Waveforms From Scattering Amplitudes

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SAGEX
Scattering Amplitudes: from Geometry to Experiment

The Niels Bohr International Academy

Outline

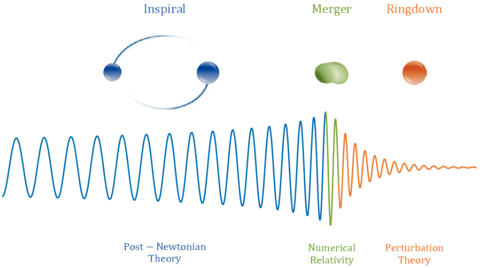
- Gravitational wave experiments $\rightarrow$ high precision
- The two-body problem and scattering amplitudes
- The KMOC approach: observables from on-shell amplitudes
- Classical wave physics from coherent states
- Time dependent observables: waveform templates

- Gravitational waves carry fingerprints of a binary motion

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G_N}{c^4} T_{\mu\nu} \quad , \quad \ddot{x}_a^\mu = -\Gamma^\mu_{\alpha\beta} \dot{x}_a^\alpha \dot{x}_a^\beta$$

... no exact solution is known!

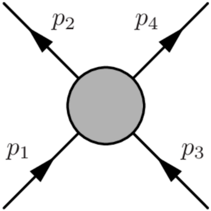
- The Effective One Body approach (Buonanno, Damour) provides an accurate solution



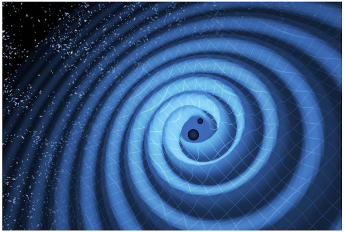
Credit: Antelis and Moreno, 1610.03567

QCD meets gravity

- Scattering amplitudes: LHC (✓), LIGO/Virgo (!)
- High precision computations: 3PM, spin, tidal effects
- Several approaches: amplitudes → observables
  (See talks by Heissenberg, Veneziano and Luna)



$\Rightarrow$



Credit: Tim Pyle, LIGO

The KMOC approach

- Binary system as superposition of single particle states

$$|\psi\rangle = \int d\Phi(p_1)\, d\Phi(p_2)\, \phi_1(p_1)\, \phi_2(p_2)\, e^{ib\cdot p_1}\, |p_1 p_2\rangle$$

- Classical limit $\leftrightarrow$ Goldilocks relations

$$\ell_c \ll \ell_w \ll \ell_s$$

- Classical observables from on-shell amplitudes

$$\underset{\checkmark}{\Delta p^\mu}, \quad \underset{\checkmark}{\Delta s^\mu}, \quad \underset{\checkmark}{R^\mu}$$

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$$\underset{\checkmark}{\Delta p^\mu}, \quad \underset{\checkmark}{\Delta s^\mu}, \quad \underset{\checkmark}{R^\mu}, \quad \underset{?}{\mathrm{W(t)}}$$

Coherent states

- Classical waves are naturally described by coherent states

$$|\alpha^{\eta}\rangle = \mathcal{N}_{\alpha} \exp\left[\int d\Phi(k)\alpha(k)a^{\dagger}_{(\eta)}(k)\right]|0\rangle$$

- Nice factorization properties in the classical limit

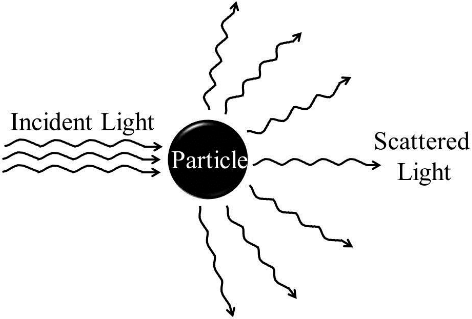
$$\langle\alpha^{-}|\,\mathbb{A}^{\mu}(x)\mathbb{A}^{\nu}(y)\,|\alpha^{-}\rangle = \left\langle\alpha^{-}\,|\mathbb{A}^{\mu}(x)|\,\alpha^{-}\right\rangle\left\langle\alpha^{-}\,|\mathbb{A}^{\nu}(y)|\,\alpha^{-}\right\rangle + \ldots$$

- Classical limit equivalent to a large occupation number

$$\langle N_{\gamma}\rangle = \frac{1}{\hbar}\int d\Phi(\bar{k})|\bar{\alpha}(\bar{k})|^{2}, \quad k^{\mu} = \hbar\bar{k}^{\mu} \quad \rightarrow \quad \langle N_{\gamma}\rangle \gg 1$$

Thomson scattering

Classical interaction between a beam of light and a charge



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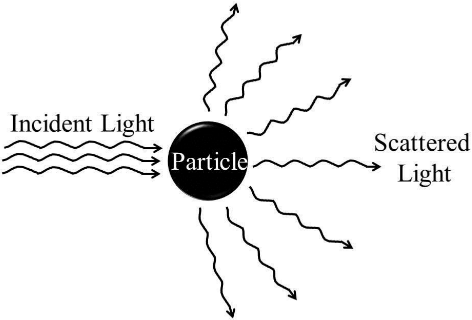

Compton scattering

Quantum interaction between a photon and an electron

- The electromagnetic field is an observable

$$\mathbb{F}^{\mu\nu} = i \int d\Phi(k) \left[a^\dagger_{(+)}(k) k^{[\mu} \varepsilon^{(+)\nu]} e^{ik\cdot x} - a_{(-)}(k) k^{[\mu} \varepsilon^{(-)\nu]*} e^{-ik\cdot x} \right]$$

- Initial state includes a coherent state

$$|\psi\rangle = \int d\Phi\,(p)\,\phi\,(p)\,e^{ib\cdot p}\,\left|p, \alpha^+\right\rangle$$

- Classical value of the scattered wave

$$\langle F^{\mu\nu}\rangle = \langle\psi|\, S^\dagger \mathbb{F}^{\mu\nu} S\, |\psi\rangle$$

- Thomson scattering at leading order in T

$$\langle F^{\mu\nu}\rangle = 2\,\mathrm{Re}\Bigg\{\int d\Phi(k,p_1,p_2)\; e^{ib\cdot(p_1-p_2)}$$
$$\times\Big[k^{[\mu}\varepsilon^{(-)\nu]*}\langle p_2\,\alpha^-|a_{(-)}(k)\;T|p_1\,\alpha^+\rangle e^{-ik\cdot x}$$
$$- k^{[\mu}\varepsilon^{(+)\nu]}\langle p_2\,\alpha^-|a^\dagger_{(+)}(k)\,T|p_1\,\alpha^+\rangle\Big]\,e^{ik\cdot x}\Bigg\}$$

- Coherent states $\rightarrow$ sum of n-point amplitudes

$$|\alpha^\eta\rangle = \sum_{n=0}^{\infty}\frac{1}{n!}\int\prod_{j=0}^{n} d\Phi(k_j)\alpha(k_j)a^\dagger_{(\eta)}(k_j)\,|0\rangle$$

Thomson meets Compton

$$F_A^{\mu\nu} = \int d\Phi(p_1, p_2, k, q)\; e^{ib\cdot(p_1 - p_2)}\; k^{[\mu}\varepsilon^{(+)\nu]} \times$$

$$\Big[\alpha(q)\langle p_2\, k^-|\, T|p_1\, q^+\rangle e^{-ix\cdot k}$$

$$+\; \alpha^*(q)\langle p_2\, q^-|\underset{(\checkmark)}{T^\dagger}|p_1\, k^+\rangle e^{ix\cdot k}\Big]$$

$$F_B^{\mu\nu} = -\frac{1}{2}\int d\Phi(k, p_1, p_1', q_1, q_1')\,\alpha(q_1)\alpha^*(q_1')\; e^{ib\cdot(p_1 - p_1')}$$

$$\times\; \Big[\,\alpha^*(k)k^{[\mu}\varepsilon^{(-)\nu]*}\langle p_1'\, q_1'^-|\, T\,|p_1\, q_1^+\rangle e^{ik\cdot x}$$

$$+\; \underset{(\checkmark)}{\alpha(k)}k^{[\mu}\varepsilon^{(+)\nu]}\langle p_1'\, q_1'^-|\, T^\dagger|p_1\, q_1^+\rangle e^{-ik\cdot x}\Big]$$

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$$\Big[\alpha(q)\langle p_2\, k^- |\, T | p_1\, q^+ \rangle e^{-ix\cdot k}$$
$$+\; \alpha^*(q)\langle p_2\, q^- | T^\dagger | p_1\, k^+ \rangle e^{ix\cdot k}\Big]$$
$$(\checkmark)$$

$$F_B^{\mu\nu} = -\frac{1}{2}\int d\Phi(k, p_1, p_1', q_1, q_1')\, \alpha(q_1)\alpha^*(q_1')\; e^{ib\cdot(p_1 - p_1')}$$
$$\times \Big[\, \alpha^*(k) k^{[\mu}\varepsilon^{(-)\nu]*} \langle p_1'\, q_1'^- | T | p_1\, q_1^+ \rangle e^{ik\cdot x}$$
$$+\; \alpha(k) k^{[\mu}\varepsilon^{(+)\nu]} \langle p_1'\, q_1'^- | T^\dagger | p_1\, q_1^+ \rangle e^{-ik\cdot x}\Big]$$
$$(\checkmark)$$

Caveat

One massless particle as initial state $\rightarrow$ Disagreement

Waveforms

- Waveforms generated by a scattering process

$$|\psi'\rangle = S\,|\psi\rangle$$

- LO contribution related to a 5-point function

$$\langle \mathbb{F}^{\mu\nu} \rangle = i \int d\Phi(p_1, p_2, p_1', p_2', k)\; e^{ib\cdot(p_1 - p_1')}$$
$$\times \Big[k^{[\mu} \varepsilon^{(-)\nu]*} e^{-ik\cdot x} \langle p_1'\, p_2'\, k^{(+)} | \, T | p_1\, p_2 \rangle$$
$$- k^{[\mu} \varepsilon^{(+)\nu]} e^{ik\cdot x} \langle p_1'\, p_2' | \, T | p_1\, p_2\, k^{(-)} \rangle \Big]$$

Waveforms

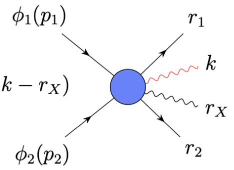
- Newman-Penrose scalar related to a waveform

$$\langle \mathbb{F}^{\mu\nu} \rangle \quad \to \quad \varphi_2 \sim \frac{W(t, \hat{n})}{|x|}$$

- Differential analogue of the total power emitted
(see talks by Ruf and Martinez)

$$R^\mu = \sum_X \int d\Phi(k) d\Phi(r_1) d\Phi(r_2)\; k_X^\mu$$

$$\times \left| \int d\Phi(p_1) d\Phi(p_2)\; e^{ib\cdot p_1/\hbar}\, \hat{\delta}^{(4)}(p_1 + p_2 - r_1 - r_2 - k - r_X) \right.$$



$$\Bigg|^2$$

KMOC, 1811.10950

Conclusions

- Classical observables $\rightarrow$ on-shell amplitudes
- Classical waves $\leftrightarrow$ coherent states
- Thomson scattering $\rightarrow$ Compton scattering
- Time dependent observables: waveforms

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Main message

New observables expressed using on-shell amplitudes