

Loop-Level Double Copy for Massive Quantum Particles

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Based on work with John Joseph Carrasco



SAGEX

Scattering Amplitudes:
from Geometry to Experiment

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No. 764850



**QCD Meets Gravity,
4 Dec 2020**

Color-kinematics and factorization
are enough to build massive
amplitudes

The diagram illustrates a factorization identity for a four-point amplitude. On the left, a horizontal red line connects two vertices. The left vertex has two incoming wavy lines labeled a (bottom) and b (top). The right vertex has two outgoing wavy lines labeled d (bottom) and c (top). This is equal to the sum of two terms. The first term shows a vertex where two red lines, labeled c (top) and b (bottom), meet. From this vertex, a wavy line labeled a goes down and another wavy line labeled d goes up. The second term shows a vertex where two red lines, labeled a (top) and c (bottom), meet. From this vertex, a wavy line labeled d goes up and another wavy line labeled b goes down.

$$\begin{array}{c} b \\ \diagup \\ \text{---} \\ \diagdown \\ a \end{array} \begin{array}{c} c \\ \diagdown \\ \text{---} \\ \diagup \\ d \end{array} = \begin{array}{c} c \\ \diagup \\ \text{---} \\ \diagdown \\ b \end{array} \begin{array}{c} d \\ \diagup \\ \text{---} \\ \diagdown \\ a \end{array} + \begin{array}{c} a \\ \diagup \\ \text{---} \\ \diagdown \\ c \end{array} \begin{array}{c} b \\ \diagdown \\ \text{---} \\ \diagup \\ d \end{array}$$

Color-kinematics and factorization
are enough to build massive
amplitudes

A Feynman diagram equation. On the left, a four-point amplitude with external legs a (bottom-left), b (top-left), c (top-right), and d (bottom-right). Internal lines are red, and wavy lines connect the vertices. This is equal to the sum of two three-point amplitudes. The first three-point amplitude has legs c (top), b (bottom), and a (bottom-right), with a wavy line connecting c and b . The second three-point amplitude has legs a (top), c (bottom), and d (bottom-right), with a wavy line connecting a and c .

A Feynman diagram equation. On the left, a four-point amplitude with external legs a (bottom-left), b (top-left), c (top-right), and d (bottom-right). Internal lines are red, and wavy lines connect the vertices. This is equal to the sum of two three-point amplitudes. The first three-point amplitude has legs b (top), a (bottom), and d (bottom-right), with a wavy line connecting b and a . The second three-point amplitude has legs c (top), d (bottom), and a (bottom-right), with a wavy line connecting c and d .

Color-kinematics duality for
massive amplitudes can be
manifest at loop level

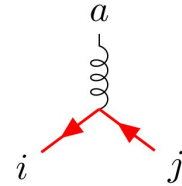
Color-kinematics and factorization
are enough to build massive
amplitudes

A Feynman diagram equation. On the left, a four-point amplitude with external legs a (red), b (red), c (red), and d (red) connected by two wavy lines. This is equal to the sum of two three-point amplitudes. The first three-point amplitude has legs b (red), c (red), and d (red) connected by a wavy line. The second three-point amplitude has legs a (red), c (red), and d (red) connected by a wavy line.

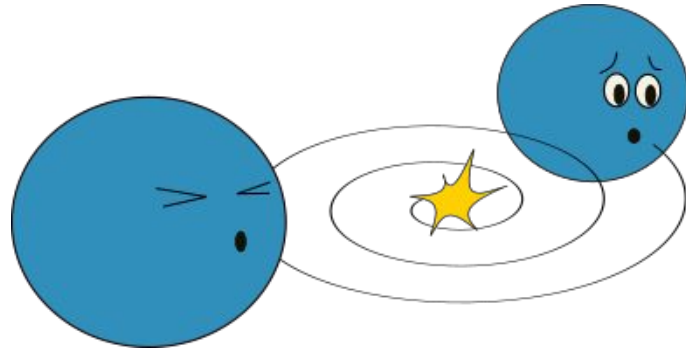
A Feynman diagram equation. On the left, a four-point amplitude with external legs a (red), b (red), c (red), and d (red) connected by two wavy lines. This is equal to the sum of two three-point amplitudes. The first three-point amplitude has legs a (red), b (red), and d (red) connected by a wavy line. The second three-point amplitude has legs a (red), c (red), and d (red) connected by a wavy line.

Color-kinematics duality for
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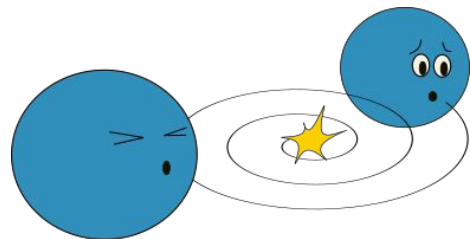
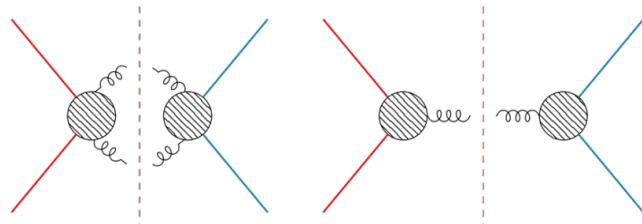
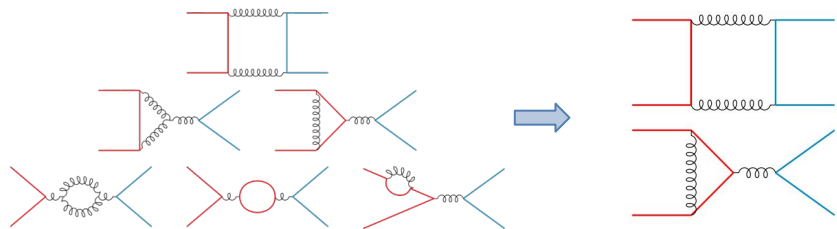
Can use the same kinematic
building blocks for massive scalars
charged in fundamental and
adjoint



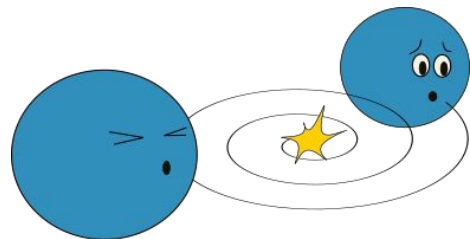
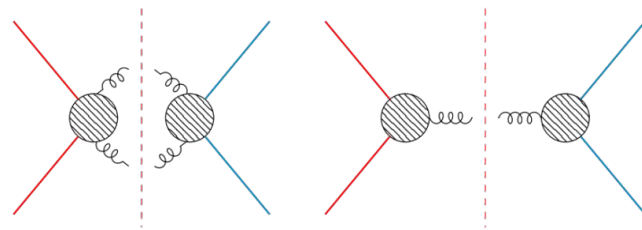
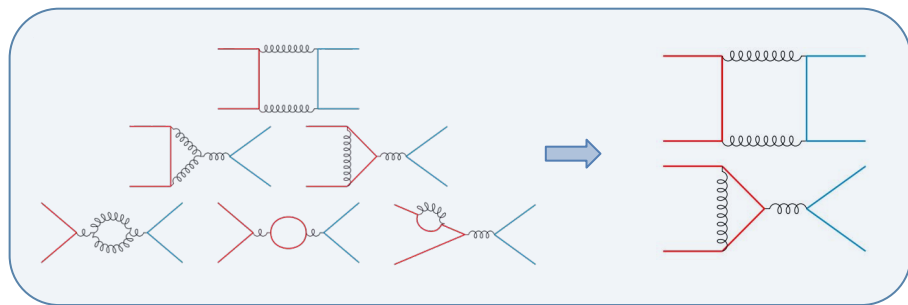
Why is gravity hard with traditional methods?



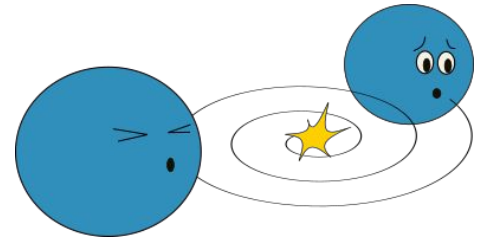
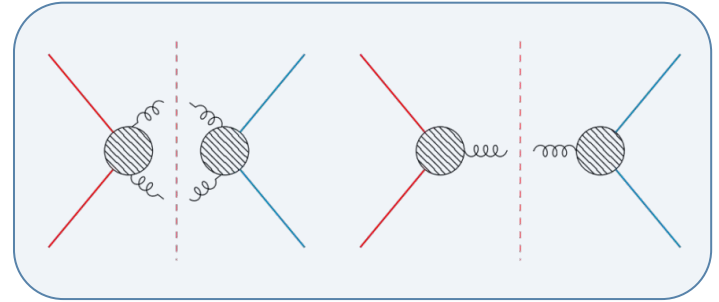
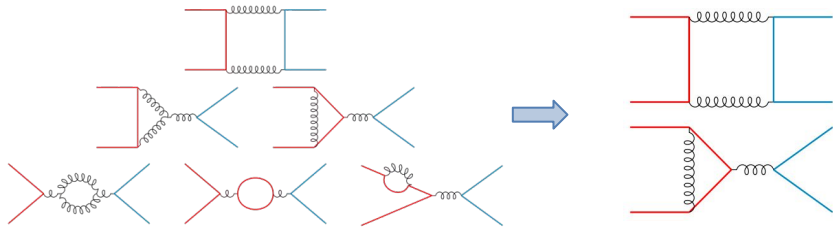
Use the color-kinematics duality and unitarity methods
to find massive amplitudes



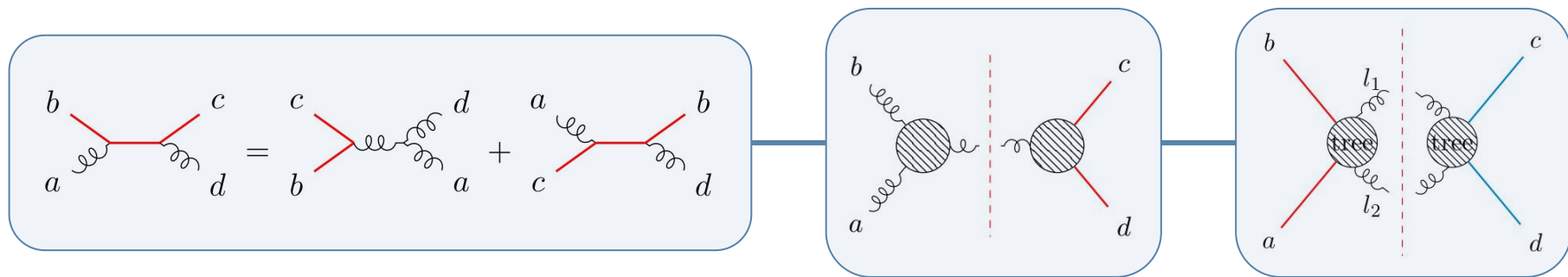
Use the **color-kinematics duality** and unitarity methods
to find massive amplitudes



Use the color-kinematics duality and **unitarity methods** to find massive amplitudes



Bootstrapping tree-level amplitudes and loop-level using the color-kinematics duality



The color-kinematics duality

The diagram shows an equation between three Feynman diagrams. The leftmost diagram has a horizontal red line with a wavy line (gluon) attached to each end. The left wavy line has labels a (bottom) and b (top), and the right wavy line has labels d (bottom) and c (top). This is equal to the sum of two diagrams. The first diagram on the right has a horizontal wavy line with a red line attached to each end. The left red line has labels b (bottom) and c (top), and the right wavy line has labels a (bottom) and d (top). The second diagram on the right has a horizontal red line with a wavy line attached to each end. The left wavy line has labels c (bottom) and a (top), and the right wavy line has labels d (bottom) and b (top).

$$\begin{array}{c} b \\ \text{red line} \\ a \end{array} \begin{array}{c} \text{wavy line} \\ \text{red line} \\ d \end{array} \begin{array}{c} c \\ \text{red line} \\ d \end{array} = \begin{array}{c} c \\ \text{red line} \\ b \end{array} \begin{array}{c} \text{wavy line} \\ \text{red line} \\ a \end{array} \begin{array}{c} d \\ \text{wavy line} \\ a \end{array} + \begin{array}{c} a \\ \text{wavy line} \\ c \end{array} \begin{array}{c} \text{red line} \\ \text{red line} \\ b \end{array} \begin{array}{c} \text{wavy line} \\ \text{red line} \\ d \end{array}$$

Color-kinematics duality

Color-dual representation: kinematic weights \sim color weights

Bern, Carrasco, Johansson '08,
Bern, Carrasco, Johansson '10

Only need small number of basis graphs (Solves combinatorics problem!)

Weaves a web of theories (Recycling is good)

See review: Bern, Carrasco,
Chiodaroli, Johansson, Roiban '19

Color-kinematics for many representations (adjoint, three-algebras,
arbitrary)

Bargheer, He, McLoughlin '12

Massive matter in the fundamental

Johansson, Ochirov '16, Plefka, Shi, Wang '19,
Bjerrum-Bohr, Cristofoli, Damgaard, Gomez '19, Luna,
Nicholson, O'Connell, White '17
Haddad, Helset '20

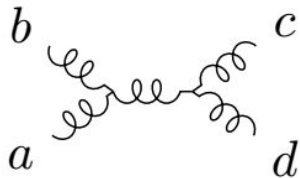
Color-kinematics duality relates kinematic weights
of graphs

$$\mathcal{A}_m^{(L)} = i^L g^{m-2+2L} \sum_{i \in \Gamma} \int \prod_{l=1}^L \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_i} \frac{n_i C_i}{\prod_{\alpha_i} (p_{\alpha_i}^2 - m_{\alpha_i}^2)}$$

Color-kinematics duality relates kinematic weights
of graphs

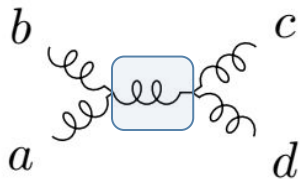
$$\frac{n_i C_i}{\prod_{\alpha_i} (p_{\alpha_i}^2 - m_{\alpha_i}^2)}$$

Color-kinematics duality relates kinematic weights
of graphs



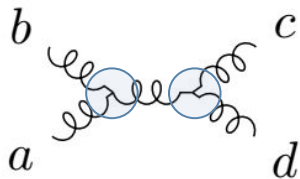
$$\frac{n_i C_i}{\prod_{\alpha_i} (p_{\alpha_i}^2 - m_{\alpha_i}^2)}$$

Color-kinematics duality relates kinematic weights of graphs



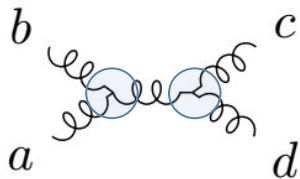
$$\frac{n_i C_i}{\prod_{\alpha_i} (p_{\alpha_i}^2 - m_{\alpha_i}^2)}$$

Color-kinematics duality relates kinematic weights of graphs



$$\frac{n_i \boxed{C_i}}{\prod_{\alpha_i} (p_{\alpha_i}^2 - m_{\alpha_i}^2)}$$

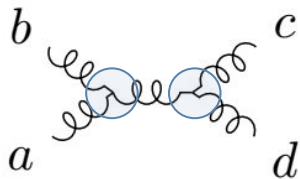
Color-kinematics duality relates kinematic weights of graphs



$$\frac{n_i \boxed{C_i}}{\prod_{\alpha_i} (p_{\alpha_i}^2 - m_{\alpha_i}^2)}$$

$$f^{abl} f^{lcd}$$

Color-kinematics duality relates kinematic weights of graphs

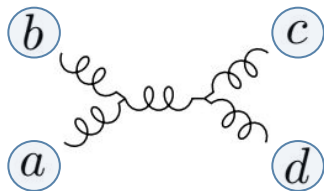


$$\frac{n_i \boxed{C_i}}{\prod_{\alpha_i} (p_{\alpha_i}^2 - m_{\alpha_i}^2)}$$

$$f^{abl} f^{lcd} = f^{bcl} f^{lda} + f^{cal} f^{lbd}$$

A diagrammatic equation representing the Jacobi identity for color factors. On the left is a box graph with external legs a , b , c , and d . The top edge is a wavy line, and the bottom edge is a wavy line. The two vertical edges are also wavy lines. The equation is set equal to the sum of two other box graphs. The first graph on the right has external legs c , b , d , and a in clockwise order. The second graph on the right has external legs a , c , b , and d in clockwise order.

Color-kinematics duality relates kinematic weights of graphs

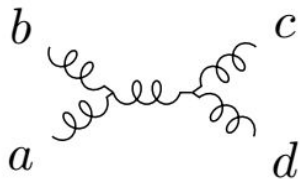


$$\frac{n_i C_i}{\prod_{\alpha_i} (p_{\alpha_i}^2 - m_{\alpha_i}^2)}$$

$$n(a, b, c, d)$$

$$\begin{array}{c} b \\ \diagup \\ a \end{array} \begin{array}{c} \diagdown \\ c \\ \diagup \\ d \end{array} = \begin{array}{c} c \\ \diagup \\ b \end{array} \begin{array}{c} \diagdown \\ d \\ \diagup \\ a \end{array} + \begin{array}{c} a \\ \diagup \\ c \end{array} \begin{array}{c} \diagdown \\ b \\ \diagup \\ d \end{array}$$

Color-kinematics duality relates kinematic weights of graphs

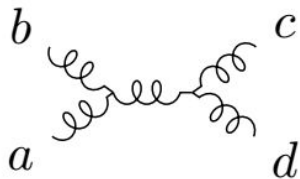


$$\frac{\boxed{n_i} C_i}{\prod_{\alpha_i} (p_{\alpha_i}^2 - m_{\alpha_i}^2)}$$

$$n(a, b, c, d) = n(b, c, d, a) + n(c, a, b, d)$$

A diagrammatic equation. On the left is a Feynman diagram with external legs labeled 'a' (bottom-left), 'b' (top-left), 'c' (top-right), and 'd' (bottom-right). This is followed by an equals sign. To the right of the equals sign are two Feynman diagrams added together. The first diagram has external legs labeled 'b' (bottom-left), 'c' (top-left), 'd' (top-right), and 'a' (bottom-right). The second diagram has external legs labeled 'c' (bottom-left), 'a' (top-left), 'b' (top-right), and 'd' (bottom-right).

Color-kinematics duality relates kinematic weights of graphs

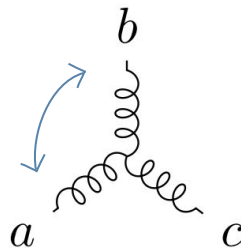


$$\frac{\boxed{n_i} C_i}{\prod_{\alpha_i} (p_{\alpha_i}^2 - m_{\alpha_i}^2)}$$

Antisymmetries of graph weights

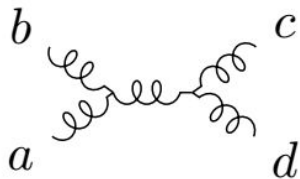
$$n(a, b, c, d) = n(b, c, d, a) + n(c, a, b, d)$$

$$= \text{graph}(b, c, d) + \text{graph}(a, c, d)$$



$$f^{abc} = -f^{bac}$$

Color-kinematics duality relates kinematic weights of graphs

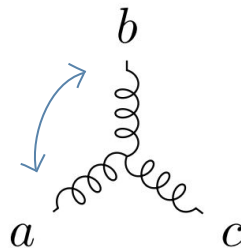


$$\frac{n_i C_i}{\prod_{\alpha_i} (p_{\alpha_i}^2 - m_{\alpha_i}^2)}$$

Antisymmetries of graph weights

$$n(a, b, c, d) = n(b, c, d, a) + n(c, a, b, d)$$

$$\text{Diagram 1} = \text{Diagram 2} + \text{Diagram 3}$$

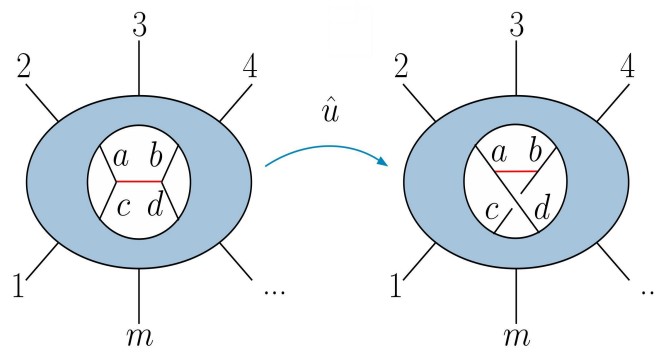
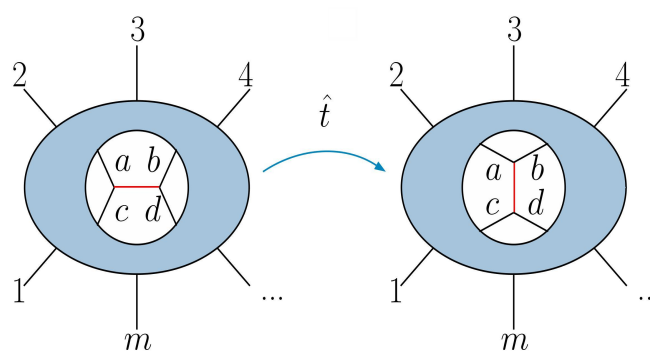


$$n(a, b, c) = -n(b, a, c)$$

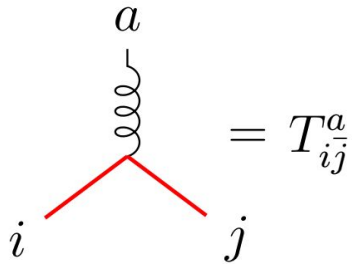
Color-kinematics duality relates kinematic weights of graphs

$$n_i - n_j = n_k \quad \Leftrightarrow \quad c_i - c_j = c_k,$$

$$n_i \rightarrow -n_i \quad \Leftrightarrow \quad c_i \rightarrow -c_i$$



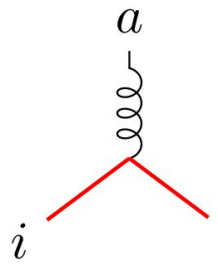
What is different about massive particles?



A Feynman diagram showing a vertex where two red lines meet. The red line on the left is labeled i and the red line on the right is labeled j . A wavy line extends upwards from the vertex and is labeled a . To the right of the diagram is an equals sign followed by the expression T_{ij}^a .

$$= T_{ij}^a$$

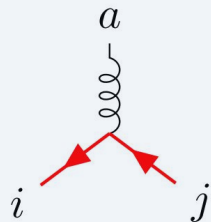
What is different about massive particles?



A Feynman diagram showing a vertex where a wavy line labeled a meets two red lines labeled i and j . The wavy line is vertical and points upwards, while the red lines branch out downwards and outwards. To the right of the diagram is an equals sign followed by the expression $T_{i\bar{j}}^a$.

$$= T_{i\bar{j}}^a$$

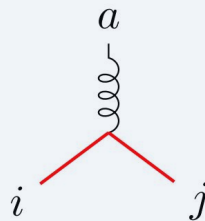
Fundamental-type



A Feynman diagram showing a vertex where a wavy line labeled a meets two red lines labeled i and j . The wavy line is vertical and points upwards. The red lines branch out downwards and outwards, with arrows on them pointing towards the vertex. To the right of the diagram is an equals sign followed by the expression $\hat{T}_{i\bar{j}}^a$.

$$= \hat{T}_{i\bar{j}}^a$$

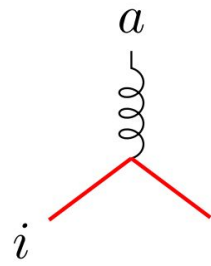
Adjoint-type



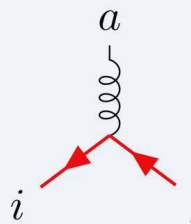
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$$= \hat{T}_{ij}^a$$

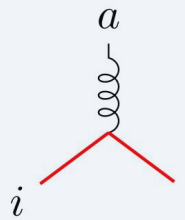
What is different about massive particles?


$$= T_{ij}^a$$

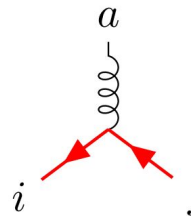
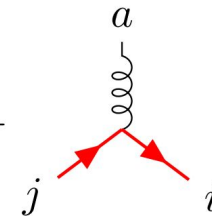
Fundamental-type


$$= \hat{T}_{ij}^a$$

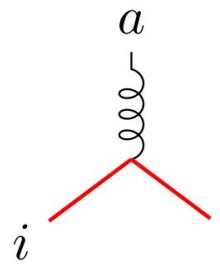
Adjoint-type


$$= \hat{T}_{ij}^a$$

Want antisymmetry:

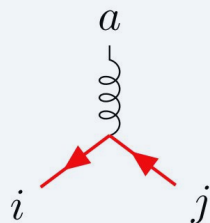

$$= -$$


What is different about massive particles?



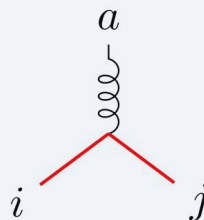
$$= T_{ij}^a$$

Fundamental-type



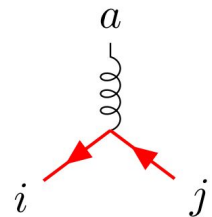
$$= \hat{T}_{ij}^a$$

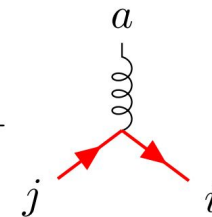
Adjoint-type



$$= \hat{T}_{ij}^a$$

Want antisymmetry:



$$= -$$


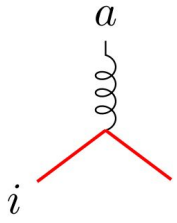
Introduce:

$$T_{ij}^a = \hat{T}_{ij}^a$$

$$T_{ji}^a = -T_{ij}^a$$

$c(\text{graph}) \times n(\text{graph})$

What is different about massive particles?

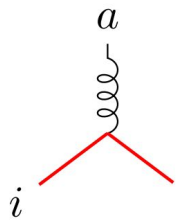


A Feynman diagram showing a vertex where two red lines meet. The left red line is labeled i and the right red line is labeled j . A wavy line extends upwards from the vertex and is labeled a .

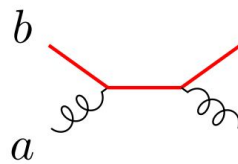
$$= T_{i\bar{j}}^a$$

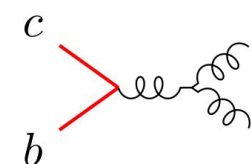
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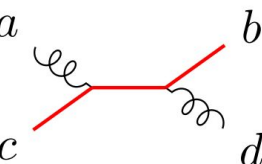
Jacobi-like relations



$$= T_{ij}^a$$



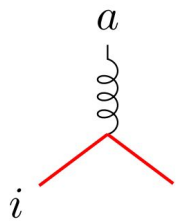
$$=$$


$$+$$


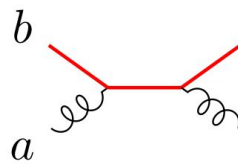
$$T_{b\bar{j}}^a T_{j\bar{c}}^d = f^{dal} T_{b\bar{c}}^l + T_{j\bar{c}}^a T_{b\bar{j}}^d$$

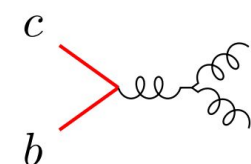
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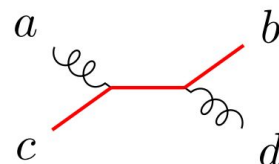
Jacobi-like relations



$$= T_{ij}^a$$

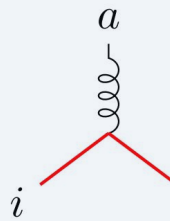


$$=$$


$$+$$


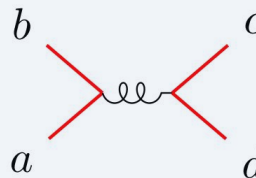
$$T_{b\bar{j}}^a T_{j\bar{c}}^d = f^{dal} T_{b\bar{c}}^l + T_{j\bar{c}}^a T_{b\bar{j}}^d$$

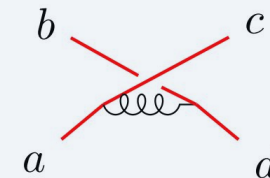
Adjoint-type

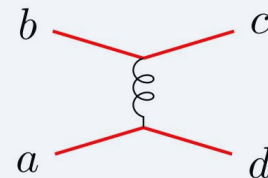


$$= \hat{T}_{ij}^a$$

Optionally



$$=$$


$$+$$


$$T_{ab}^g T_{cd}^g = T_{ac}^g T_{bd}^g + T_{ad}^g T_{bc}^g$$

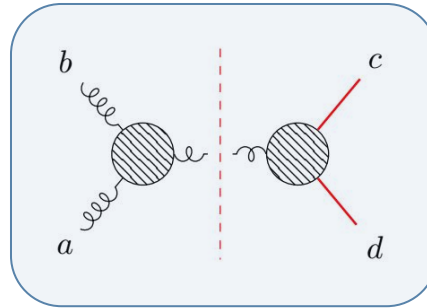
Color-kinematics duality relates kinematic weights of graphs

$$\begin{array}{c} b \\ \diagup \\ a \end{array} \begin{array}{c} \diagdown \\ c \\ \diagup \\ d \end{array} = \begin{array}{c} c \\ \diagup \\ b \end{array} \begin{array}{c} \diagdown \\ d \\ \diagup \\ a \end{array} + \begin{array}{c} a \\ \diagup \\ c \end{array} \begin{array}{c} \diagdown \\ b \\ \diagup \\ d \end{array}$$

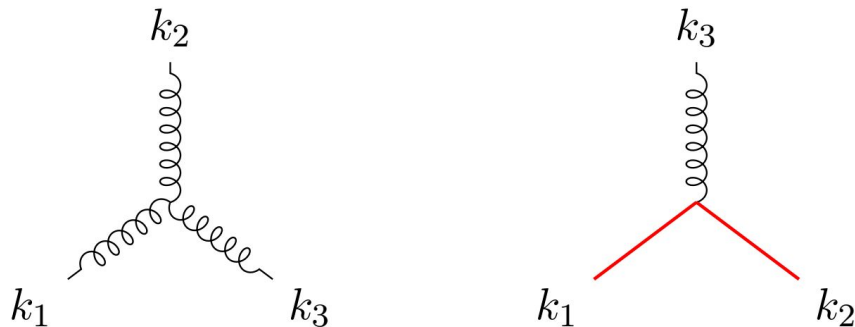
$$\begin{array}{c} b \\ \diagup \\ a \end{array} \begin{array}{c} \diagdown \\ c \\ \diagup \\ d \end{array} = \begin{array}{c} c \\ \diagup \\ b \end{array} \begin{array}{c} \diagdown \\ d \\ \diagup \\ a \end{array} + \begin{array}{c} a \\ \diagup \\ c \end{array} \begin{array}{c} \diagdown \\ b \\ \diagup \\ d \end{array}$$

$$\left[\begin{array}{c} b \\ \diagup \\ a \end{array} \begin{array}{c} \diagdown \\ c \\ \diagup \\ d \end{array} \right] = \left[\begin{array}{c} c \\ \diagup \\ a \end{array} \begin{array}{c} \diagdown \\ b \\ \diagup \\ d \end{array} \right] + \left[\begin{array}{c} a \\ \diagup \\ d \end{array} \begin{array}{c} \diagdown \\ b \\ \diagup \\ c \end{array} \right]$$

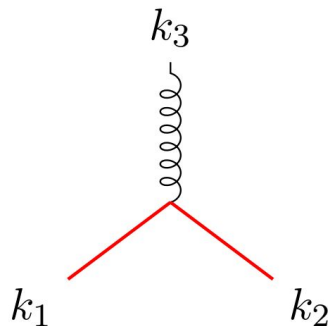
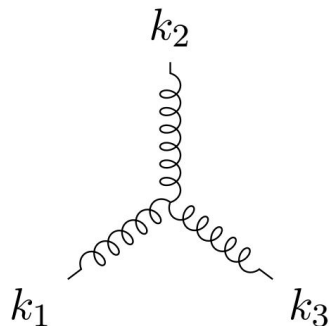
Construction



3-point amplitudes are completely determined by color-kinematics and symmetries



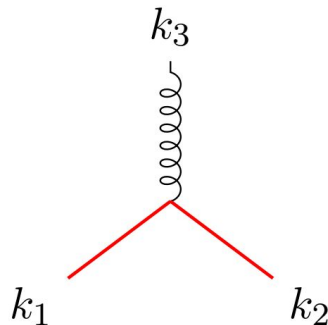
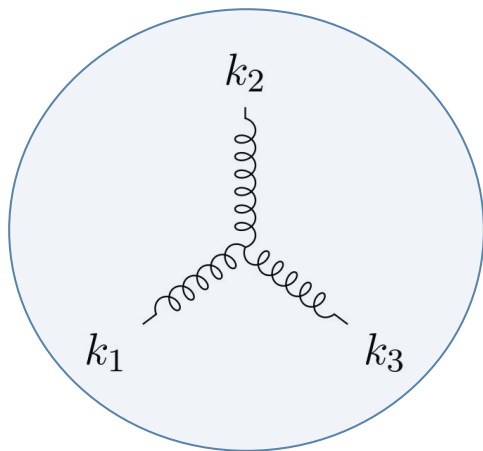
3-point amplitudes are completely determined by color-kinematics and symmetries



$$n_3(k_1, k_2, k_3)$$

$$n_{3,2}(k_1^m, k_2^m, k_3)$$

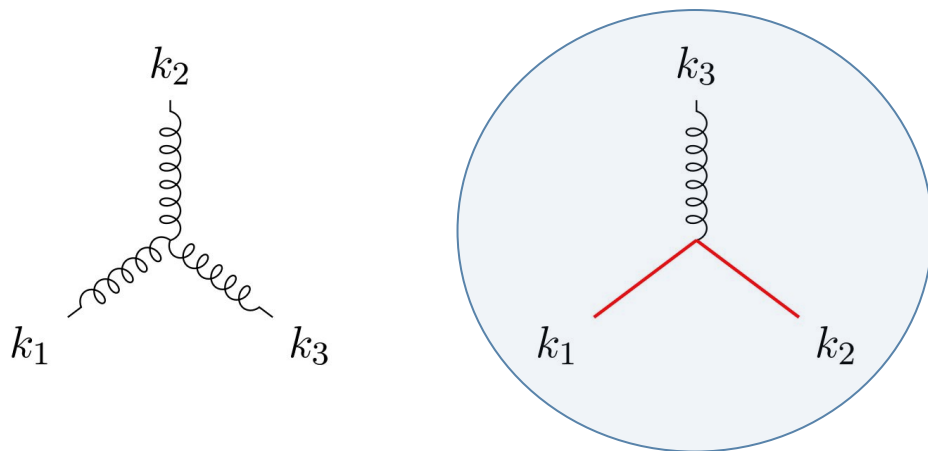
3-point amplitudes are completely determined by color-kinematics and symmetries



$$n_3(k_1, k_2, k_3) = \alpha_1(k_3 \cdot \epsilon_1)(\epsilon_2 \cdot \epsilon_3) + \alpha_2(k_3 \cdot \epsilon_2)(\epsilon_1 \cdot \epsilon_3) + \alpha_3(k_2 \cdot \epsilon_3)(\epsilon_1 \cdot \epsilon_2)$$

$$n_{3,2}(k_1^m, k_2^m, k_3)$$

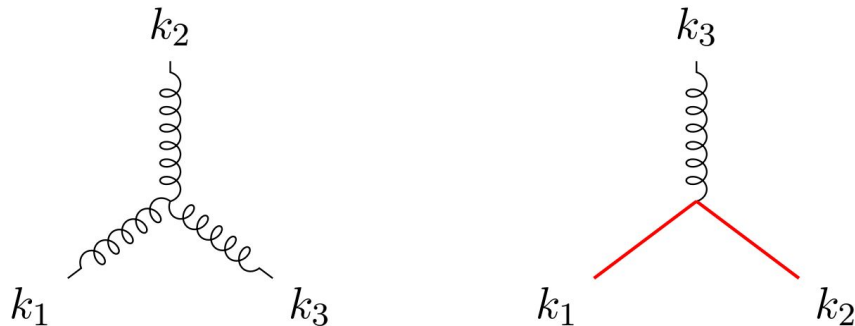
3-point amplitudes are completely determined by color-kinematics and symmetries



$$n_3(k_1, k_2, k_3) = \alpha_1(k_3 \cdot \epsilon_1)(\epsilon_2 \cdot \epsilon_3) + \alpha_2(k_3 \cdot \epsilon_2)(\epsilon_1 \cdot \epsilon_3) + \alpha_3(k_2 \cdot \epsilon_3)(\epsilon_1 \cdot \epsilon_2)$$

$$n_{3,2}(k_1^m, k_2^m, k_3) = \alpha_1(k_1 \cdot \epsilon_3)$$

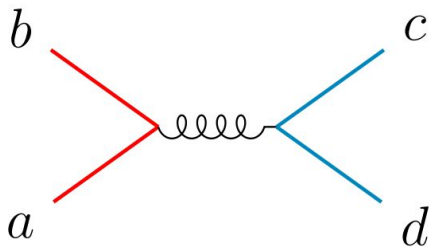
3-point amplitudes are completely determined by color-kinematics and symmetries



$$n_3(k_1, k_2, k_3) = \alpha_1 \left((k_3 \cdot \epsilon_1)(\epsilon_2 \cdot \epsilon_3) - (k_3 \cdot \epsilon_2)(\epsilon_1 \cdot \epsilon_3) + (k_2 \cdot \epsilon_3)(\epsilon_1 \cdot \epsilon_2) \right)$$

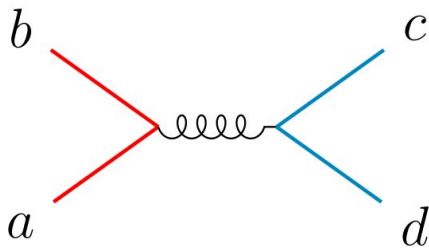
$$n_{3,2}(k_1^m, k_2^m, k_3) = \alpha_1(k_1 \cdot \epsilon_3)$$

Tree-level amplitudes are completely determined by color-kinematics and factorization



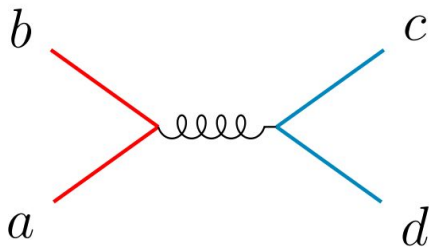
$$n_{4,2}(a, b, c, d) = \alpha_1(a \cdot b) + \alpha_2(b \cdot b) + \alpha_3(b \cdot c) + \alpha_4(c \cdot c)$$

Tree-level amplitudes are completely determined by
color-kinematics and factorization

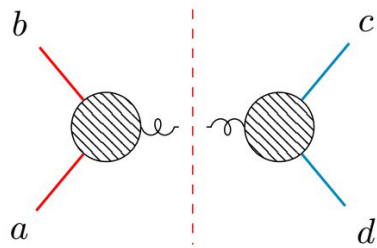


$$n_{4,2}(a, b, c, d) = \frac{\alpha_3}{2} ((a \cdot b) + (b \cdot b) + 2(b \cdot c))$$

Tree-level amplitudes are completely determined by color-kinematics and **factorization**

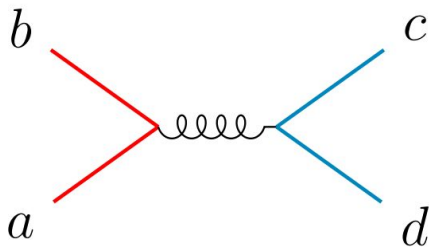


$$n_{4,2}(a, b, c, d) = \frac{\alpha_3}{2} ((a \cdot b) + (b \cdot b) + 2(b \cdot c))$$



$$\sum_{s \in \text{states}} A_{3,2}(a, b, l^s) A_{3,2}(-l^{\bar{s}}, c, d) = n_{4,2}(a, b, c, d)|_{(a+b)^2=0}$$

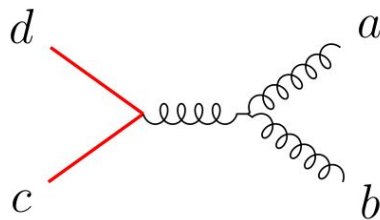
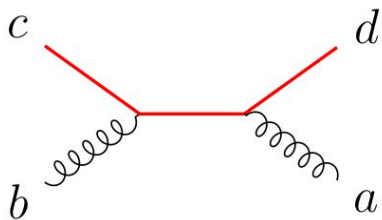
Tree-level amplitudes are completely determined by color-kinematics and factorization



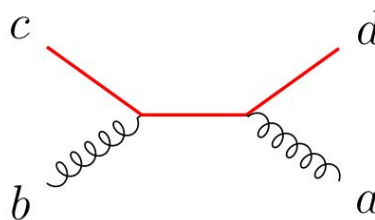
$$n_{4,2}(a, b, c, d) = \frac{\alpha_3}{2} ((a \cdot b) + (b \cdot b) + 2(b \cdot c))$$

$$A_{4,2}^{\text{tree}}(a, b, c, d) = -\frac{(a \cdot b) + (b \cdot b) + 2(b \cdot c)}{2[(a \cdot b) + (b \cdot b)]}$$

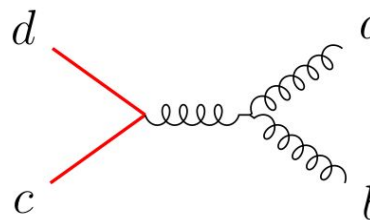
Tree-level amplitudes are completely determined by color-kinematics and factorization



Tree-level amplitudes are completely determined by color-kinematics and factorization

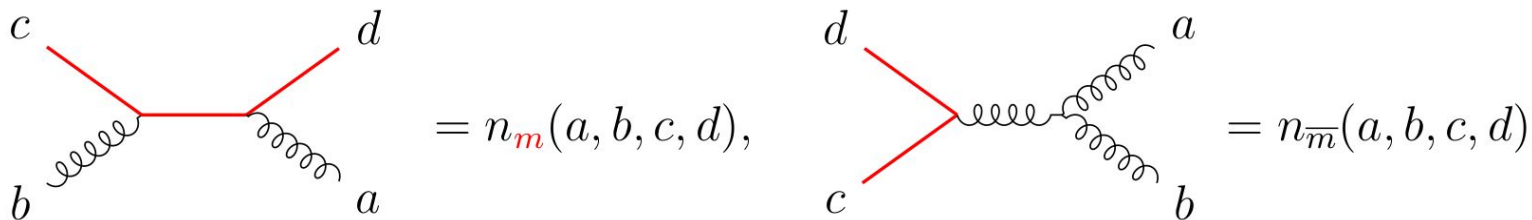


$$= n_{\textcolor{red}{m}}(a, b, c, d),$$



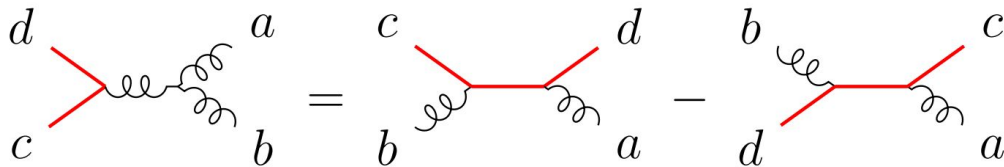
$$= n_{\overline{m}}(a, b, c, d)$$

Tree-level amplitudes are completely determined by
color-kinematics and factorization



$$\begin{aligned}
 & \text{Diagram 1: } c \text{ (red) and } b \text{ (black) enter from left, } d \text{ (red) and } a \text{ (black) exit to right.} \\
 & = n_{\textcolor{red}{m}}(a, b, c, d), \\
 & \text{Diagram 2: } d \text{ (red) and } c \text{ (black) enter from left, } a \text{ (black) and } b \text{ (black) exit to right.} \\
 & = n_{\overline{m}}(a, b, c, d)
 \end{aligned}$$

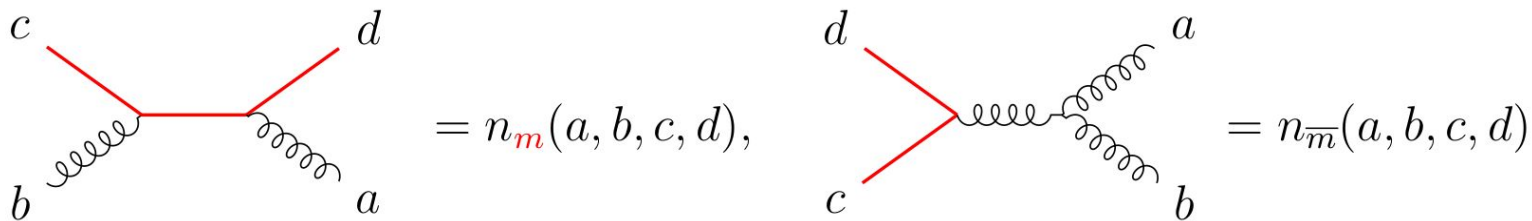
Color-kinematics



$$\begin{aligned}
 & \text{Diagram 1: } d \text{ (red) and } c \text{ (black) enter from left, } a \text{ (black) and } b \text{ (black) exit to right.} \\
 & = \text{Diagram 2: } c \text{ (red) and } b \text{ (black) enter from left, } d \text{ (red) and } a \text{ (black) exit to right.} \\
 & \quad - \text{Diagram 3: } b \text{ (black) and } d \text{ (red) enter from left, } c \text{ (red) and } a \text{ (black) exit to right.}
 \end{aligned}$$

$$n_{\overline{m}}(a, b, c, d) = n_{\textcolor{red}{m}}(a, b, c, d) - n_{\textcolor{red}{m}}(b, a, c, d)$$

Tree-level amplitudes are completely determined by color-kinematics and factorization

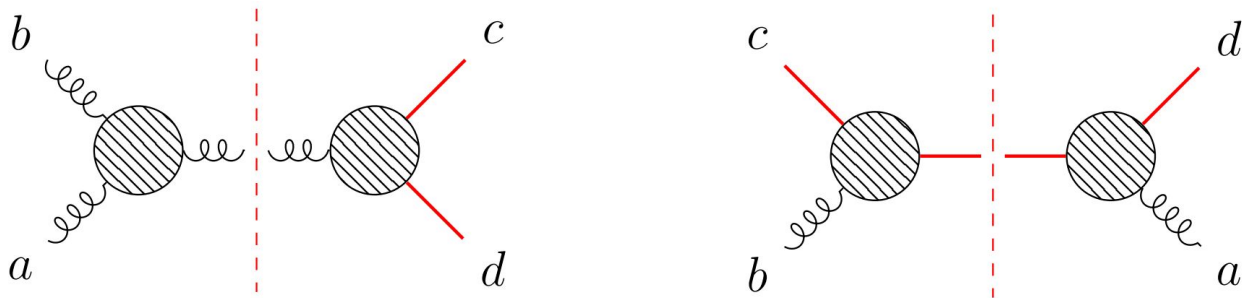


$$= n_{\textcolor{red}{m}}(a, b, c, d), \quad = n_{\overline{m}}(a, b, c, d)$$

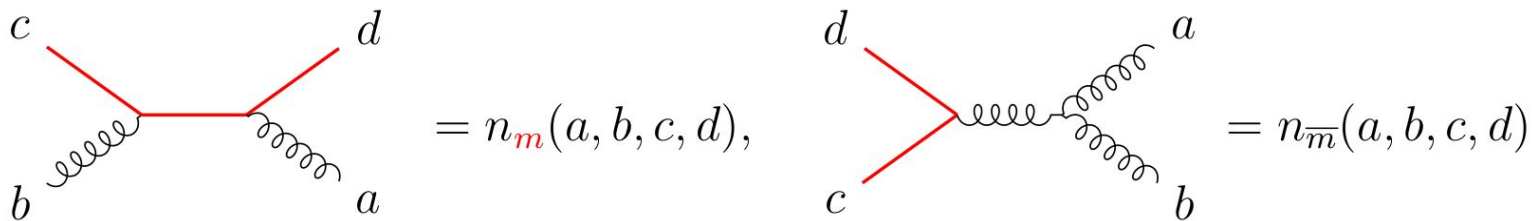
$$\begin{aligned} n_{\textcolor{red}{m}}(a, b, c, d) = & (\alpha_1(a \cdot b) + \alpha_2(b \cdot c) + \alpha_3(c \cdot c)) (\varepsilon_a \cdot \varepsilon_b) + \alpha_4(b \cdot \varepsilon_a)(a \cdot \varepsilon_b) \\ & + \alpha_5(c \cdot \varepsilon_a)(a \cdot \varepsilon_b) + \alpha_6(b \cdot \varepsilon_a)(c \cdot \varepsilon_b) + \alpha_7(c \cdot \varepsilon_a)(c \cdot \varepsilon_b) \end{aligned}$$

Tree-level amplitudes are completely determined by color-kinematics and **factorization**

Factorization



Tree-level amplitudes are completely determined by color-kinematics and factorization

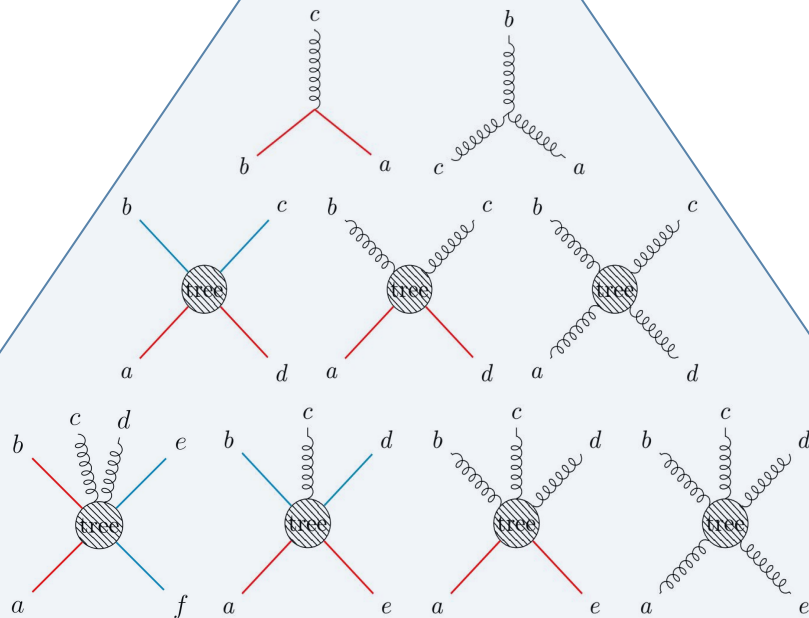


$$= n_{\textcolor{red}{m}}(a, b, c, d), \quad = n_{\overline{m}}(a, b, c, d)$$

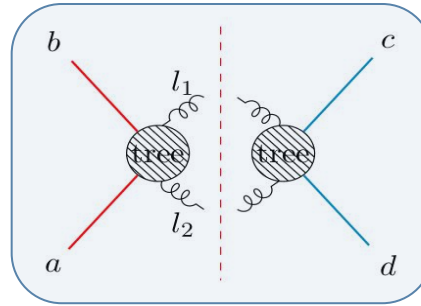
$$n_{\textcolor{red}{m}}(a, b, c, d) = (c \cdot \varepsilon_b) ((b \cdot \varepsilon_a) + (c \cdot \varepsilon_a)) - \frac{1}{2}(b \cdot c)(\varepsilon_a \cdot \varepsilon_b)$$

$$n_{\overline{m}}(a, b, c, d) = n_{\textcolor{red}{m}}(a, b, c, d) - n_{\textcolor{red}{m}}(b, a, c, d)$$

Tree-level amplitudes are completely determined by color-kinematics and factorization

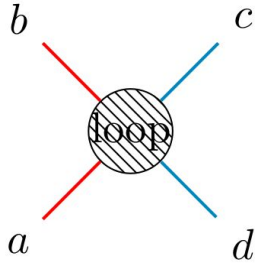


Construction loop-level



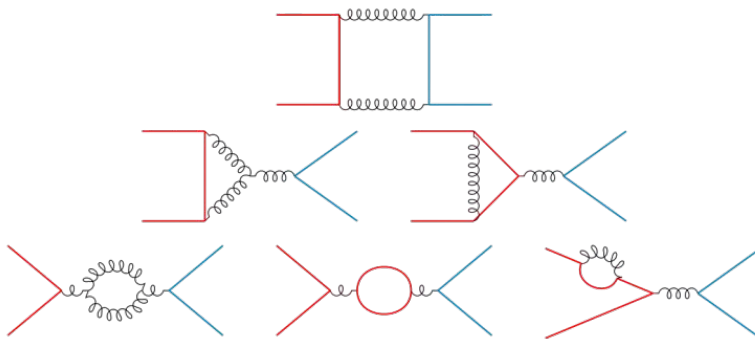
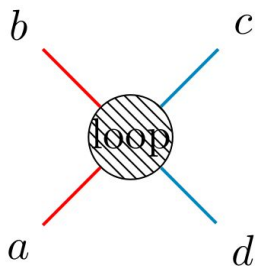
Loop-level amplitudes are determined by
color-kinematics and unitarity cuts

4-point one-loop



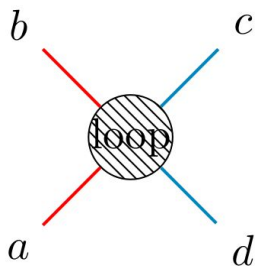
Loop-level amplitudes are determined by color-kinematics and unitarity cuts

4-point one-loop

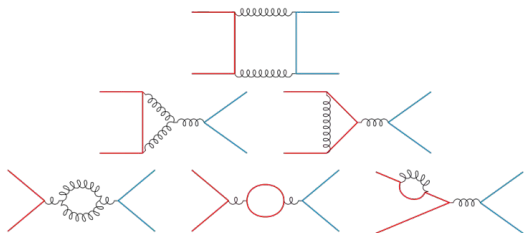
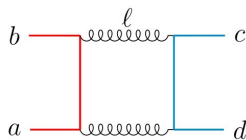


Loop-level amplitudes are determined by color-kinematics and unitarity cuts

4-point one-loop

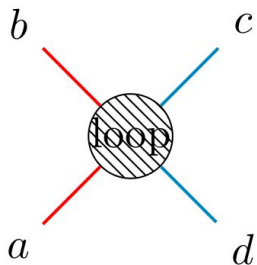


Loop level color-kinematics

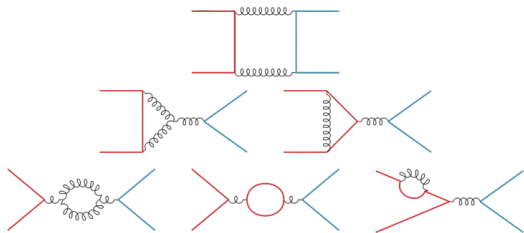
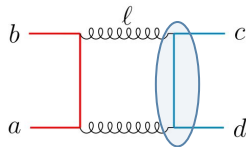


Loop-level amplitudes are determined by color-kinematics and unitarity cuts

4-point one-loop

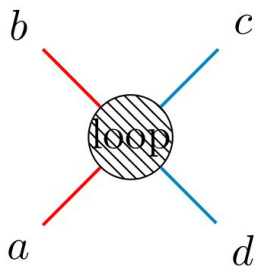


Loop level color-kinematics

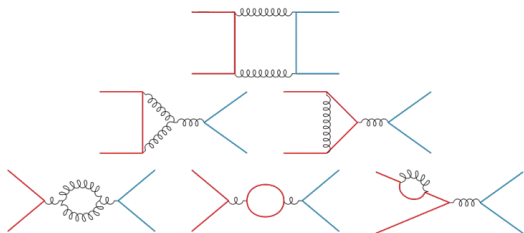
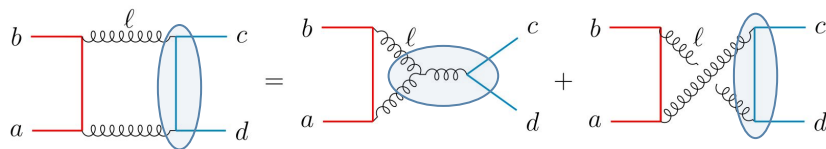


Loop-level amplitudes are determined by color-kinematics and unitarity cuts

4-point one-loop

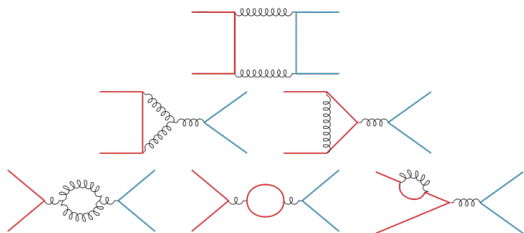
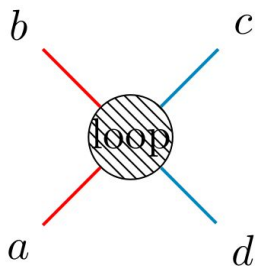


Loop level color-kinematics



Loop-level amplitudes are determined by color-kinematics and unitarity cuts

4-point one-loop



Loop level color-kinematics

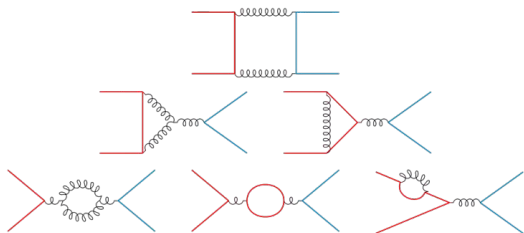
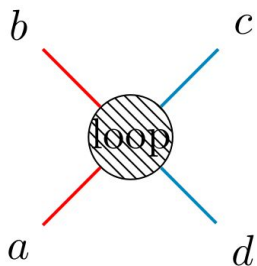
$$\begin{array}{c} b \\ \text{---} \end{array} \begin{array}{c} \ell \\ \text{---} \end{array} \begin{array}{c} c \\ \text{---} \end{array} = \begin{array}{c} b \\ \text{---} \end{array} \begin{array}{c} \ell \\ \text{---} \end{array} \begin{array}{c} c \\ \text{---} \end{array} + \begin{array}{c} b \\ \text{---} \end{array} \begin{array}{c} \ell \\ \text{---} \end{array} \begin{array}{c} c \\ \text{---} \end{array}$$



$$\begin{array}{c} b \\ \text{---} \end{array} \begin{array}{c} \ell \\ \text{---} \end{array} \begin{array}{c} c \\ \text{---} \end{array} = \begin{array}{c} b \\ \text{---} \end{array} \begin{array}{c} \ell \\ \text{---} \end{array} \begin{array}{c} c \\ \text{---} \end{array} - \begin{array}{c} b \\ \text{---} \end{array} \begin{array}{c} \ell \\ \text{---} \end{array} \begin{array}{c} c \\ \text{---} \end{array}$$

Loop-level amplitudes are determined by color-kinematics and unitarity cuts

4-point one-loop



Loop level color-kinematics

$$\begin{array}{c} b \\ \text{---} \end{array} \begin{array}{c} \ell \\ \text{---} \end{array} \begin{array}{c} c \\ \text{---} \end{array} = \begin{array}{c} b \\ \text{---} \end{array} \begin{array}{c} \ell \\ \text{---} \end{array} \begin{array}{c} c \\ \text{---} \end{array} + \begin{array}{c} b \\ \text{---} \end{array} \begin{array}{c} \ell \\ \text{---} \end{array} \begin{array}{c} c \\ \text{---} \end{array}$$

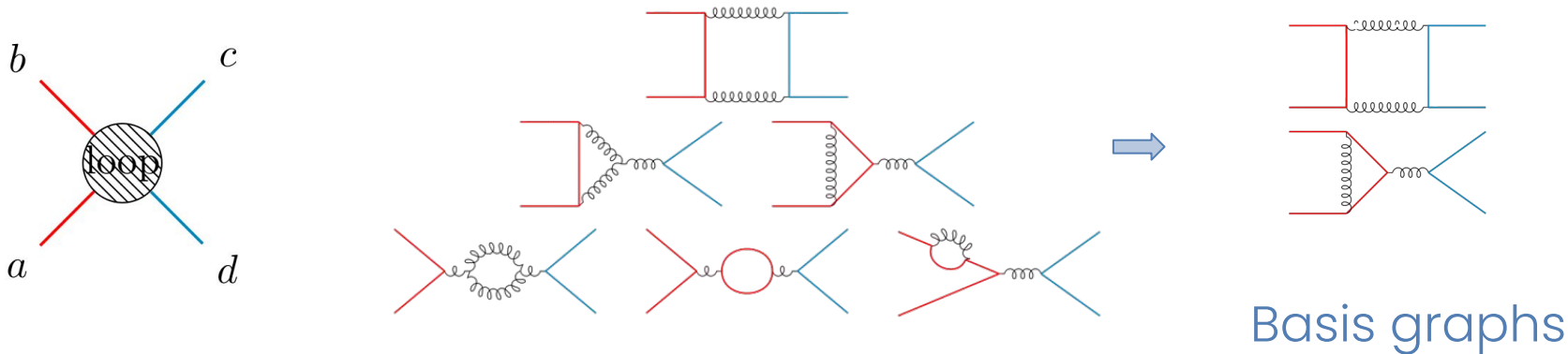


$$\begin{array}{c} b \\ \text{---} \end{array} \begin{array}{c} \ell \\ \text{---} \end{array} \begin{array}{c} c \\ \text{---} \end{array} = \begin{array}{c} b \\ \text{---} \end{array} \begin{array}{c} \ell \\ \text{---} \end{array} \begin{array}{c} c \\ \text{---} \end{array} - \begin{array}{c} b \\ \text{---} \end{array} \begin{array}{c} \ell \\ \text{---} \end{array} \begin{array}{c} c \\ \text{---} \end{array}$$

$$\begin{array}{c} b \\ \text{---} \end{array} \begin{array}{c} \ell \\ \text{---} \end{array} \begin{array}{c} c \\ \text{---} \end{array} = \begin{array}{c} b \\ \text{---} \end{array} \begin{array}{c} \ell \\ \text{---} \end{array} \begin{array}{c} c \\ \text{---} \end{array} + \begin{array}{c} b \\ \text{---} \end{array} \begin{array}{c} \ell \\ \text{---} \end{array} \begin{array}{c} c \\ \text{---} \end{array}$$

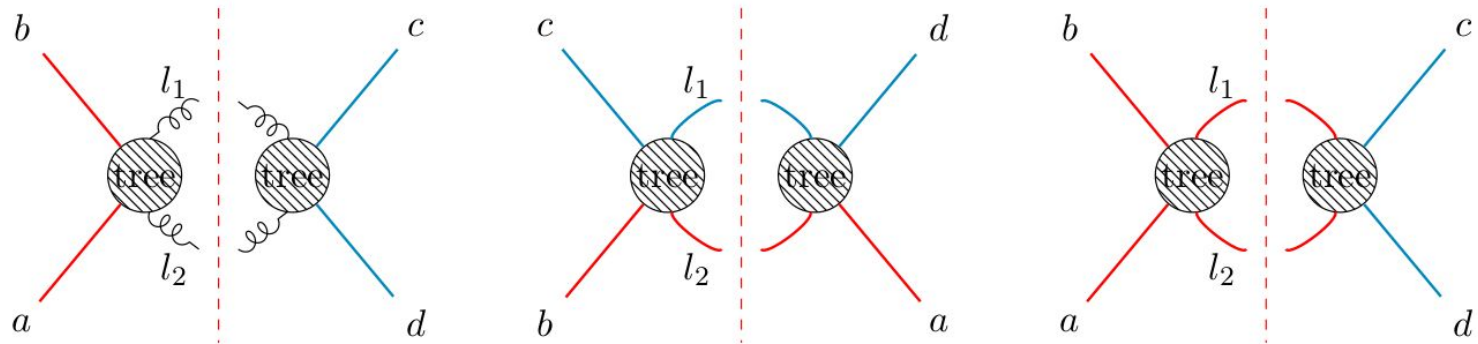
Loop-level amplitudes are determined by color-kinematics and unitarity cuts

4-point one-loop



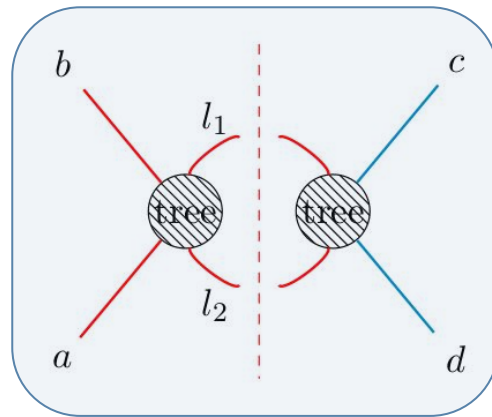
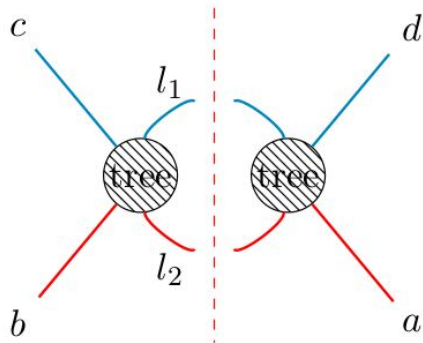
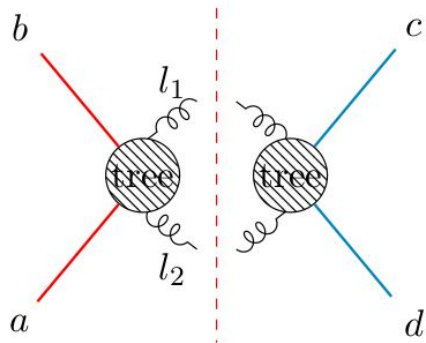
Loop-level amplitudes are determined by
color-kinematics and **unitarity cuts**

4-point one-loop



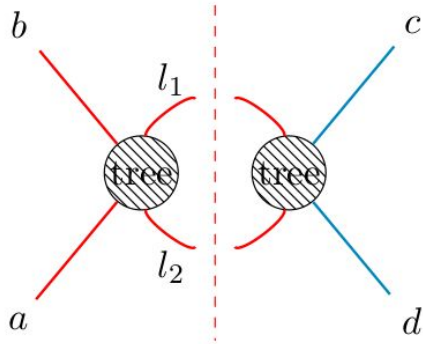
Loop-level amplitudes are determined by
color-kinematics and **unitarity cuts**

4-point one-loop

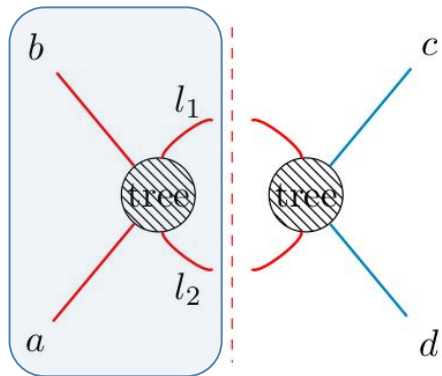


Ordered cut?

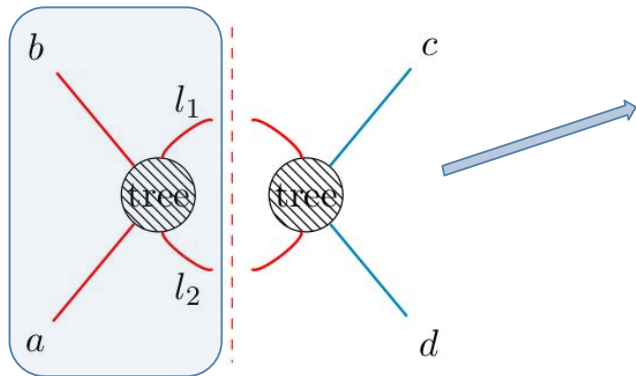
What does *ordered cut* mean?



What does *ordered cut* mean?



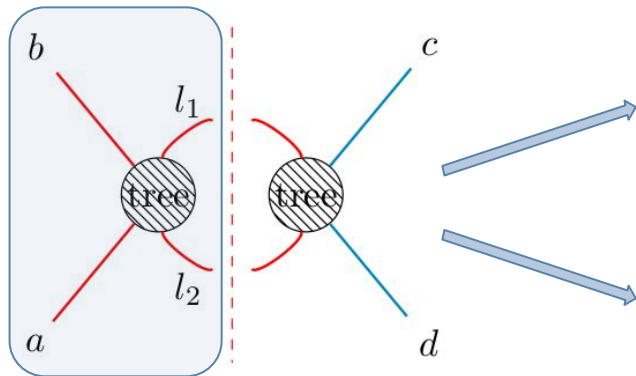
What does *ordered cut* mean?



Adjoint-type ordered cut

$$A^{\text{tree}}(a, b, c, d) = \begin{array}{c} b \\ \diagdown \\ \text{---} \text{---} \text{---} \\ \diagup \\ a \end{array} \begin{array}{c} c \\ \diagup \\ \text{---} \text{---} \text{---} \\ \diagdown \\ d \end{array} + \begin{array}{c} b \\ \diagdown \\ \text{---} \text{---} \text{---} \\ \diagup \\ a \end{array} \begin{array}{c} c \\ \diagdown \\ \text{---} \text{---} \text{---} \\ \diagup \\ d \end{array} = \frac{n_s}{s} + \frac{n_t}{t}$$

What does *ordered cut* mean?



Adjoint-type ordered cut

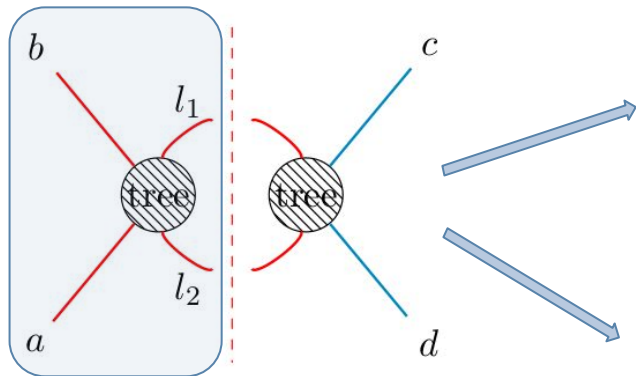
$$A^{\text{tree}}(a, b, c, d) = \begin{array}{c} b \\ \diagdown \\ \text{---} \text{wavy line} \text{---} \\ \diagup \\ a \end{array} \begin{array}{c} c \\ \diagup \\ \text{---} \text{wavy line} \text{---} \\ \diagdown \\ d \end{array} + \begin{array}{c} b \\ \text{---} \text{wavy line} \text{---} \\ \diagdown \\ a \end{array} \begin{array}{c} c \\ \text{---} \text{wavy line} \text{---} \\ \diagup \\ d \end{array} = \frac{n_s}{s} + \frac{n_t}{t}$$

Fundamental-type ordered cut

$$A^{\text{tree}}(\bar{a}, b, \bar{c}, d) = \begin{array}{c} b \\ \diagdown \\ \text{---} \text{wavy line} \text{---} \\ \diagup \\ a \end{array} \begin{array}{c} c \\ \diagup \\ \text{---} \text{wavy line} \text{---} \\ \diagdown \\ d \end{array} = \frac{n_s}{s}$$

$$A^{\text{tree}}(\bar{a}, d, \bar{c}, b) = \begin{array}{c} b \\ \text{---} \text{wavy line} \text{---} \\ \diagdown \\ a \end{array} \begin{array}{c} c \\ \text{---} \text{wavy line} \text{---} \\ \diagup \\ d \end{array} = \frac{n_t}{t}$$

What does *ordered cut* mean?



Adjoint-type ordered cut

$$A^{\text{tree}}(a, b, c, d) = \begin{array}{c} b \quad c \\ \diagdown \quad \diagup \\ \text{---} \text{---} \text{---} \\ \diagup \quad \diagdown \\ a \quad d \end{array} + \begin{array}{c} b \quad c \\ \diagdown \quad \diagup \\ \text{---} \text{---} \text{---} \\ \diagup \quad \diagdown \\ a \quad d \end{array} = \frac{n_s}{s} + \frac{n_t}{t}$$

$$\begin{array}{c} b \quad c \\ \diagdown \quad \diagup \\ \text{---} \text{---} \text{---} \\ \diagup \quad \diagdown \\ a \quad d \end{array} = \begin{array}{c} c \quad b \\ \diagdown \quad \diagup \\ \text{---} \text{---} \text{---} \\ \diagup \quad \diagdown \\ a \quad d \end{array} + \begin{array}{c} a \quad b \\ \diagdown \quad \diagup \\ \text{---} \text{---} \text{---} \\ \diagup \quad \diagdown \\ d \quad c \end{array}$$

Fundamental-type ordered cut

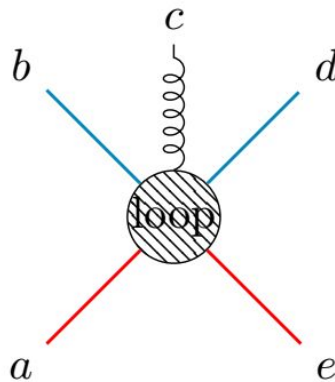
$$A^{\text{tree}}(\bar{a}, b, \bar{c}, d) = \begin{array}{c} b \quad c \\ \diagdown \quad \diagup \\ \text{---} \text{---} \text{---} \\ \diagup \quad \diagdown \\ a \quad d \end{array} = \frac{n_s}{s}$$

$$A^{\text{tree}}(\bar{a}, d, \bar{c}, b) = \begin{array}{c} b \quad c \\ \diagdown \quad \diagup \\ \text{---} \text{---} \text{---} \\ \diagup \quad \diagdown \\ a \quad d \end{array} = \frac{n_t}{t}$$

Loop-level amplitudes are determined by
color-kinematics and unitarity cuts

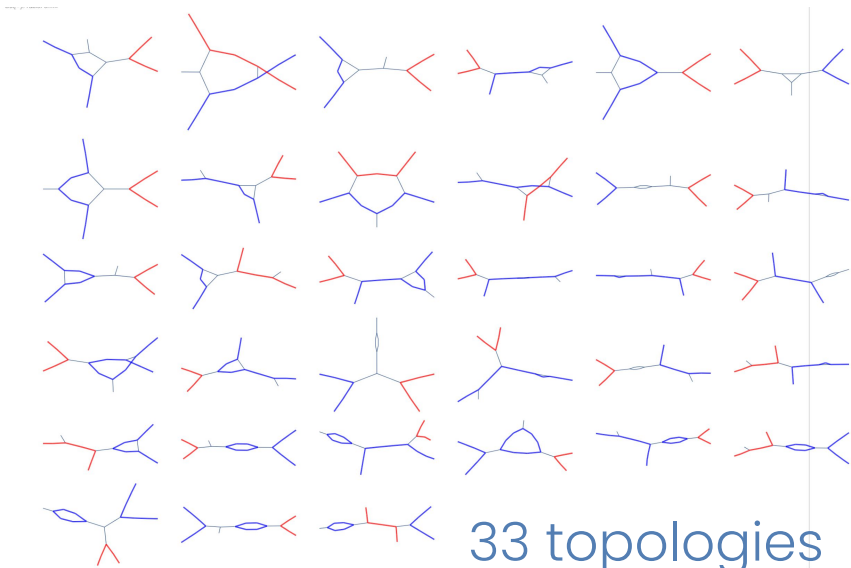
5-point one-loop

Encodes first correction to radiation

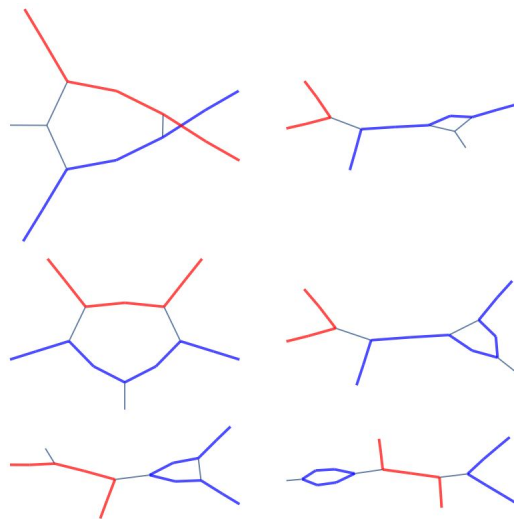


Loop-level amplitudes are determined by color-kinematics and unitarity cuts

5-point one-loop



6 basis graphs



Loop-level amplitudes are determined by color-kinematics and unitarity cuts

5-point one-loop

33 topologies

10 296
parameters

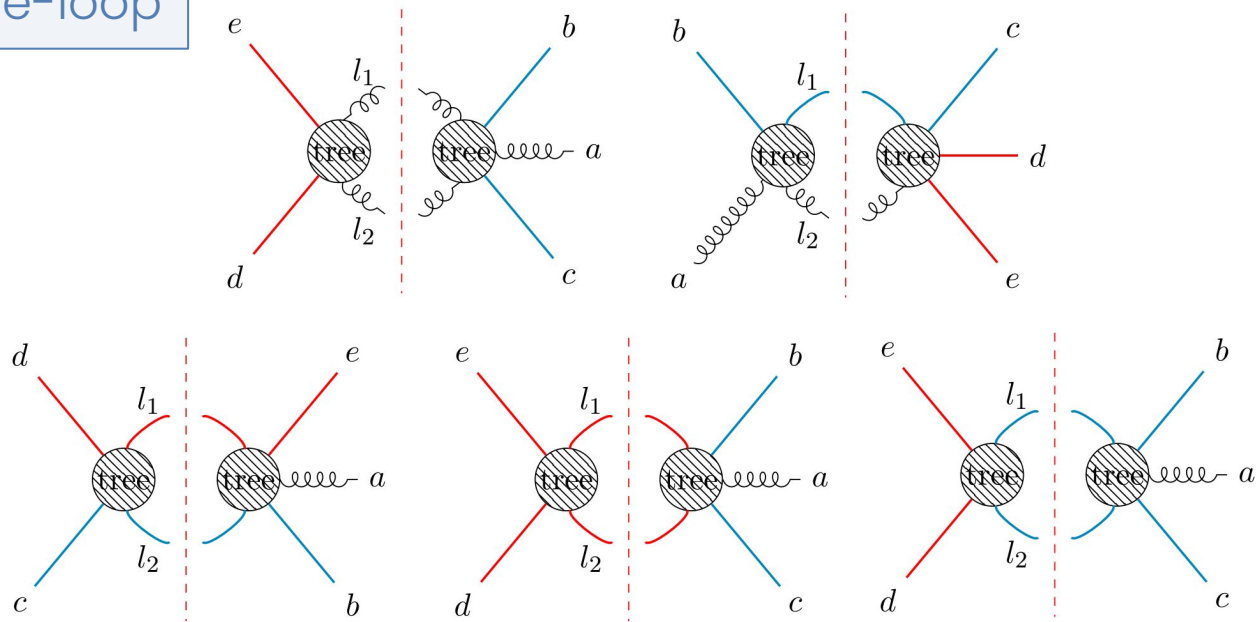


6 basis graphs

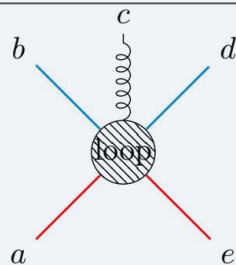
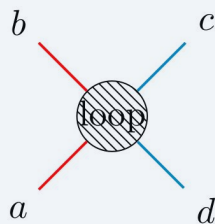
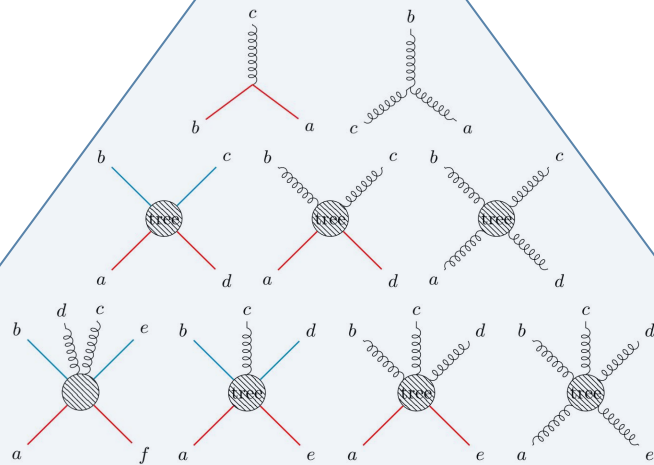
1872
parameters

Loop-level amplitudes are determined by color-kinematics and unitarity cuts

5-point one-loop



Amplitudes are fully
determined by
color-kinematics and
unitarity methods



1 loop

Double copy to obtain N=0 supergravity predictions

$$S = \int d^D x \sqrt{-g} \left[-\frac{1}{2} R + \frac{1}{2(D-2)} \partial^\mu \phi \partial_\mu \phi + \frac{1}{6} e^{-4\phi/(D-2)} H^{\lambda\mu\nu} H_{\lambda\mu\nu} \right]$$

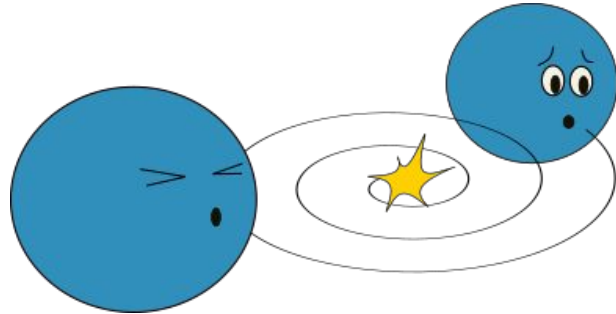
Scherk, Schwarz '74
Gross, Sloan '87

+ Matter terms

Johansson, Ochirov '16,'19
Bautista, Guevara '19
Plefka, Shi, Wang '19

Future work includes

exploring double copies directly to pure gravity predictions,
massive higher-spins in arbitrary rep,
generating classical observables,
extending to higher loop order (2-loops and more)!



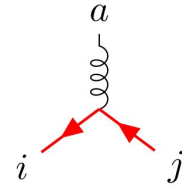
Color-kinematics and factorization
are enough to build massive
amplitudes

A Feynman diagram equation. On the left, a four-point amplitude with external legs a (red), b (red), c (red), and d (red) connected by two wavy lines. This is equal to the sum of two three-point amplitudes. The first three-point amplitude has legs b (red), c (red), and d (red) connected by a wavy line. The second three-point amplitude has legs a (red), c (red), and d (red) connected by a wavy line.

A Feynman diagram equation. On the left, a four-point amplitude with external legs a (red), b (red), c (red), and d (red) connected by two wavy lines. This is equal to the sum of two three-point amplitudes. The first three-point amplitude has legs a (red), b (red), and d (red) connected by a wavy line. The second three-point amplitude has legs a (red), c (red), and d (red) connected by a wavy line.

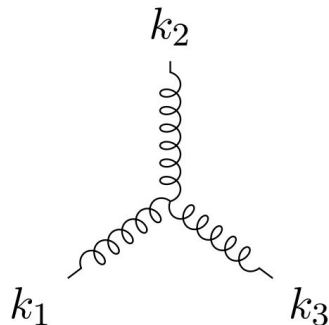
Color-kinematics duality for
massive amplitudes can be
manifest at loop level

Can use the same kinematic
building blocks for massive scalars
charged in fundamental and
adjoint



Extra

3-point amplitudes are completely determined by color-kinematics and symmetries



Build the ansatz from a kinematic basis

$$n_3(k_1, k_2, k_3) = \alpha_1(k_3 \cdot \epsilon_1)(\epsilon_2 \cdot \epsilon_3) + \alpha_2(k_3 \cdot \epsilon_2)(\epsilon_1 \cdot \epsilon_3) + \alpha_3(k_2 \cdot \epsilon_3)(\epsilon_1 \cdot \epsilon_2)$$

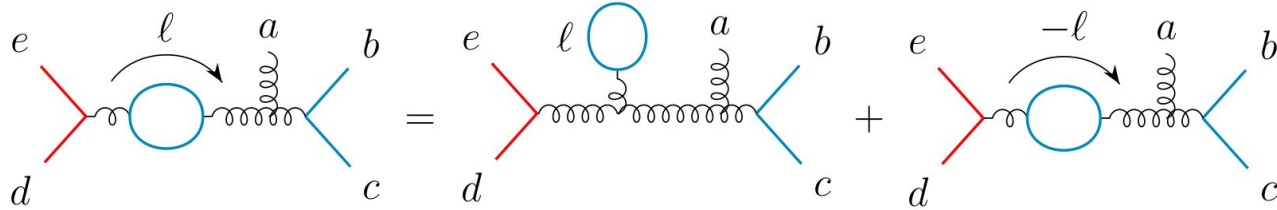
Momentum conservation
Transversality

$$k_1 + k_2 + k_3 = 0$$

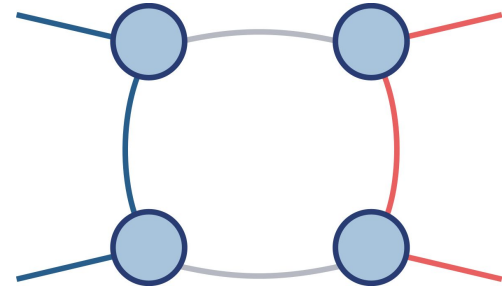
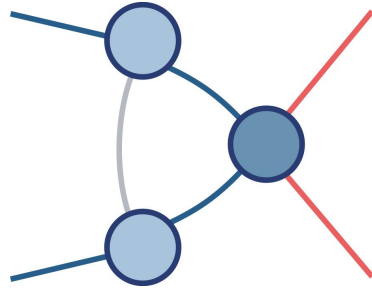
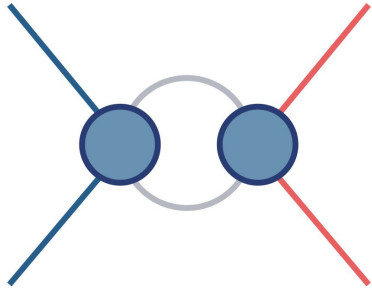
$$\epsilon_i \cdot k_i = 0$$

$$k_2 \cdot \epsilon_1 = (-k_1 - k_3) \cdot \epsilon_1 = -k_3 \cdot \epsilon_1$$

Tadpoles can be reached using color-kinematics

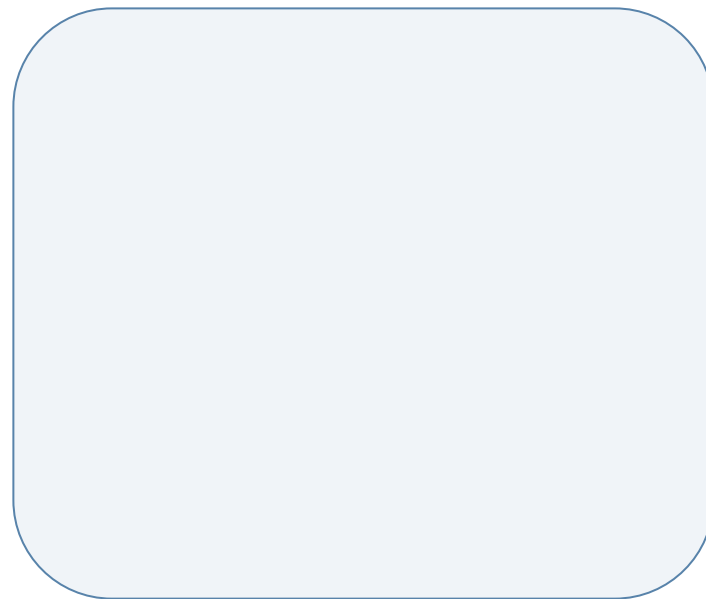
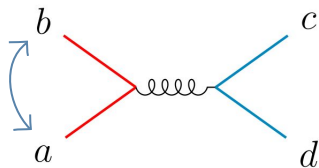


We verified on bubble, triangle and box cuts



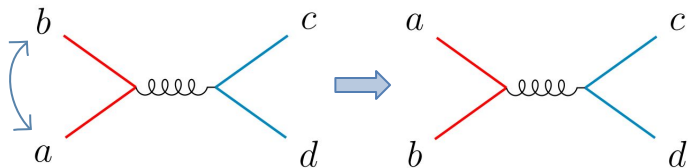
Tree-level amplitudes are completely determined by color-kinematics and factorization

What are symmetries of the graph?



Tree-level amplitudes are completely determined by color-kinematics and factorization

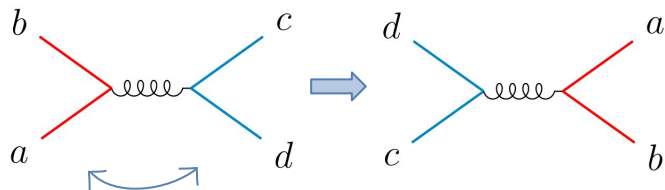
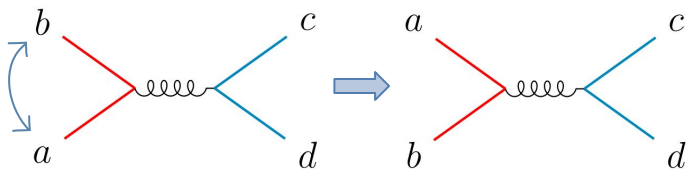
What are symmetries of the graph?



$$n_{4,2}(b, a, c, d) = -n_{4,2}(a, b, c, d)$$

Tree-level amplitudes are completely determined by color-kinematics and factorization

What are symmetries of the graph?



$$n_{4,2}(b, a, c, d) = -n_{4,2}(a, b, c, d)$$

$$n_{4,2}(c, d, a, b) = n_{4,2}(a, b, c, d)$$

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