From symmetries to generalised color-kinematics rules

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Table of Contents

How general is the double copy?

- Can we construct the full space of the gravity theory without resorting to special gauge choices and coordinate choices ?
- We are seeking a *dictionary that continues to hold when we perfom gauge/coordinate transformations*.
- Schematically, for the minimal YM^2:

$$A_\mu * \tilde{A}_\nu \equiv h_{\mu\nu} + B_{\mu\nu} + \eta_{\mu\nu}\phi$$

- Want to construct the ***full theory*** with ***arbitrary boundary conditions*** (i.e. sources) of graviton, dilaton and two-form from the double copy.
- If this is achievable, it should be possible to extract (pure) gravity theory by setting $B_{\mu\nu} = \phi = 0$.

Table of Contents

BRST formulation of the double copy

- We work in the Becchi-Rouet-Stora-Tyutin (BRST) formulation for both the gauge theory and gravity, where the gauge parameters are replaced by ghost fields.
- Benefits:
  - ✓ gives a gauge mapping algorithm between YM and gravity
  - ✓ disentangles the graviton and dilaton
  - ✓ extends the color-kinematic replacement rules to the ghost-sector
- Conjectured by [Siegel'96].

BRST intro

- Introduced to avoid issues caused by gauge symmetry in path integrals.
- Schematically

$$S_{BRST} = \int d^Dx \left(\mathcal{L}_0[f] + b\left(G[f] - \tfrac{\xi}{2} b\right) - \bar{c}Q\left(G[f]\right) - fj^{(f)} + \bar{j}c + \bar{c}j \right) ,$$

  where $\mathcal{L}_0[f]$ is the classical action for the field f, $G[f]$ is the gauge-fixing functional and b is the Lautrup-Nakanishi Lagrange multiplier field.
- Note that, *unlike in the standard treatment, we have coupled sources to the ghost and anti-ghost*.
- We require that the BRST symmetries of YM system:

$$QA_\mu = \partial_\mu c, \quad Qc = 0, \quad Q\bar{c} = \tfrac{1}{\xi} G(A)$$

  induce the correct symmetries for the gravitational fields:

$$\begin{aligned} Qh_{\mu\nu} &= 2\partial_{(\mu}c_{\nu)}, & Qc_\mu &= 0, & Q\bar{c}_\mu &= \tfrac{1}{\xi^{(h)}} G_\mu[h,\varphi], \\ QB_{\mu\nu} &= 2\partial_{[\mu}d_{\nu]}, & Qd_\mu &= \partial_\mu d, & Q\bar{d}_\mu &= \tfrac{1}{\xi^{(B)}} G_\mu[B,\eta], \\ Q\varphi &= 0. \end{aligned}$$

BRST dictionary [Anastasiou,Borsten,Duff,SN,Zoccali'18]

The most general dictionary in the absence of $\square^{-1}$ terms and compatible with symmetries is:

$$
\begin{aligned}
h_{\mu\nu} =& A_\mu \circ \tilde{A}_\nu + A_\nu \circ \tilde{A}_\mu + a\eta_{\mu\nu}\left(A^\rho \circ \tilde{A}_\rho + \xi c^\alpha \circ \tilde{c}_\alpha\right),\\
B_{\mu\nu} =& A_\mu \circ \tilde{A}_\nu - A_\nu \circ \tilde{A}_\mu,\\
\varphi =& A^\rho \circ \tilde{A}_\rho + \xi c^\alpha \circ \tilde{c}_\alpha.
\end{aligned}
$$

where we have introduced the OSp(2) ghost singlet

$$c^\alpha \circ \tilde{c}_\alpha = c \circ \tilde{\bar{c}} - \bar{c} \circ \tilde{\bar{c}}.$$

where

$$A_\mu \circ \tilde{A}_\nu \equiv [A_\mu{}^i \star \Phi^{-1}_{ii'} \star \tilde{A}_\nu{}^{i'}](x)$$

where $\Phi_{ii'}$ is a bi-adjoint scalar field. Here $\star$ denotes the convolution is defined as

$$[f \star g](x) = \int d^4y f(y) g(x-y).$$

and is a consequence of the momentum-space origin of squaring: product in momentum space is convolution in position space! Importantly, is **doesn't** obey the Leibnitz rule:

$$\partial_\mu(f \star g) = (\partial_\mu f) \star g = f \star (\partial_\mu g)$$

Gauge mapping algorithm

- BRST gives an algorithm for mapping YM gauge choice to gravity gauge choice. Illustrate with simple, covariant example.
- Choose YM gauge fixing functional

$$G[A] \equiv \partial^\mu A_\mu, \quad G[\tilde{A}] \equiv \partial^\mu \tilde{A}_\mu$$

- From the graviton dictionary

$$h_{\mu\nu} = A_\mu \circ \tilde{A}_\nu + A_\nu \circ \tilde{A}_\mu + a\eta_{\mu\nu}\left(A^\rho \circ \tilde{A}_\rho + \xi c^\alpha \circ \tilde{\bar{c}}_\alpha\right),$$

we read off the gravitational ghost dictionary

$$c_\mu = c \circ \tilde{A}_\mu + A_\mu \circ \tilde{c},$$

- Conjugation immediately gives us the anti-ghost

$$\bar{c}_\mu = \bar{c} \circ \tilde{A}_\mu + A_\mu \circ \tilde{\bar{c}},$$

- We know that the BRST transformation of this should be

$$Q\bar{c}_\mu = \frac{1}{\xi} G_\mu[h, \varphi],$$

BUT we can compute $Q\bar{c}_\mu$ directly, using the the YM transformation rules:

$$Q\bar{c}_\mu = \frac{1}{\xi}\left[\partial^\rho A_\rho \circ \tilde{A}_\mu + A_\mu \circ \partial^\rho \tilde{A}_\rho\right] + \partial_\mu c^\alpha \circ \tilde{\bar{c}}_\alpha$$

Then, inverting the graviton and dilaton dictionaries, we read off

$$G_\mu[h, \varphi] = \partial^\nu h_{\nu\mu} - \frac{1}{2}\partial_\mu h + \left(1 + \frac{D-2}{2}a\right)\partial_\mu \varphi$$

Disentangling the graviton and dilaton: JNW solution [Luna,SN,White'20]

- What is the double copy of a point charge in pure (non-supersymmetric) YM ?
- Original Kerr-Schild(KS) approach [Monteiro, O'Connell, White'14] identifies it at Schwarzschild. Subsequent work, either in the perturbative [Goldberger, Ridgway'17, Luna, Monteiro, Nicholson, Ochirov, O'Connell, Westberger, White'17], or the exact (generalised) KS approach [Kim, Lee, Monteiro, Nicholson, Peinador Veiga'19] show that a dilaton must be turned on and one more generally expects the Janis-Newman-Winicour (JNW) solution.
- JNW solution:
$$\bar{h}_{\mu\nu} = \frac{\kappa}{2}\frac{M}{4\pi r}u_\mu u_\nu, \quad \varphi = -\frac{\kappa}{2}\frac{Y}{4\pi r},$$
where
$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{h}{2}\eta_{\mu\nu}, \qquad u_\mu = (1,0,0,0)$$
and M, Y are arbitrary parameters.
- What is the YM origin of M,Y ?
- The BRST framework gives a way to construct the full JNW solution + we learn some new lessons from studying solutions !

Case study: JNW solution [Luna, SN, White'20]

- The dictionary mapping YM solutions in Lorenz gauge to gravitational solutions in De Donder is obtained (via a gauge mapping procedure for solutions):

$$\bar{h}_{\mu\nu} = 2A_\mu \circ A_\nu - \tfrac{2}{\Box}\partial_{(\mu}A_{\nu)} \circ \partial A, \quad \varphi = 2A^\rho \circ A_\rho + 4\xi c \circ \bar{c}$$

- Then from

$$A^a_\mu = \tfrac{g\alpha^a}{4\pi r} u_\mu, \quad u_\mu = (1,0,0,0)$$

and

$$c = \tfrac{gD}{4\pi r}, \quad \bar{c} = \tfrac{g\bar{D}}{4\pi r}, \qquad \Phi^{aa'} = \tfrac{g\delta^{aa'}}{4\pi r}.$$

we get

$$\bar{h}_{\mu\nu} = \tfrac{g(\alpha\cdot\alpha)}{4\pi r} u_\mu u_\nu, \quad \varphi = \tfrac{g}{4\pi r}\left[(\alpha\cdot\alpha) + 4\xi D\cdot\bar{D}\right]$$

with the free parameters mapping as

$$g \to \tfrac{\kappa}{2}, \quad (\alpha\cdot\alpha) \to M, \quad [(\alpha\cdot\alpha) + 4g\xi D\cdot\bar{D}] \to -Y$$

- BRST thus gives a key insight into where the two parameters of the JNW solution come from.

EH Lagrangian at cubic order [Borsten,SN'20]

- Construct the action at higher orders in perturbation theory by employing a result connecting perturbative expansions of classical fields to amplitudes of increasing order [Boulware,Brown'68,Luna,Monteiro,Nicholson,Ochirov, O'Connell,Westerberg,White'17].
- This allows us to make use of the BCJ duality and corresponding double copy prescription.
- Remember

$$h_{\mu\nu} = 2A_{(\mu} \circ \tilde{A}_{\nu)} + a\eta_{\mu\nu}\left[A_\rho \circ \tilde{A}^\rho + \xi c^\alpha \circ \tilde{\bar{c}}_\alpha\right]$$
$$B_{\mu\nu} = 2A_{[\mu} \circ \tilde{A}_{\nu]}$$
$$\varphi = A^\rho \circ \tilde{A}_\rho + \xi c^\alpha \circ \tilde{\bar{c}}_\alpha$$

toghether with ghost dictionaries. We can remove the entire Kalb-Ramond sector by setting: $A_\mu = \tilde{A}_\mu,\ c = \tilde{c},\ \bar{c} = \tilde{\bar{c}}$ and further kill the dilaton by picking:

$$c \circ \bar{c} = -\frac{1}{2\xi} A^\rho \circ A_\rho \quad \Rightarrow \quad \varphi = 0$$

- Note we have not constrained A_μ!
- Finally, we can read off the inverse dictionaries

$$A_\mu \circ A_\nu = \tfrac{1}{2} h_{\mu\nu}, \quad c \circ A_\mu = \tfrac{1}{2} c_\mu, \quad \bar{c} \circ A_\mu = \tfrac{1}{2}\bar{c}_\mu, \quad c \circ \bar{c} = -\tfrac{1}{4\xi} h$$

which will be needed for the cubic order construction.

Cubic order [Borsten,SN'20]

- We work with the standard Yang-Mills BRST action

$$\mathcal{L}_{\text{YM}} = -\tfrac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} + \tfrac{1}{2\xi} G[A]^a G[A]^a - \bar{c}^a \partial^\mu D^{ac}_\mu c^c ,$$

  where $D^{ac}_\mu = \delta^{ac}\partial_\mu + g f^{abc} A^b_\mu$ and $G[A]^a = \partial^\rho A^a_\rho$.
- We can write the cubic terms as:

$$\mathcal{L}^{(3)}_{\text{YM}} = i g f^{abc} \int đp_1 đp_2 đp_3 \; e^{-i(p_1+p_2+p_3)x} \Big[\tfrac{1}{6} n^{\mu\nu\rho}(p_i) A^a_\mu(p_1) A^b_\nu(p_2) A^c_\rho(p_3) + n^{\mu\alpha\beta}(p_1) c^a_\alpha(p_1) A^b_\mu(p_2) c^c_\beta(p_3) \Big],$$

- The color factor at cubic order is just $c = i f^{abc}$, and the kinematic factors are

$$n^{\mu_1\mu_2\mu_3}(p_i) = -(p^{\mu_3}_{12}\eta^{\mu_1\mu_2} + p^{\mu_1}_{23}\eta^{\mu_2\mu_3} + p^{\mu_2}_{31}\eta^{\mu_3\mu_1}),$$
$$n^{\mu\alpha\beta}(p) = -p^\mu \sigma^{\alpha\beta}_+ , \qquad \text{with } \sigma_\pm = \tfrac{1}{2}(\sigma_x \pm i\sigma_y)$$

  with $p_{ij} = p_i - p_j$.
- Note that $n^{\mu_1\mu_2\mu_3}(p_i)$ is the standard YM kinematic factor, whereas $n^{\mu\alpha\beta}(p)$ is the novel one brought in by BRST. It characterises the gluon-ghost-antighost interaction.

Cubic order[Borsten,SN'20]

- When performing the double-copy, we must take all possible combinations: the gluon-gluon-gluon term with itself will contribute graviton-graviton-graviton interactions, same as the ghost-antighost-gluon term with itself, while the cross terms will contribute the graviton-ghost-antighost interactions.
- We get the momentum space double copy Lagrangian

$$\hat{\mathcal{L}}^{(3,dc)}_{(grav)} = \frac{1}{6} n^{\mu\nu\rho} n^{\tilde{\mu}\tilde{\nu}\tilde{\rho}} A_{\mu\tilde{\mu}}(p) A_{\nu\tilde{\nu}}(k) A_{\rho\tilde{\rho}}(q) + \xi^2 p^{\mu} p^{\tilde{\mu}} \bar{C}^{(0)}(p) A_{\mu\tilde{\mu}}(k) C^{(0)}(q) \\ - 2 n^{\mu\nu\rho} p^{\tilde{\nu}} \bar{C}_{\mu}(p) A_{\nu\tilde{\nu}}(k) C_{\rho}(q),$$

- Now we get to use our linearised convolution dictionary:

$$\mathcal{F}[A_{\mu\nu}] = A_{\mu} \circ A_{\nu} = \frac{1}{2} h_{\mu\nu}, \quad \mathcal{F}[C_{\mu}] = c \circ A_{\mu} = \frac{1}{2} c_{\mu} \\ \mathcal{F}[\bar{C}_{\mu}] = \bar{c} \circ A_{\mu} = \frac{1}{2} \bar{c}_{\mu}, \quad \mathcal{F}[C^{(0)}] = c \circ \bar{c} = -\frac{1}{4\xi} h$$

Cubic order

- Finally, we get the action

$$\mathcal{L}^{(3,dc)}_{(grav)} = -\frac{1}{8}h^{\mu\nu}\left(h^{\rho\sigma}\partial_\rho\partial_\sigma h_{\mu\nu} - \partial_\mu h^{\rho\sigma}\partial_\nu h_{\rho\sigma} - h^{\rho\sigma}\partial_\sigma\partial_\nu h_{\mu\rho}\right.$$
$$\left. + 2\partial_\nu h_{\rho\sigma}\partial^\sigma h_\mu^{\ \rho} - \partial_\rho h_{\nu\sigma}\partial^\sigma h_\mu^{\ \rho} - \frac{1}{4}h\partial_\mu\partial_\nu h\right).$$

- Note that this is simpler than the EH action at cubic order, thus revealing one of the advantages of the double copy. We can map between the two via the (unique local) field redefinition

$$h_{\mu\nu} \to h_{\mu\nu} - \frac{1}{4}h_{\mu\nu}h + \frac{1}{2}h_\mu^{\ \rho}h_{\nu\rho} - \frac{1}{16}\eta_{\mu\nu}\left(h_{\rho\sigma}h^{\rho\sigma} - \frac{3}{4}h^2\right).$$

- Read off 2nd order gauge-fixing functional

$$G_\mu[h]^{(2)} = \frac{3}{8}h^{\nu\rho}\partial_\mu h_{\nu\rho} - \frac{5+2\xi}{32}h\partial_\mu h + \frac{4-3\xi}{16}h_\mu^{\ \rho}\partial_\rho h$$
$$- \frac{1}{2}h^{\nu\rho}\partial_\rho h_{\mu\nu} + \frac{1}{4}h\partial^\rho h_{\mu\rho} - \frac{4-\xi}{4}h_\mu^{\ \nu}\partial^\rho h_{\nu\rho}.$$

- Graviton-ghost-antighost terms work out similarly.
- In the BRST framework, it is always possible to find *local* field redefinitions between the standard theory and the double copy theory at any order [Borsten,Jurco,Kim,Macrelli,Saemann,Wolf'20]
- See also [Ferrero,Francia'20] for Lagrangian constructions based on the convolution dictionary.

- BRST gives nice map off-shell.
- We also have more control over the gauge choices.
- Need for redefinitions [simplified by the presence of the ghosts but still there].
- A lot in our construction was dependent on symmetries (in particular the gauge mapping) .
- A direct double copy construction for symmetries (either order-by-order in perturbation theory or even non-perturbative in simple cases) could feed back into the BRST formalism and give us a simpler way to construct off-shell quentities.
- Is it possible to move the problem of the field redefinitions from the level of the action (or eom, or solutions), to the level of transformations rules on fields (where it looks simpler) ?

Table of Contents

Asymptotic symmetries [Campiglia,SN - to appear]

- Appear in solution-generating techniques studied in the context of the double copy [Huang,Kol,O'Connell'20,Emond,Huang,Kol,Moynihan,O'Connell'20], celestial amplitudes [Casali,Puhm'20,Casali,Sharma'20,Kalyanapuram'20].
- Certain simplifications afforded by the $1/r$ expansion make it a good place to start looking at a more general dictionary.
- A further simplification: look at the self-dual sector, where the story can be understood at all orders in g and k (the kinematic algebra was understood in [Monteiro,O'Connell'11]).
- *How do the transformation rules of the fields double copy?*

Set-up [Campiglia,SN - to appear]

- We work with light-cone coordinates in the bulk $U = \frac{X^0 - X^3}{\sqrt{2}}$, $V = \frac{X^0 + X^3}{\sqrt{2}}$, $Z = \frac{X^1 + iX^2}{\sqrt{2}}$ and $\bar{Z} = \frac{X^1 - iX^2}{\sqrt{2}}$. We will split these into: $x^i := (U, \bar{Z})$ and $y^\alpha := (V, Z)$. The spacetime metric then takes the form

$$ds^2 = 2\eta_{i\alpha} dx^i dy^\alpha = -2dUdV + 2dZd\bar{Z}.$$

We also introduce the objects

$$\Omega_{ij} dx^i dx^j = dUd\bar{Z} - d\bar{Z}dU, \quad \Pi_{\alpha\beta} dy^\alpha dy^\beta = dVdZ - dZdV$$

By raising indices with the inverse spacetime metric $\eta^{i\alpha}$, and:

$$\Omega_i{}^\alpha \Pi_\alpha^j = \delta_i^j \Pi_\alpha{}^i \Omega_i{}^\beta = \delta_\alpha^\beta$$

The non-zero components of the self-dual YM and metric fields will be described in terms of scalar fields Φ and ϕ according to

$$A_\alpha = \Pi_\alpha{}^i \partial_i \Phi, \quad h_{\alpha\beta} = \Pi_\alpha{}^i \Pi_\beta{}^j \partial_i \partial_j \phi,$$

with the scalar fields satisfying:

$$\begin{aligned} \Box\Phi &= -i\Pi^{ij}[\partial_i\Phi, \partial_j\Phi] = f^{abc}\Pi^{ij}\partial_i\Phi^b\partial_j\Phi^c \\ \Box\phi &= \tfrac{1}{2}\Pi^{ij}\{\partial_i\phi, \partial_j\phi\} = \tfrac{1}{2}\Pi^{ij}\Pi^{kl}\partial_i\partial_k\phi\partial_j\partial_l\phi \end{aligned}$$

where $\Box \equiv 2\eta^{i\alpha}\partial_i\partial_\alpha$ is the wave operator and $\{f, g\} := \Pi^{ij}\partial_i f \partial_j g$ is the Poisson bracket, acting as a kinematic numerator.

Symmetries - 1^{st} family [Campiglia,SN - to appear]

- We are looking for symmetries preserving the self-duality condition
- The 1^{st} family is given by

$$\text{YM:} \quad \delta_\lambda A_\mu = \partial_\mu \Lambda + i[\Lambda, A_\mu], \qquad \text{with} \quad \Lambda = \Lambda(y)$$
$$\text{gravity:} \quad \delta_\xi h_{\alpha\beta} = 2\partial_{(\alpha}\xi_{\beta)} + \xi^i \partial_i h_{\alpha\beta}, \quad \text{with} \quad \xi^i = \eta^{i\alpha}\xi_\alpha(y), \quad \xi^\alpha = 0$$

- We define

$$\boxed{\lambda = 2\Omega_i{}^\alpha x^i \xi_\alpha = 2\Pi_{ij} x^i \xi^j.}$$

where λ can be thought of as the “Hamiltonian” of the vector field $\xi^i = \eta^{i\alpha} b_\alpha$ with respect to the Poisson Bracket .

- It is instructive to write the transformation rules in terms of the scalars

$$\delta A_\alpha = \quad \partial_\alpha \Lambda - i\Pi_\alpha^{\ i}[\Phi, \Lambda]$$
$$\delta h_{\alpha\beta} = \Pi_{(\alpha}^{\ i}\partial_i \left[\partial_{\beta)}\lambda + \tfrac{1}{2}\Pi_{\beta)}^{\ j}\partial_j\{\phi, \lambda\}\right]$$

- We then get the duble-copy replacement rules

$$\boxed{\Phi \to \phi, \quad \Lambda \to \lambda, \quad -i[,] \to \tfrac{1}{2}\{,\}}$$

- These are *exact* symmetries !

Asymptotics - 1^{st} family [Campiglia,SN - to appear]

- Work in the usual Bondi coordinates $(u, r, z, \bar{z})$, related to the light-cone coordinates via $U = u + r\frac{2z\bar{z}}{1+z\bar{z}}$, $V = u + r\frac{2}{1+z\bar{z}}$, $W = r\frac{2z}{1+z\bar{z}}$, $\bar{W} = r\frac{2\bar{z}}{1+z\bar{z}}$ and the usual metric

$$ds^2 = -du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z} + \frac{2m_B}{r}du^2 + rC_{zz}dz^2 + rC_{\bar{z}\bar{z}}d\bar{z}^2 + \dots$$

  In the self-dual sector, the only non-vanishing components at leading order in r are $C_{\bar{z}\bar{z}}$ on the gravity side and $A_{\bar{z}}$ on the YM side.

- We learn that we have obtained a subset of the supertranslations

$$\begin{aligned}\xi_{(f)} =& f\partial_u + D^z D_z f\partial_r - \frac{1}{r}\left(D^z f\partial_z + D^{\bar{z}} f\partial_{\bar{z}}\right) + \dots,\\ \mathcal{L}_f C_{\bar{z}\bar{z}} =& f\partial_u C_{\bar{z}\bar{z}} - 2D_{\bar{z}}^2 f\end{aligned}$$

  with

$$f(z, \bar{z}) = \frac{g(\bar{z})}{1+z\bar{z}}$$

  from the anti-holomorphic YM parameter

$$\Lambda = \Lambda(\bar{z})$$

Symmetries - 2^{nd} family [Campiglia,SN - to appear]

- We start on the gravity side, where we find a second family of symmetries, given by

$$\delta_{(c)} h_{\alpha\beta} = 2\partial_{(\alpha}\tilde{\xi}_{\beta)} + \Pi_\alpha^{\ i}\Pi_\beta^{\ j}\partial_i\partial_j\left[\tilde{\xi}^i\partial_i\phi + \tilde{\xi}^\alpha\partial_\alpha\phi\right]$$

with

$$\tilde{\xi}^i = \eta^{i\alpha}\Omega_j^{\ \beta}x^j\partial_\alpha\partial_\beta c, \quad \tilde{\xi}^\alpha = -\Omega^{\alpha\beta}\partial_\beta c, \quad c = c(y)$$

We can again define a 'Hamiltonian' $\boxed{\tilde{\lambda} = \Omega_i^{\ \alpha}\Omega_j^{\ \beta}x^i x^j\partial_\alpha\partial_\beta c}$ in terms of which the gravity transformation can be written as

$$\delta_{(c)} h_{\alpha\beta} = \Pi_{(\alpha}^{\ i}\partial_i\left[\partial_{\beta)}\lambda + \tfrac{1}{2}\Pi_\beta^{\ j}\partial_j\left[\{\phi,\tilde{\lambda}\} + \tfrac{1}{\Box}\eta^{\alpha i}\{\partial_\alpha\phi,\partial_i\tilde{\lambda}\}\right]\right]$$

Using exactly the same replacement rules as before

$$\Phi \leftarrow \phi, \quad \tilde{\Lambda} \leftarrow \tilde{\lambda}, \quad -i[,] \leftarrow \tfrac{1}{2}\{,\}$$

we read off a transformation rule for the self-dual YM scalar

$$\delta_{\tilde{\Lambda}}\Phi = x^i\Omega_i^\alpha\partial_\alpha\tilde{\Lambda} - i[\Phi,\tilde{\Lambda}] - i\tfrac{1}{\Box}\eta^{\alpha i}[\partial_\alpha\Phi,\partial_i\tilde{\Lambda}]$$

which turns out to be a symmetry of the YM self-dual equation !

Asymptotics - 2^{nd} family [Campiglia,SN - to appear]

- We learn that we have obtained the anti-holomorphic superrotations

$$\xi_{(Y)} = \left(1 + \tfrac{u}{2r}\right) Y^{\bar z}\partial_{\bar z} - \tfrac{u}{2r} D^z D_{\bar z} Y^{\bar z}\partial_z - \tfrac{1}{2}(u+r) D_{\bar z} Y^{\bar z}\partial_r + \tfrac{u}{2} D_{\bar z} Y^{\bar z}\partial_u$$
$$\mathcal{L}_Y C_{\bar z\bar z} = \tfrac{u}{2} D\cdot Y N_{\bar z\bar z} + Y\cdot D C_{\bar z\bar z} - \tfrac{1}{2} D\cdot Y C_{\bar z\bar z} + 2 D_{\bar z} Y^{\bar z} C_{\bar z\bar z} - u D^3_{\bar z} Y^{\bar z}$$

  with

$$Y^{\bar z} = Y^{\bar z}(\bar z), \quad Y^z = 0$$

  comes from a transformation on the YM self-dual scalar with parameter

$$\tilde\Lambda = r\,\tfrac{z}{1+z\bar z}\,\tilde g(\bar z)$$

- Possible connection to $\mathcal{O}(r^1)$ large gauge symmetry, related to the sub-leading soft theorem (studied at $\mathcal{O}(g^0)$ in [Campiglia,Laddha'16]).

Future work

- Hopefully this is a first step to understanding the full set of asymptotic symmetries form the double copy (adding together the self-dual and anti-selfdual sector gets us some of the way there).
- More generally, having a simple map for symmetries at higher orders in perturbation theory could help simplify the derivation of the field redefinition currently needed when looking at off-shell objects.
- Could feed back into the BRST gauge fixing maping procedure.

Thank You !