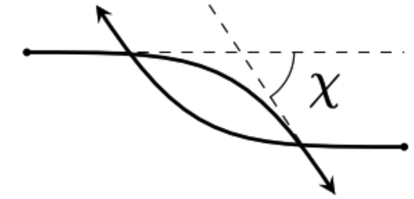
2006.01184

PM EFT for scattering



$$e^{iS_{\rm eff}[x_a]} = \int \mathcal{D}h_{\mu\nu}\, e^{iS_{\rm EH}[h]+iS_{\rm GF}[h]+iS_{\rm pp}[x_a,h]}\,,$$

Potential Modes
$(k_0 \ll |\boldsymbol{k}|)$

Radiation Modes
$(k_0 \sim |\boldsymbol{k}|)$

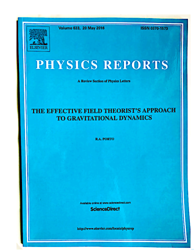
Dissipative/Flux
(Imaginary part with Feynman ieps) *

Conservative
(Real-part time-reversal invariant)

NRGR

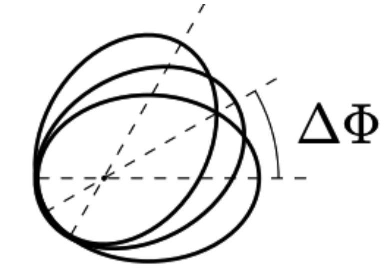
(Goldberger & Rothstein,…)

The effective field theorist's approach to gravitational dynamics
Rafael A. Porto
Physics Reports
Volume 633, 20 May 2016, Pages 1-104



* *To comute dissipative RR-force we need the in-in computation (retarded b.c.)*

PN EFT for bound states



$$e^{iS_{\rm eff}[x_a]} = \int \mathcal{D}h_{\mu\nu}\, e^{iS_{\rm EH}[h]+iS_{\rm GF}[h]+iS_{\rm pp}[x_a,h]}\,,$$

Potential Modes ($k_0 \ll |\boldsymbol{k}|$)

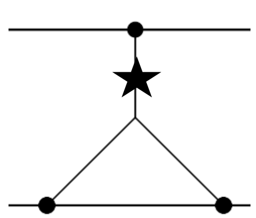
$(v \ll 1)$

Radiation Modes ($k_0 \sim |\boldsymbol{k}|$)

Conservative (Real-part time-reversal invariant)

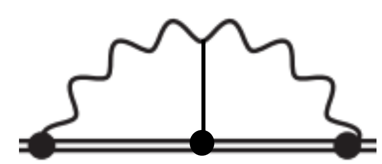
NRGR

Integrals are much easier BUT screws the IR!



$$\frac{1}{p_0^2 - \boldsymbol{p}^2} \simeq -\frac{1}{\boldsymbol{p}^2}\left(1 + \frac{p_0^2}{\boldsymbol{p}^2} + \cdots\right).$$

Integrals are much easier BUT screws the UV!



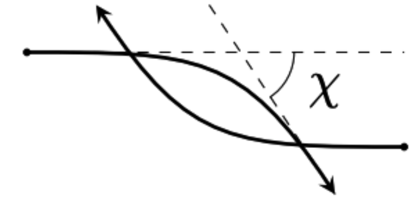
$$e^{i\mathbf{k}\cdot\mathbf{x}} = 1 + i\mathbf{k}\cdot\mathbf{x} + \cdots$$

PHYSICAL REVIEW D **100**, 024048 (2019)

Conservative dynamics of binary systems to fourth post-Newtonian order in the EFT approach. II. Renormalized Lagrangian

Stefano Foffa,[1] Rafael A. Porto,[2,3] Ira Rothstein,[4] and Riccardo Sturani[5]

PM EFT for scattering



$$e^{iS_{\rm eff}[x_a]} = \int \mathcal{D}h_{\mu\nu}\, e^{iS_{\rm EH}[h]+iS_{\rm GF}[h]+iS_{\rm pp}[x_a,h]}\,,$$

Potential Modes ($k_0 \ll |\boldsymbol{k}|$)

Radiation Modes ($k_0 \sim |\boldsymbol{k}|$)

(relativistic) *"Soft"* Region

Conservative (Real-part time-reversal invariant)

PM EFT for scattering

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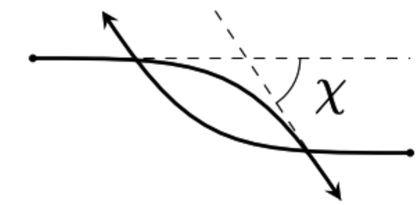
2007.04977



$$e^{iS_{\rm eff}[x_a]} = \int \mathcal{D}h_{\mu\nu}\, e^{iS_{\rm EH}[h]+iS_{\rm GF}[h]+iS_{\rm pp}[x_a,h]}\,,$$

Potential Modes $(k_0 \ll |\boldsymbol{k}|)$

Radiation Modes $(k_0 \sim |\boldsymbol{k}|)$

(relativistic) "*Soft*" Region

IR/UV finite!*

Conservative (Real-part time-reversal invariant) *

NNNLO
(See Gregor's talk)

See also Michael's Julio's, Gabriele's and Carlo's talks

Parra-Martinez
Ruff and Zeng
2005.04236

Differential Equations
b.c. from **entire region**

$$\partial_x \vec{h}(x,\epsilon) = \epsilon\, \mathbb{M}(x)\, \vec{h}(x,\epsilon)$$

Single scale!

$$\gamma = \frac{1+x^2}{2x}, \quad \gamma \equiv u_1 \cdot u_2$$

***UV from finite-size only**

***Caveat: need to extend B2B to "non-local" terms**

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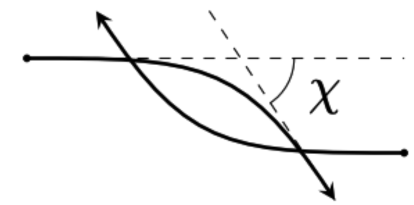
PM EFT for scattering



$$e^{iS_{\text{eff}}[x_a]} = \int \mathcal{D}h_{\mu\nu}\, e^{iS_{\text{EH}}[h]+iS_{\text{GF}}[h]+iS_{\text{pp}}[x_a,h]}\,,$$

Potential Modes
$(k_0 \ll |\boldsymbol{k}|)$

Sufficient for real part NNLO

<u>keep the (special-) relativistic corrections</u>

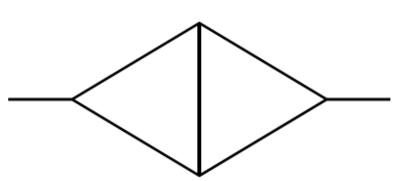
Differential Equations
b.c. from **potentials**

$$\partial_x \vec{h}(x,\epsilon) = \epsilon\, \mathbb{M}(x)\, \vec{h}(x,\epsilon)$$

Single scale!

eight elements to NNLO

b.c:



$(\gamma \to 1)$

$$\gamma = \frac{1+x^2}{2x}, \quad \gamma \equiv u_1 \cdot u_2$$

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2007.04977

PM EFT for scattering : NNLO

Integrals (one family!):

$$M^{(a,\tilde{a})}_{n_1 n_2; i_1 \cdots i_5}(q,\gamma) \equiv \int_{k_1,k_2} \frac{\hat{\delta}(k_1 \cdot u_a)\hat{\delta}(k_2 \cdot u_{\tilde{a}})}{A^{n_1}_{1,\not{a}} A^{n_2}_{2,\not{\tilde{a}}} D_1^{i_1} \cdots D_5^{i_5}} \,,$$

$$A_{1,\not{a}} = k_1 \cdot u_{\not{a}},\; A_{2,\not{\tilde{a}}} = k_2 \cdot u_{\not{\tilde{a}}},\; D_1 = k_1^2,\; D_2 = k_2^2\,,$$
$$D_3 = (k_1+k_2-q)^2,\; D_4 = (k_1-q)^2,\; D_5 = (k_2-q)^2.$$

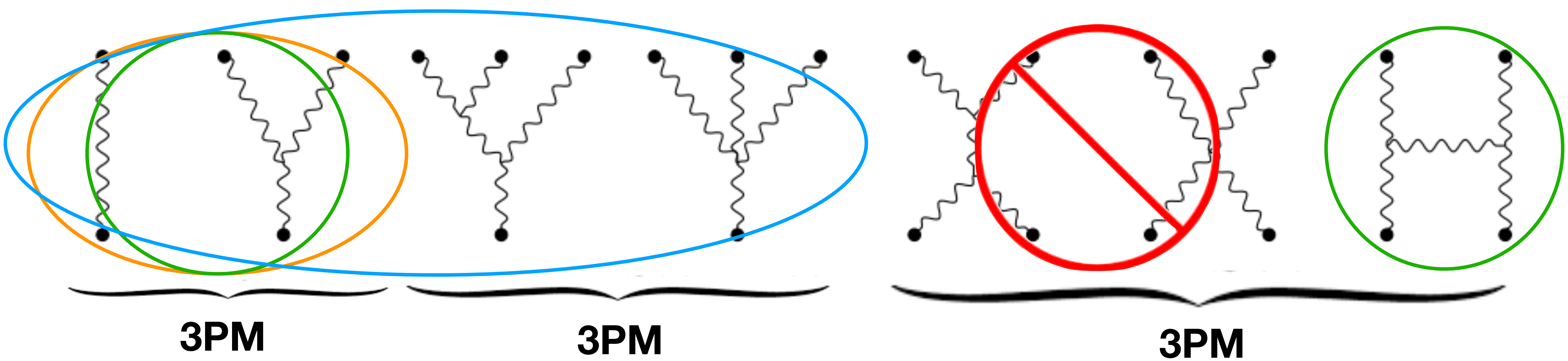
POTENTIAL REGION: **DFQ with b.c. from the static limit of NRGR!**

$$\Delta^{(3)} p_1^\mu = \frac{G^3 b^\mu}{|b^2|^2}\left(\frac{16 m_1^2 m_2^2 (4\gamma^4 - 12\gamma^2 - 3)\sinh^{-1}\sqrt{\frac{\gamma-1}{2}}}{(\gamma^2-1)} - \frac{4 m_1^2 m_2^2 \gamma (20\gamma^6 - 90\gamma^4 + 120\gamma^2 - 53)}{3(\gamma^2-1)^{5/2}} - \frac{2 m_1 m_2 (m_1^2 + m_2^2)(16\gamma^6 - 32\gamma^4 + 16\gamma^2 - 1)}{(\gamma^2-1)^{5/2}} \right)$$
$$+ \frac{3\pi}{2} \frac{(2\gamma^2-1)(5\gamma^2-1)}{(\gamma^2-1)^2} \frac{G^3 M^2 \mu}{|b^2|^{3/2}} \times \left((\gamma m_2 + m_1) u_2^\mu - (\gamma m_1 + m_2) u_1^\mu \right).$$

related to Schw. via b.c. of DFQ

fixed by on-shell condition

Schwarzschild + mirror image



PM EFT for scattering : NNLO

Integrals (one family!):

$$M^{(a,\tilde{a})}_{n_1 n_2; i_1 \cdots i_5}(q,\gamma) \equiv \int_{k_1,k_2} \frac{\hat{\delta}(k_1 \cdot u_a)\hat{\delta}(k_2 \cdot u_{\tilde{a}})}{A^{n_1}_{1,\not{a}} A^{n_2}_{2,\not{\tilde{a}}} D_1^{i_1} \cdots D_5^{i_5}} ,$$

$$A_{1,\not{a}} = k_1 \cdot u_{\not{a}},\ A_{2,\not{\tilde{a}}} = k_2 \cdot u_{\not{\tilde{a}}},\ D_1 = k_1^2,\ D_2 = k_2^2 ,$$
$$D_3 = (k_1+k_2-q)^2,\ D_4 = (k_1-q)^2,\ D_5 = (k_2-q)^2 .$$

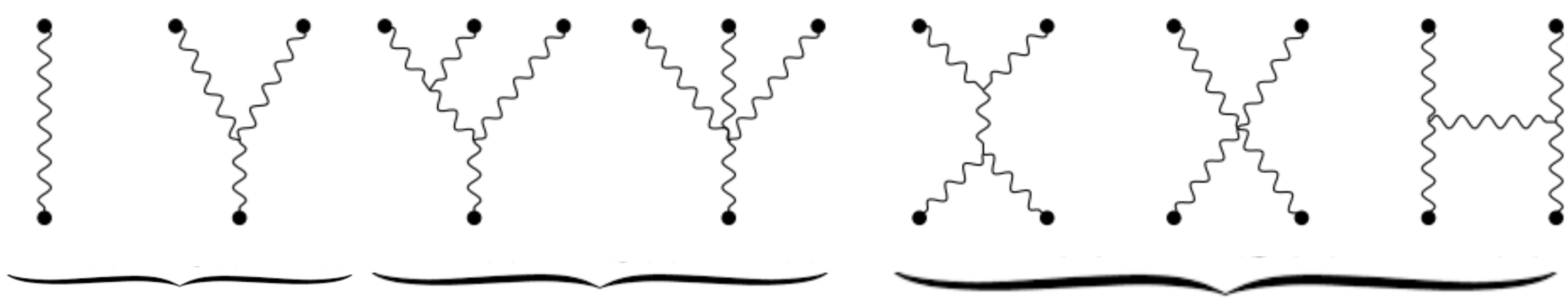
***POTENTIAL REGION*: DFQ with b.c. from the static limit of NRGR!**

Advantages wrt amplitudes:

- ***We land in the soft-expanded cut-version of the integrand***
- No ("super-classical") divergences (e.g. "box")
- ***On-shell philosophy***: No potential!, extra matching nor Born iterations!

Main Drawback:

- Feynman diagrams (***significantly fewer*** than in PN and ***simpler rules***!)



3PM 3PM 3PM

(See also Gregor's talk)

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2007.04977

B2B dictionary : NNLO

$$i_r = \frac{p_\infty}{\sqrt{-p_\infty^2}}\chi_j^{(1)} - j\left(1 - \frac{2}{\pi}\left(\frac{\chi_j^{(2)}}{j^2} + \frac{\chi_j^{(4)}}{3j^4}\right) + \cdots\right)$$

We have the 3PM impulse/angle

$$\frac{\chi_b^{(3)}}{\Gamma} = \frac{1}{(\gamma^2-1)^{3/2}}\Bigg[-\frac{4\nu}{3}\gamma\sqrt{\gamma^2-1}(14\gamma^2+25) + \frac{(64\gamma^6 - 120\gamma^4 + 60\gamma^2 - 5)(1+2\nu(\gamma-1))}{3(\gamma^2-1)^{3/2}} - 8\nu(4\gamma^4 - 12\gamma^2 - 3)\sinh^{-1}\sqrt{\frac{\gamma-1}{2}}\Bigg],$$

BUT WE DO NOT HAVE THE 4PM!

	0PN	1PN	2PN	3PN	4PN	5PN	6PN	7PN	
1PM	$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+v^{12}$	$+v^{14}$	$+\cdots)\,G$
2PM		$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+v^{12}$	$+\cdots)\,G^2$
3PM			$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+\cdots)\,G^3$
4PM				$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+\cdots)\,G^4$
5PM					$(1$	$+v^2$	$+v^4$	$+v^6$	$+\cdots)\,G^5$

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B2B dictionary : NNLO

$$i_r = \frac{p_\infty}{\sqrt{-p_\infty^2}}\chi_j^{(1)} - j\left(1 - \frac{2}{\pi}\left(\frac{\chi_j^{(2)}}{j^2} + \frac{\chi_j^{(4)}}{3j^4}\right) + \cdots\right)$$

$$\chi_j^{(3)} = \frac{1}{M^3\mu^3 p_\infty^3}\left(-\frac{P_1^3}{24} + p_\infty^2\frac{P_1P_2}{2} + p_\infty^4 P_3\right)$$

Everything * you need to know about 3PM (coincides with M3 thru impetus)

$$\frac{P_3}{M^3\mu^2} = \left(\frac{18\gamma^2-1}{2\Gamma} + \frac{8\nu}{\Gamma}(3+12\gamma^2-4\gamma^4)\frac{\sinh^{-1}\sqrt{\frac{\gamma-1}{2}}}{\sqrt{\gamma^2-1}} + \frac{\nu}{6\Gamma}\left(6-206\gamma-108\gamma^2-4\gamma^3 + \frac{18\Gamma(1-2\gamma^2)(1-5\gamma^2)}{(1+\Gamma)(1+\gamma)}\right)\right).$$

$$\chi_b^{(n)} = \frac{\sqrt{\pi}}{2}\Gamma\left(\frac{n+1}{2}\right)\sum_{\sigma\in\mathcal{P}(n)}\frac{1}{\Gamma\left(1+\frac{n}{2}-\Sigma^\ell\right)}\prod_\ell \frac{f_{\sigma_\ell}^{\sigma^\ell}}{\sigma^\ell!},$$

$$\boldsymbol{p}^2(r,E) = p_\infty^2(E) + \sum_i P_i(E)\left(\frac{G}{r}\right)^i$$

$$= p_\infty^2(E)\left(1 + \sum_i f_i(E)\left(\frac{GM}{r}\right)^i\right)$$

	0PN	1PN	2PN	3PN	4PN	5PN	6PN	7PN	
1PM	$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+v^{12}$	$+v^{14}$	$+\cdots)G$
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3PM			$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+\cdots)G^3$
4PM				$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+\cdots)G^4$
5PM					$(1$	$+v^2$	$+v^4$	$+v^6$	$+\cdots)G^5$

*We also reconstructed much lengthier PM Hamiltonian

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2007.04977

B2B dictionary : NNLO

$$i_r = \frac{p_\infty}{\sqrt{-p_\infty^2}}\chi_j^{(1)} - j\left(1-\frac{2}{\pi}\left(\frac{\chi_j^{(2)}}{j^2}+\frac{\chi_j^{(4)}}{3j^4}\right)+\cdots\right)$$

$$\chi_j^{(4)} = \frac{3\pi}{8M^4\mu^4}\left(\underbrace{P_1P_3+\frac{1}{2}P_2^2}+p_\infty^2 P_4\right),$$

$$\frac{P_3}{M^3\mu^2} = \left(\frac{18\gamma^2-1}{2\Gamma}+\frac{8\nu}{\Gamma}(3+12\gamma^2-4\gamma^4)\frac{\sinh^{-1}\sqrt{\frac{\gamma-1}{2}}}{\sqrt{\gamma^2-1}}+\right.$$
$$\left.\frac{\nu}{6\Gamma}\left(6-206\gamma-108\gamma^2-4\gamma^3+\frac{18\Gamma(1-2\gamma^2)(1-5\gamma^2)}{(1+\Gamma)(1+\gamma)}\right)\right).$$

lower-order P_n's
enter in the 4PM angle

$$\chi_b^{(n)} = \frac{\sqrt{\pi}}{2}\Gamma\left(\frac{n+1}{2}\right)\sum_{\sigma\in\mathcal{P}(n)}\frac{1}{\Gamma\left(1+\frac{n}{2}-\Sigma^\ell\right)}\prod_\ell\frac{f_{\sigma_\ell}^{\sigma^\ell}}{\sigma^\ell!},$$

$$\boldsymbol{p}^2(r,E) = p_\infty^2(E)+\sum_i P_i(E)\left(\frac{G}{r}\right)^i$$
$$= p_\infty^2(E)\left(1+\sum_i f_i(E)\left(\frac{GM}{r}\right)^i\right)$$

	0PN	1PN	2PN	3PN	4PN	5PN	6PN	7PN	
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2PM		$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+v^{12}$	$+\cdots)\,G^2$
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B2B dictionary : NNLO

$$i_r = \frac{p_\infty}{\sqrt{-p_\infty^2}}\chi_j^{(1)} - j\left(1 - \frac{2}{\pi}\left(\frac{\chi_j^{(2)}}{j^2} + \frac{\chi_j^{(4)}}{3j^4}\right) + \cdots\right)$$

$$\chi_j^{(4)} = \frac{3\pi}{8M^4\mu^4}\left(P_1P_3 + \frac{1}{2}P_2^2 + p_\infty^2 P_4\right)^*, \quad \mathcal{O}(G/J)^6$$

Missing! **BUT** PN-suppressed (after analytic continuation)

$1/J \sim |p_\infty|$

$p_\infty^2 \sim \mathcal{E}$

This pattern is generic! and allows us to perform a **consistent truncation**

***P_n has well-defined static limit**

$$\chi_b^{(n)} = \frac{\sqrt{\pi}}{2}\Gamma\left(\frac{n+1}{2}\right)\sum_{\sigma\in\mathcal{P}(n)}\frac{1}{\Gamma\left(1+\frac{n}{2}-\Sigma^\ell\right)}\prod_\ell \frac{f_{\sigma_\ell}^{\sigma^\ell}}{\sigma^\ell!},$$

$$\boldsymbol{p}^2(r,E) = p_\infty^2(E) + \sum_i P_i(E)\left(\frac{G}{r}\right)^i$$

$$= p_\infty^2(E)\left(1 + \sum_i f_i(E)\left(\frac{GM}{r}\right)^i\right)$$

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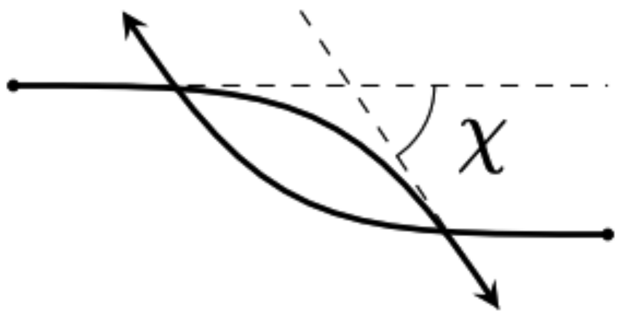
5PM					$(1$	$+v^2$	$+v^4$	$+v^6+\cdots)G^5$

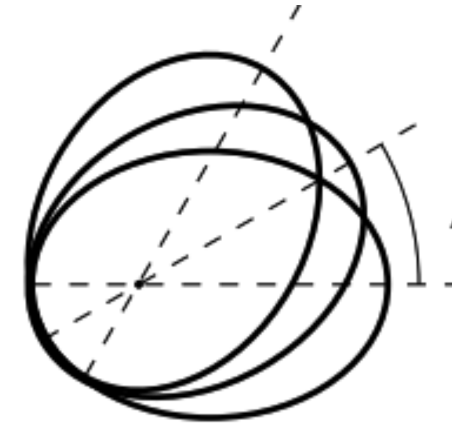
Kalin Liu **RAP** 2007.04977 Kalin **RAP** 1911.09130

B2B dictionary : NNLO



Map from angle to periastron advance

Full result agrees with literature to 2PN



✓ 1PN ✓ 2PN ✓ 2PN ✓ 3PN ✓ 4PN

$$\left(\frac{\Delta\Phi}{2\pi}\right)_{\text{2-loop}} = \frac{3}{j^2} + \frac{3(35-10\nu)}{4j^4} + \frac{3}{4j^2}\left(10 - 4\nu + \frac{194 - 184\nu + 23\nu^2}{j^2}\right)\mathcal{E}$$
$$+ \frac{3}{4j^2}\left(5 - 5\nu + 4\nu^2 + \frac{3535 - 6911\nu + 3060\nu^2 - 375\nu^3}{10j^2}\right)\mathcal{E}^2$$
$$+ \frac{3}{4j^2}\left((5-4\nu)\nu^2 + \frac{35910 - 126347\nu + 125559\nu^2 - 59920\nu^3 + 7385\nu^4}{140j^2}\right)\mathcal{E}^3$$
$$+ \frac{3}{4j^2}\left((5 - 20\nu + 16\nu^2)\frac{\nu^2}{4}\right)\mathcal{E}^4 + \cdots ,$$

The 5PN prediction (confirmed by tutifruti at 6PN too!)

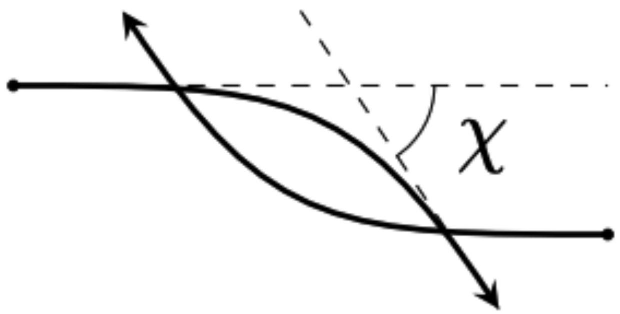
ONE-LOOP EXACT! $\frac{\Delta\Phi}{2\pi} = \frac{\widetilde{\mathcal{M}}_2 G^2}{2J^2}$

BUT: 1/j^2 is predicted *to all orders in v*
4PM will complete 1/j^4

$$\chi_j^{(2)} \propto \widetilde{\mathcal{M}}_2 = \frac{3M^2\mu^2}{2}\left(\frac{5\gamma^2-1}{\Gamma}\right)$$

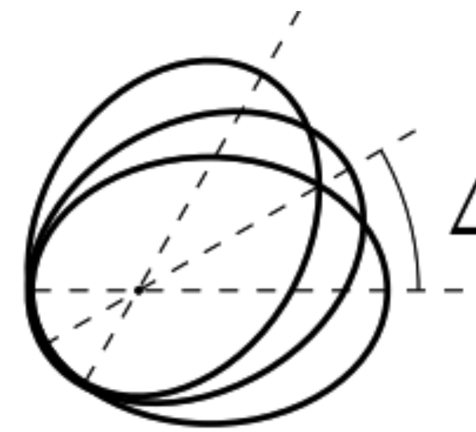
Kalin Liu **RAP** 2007.04977 Kalin **RAP** 1911.09130

B2B dictionary : NNLO



Map for *ALL* Dynamical Invariants!

Without HAMILTONIAN!



$$\delta\mathcal{S}_r(J,\mathcal{E},m_a) = -\left(1+\frac{\Delta\Phi}{2\pi}\right)\delta J + \frac{\mu}{\Omega_r}\delta\mathcal{E} - \sum_a \frac{1}{\Omega_r}\left(\langle z_a\rangle - \frac{\partial E(\mathcal{E},m_a)}{\partial m_a}\right)\delta m_a$$

$$\Omega_r(j,\mathcal{E}) \equiv \frac{2\pi}{T_p}, \quad \Omega_p(j,\mathcal{E}) \equiv \frac{\Delta\Phi}{T_p},$$

$$\Omega_\phi \equiv \Omega_r + \Omega_p = \frac{2\pi}{T_p}\left(1+\frac{\Delta\Phi}{2\pi}\right).$$

$$\frac{GM\Omega_r^{(L=2)}}{\epsilon^{\frac{3}{2}}} = 1 - \frac{(15-\nu)}{8}\epsilon + \frac{555+30\nu+11\nu^2}{128}\epsilon^2 + \left(\frac{3(2\nu-5)}{2j} - \frac{194-184\nu+23\nu^2}{4j^3}\right)\epsilon^{\frac{3}{2}} + \left(\frac{15(17-9\nu+2\nu^2)}{8j} + \frac{21620-28592\nu+8765\nu^2-865\nu^3}{80j^3}\right)\epsilon^{\frac{5}{2}} + \cdots$$

$$\frac{GM\Omega_\phi^{(L=2)}}{\epsilon^{\frac{3}{2}}} = 1 + \frac{3}{j^2} - \frac{15(2\nu-7)}{4j^4} + \left(\frac{1}{8}(\nu-15) + \frac{15(\nu-5)}{8j^2} - \frac{3(1301-921\nu+102\nu^2)}{32j^4}\right)\epsilon + \left(\frac{3(2\nu-5)}{2j} + \frac{-284+220\nu-23\nu^2}{4j^3} + \frac{3(913-728\nu+106\nu^2)}{j^5}\right)\epsilon^{\frac{3}{2}} + \left(\frac{1}{128}(555+30\nu+11\nu^2) + \frac{3(895-150\nu+51\nu^2)}{128j^2} - \frac{3(-270085+251236\nu-70545\nu^2+7470\nu^3)}{2560j^4}\right)\epsilon^2 + \left(\frac{15(17-9\nu+2\nu^2)}{8j} + \frac{31520-34442\nu+10025\nu^2-865\nu^3}{80j^3}\right)\epsilon^{\frac{5}{2}}.$$

$$\epsilon = -2\mathcal{E}$$

One loop exact (Missing three loop!)

3PN match

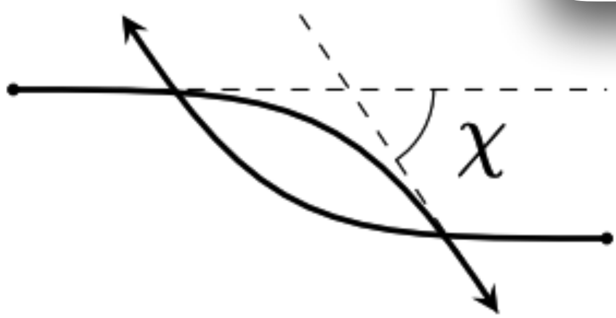
3PN missmatch

Higher orders velocity

Kalin **RAP**
1911.09130

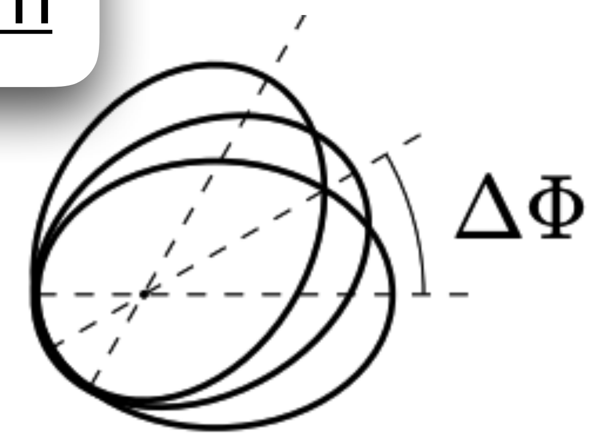
Valid for (aligned) spin!

J=canonical **total** ang. momentum



Angle from
Vines Steinhoff Buonanno
1812.00956.

$$\Delta\Phi(J,\mathcal{E}) = \chi(J,\mathcal{E}) + \chi(-J,\mathcal{E})$$



Periastron from
Tessmer Hartung Schaefer
1207.6961

$$\begin{aligned}
\frac{\chi(\ell,a,\epsilon)}{2\pi} = &\left[\frac{1}{\pi}(-\epsilon)^{-\frac{1}{2}} - \frac{(\nu-15)}{8\pi}(-\epsilon)^{\frac{1}{2}} + \frac{35+30\nu+3\nu^2}{128\pi}(-\epsilon)^{\frac{3}{2}}\right]\frac{1}{\ell} \\
&+ \left[3 + \frac{3(2\nu-5)}{4}\epsilon + \frac{3(5-5\nu+4\nu^2)}{16}\epsilon^2 - \frac{7\tilde{a}_+ + \Delta\tilde{a}_-}{2\pi}\epsilon^{-\frac{1}{2}} \right. \\
&\left. + \frac{5\Delta(\nu-3)\tilde{a}_- + (23\nu-25)\tilde{a}_+}{16\pi}(-\epsilon)^{\frac{3}{2}}\right]\frac{1}{2\ell^2} \\
&+ \left[-\frac{7\tilde{a}_+ + \Delta\tilde{a}_-}{2} - \frac{(\nu-6)\Delta\tilde{a}_- + (7\nu-18)\tilde{a}_+}{2}\epsilon \right. \\
&- \frac{3\left((15-14\nu+2\nu^2)\Delta\tilde{a}_- + (25-38\nu+14\nu^2)\tilde{a}_+\right)}{16}\epsilon^2 \\
&\left. - \frac{2}{3\pi}(-\epsilon)^{-\frac{3}{2}} + \frac{33+\nu}{4\pi}(-\epsilon)^{-\frac{1}{2}} + \frac{3003-1090\nu-5\nu^2+128\tilde{a}_+^2}{64\pi}(-\epsilon)^{\frac{1}{2}}\right]\frac{1}{2\ell^3} \\
&+ \left[\frac{3(35+2\tilde{a}_+^2-10\nu)}{4} - \frac{10080-13952\nu+123\pi^2\nu+1440\nu^2}{128}\epsilon \right. \\
&\left. - \frac{624\Delta\tilde{a}_-\tilde{a}_+ + 24(1-8\nu)\tilde{a}_-^2 - 24(12\nu-61)\tilde{a}_+^2}{\ldots}\epsilon + \cdots\right]\frac{1}{2\ell^4} + \cdots.
\end{aligned}$$

We have just computed it to all orders in velocity!

STAY TUNED!

Liu **RAP** Yang
2012.XXXX

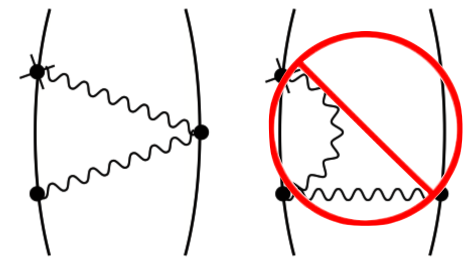
PM EFT for scattering: spins

COMING SOON

$$e^{iS_{\rm eff}[x_a]} = \int \mathcal{D}h_{\mu\nu}\, e^{iS_{\rm EH}[h]+iS_{\rm GF}[h]+iS_{\rm pp}[x_a,h]}\,,$$

$$S_{\rm pp} = -\sum_A \frac{m_A}{2}\int {\rm d}\tau_A g_{\mu\nu}\left(x_A\left(\tau_A\right)\right) v_A^\mu\left(\tau_A\right) v_A^\nu\left(\tau_A\right) - \frac{1}{2}\int {\rm d}\tau_A S_{ab}(\tau_A)\omega_\mu^{ab}(\tau_A) v_A^\mu(\tau_A).$$

spin coupling



New features

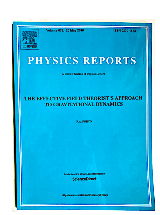
SSC preserving $$S^{ab}p_b = 0 \rightarrow\; -\frac{1}{2m_A}\int d\tau_A R_{deab} S_A^{cd} S_A^{ab} u_A^e u_{cA}$$

finite size effects $$\frac{C_{ES^2}}{2m_A}\int d\tau_A E_{ab} S_A^{ac} S_{cA}^b$$

EOM $$\dot{S}^{ab} = \{S^{ab}, S_{\rm pp}\}$$

Routhian (locally Lorentz spin algebra)

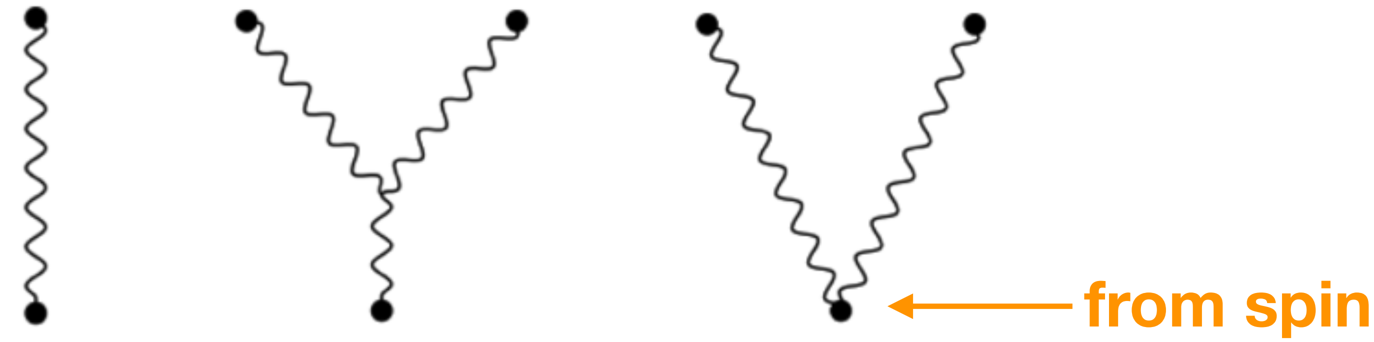
The effective field theorist's approach to gravitational dynamics
Rafael A. Porto
Physics Reports
Volume 633, 20 May 2016, Pages 1-104



Liu **RAP** Yang
2012.XXXX

PM EFT for scattering: spins

TOPOLOGIES at one loop



Spin-Orbit to NLO in ***covariant*** gauge

$$\Delta^{(2)}_{S_1} p_1^\mu = \frac{G_N^2 m_2}{4b^6}\frac{1}{(\gamma^2-1)^{5/2}}\Bigg[-\left(b^\alpha {s_1}^\rho {u_1}^\beta {u_2}^\sigma \epsilon_{\alpha\rho\beta\sigma}\right)\Bigg(3\pi\sqrt{-b^2}\gamma\left(5\gamma^4-8\gamma^2+3\right)(4m_1+3m_2)\,b^\mu$$
$$+8b^2\sqrt{\gamma^2-1}\Big(\gamma\left(8\gamma\left(1-2\gamma^2\right)m_1+\left(5-12\gamma^2\right)m_2\right){u_1}^\mu$$
$$+\left(\left(8\gamma^4-1\right)m_2+8\gamma\left(2\gamma^2-1\right)m_1\right){u_2}^\mu\Big)\Bigg)$$
$$+8b^2\left(\gamma^2-1\right)^{3/2}\left(8\gamma^3 m_1+4\gamma^2 m_2-4\gamma m_1-m_2\right)b^\alpha {s_1}^\rho {u_1}^\beta {\epsilon^\mu}_{\alpha\rho\beta}$$
$$-\pi\left(-b^2\right)^{3/2}\gamma\left(5\gamma^4-8\gamma^2+3\right)(4m_1+3m_2)\,{s_1}^\alpha {u_1}^\rho {u_2}^\beta {\epsilon^\mu}_{\alpha\rho\beta}\Bigg].$$

Bern et al.
2005.03071

We also computed spin1-spin2 and spin1^2 **general-orientation**

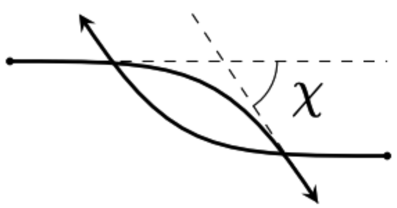
$$\frac{C_{ES^2}}{2m_A}\int d\tau_A E_{ab} S_A^{ac} S_{cA}^b$$

Liu **RAP** Yang
2012.XXXX

PM EFT for scattering: aligned-spins

$\tilde{a}_\pm \equiv a_\pm/(GM)$, with $a_\pm = a_1 \pm a_2$

deflection angle to NLO



$$
\begin{aligned}
\frac{\chi_2}{\Gamma} = & -\pi \left(\frac{G_N^3 M^3}{|b|^3}\right) \frac{\gamma\left(5\gamma^2-3\right)}{4\left(\gamma^2-1\right)^{3/2}} \left(7\tilde{a}_+ + \tilde{a}_-\delta\right) \\
& +\pi \left(\frac{G_N^4 M^4}{|b|^4}\right) \frac{3}{256\left(\gamma^2-1\right)^2} \Bigg[\tilde{a}_-^2 \Big(5\gamma^4 (7\delta \left(C_{ES^2}^{(1)} - C_{ES^2}^{(2)}\right) - 90) + \gamma^2 \Big(468 - 54\delta \left(C_{ES^2}^{(1)} - C_{ES^2}^{(2)}\right)\Big) \\
& \qquad + 19\delta \left(C_{ES^2}^{(1)} - C_{ES^2}^{(2)}\right) + \left(215\gamma^4 - 222\gamma^2 + 39\right)\left(C_{ES^2}^{(1)} + C_{ES^2}^{(2)}\right) - 82 \Big) \\
& + 2\tilde{a}_+\tilde{a}_- \Big(\left(215\gamma^4 - 222\gamma^2 + 39\right)\left(C_{ES^2}^{(1)} - C_{ES^2}^{(2)}\right) \\
& \qquad + \left(\gamma^2-1\right)\delta \left(70\gamma^2 + \left(35\gamma^2 - 19\right)\left(C_{ES^2}^{(1)} + C_{ES^2}^{(2)}\right) + 10\right) \Big) \\
& + \tilde{a}_+^2 \Big(5\gamma^4 \left(7\delta\left(C_{ES^2}^{(1)} - C_{ES^2}^{(2)}\right) + 166\right) - 6\gamma^2 \left(9\delta \left(C_{ES^2}^{(1)} - C_{ES^2}^{(2)}\right) + 146\right) \\
& \qquad + 19\delta \left(C_{ES^2}^{(1)} - C_{ES^2}^{(2)}\right) + \left(215\gamma^4 - 222\gamma^2 + 39\right)\left(C_{ES^2}^{(1)} + C_{ES^2}^{(2)}\right) + 110 \Big) \Bigg].
\end{aligned}
$$

For Kerr (CES2=1) explicitly confirms the result in

Vines et al. 1812.00956. Guevara et al. 1812.06895.

B2B: **FULL** periastron including finite-size effects

Tessmer Hartung Schaefer 1207.6961

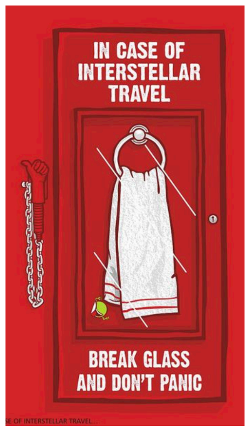
B2B dictionary *for aligned-spins* → *ALL the observables!*

$$\Delta\Phi(J,\mathcal{E}) = \chi(J,\mathcal{E}) + \chi(-J,\mathcal{E})$$

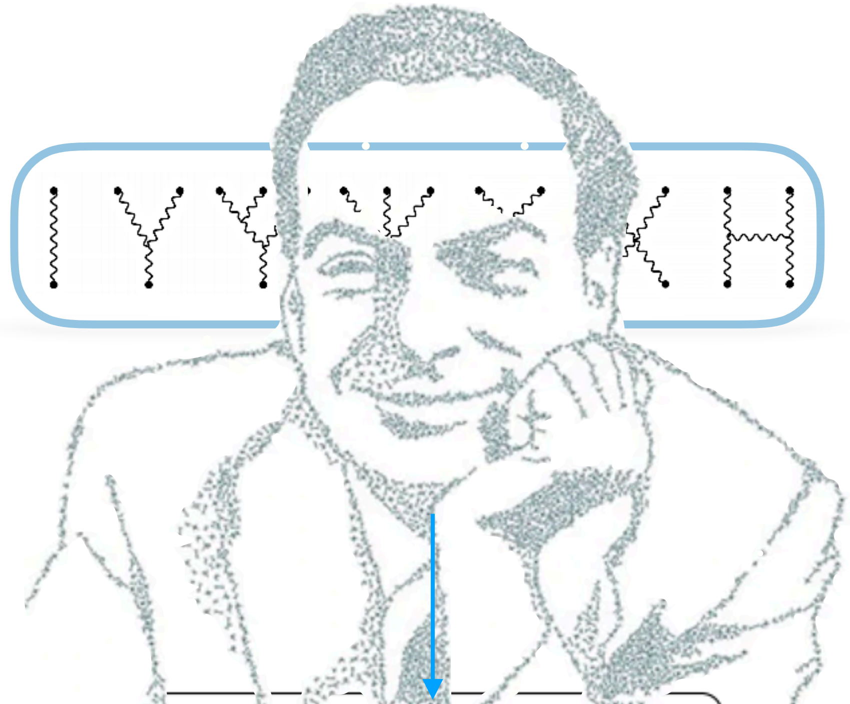
(J total canonical angular momentum)

Kalin **RAP**
1911.09130

Thank you!



PMEFT
2006.01184
2007.04977
2008.06047



Foffa **RAP** Rothstein Sturani
1903.05118

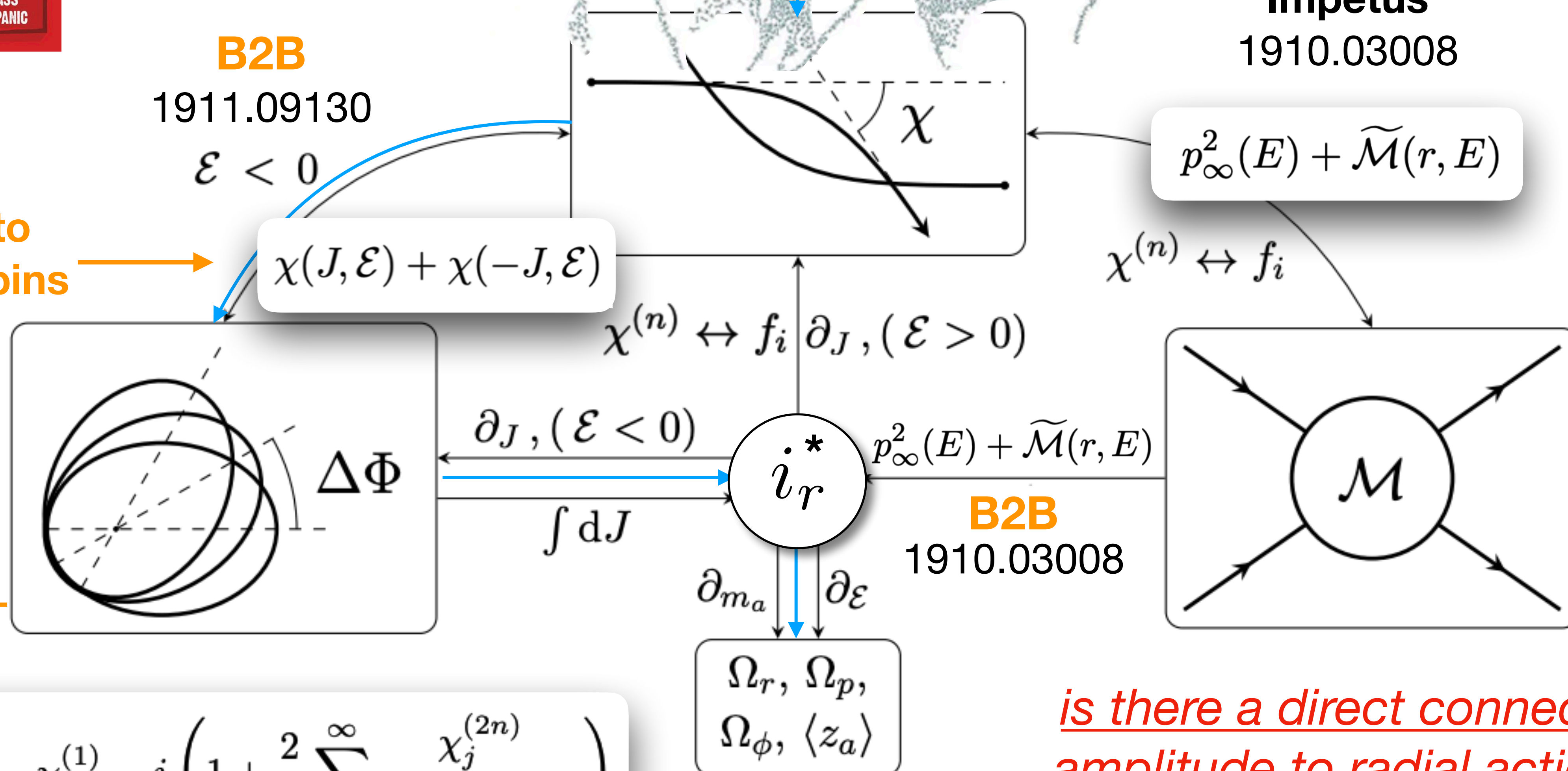
	0PN	1PN	2PN	3PN	4PN	5PN	6PN	7PN	
1PM	$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+v^{12}$	$+v^{14}$	$+\cdots)G$
2PM		$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+v^{12}$	$+\cdots)G^2$
3PM			$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+\cdots)G^3$
4PM				$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+\cdots)G^4$
5PM					$(1$	$+v^2$	$+v^4$	$+v^6$	$+\cdots)G^5$

Impetus
1910.03008

B2B
1911.09130

applies to aligned-spins

B2B



B2B
1910.03008

<u>*is there a direct connection amplitude to radial action??*</u>

$$* \quad i_r \equiv \frac{p_\infty}{\sqrt{-p_\infty^2}}\chi_j^{(1)} - j\left(1 + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\chi_j^{(2n)}}{(1-2n)j^{2n}}\right)$$

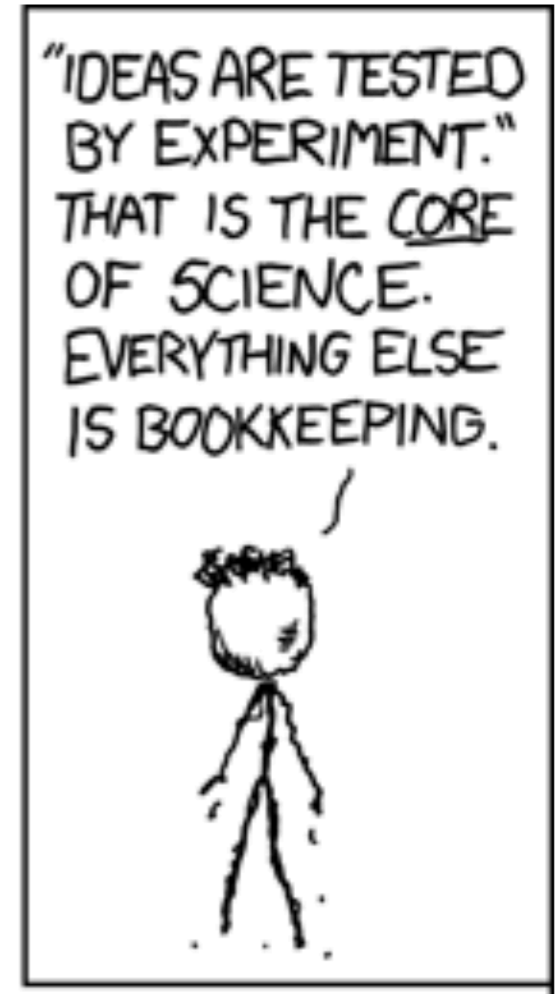
$$* \quad \frac{1}{2\sqrt{-p_\infty^2}}\frac{\widetilde{\mathcal{M}}_1}{M\mu} + \frac{j}{2\sqrt{\pi}}\sum_{n=0}^{\infty}\left(\frac{\Gamma\left(n-\frac{1}{2}\right)}{(\mu M j)^{2n}}\sum_{\sigma\in\mathcal{P}(2n)}\frac{p_\infty^{2(n-\Sigma^\ell)}}{\Gamma\left(1+n-\Sigma^\ell\right)}\prod_\ell\frac{\widetilde{\mathcal{M}}_{\sigma_\ell}^{\sigma^\ell}}{\sigma^\ell!}\right)$$

Through Impetus/Firsov

The most exciting phrase
to hear in science,
the one that heralds
new discoveries, is not
"**EUREKA!**"
but, "**that's funny...**"

—Isaac Asimov

Extra Slides



"A method is more important than a discovery, since the right method will lead to new and even more important discoveries."

Lev Landau

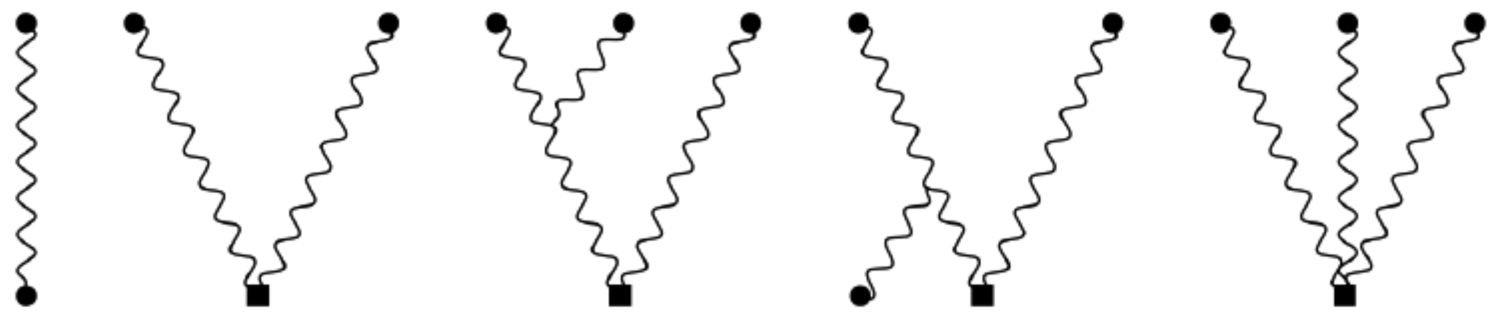
Kalin Liu **RAP**
2008.06047

Tidal Effects: NLO

$$\Delta p_a^\mu = -\eta^{\mu\nu} \int_{-\infty}^{+\infty} \mathrm{d}\tau_a \frac{\partial \mathcal{L}_{\text{eff}}}{\partial x_a^\nu}(x_a(\tau_a))$$

Quadrupole to NLO agrees with Cheng-Solon

2006.06665



$$S_{\text{pp}} = \sum_{a=1,2} \int \mathrm{d}\tau_a \Big(-\frac{m_a}{2} g_{\mu\nu} v_a^\mu v_a^\nu + c_{E^2}^{(a)} E_{\mu\nu} E^{\mu\nu} + c_{B^2}^{(a)} B_{\mu\nu} B^{\mu\nu} - c_{\tilde{E}^2}^{(a)} E_{\mu\nu\alpha} E^{\mu\nu\alpha} - c_{\tilde{B}^2}^{(a)} B_{\mu\nu\alpha} B^{\mu\nu\alpha} .$$

$$\lambda_{E^2} \equiv \frac{1}{G^4 M^5} \left(m_2 \frac{c_{E^2}^{(1)}}{m_1} + m_1 \frac{c_{E^2}^{(2)}}{m_2} \right) ,$$

$$\kappa_{E^2} \equiv \lambda_{E^2} + \frac{c_{E^2}^{(1)} + c_{E^2}^{(2)}}{G^4 M^5} = \frac{1}{G^4 M^4} \left(\frac{c_{E^2}^{(1)}}{m_1} + \frac{c_{E^2}^{(2)}}{m_2} \right)$$

$$\begin{aligned}
\frac{\Delta\chi_{(E,B)}}{\Gamma} = {} & \frac{45\pi}{64} \frac{(\gamma^2-1)^2}{(\Gamma j)^6} \Big[\big(35\gamma^4 - 30\gamma^2 - 5\big) \lambda_{B^2} + \big(35\gamma^4 - 30\gamma^2 + 11\big) \lambda_{E^2} \Big] \\
& + \frac{192}{35} \frac{(\gamma^2-1)^{3/2}}{(\Gamma j)^7} \Big[\big(160\gamma^6 - 192\gamma^4 + 30\gamma^2 + 2\big) \lambda_{B^2} + \big(160\gamma^6 - 192\gamma^4 + 72\gamma^2 - 5\big) \lambda_{E^2} \Big] \\
& + \frac{96\nu}{35} \frac{\sqrt{\gamma^2-1}}{(\Gamma j)^7} \kappa_{B^2} \Big[224\gamma^9 - 320\gamma^8 - 728\gamma^7 + 704\gamma^6 + 5488\gamma^5 - 444\gamma^4 + 66262\gamma^3 + 56\gamma^2 + 28084\gamma + 4 \Big] \\
& + \frac{96\nu}{35} \frac{\sqrt{\gamma^2-1}}{(\Gamma j)^7} \kappa_{E^2} \Big[224\gamma^9 - 320\gamma^8 - 728\gamma^7 + 704\gamma^6 + 5628\gamma^5 - 528\gamma^4 + 65982\gamma^3 + 154\gamma^2 + 28329\gamma - 10 \Big] \\
& - \frac{576\nu\sqrt{\gamma^2-1}}{(\Gamma j)^7} \Big[\big(440\gamma^4 + 474\gamma^2 + 32\big)\, \kappa_{B^2} + \big(440\gamma^4 + 474\gamma^2 + 33\big)\, \kappa_{E^2} \Big] a\text{sh}(\gamma) ,
\end{aligned}$$

*Confirmed observed high-energy pattern

$$a\text{sh}(\gamma) \equiv (\gamma^2-1)^{-1/2} \sinh^{-1} \sqrt{\frac{\gamma-1}{2}}$$

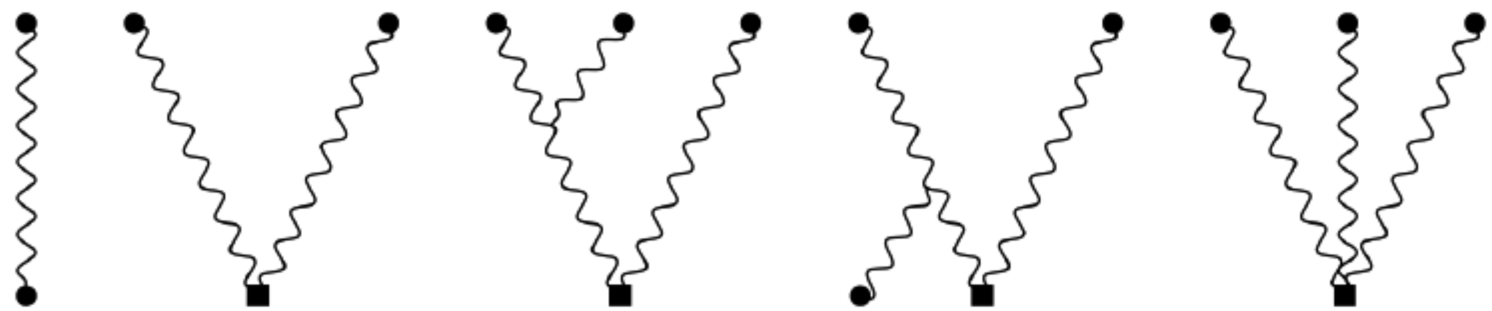
Kalin Liu **RAP**
2008.06047

Tidal Effects: NLO

Bound orbits
$$x = (GM\omega)^{2/3} \sim v^2$$

Quadrupole to NLO agrees with Cheng-Solon

2006.06665



Quadrupole/Octupole TLN in binding energy to O(G^3)

$$\Delta\mathcal{E}_{\rm T} = x\Big[18\,\lambda_{E^2}x^5 + 11\Big(3(1-\nu)\lambda_{E^2} + 6\,\lambda_{B^2} + 5\nu\,\kappa_{E^2}\Big)x^6 + \Big(390\lambda_{\tilde{E}^2} - \frac{13}{28}(161\nu^2 - 161\nu - 132)\lambda_{E^2} - \frac{1326\nu}{7}\kappa_{B^2}$$
$$+ \frac{13}{28}(616\nu + 699)\lambda_{B^2} + \frac{13\nu}{84}(490\nu - 729)\kappa_{E^2} + \frac{13}{6}\Delta\bar{P}^{(E,B)}_{8,\rm stc}\Big)x^7 + 75\big(45\nu\kappa_{\tilde{E}^2} - (13\nu+3)\lambda_{\tilde{E}^2} + 16\lambda_{\tilde{B}^2}\big)x^8$$
$$-\Big(\frac{85}{36}\left(1083\nu^2 + 1539\nu + 163\right)\lambda_{\tilde{E}^2} + \frac{27200\nu}{3}\kappa_{\tilde{B}^2} - \frac{85}{4}(270\nu + 383)\nu\kappa_{\tilde{E}^2} - \frac{680}{9}(90\nu + 173)\lambda_{\tilde{B}^2} - \frac{17}{6}\Delta\bar{P}^{(\tilde{E},\tilde{B})}_{10,\rm stc}\Big)x^9\Big]$$

$$\Delta\bar{P}^{(E,B)}_{8,\rm stc} = \frac{1326}{7}\nu\kappa_{B^2} + (243 - 90\nu)\nu\kappa_{E^2} + \Big(45\nu^2 - \frac{885\nu}{7} + \frac{675}{14}\Big)\lambda_{E^2} - \Big(234\nu + \frac{837}{14}\Big)\lambda_{B^2}\,.$$

$$\Delta\bar{P}^{(\tilde{E},\tilde{B})}_{10,\rm stc} = \frac{1}{3}\big(2050\lambda_{\tilde{E}^2} - 13120\lambda_{\tilde{B}^2}\big) + \mathcal{O}(\nu)\,.$$

*We also reconstructed the full PM Hamiltonian to NLO

Radiation-Reaction

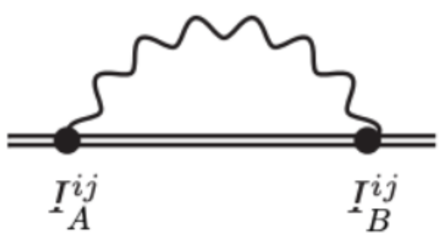
in-in b.c.
cons. vs dissip. from symmetry in w->-w

PHYSICAL REVIEW D **93**, 124010 (2016)

Tail effect in gravitational radiation reaction: Time nonlocality and renormalization group evolution

Chad R. Galley,[1] Adam K. Leibovich,[2] Rafael A. Porto,[3] and Andreas Ross[4]

The radiation-reaction force at LO (multipole expansion)

$iW[\boldsymbol{x}_a^{\pm}] =$ 

$\longrightarrow$

$$W[\boldsymbol{x}_a^{\pm}] = -\frac{G_N}{5}\int dt\, I_{-}^{ij}(t) I_{+ij}^{(5)}(t)$$

$$(\boldsymbol{a}_a^i)_{\rm rr}(t) = -\frac{2G_N}{5} I^{(5)ij}(t)\boldsymbol{x}_a^j(t). \longrightarrow$$

$$\dot{M} = -\frac{G_N}{5} I^{(1)ij}(t) I^{(5)ij}(t)\,,$$

$$\dot{\mathcal{E}}_N = \frac{8}{15}\frac{G^3 m^2\mu^2}{c^5 r^4}\left\{12v^2 - 11\dot{r}^2\right\}$$

leading cross-term

BUT! we get the angle from the impulse **(integrated in time):**

$$\sqrt{\Delta p^2} \to G\int dt\, I^{(5)ij} x^i x^j \sim G\int dt I^{(5)ij} I^{ij}$$

$$\dot{\mathcal{J}}_N = \frac{8}{5}\frac{G^2 m\,\mu^2}{c^5 r^3}\tilde{\mathbf{L}}_{\mathbf{N}}\left\{2v^2 - 3\dot{r}^2 + 2\frac{Gm}{r}\right\}$$

$$\sqrt{\Delta p^2} \sim G\int dt I^{(3)ij} I^{(2)ij} \sim G\int dt\frac{dL}{dt} \sim G\Delta L \sim G^3$$

$$L^{ij} \equiv -\int d^3\mathbf{x}\,(T^{0i}x^j - T^{0j}x^i)$$

Radiation-Reaction

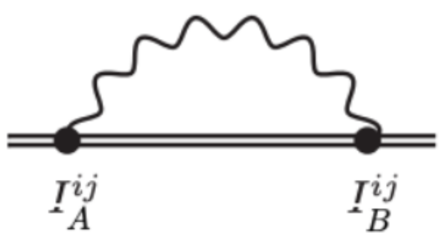
in-in b.c.
cons. vs dissip. from symmetry in w->-w

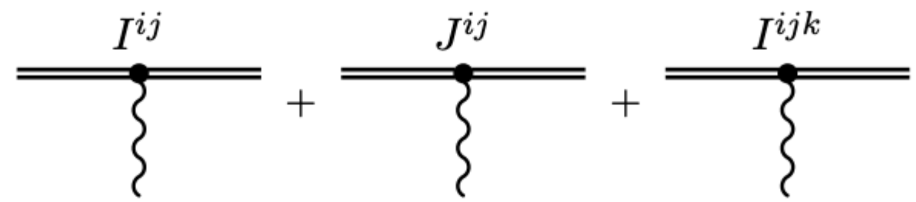
PHYSICAL REVIEW D **93**, 124010 (2016)

Tail effect in gravitational radiation reaction: Time nonlocality and renormalization group evolution

Chad R. Galley,[1] Adam K. Leibovich,[2] Rafael A. Porto,[3] and Andreas Ross[4]

The radiation-reaction force at all orders in the multipole expansion

$iW[\boldsymbol{x}_a^{\pm}] =$ 

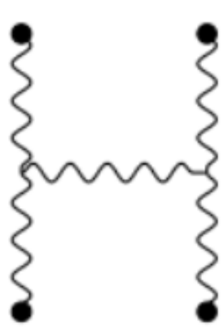
$i\mathcal{A}_h(\omega, \boldsymbol{k}) =$ 

$$= \frac{i}{4M_{\rm Pl}} \epsilon^*_{ij}(\boldsymbol{k}, h) \left[\omega^2 I^{ij}(\omega) + \frac{4}{3}\omega\, \boldsymbol{k}^l \epsilon^{ikl} J^{jk}(\omega) - \frac{i}{3}\omega^2 \boldsymbol{k}^l I^{ijl}(\omega) + \cdots \right],$$

The energy would also follow directly by squaring:

$$d\Gamma_h(\boldsymbol{k}) = \frac{1}{T} \frac{d^3\boldsymbol{k}}{(2\pi)^3 2|\boldsymbol{k}|} |\mathcal{A}_h(|\boldsymbol{k}|, \boldsymbol{k})|^2 \;\rightarrow\; P\,\Big|_{h=\pm 2} = \int_{\boldsymbol{k}} |\boldsymbol{k}| d\Gamma_h(\boldsymbol{k})$$

In PMEFT we should re-compute the **soft part** of the H-diagram in the in-in formalism:



$$(I_{\rm H} + I_{\overline{\rm H}})\,\big|_{\ln(-t)} = \frac{1}{64\pi^3} \frac{1}{m^2 t^2} \frac{1}{\sqrt{\sigma^2 - 1}} \operatorname{arcsinh} \sqrt{\frac{\sigma - 1}{2}} \left[\pi + 2i \operatorname{arcsinh} \sqrt{\frac{\sigma - 1}{2}} \right]. \quad \text{(D.3)}$$

Bern et al.
1908.01493

Conservative

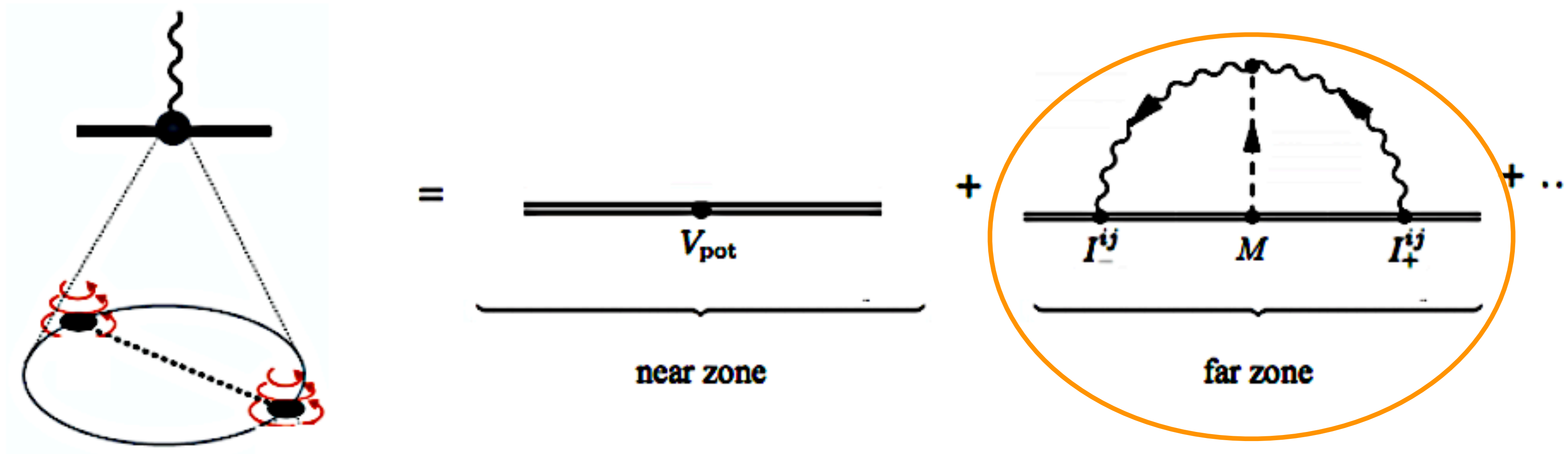
Radiation-Reaction

in-in b.c.
cons. vs dissip. from
symmetry in w->-w

PHYSICAL REVIEW D **93**, 124010 (2016)

Tail effect in gravitational radiation reaction: Time nonlocality and renormalization group evolution

Chad R. Galley,[1] Adam K. Leibovich,[2] Rafael A. Porto,[3] and Andreas Ross[4]



Similarly to NRQCD:

$$\frac{2G_N^2 M}{5} I^{(3)ij} I^{(3)ij} \left(-\frac{1}{\epsilon_{\rm IR}} + 2\log(\mu r) + \cdots\right) + \left(\frac{1}{\epsilon_{\rm UV}} + 2\log(\Omega/\mu) + \cdots\right)$$

$$W_{\rm tail}[\boldsymbol{x}_a^{\pm}] = \frac{2G_N^2 M}{5} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi}\, \omega^6\, I^{ij}_-(-\omega) I^{ij}_+(\omega) \left[-\frac{1}{(d-4)_{\rm UV}} - \gamma_E + \log\pi - \log\frac{\omega^2}{\mu^2} + \frac{41}{30} + i\pi\,{\rm sign}(\omega) \right].$$

$$\mu \frac{d}{d\mu} V_{\rm ren}(\mu) = \frac{2G_N^2 M}{5} I^{ij(3)}(t) I^{ij(3)}(t)$$

dissipative part

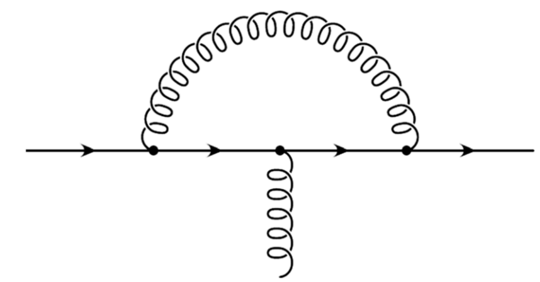
Radiation-Reaction

PHYSICAL REVIEW D **96,** 024063 (2017)

Lamb shift and the gravitational binding energy for binary black holes

Rafael A. Porto

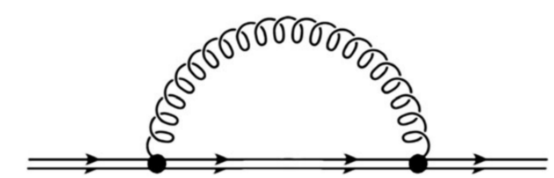
Computation in NRQED



$$\delta E_{n,\ell} = (\delta E_{n,\ell})_{US} + (\delta E_{n,\ell})_{cV} + \cdots$$

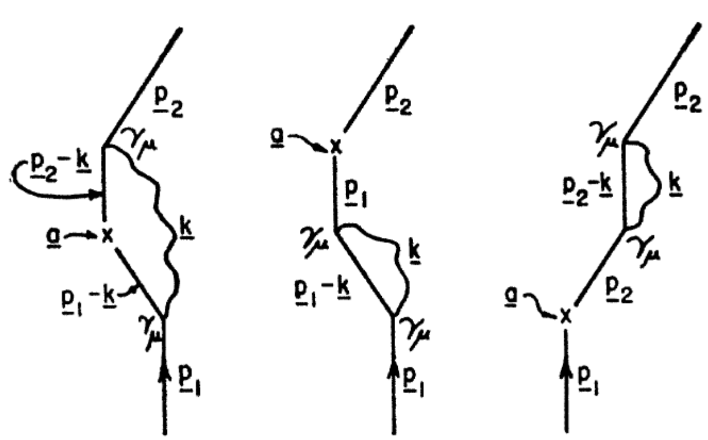
$$= \frac{2\alpha_e}{3\pi}\left[\frac{5}{6}e^2\frac{|\psi_{n,\ell}(\boldsymbol{x}=0)|^2}{2m_e^2} - \sum_{m\neq n,\ell}\left\langle n,\ell\left|\frac{\boldsymbol{p}}{m_e}\right|m,\ell\right\rangle^2 (E_m - E_n)\log\frac{2|E_n - E_m|}{m_e}\right] +$$

$$+\frac{4\alpha_e^2}{3m_e^2}\left(\frac{1}{\epsilon_{\rm UV}} - \frac{1}{\epsilon_{\rm IR}}\right)|\psi_{n,\ell}(\boldsymbol{x}=0)|^2 .$$



$$(p^0_{US}, \boldsymbol{p}_{US}) \sim (m_e v^2, m_e v^2)$$

only cancel explicitly in dim. reg.!
(zero-bin subtraction)



a. Eq.12 b. Eq.13 c. Eq.14

Space-Time Approach to Quantum Electrodynamics

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(Received May 9, 1949)

Lamb shift as interpreted in more detail in B.[13]

[13] That the result given in B in Eq. (19) was in error was repeatedly pointed out to the author, in private communication, by V. F. Weisskopf and J. B. French, as their calculation, completed simultaneously with the author's early in 1948, gave a different result. French has finally shown that although the expression for the radiationless scattering B, Eq. (18) or (24) above is correct, it was incorrectly joined onto Bethe's non-relativistic result. He shows that the relation $\ln 2k_{\max} - 1 = \ln\lambda_{\min}$ used by the author should have been $\ln 2k_{\max} - 5/6 = \ln\lambda_{\min}$. This results in adding a term $-(1/6)$ to the logarithm in B, Eq. (19) so that the result now agrees with that of J. B. French and V. F. Weisskopf,