Loop-Level Double Copy for Massive Quantum Particles

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Based on work with John Joseph Carrasco

IPhT SACLAY

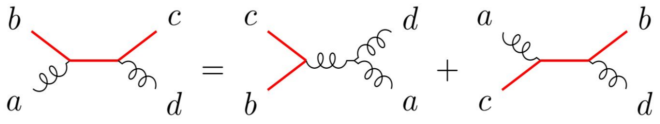
SAGEX

Scattering Amplitudes: from Geometry to Experiment

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No. 764850

QCD Meets Gravity, 4 Dec 2020

Color-kinematics and factorization are enough to build massive amplitudes



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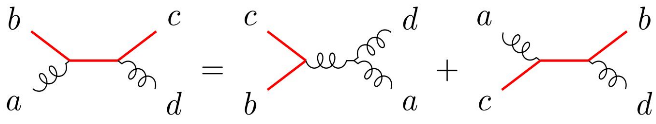
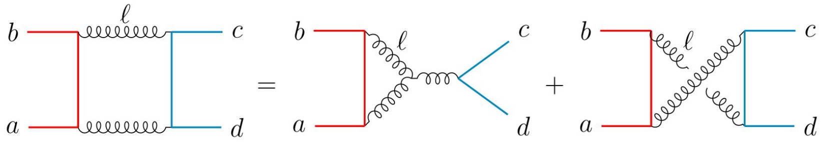
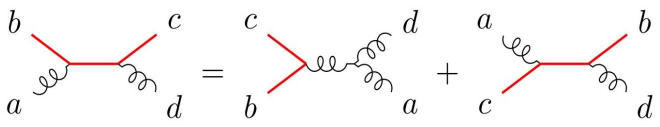
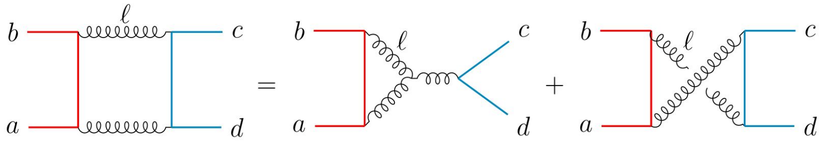




Color-kinematics duality for massive amplitudes can be manifest at loop level

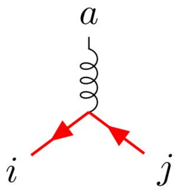
Color-kinematics and factorization are enough to build massive amplitudes



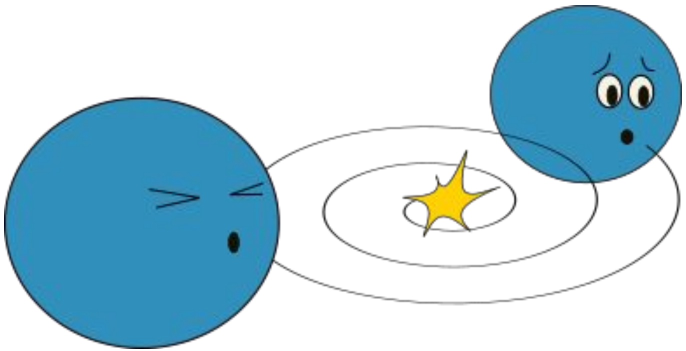


Color-kinematics duality for massive amplitudes can be manifest at loop level

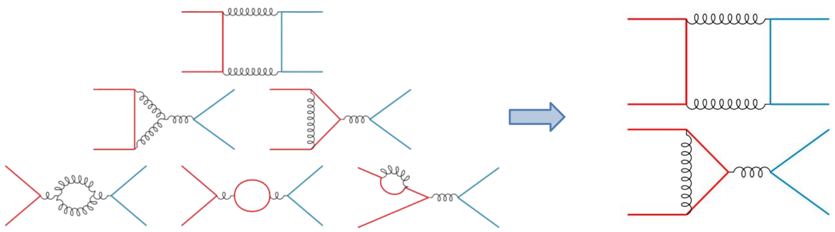
Can use the same kinematic building blocks for massive scalars charged in fundamental and adjoint

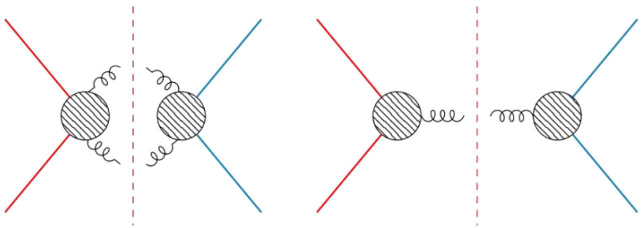


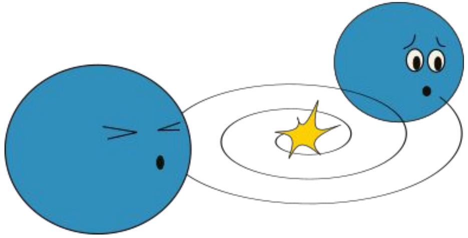
Why is gravity hard with traditional methods?



Use the color-kinematics duality and unitarity methods to find massive amplitudes







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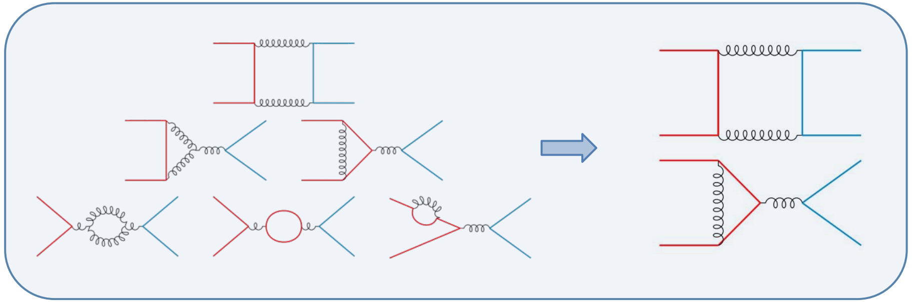

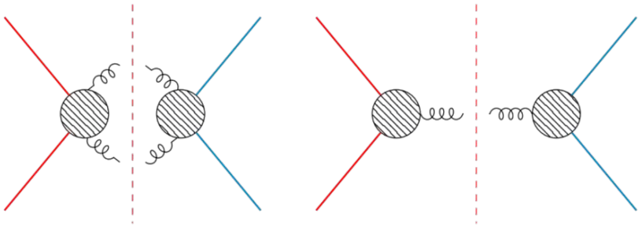

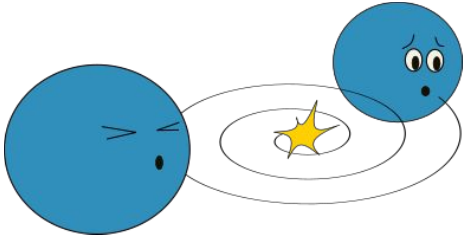

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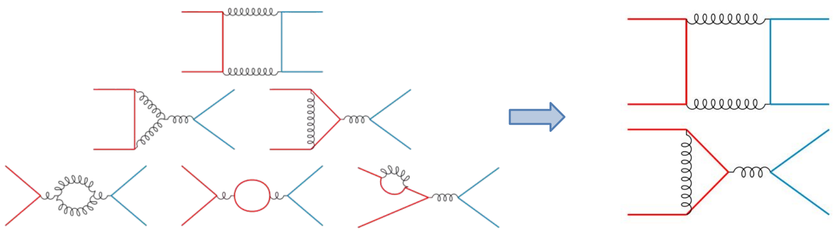
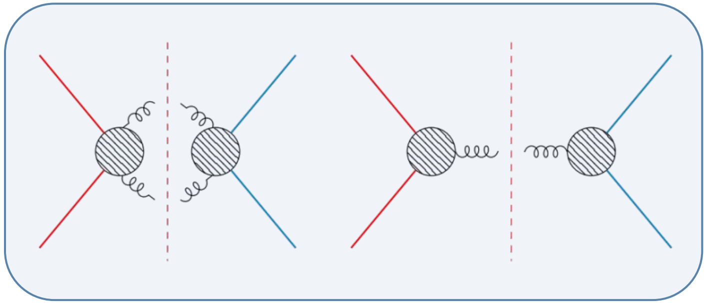
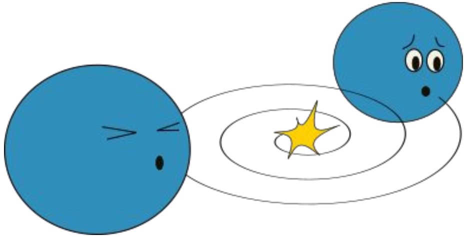
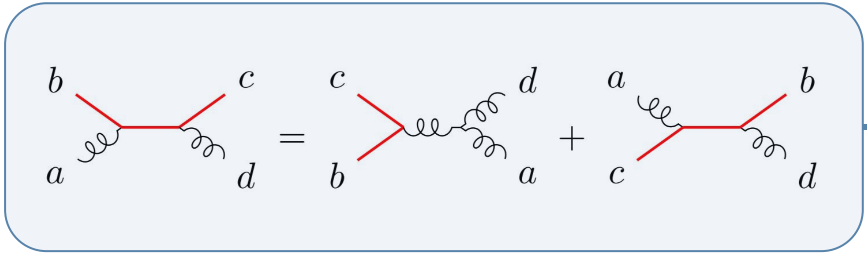
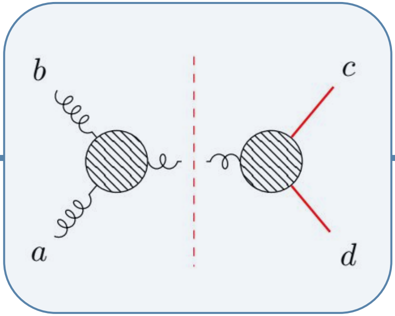


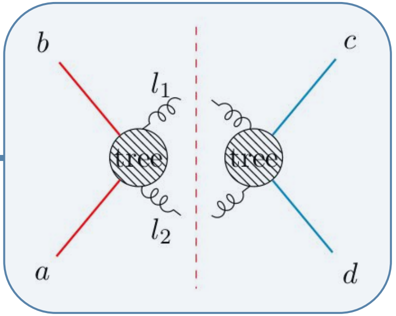




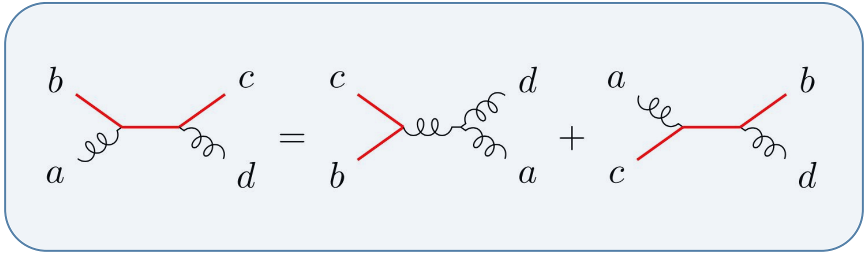
Bootstrapping tree-level amplitudes and loop-level using the color-kinematics duality







The color-kinematics duality



Color-kinematics duality

Color-dual representation: kinematic weights ~ color weights

Bern, Carrasco, Johansson '08,
Bern, Carrasco, Johansson '10

Only need small number of basis graphs (Solves combinatorics problem!)

Weaves a web of theories (Recycling is good)

See review: Bern, Carrasco, Chiodaroli, Johansson, Roiban '19

Color-kinematics for many representations (adjoint, three-algebras, arbitrary)

Bargheer, He, McLoughlin '12

Massive matter in the fundamental

Johansson, Ochirov '16, Plefka, Shi, Wang '19, Bjerrum-Bohr, Cristofoli, Damgaard, Gomez '19, Luna, Nicholson, O'Connell, White '17
Haddad, Helset '20

Color-kinematics duality relates kinematic weights of graphs

$$\mathcal{A}_m^{(L)} = i^L g^{m-2+2L} \sum_{i\in\Gamma} \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_i} \frac{n_i C_i}{\prod_{\alpha_i}(p_{\alpha_i}^2 - m_{\alpha_i}^2)}$$

Color-kinematics duality relates kinematic weights of graphs

$$\frac{n_i C_i}{\prod_{\alpha_i}(p_{\alpha_i}^2 - m_{\alpha_i}^2)}$$

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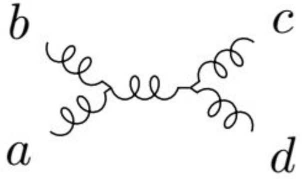

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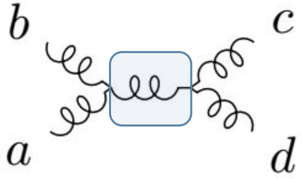

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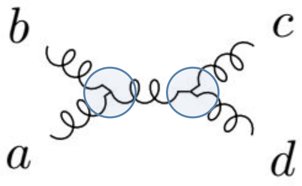

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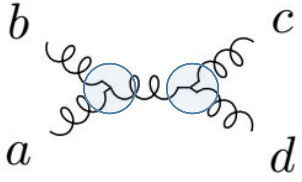
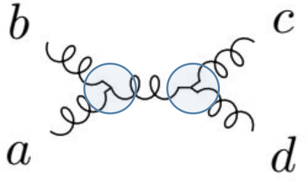


$$\frac{n_i C_i}{\prod_{\alpha_i}(p_{\alpha_i}^2 - m_{\alpha_i}^2)}$$

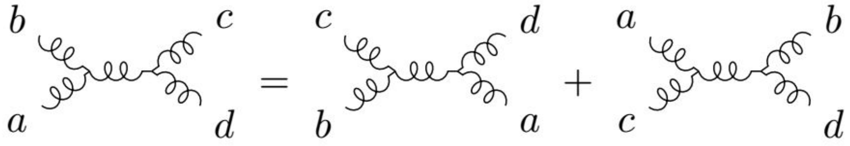
$f^{abl} f^{lcd}$

Color-kinematics duality relates kinematic weights of graphs



$$\frac{n_i C_i}{\prod_{\alpha_i}(p_{\alpha_i}^2 - m_{\alpha_i}^2)}$$

$$f^{abl} f^{lcd} = f^{bcl} f^{lda} + f^{cal} f^{lbd}$$



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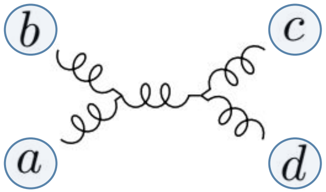
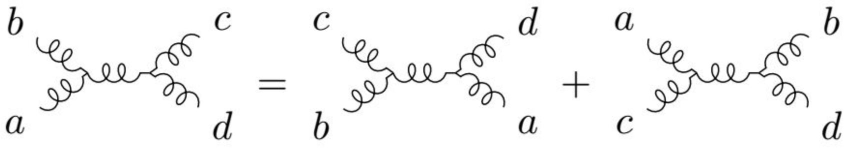


$$\frac{n_i C_i}{\prod_{\alpha_i}(p_{\alpha_i}^2 - m_{\alpha_i}^2)}$$

$n(a, b, c, d)$



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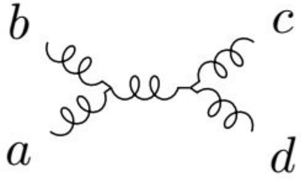
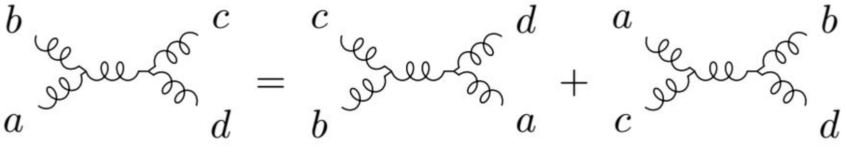


$$\frac{n_i C_i}{\prod_{\alpha_i}(p_{\alpha_i}^2 - m_{\alpha_i}^2)}$$

$$n(a,b,c,d) = n(b,c,d,a) + n(c,a,b,d)$$



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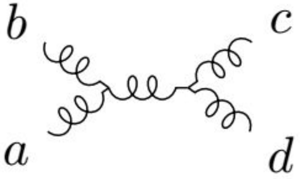

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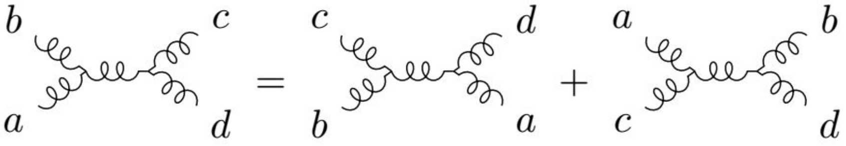
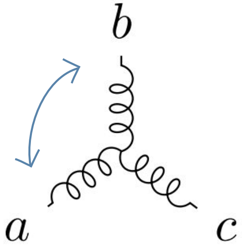
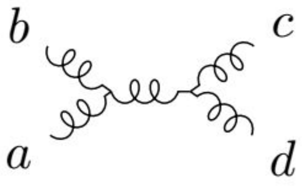


Antisymmetries of graph weights



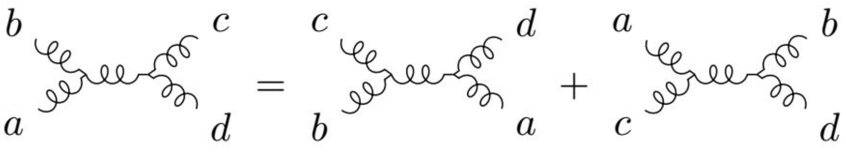
$$f^{abc} = -f^{bac}$$

Color-kinematics duality relates kinematic weights of graphs

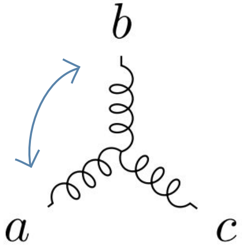


$$\frac{n_i C_i}{\prod_{\alpha_i}(p_{\alpha_i}^2 - m_{\alpha_i}^2)}$$

$$n(a,b,c,d) = n(b,c,d,a) + n(c,a,b,d)$$



Antisymmetries of graph weights

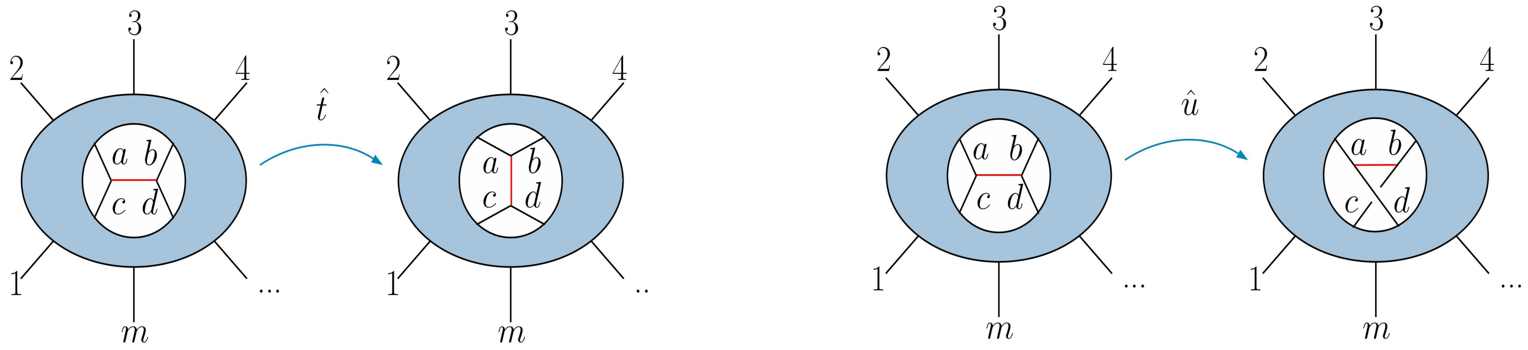


$$n(a,b,c) = -n(b,a,c)$$

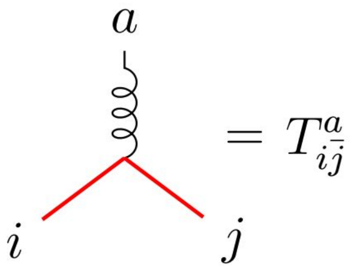
Color-kinematics duality relates kinematic weights of graphs

$$n_i - n_j = n_k \qquad \Leftrightarrow \qquad c_i - c_j = c_k,$$

$$n_i \to -n_i \qquad \Leftrightarrow \qquad c_i \to -c_i$$



What is different about massive particles?



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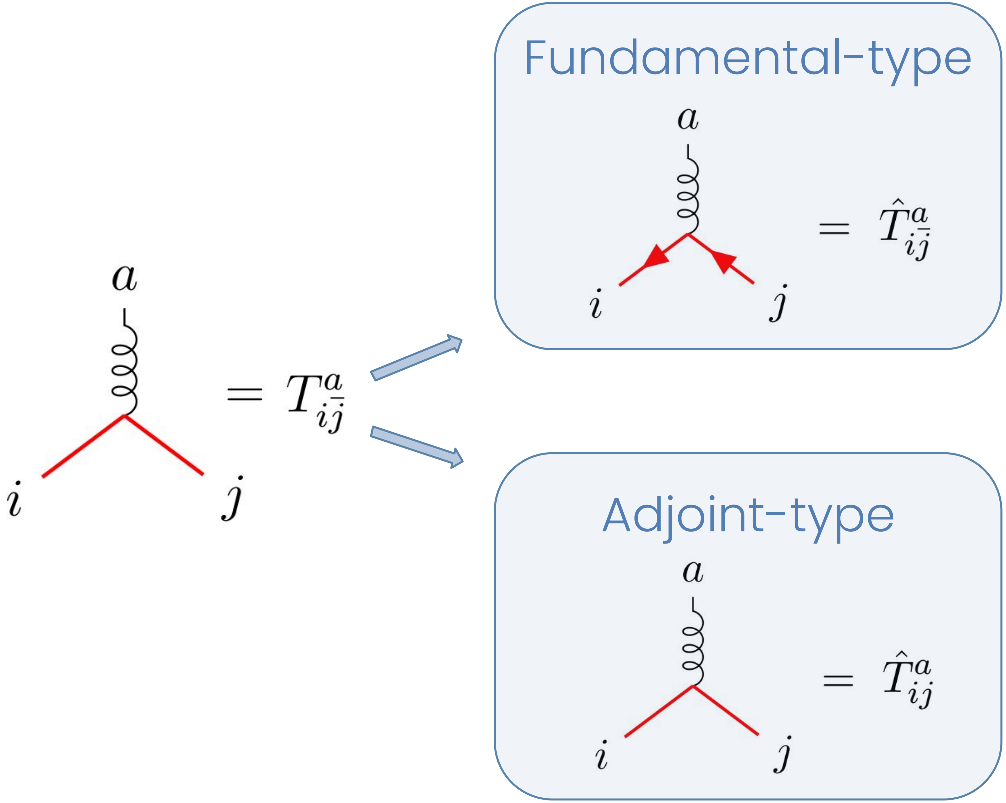

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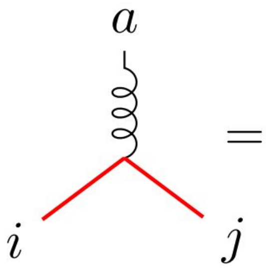
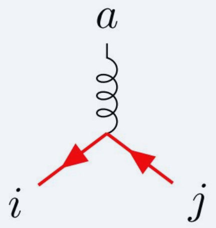


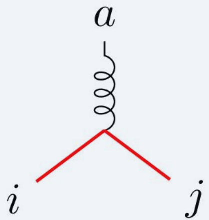
$$= T^a_{i\bar{j}}$$

Fundamental-type



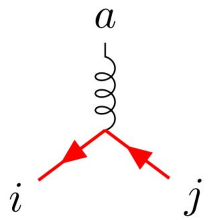
$$= \hat{T}^a_{i\bar{j}}$$

Adjoint-type

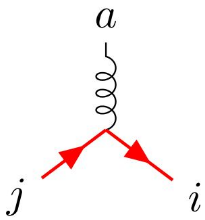


$$= \hat{T}^a_{ij}$$

Want antisymmetry:



$$= -$$



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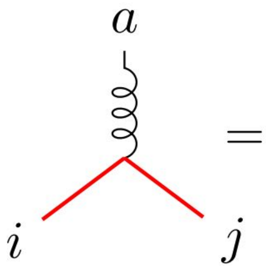
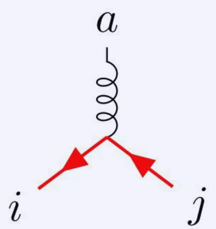


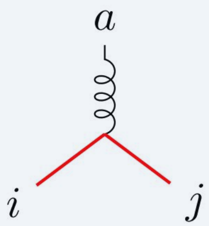
$= T^a_{i\bar{j}}$

Fundamental-type



$= \hat{T}^a_{i\bar{j}}$

Adjoint-type



$= \hat{T}^a_{ij}$

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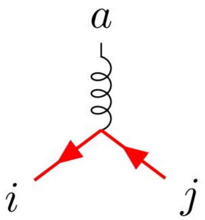
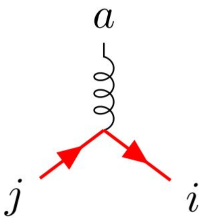


$= -$



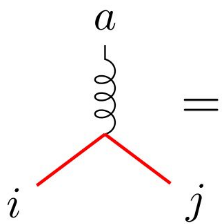
Introduce:

$$T^a_{i\bar{j}} = \hat{T}^a_{i\bar{j}}$$

$$T^a_{\bar{j}i} = -T^a_{i\bar{j}}$$

c(graph) x n(graph)

What is different about massive particles?



$$= T^a_{i\bar{j}}$$

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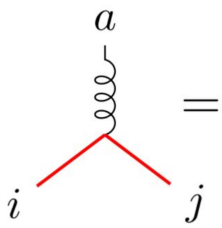
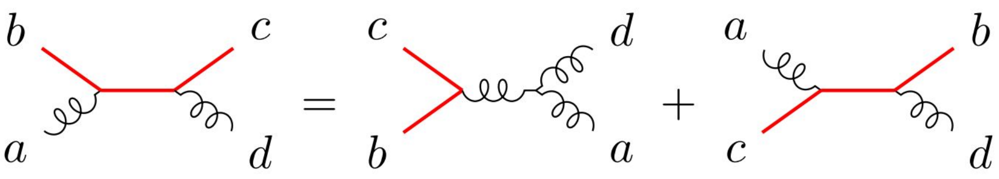


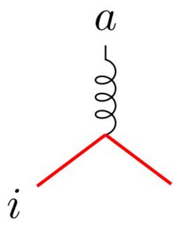
$= T^a_{i\bar{j}}$

Jacobi-like relations



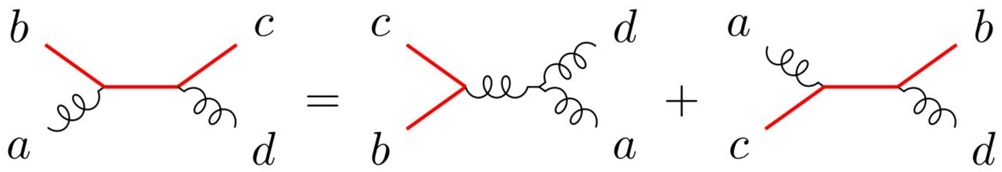
$$T^a_{b\bar{j}}T^d_{j\bar{c}} = f^{dal}T^l_{b\bar{c}} + T^a_{j\bar{c}}T^d_{b\bar{j}}$$

What is different about massive particles?



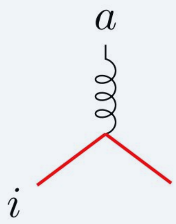
$= T^a_{i\bar{j}}$

Jacobi-like relations



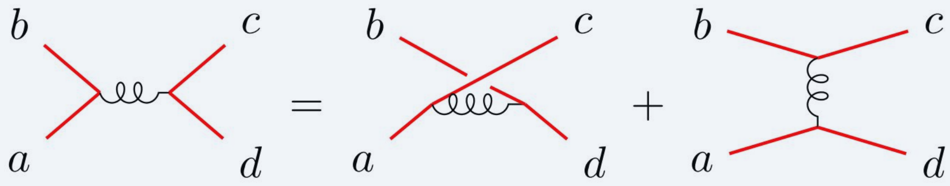
$$T^a_{b\bar{j}} T^d_{j\bar{c}} = f^{dal} T^l_{b\bar{c}} + T^a_{j\bar{c}} T^d_{b\bar{j}}$$

Adjoint-type



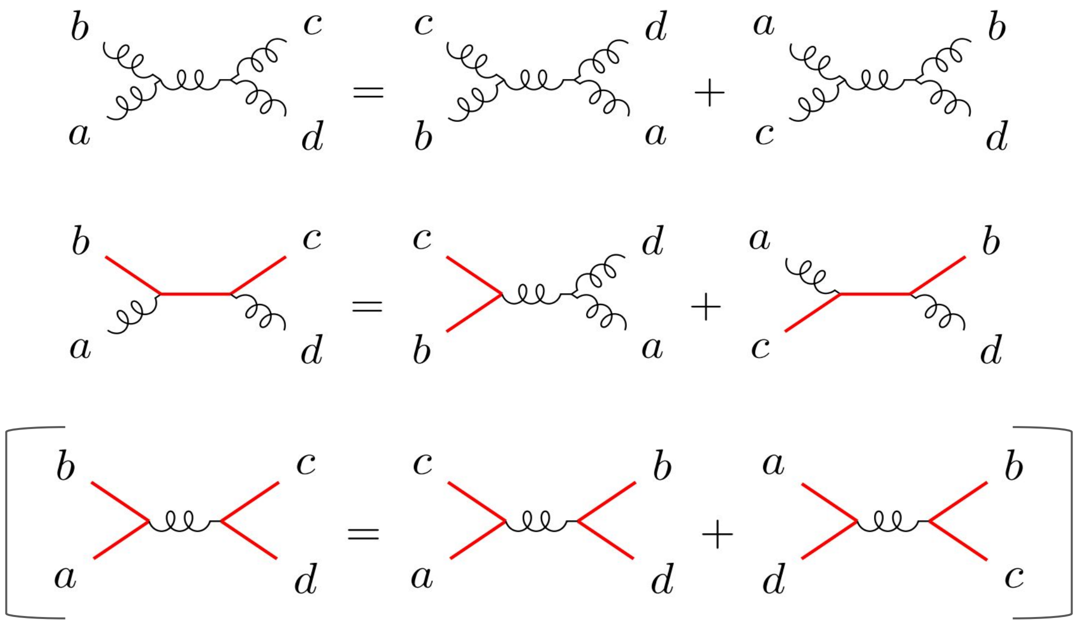
$= \hat{T}^a_{ij}$

Optionally

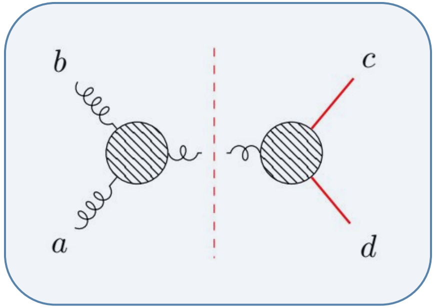


$$T^g_{ab} T^g_{cd} = T^g_{ac} T^g_{bd} + T^g_{ad} T^g_{bc}$$

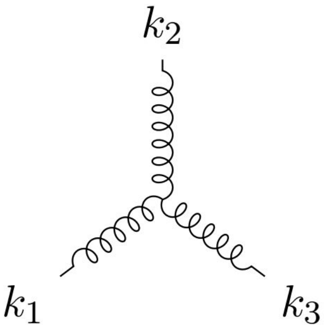
Color-kinematics duality relates kinematic weights of graphs

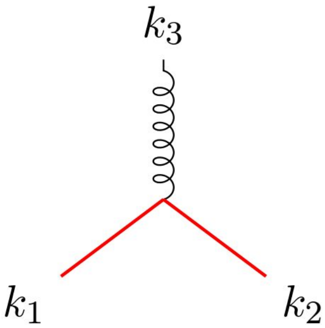


Construction

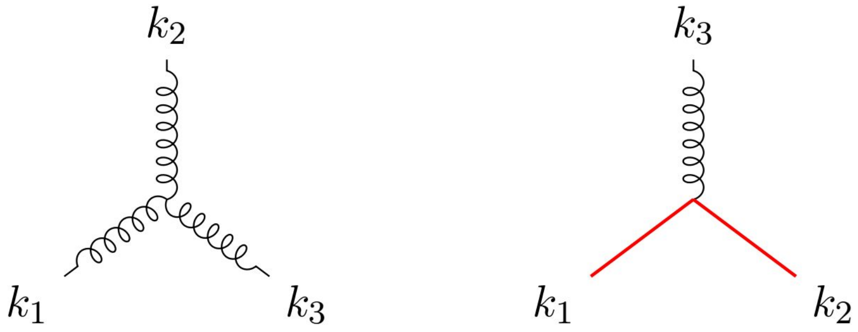


3-point amplitudes are completely determined by color-kinematics and symmetries





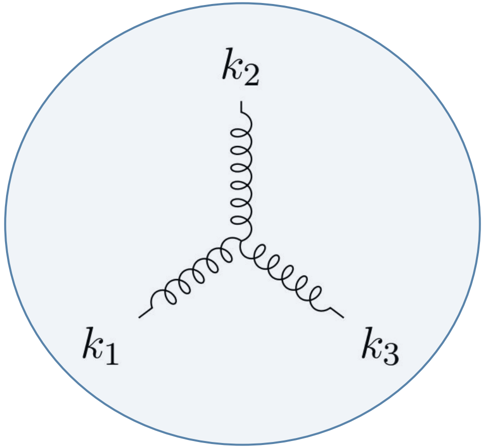
3-point amplitudes are completely determined by color-kinematics and symmetries

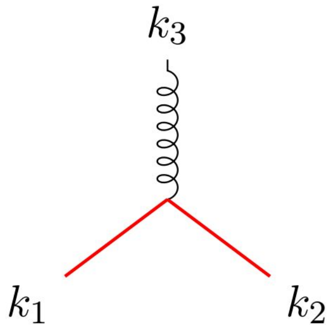


$$n_3(k_1, k_2, k_3)$$

$$n_{3,2}(k_1^m, k_2^m, k_3)$$

3-point amplitudes are completely determined by color-kinematics and symmetries

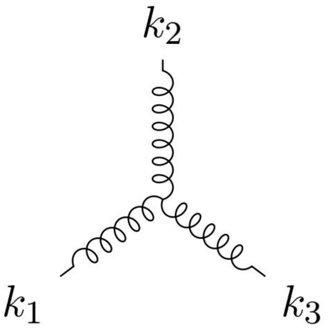


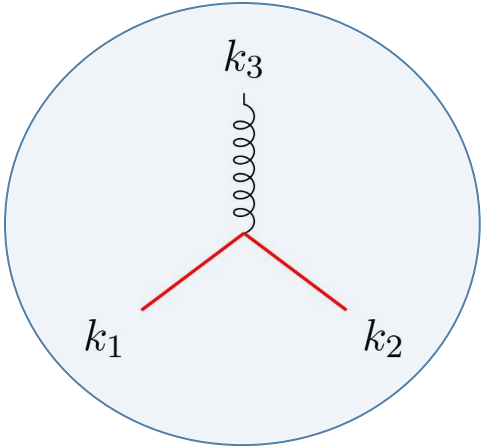


$$n_3(k_1, k_2, k_3) = \alpha_1(k_3 \cdot \epsilon_1)(\varepsilon_2 \cdot \varepsilon_3) + \alpha_2(k_3 \cdot \varepsilon_2)(\varepsilon_1 \cdot \varepsilon_3) + \alpha_3(k_2 \cdot \varepsilon_3)(\varepsilon_1 \cdot \varepsilon_2)$$

$$n_{3,2}(k_1^m, k_2^m, k_3)$$

3-point amplitudes are completely determined by color-kinematics and symmetries

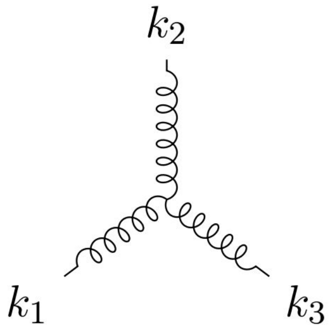


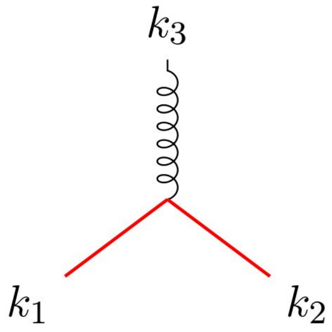


$$n_3(k_1, k_2, k_3) = \alpha_1(k_3 \cdot \epsilon_1)(\varepsilon_2 \cdot \varepsilon_3) + \alpha_2(k_3 \cdot \varepsilon_2)(\varepsilon_1 \cdot \varepsilon_3) + \alpha_3(k_2 \cdot \varepsilon_3)(\varepsilon_1 \cdot \varepsilon_2)$$

$$n_{3,2}(k_1^m, k_2^m, k_3) = \alpha_1(k_1 \cdot \varepsilon_3)$$

3-point amplitudes are completely determined by color-kinematics and symmetries

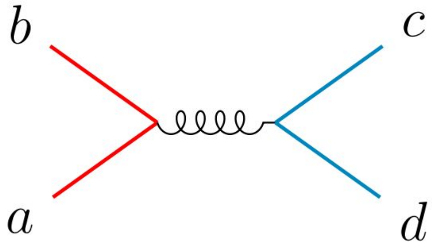




$$n_3(k_1, k_2, k_3) = \alpha_1\Big((k_3 \cdot \epsilon_1)(\varepsilon_2 \cdot \varepsilon_3) - (k_3 \cdot \varepsilon_2)(\varepsilon_1 \cdot \varepsilon_3) + (k_2 \cdot \varepsilon_3)(\varepsilon_1 \cdot \varepsilon_2)\Big)$$

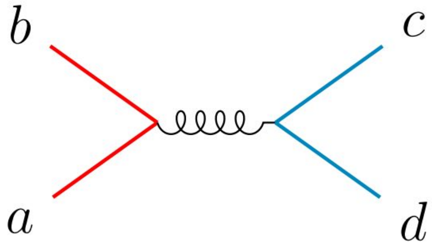
$$n_{3,2}(k_1^m, k_2^m, k_3) = \alpha_1(k_1 \cdot \varepsilon_3)$$

Tree-level amplitudes are completely determined by color-kinematics and factorization



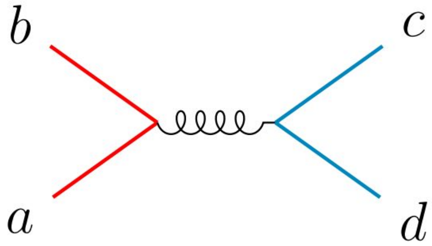
$$n_{4,2}(a,b,c,d) = \alpha_1(a\cdot b) + \alpha_2(b\cdot b) + \alpha_3(b\cdot c) + \alpha_4(c\cdot c)$$

Tree-level amplitudes are completely determined by color-kinematics and factorization

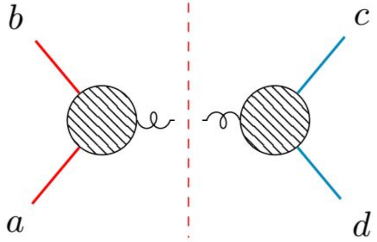


$$n_{4,2}(a,b,c,d) = \frac{\alpha_3}{2}\Big((a\cdot b) + (b\cdot b) + 2(b\cdot c)\Big)$$

Tree-level amplitudes are completely determined by color-kinematics and factorization

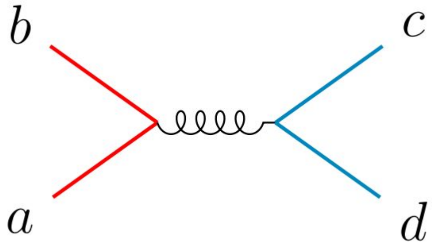


$$n_{4,2}(a,b,c,d) = \frac{\alpha_3}{2}\Big((a\cdot b) + (b\cdot b) + 2(b\cdot c)\Big)$$



$$\sum_{s\in \text{states}} A_{3,2}(a,b,l^s)A_{3,2}(-l^{\bar{s}},c,d) = n_{4,2}(a,b,c,d)|_{(a+b)^2=0}$$

Tree-level amplitudes are completely determined by color-kinematics and factorization

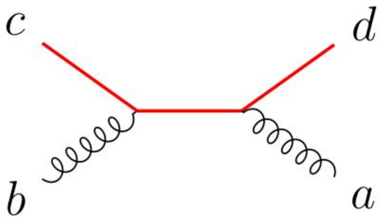


$$n_{4,2}(a,b,c,d) = \frac{\alpha_3}{2}\Big((a\cdot b) + (b\cdot b) + 2(b\cdot c)\Big)$$

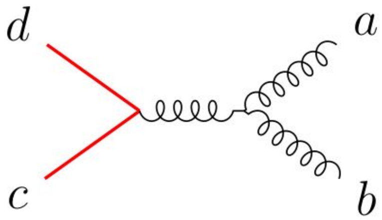
$$A^{\text{tree}}_{4,2}(a,b,c,d) = -\frac{(a\cdot b) + (b\cdot b) + 2(b\cdot c)}{2\big[(a\cdot b) + (b\cdot b)\big]}$$

Tree-level amplitudes are completely determined by color-kinematics and factorization

Tree-level amplitudes are completely determined by color-kinematics and factorization

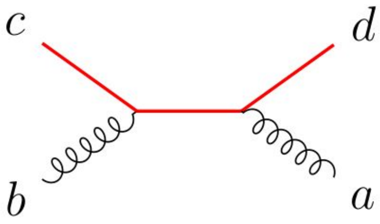


$= n_{\color{red}{m}}(a, b, c, d),$

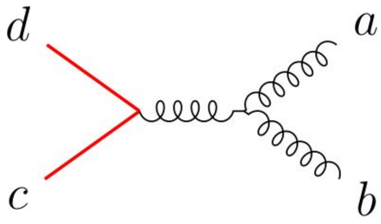


$= n_{\overline{m}}(a, b, c, d)$

Tree-level amplitudes are completely determined by color-kinematics and factorization

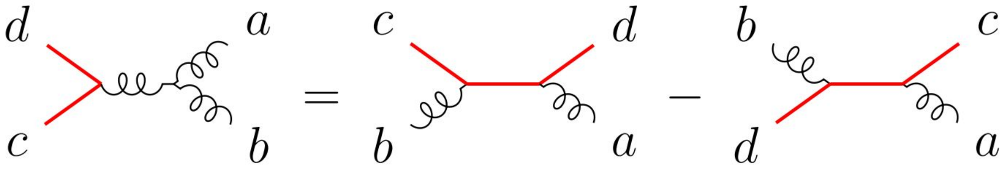


$$= n_m(a,b,c,d),$$



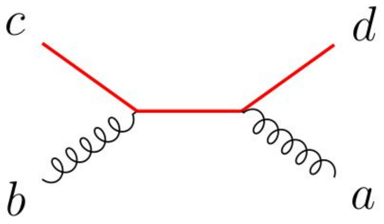
$$= n_{\overline{m}}(a,b,c,d)$$

Color-kinematics

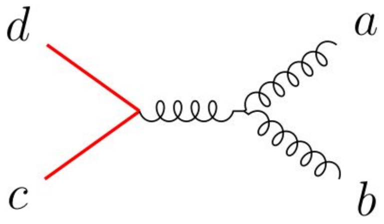


$$n_{\overline{m}}(a,b,c,d) = n_m(a,b,c,d) - n_m(b,a,c,d)$$

Tree-level amplitudes are completely determined by color-kinematics and factorization



$= n_{m}(a,b,c,d),$

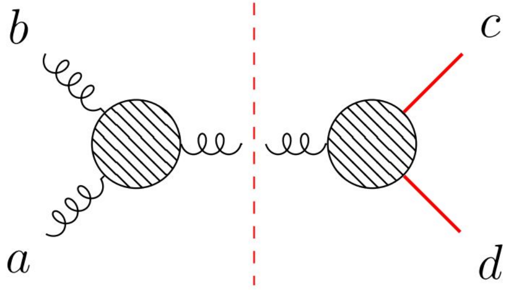


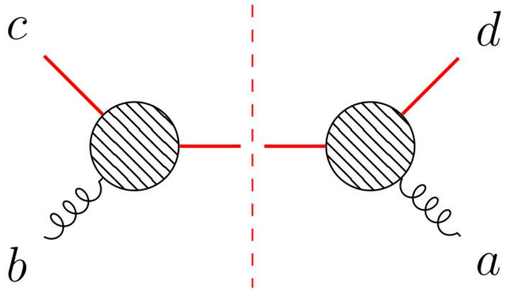
$= n_{\overline{m}}(a,b,c,d)$

$$
\begin{aligned}
n_{m}(a,b,c,d) = &\left(\alpha_1(a\cdot b) + \alpha_2(b\cdot c) + \alpha_3(c\cdot c)\right)(\varepsilon_a\cdot\varepsilon_b) + \alpha_4(b\cdot\varepsilon_a)(a\cdot\varepsilon_b) \\
&+ \alpha_5(c\cdot\varepsilon_a)(a\cdot\varepsilon_b) + \alpha_6(b\cdot\varepsilon_a)(c\cdot\varepsilon_b) + \alpha_7(c\cdot\varepsilon_a)(c\cdot\varepsilon_b)
\end{aligned}
$$

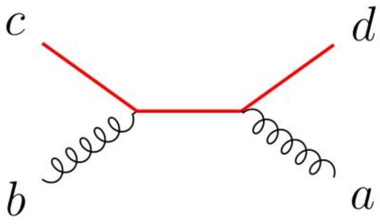
Tree-level amplitudes are completely determined by color-kinematics and factorization

Factorization

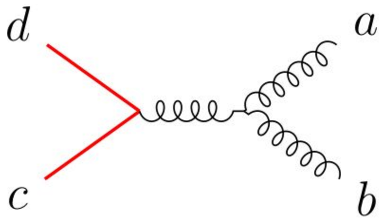




Tree-level amplitudes are completely determined by color-kinematics and factorization



$= n_m(a,b,c,d),$

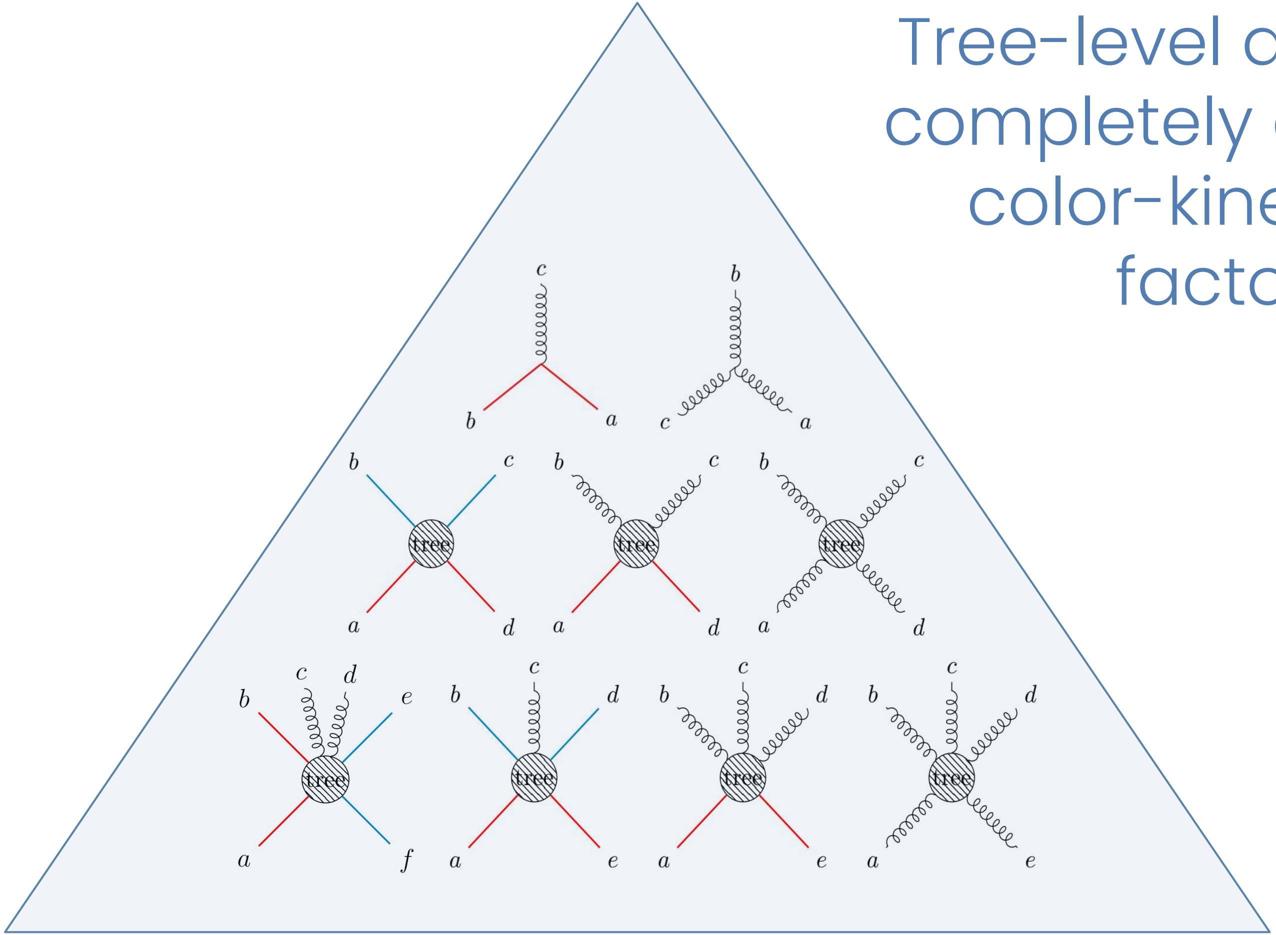


$= n_{\overline{m}}(a,b,c,d)$

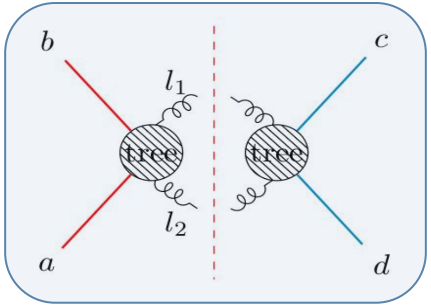
$$n_m(a,b,c,d) = (c \cdot \varepsilon_b)\left((b \cdot \varepsilon_a) + (c \cdot \varepsilon_a)\right) - \frac{1}{2}(b \cdot c)(\varepsilon_a \cdot \varepsilon_b)$$

$$n_{\overline{m}}(a,b,c,d) = n_m(a,b,c,d) - n_m(b,a,c,d)$$

Tree-level amplitudes are completely determined by color-kinematics and factorization

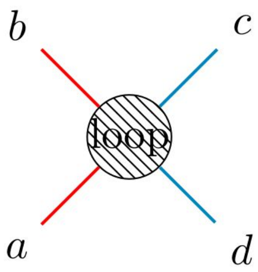


Construction loop-level



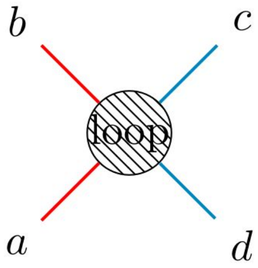
Loop-level amplitudes are determined by color-kinematics and unitarity cuts

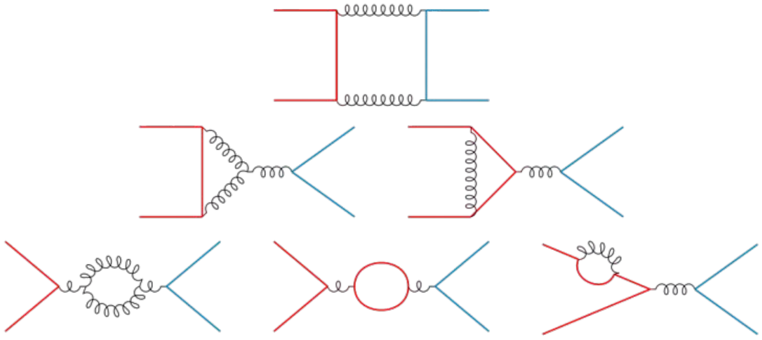
4-point one-loop



Loop-level amplitudes are determined by color-kinematics and unitarity cuts

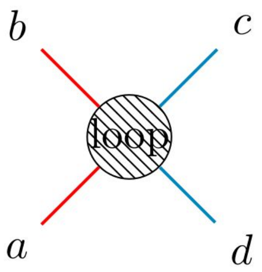
4-point one-loop

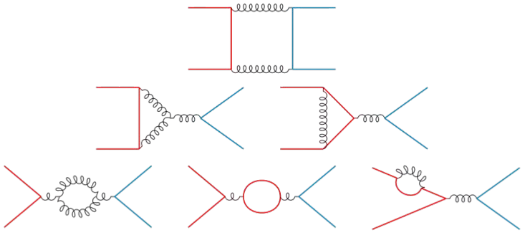




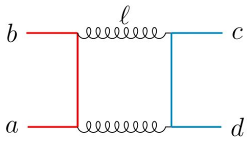
Loop-level amplitudes are determined by color-kinematics and unitarity cuts

4-point one-loop



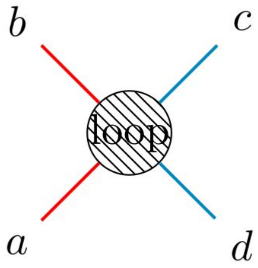


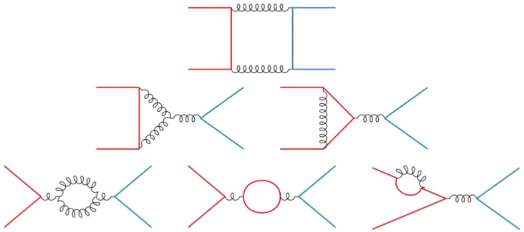
Loop level color-kinematics



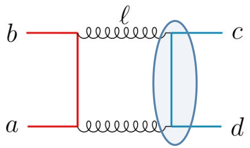
Loop-level amplitudes are determined by color-kinematics and unitarity cuts

4-point one-loop



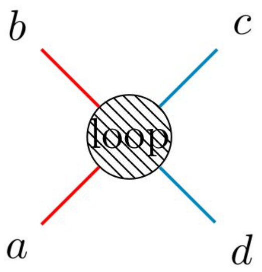


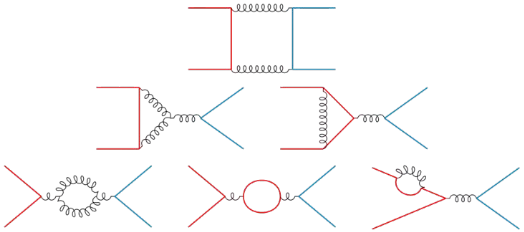
Loop level color-kinematics



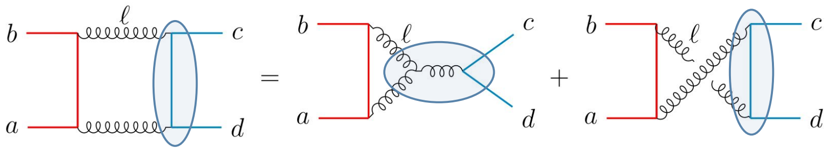
Loop-level amplitudes are determined by color-kinematics and unitarity cuts

4-point one-loop



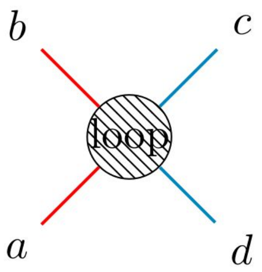


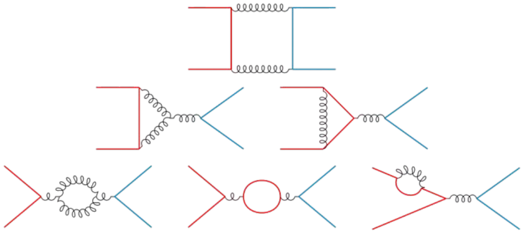
Loop level color-kinematics



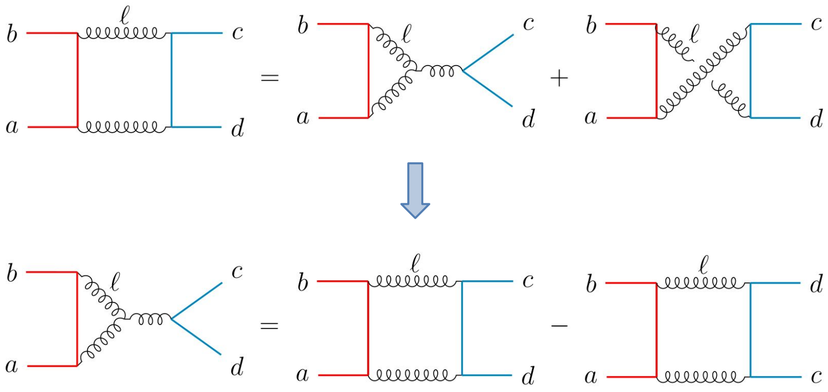
Loop-level amplitudes are determined by color-kinematics and unitarity cuts

4-point one-loop



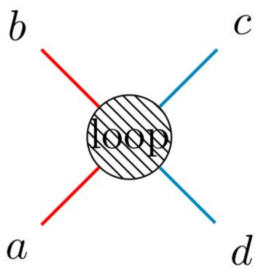


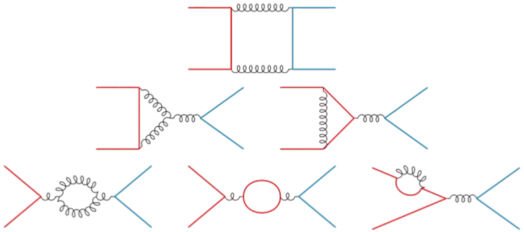
Loop level color-kinematics



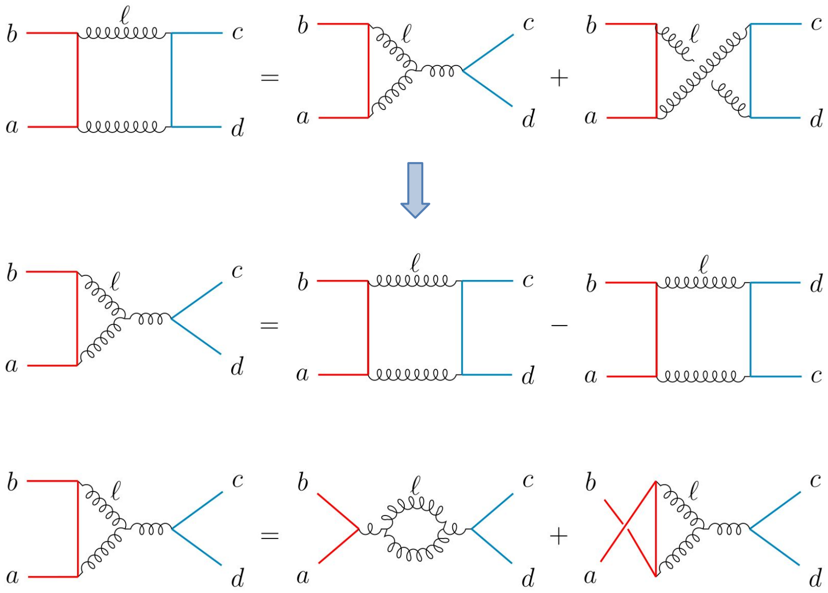
Loop-level amplitudes are determined by color-kinematics and unitarity cuts

4-point one-loop



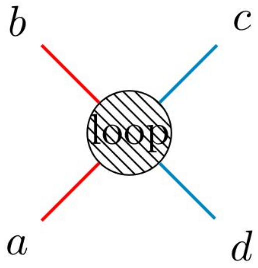


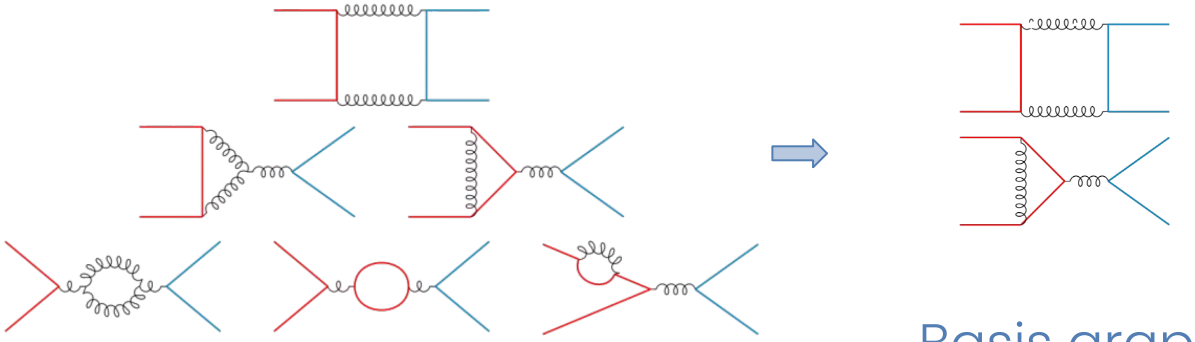
Loop level color-kinematics



Loop-level amplitudes are determined by color-kinematics and unitarity cuts

4-point one-loop

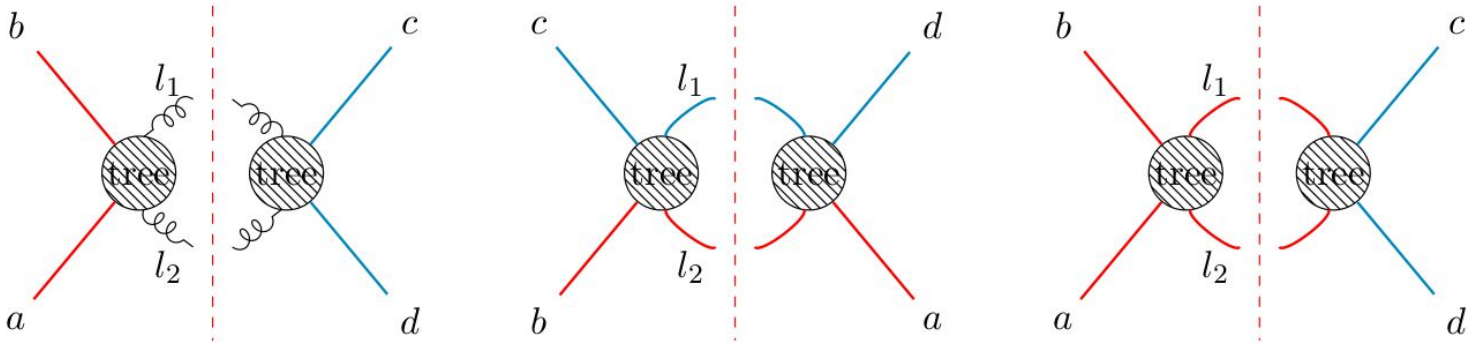




Basis graphs

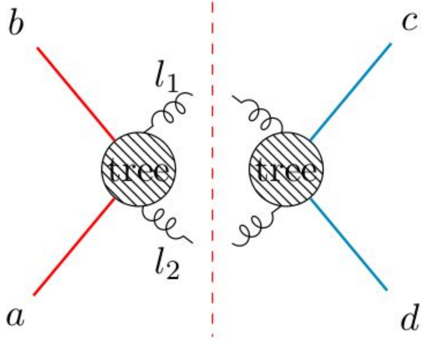
Loop-level amplitudes are determined by color-kinematics and unitarity cuts

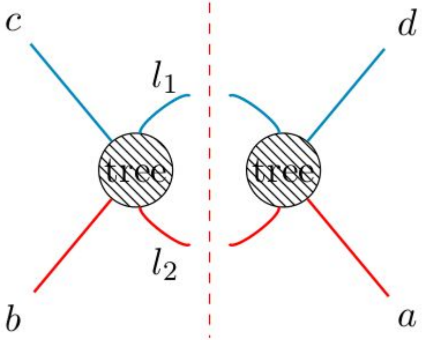
4-point one-loop

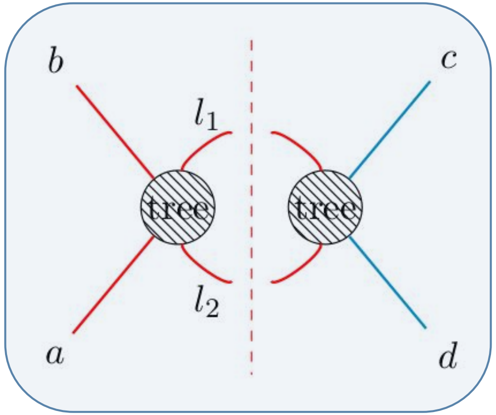


Loop-level amplitudes are determined by color-kinematics and unitarity cuts

4-point one-loop

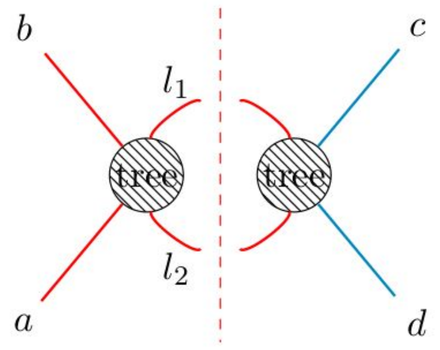




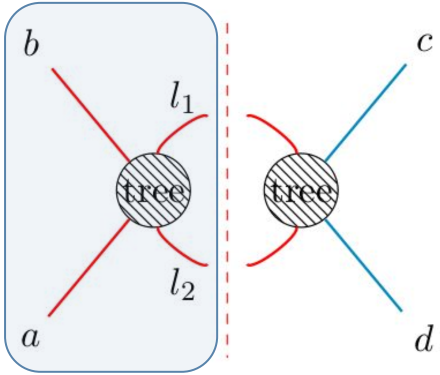


Ordered cut?

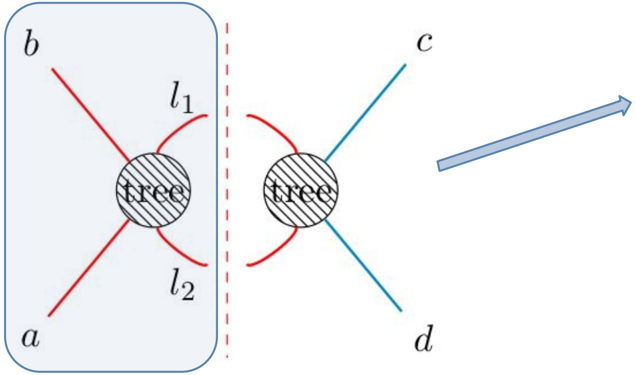
What does *ordered cut* mean?



What does *ordered cut* mean?

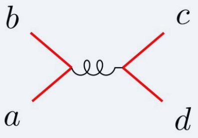


What does *ordered cut* mean?

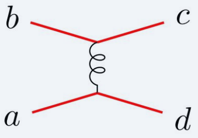


Adjoint-type ordered cut

$$A^{\text{tree}}(a,b,c,d) = $$

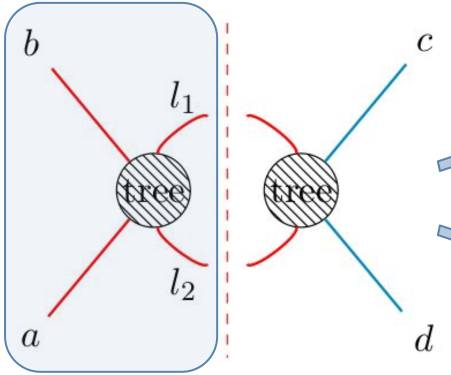


$+$

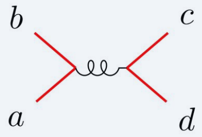
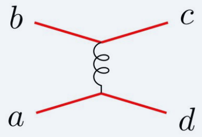


$$= \frac{n_s}{s} + \frac{n_t}{t}$$

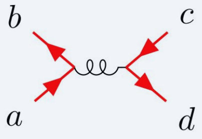
What does *ordered cut* mean?

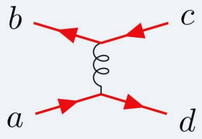


Adjoint-type ordered cut

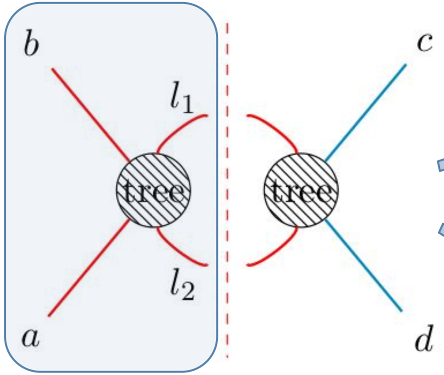
$A^{\text{tree}}(a,b,c,d) =$  $+$  $= \frac{n_s}{s} + \frac{n_t}{t}$

Fundamental-type ordered cut

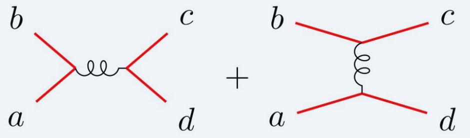
$A^{\text{tree}}(\overline{a},b,\overline{c},d) =$  $= \frac{n_s}{s}$

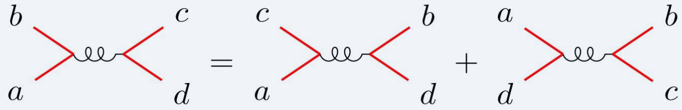
$A^{\text{tree}}(\overline{a},d,\overline{c},b) =$  $= \frac{n_t}{t}$

What does *ordered cut* mean?

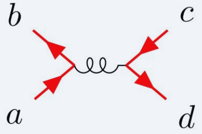


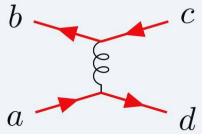
Adjoint-type ordered cut

$A^{\text{tree}}(a,b,c,d) =$  $= \frac{n_s}{s} + \frac{n_t}{t}$



Fundamental-type ordered cut

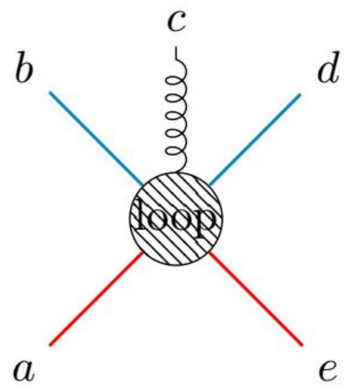
$A^{\text{tree}}(\overline{a},b,\overline{c},d) =$  $= \frac{n_s}{s}$

$A^{\text{tree}}(\overline{a},d,\overline{c},b) =$  $= \frac{n_t}{t}$

Loop-level amplitudes are determined by color-kinematics and unitarity cuts

5-point one-loop

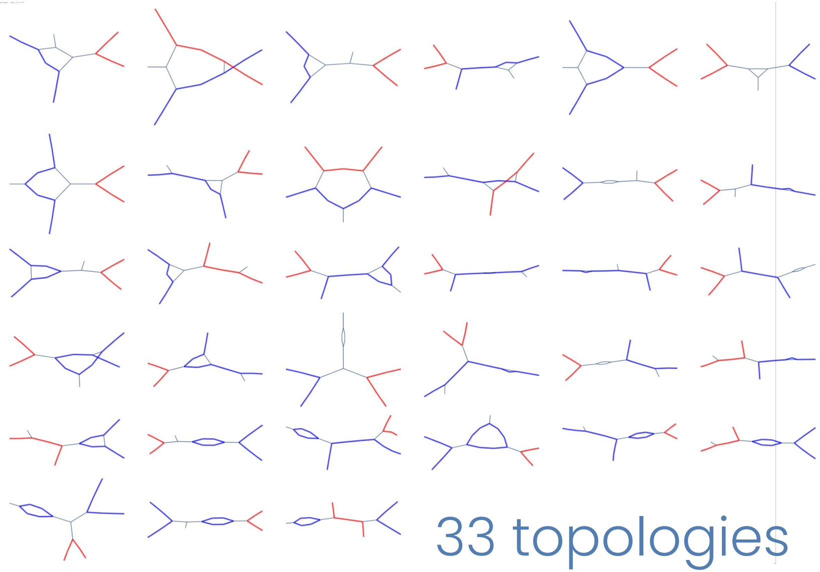
Encodes first correction to radiation



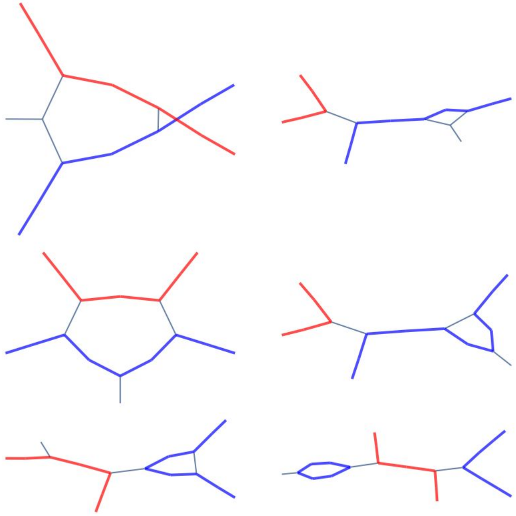
Loop-level amplitudes are determined by color-kinematics and unitarity cuts

5-point one-loop

6 basis graphs



33 topologies



Loop-level amplitudes are determined by color-kinematics and unitarity cuts

5-point one-loop

33 topologies

10 296
parameters

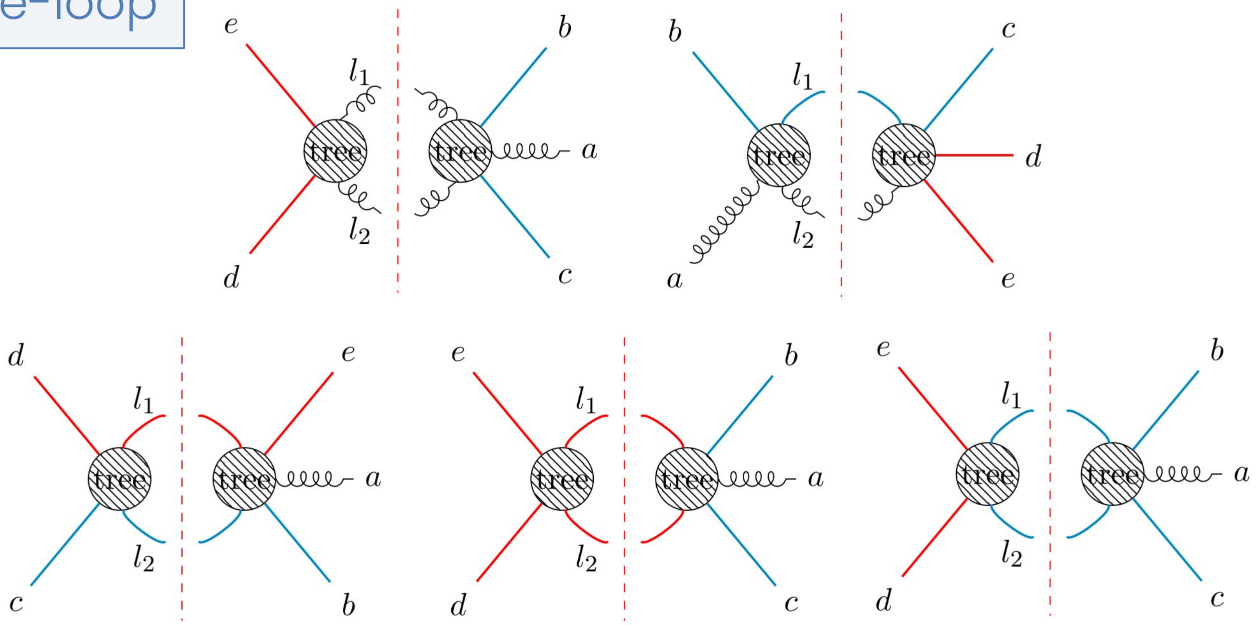
→

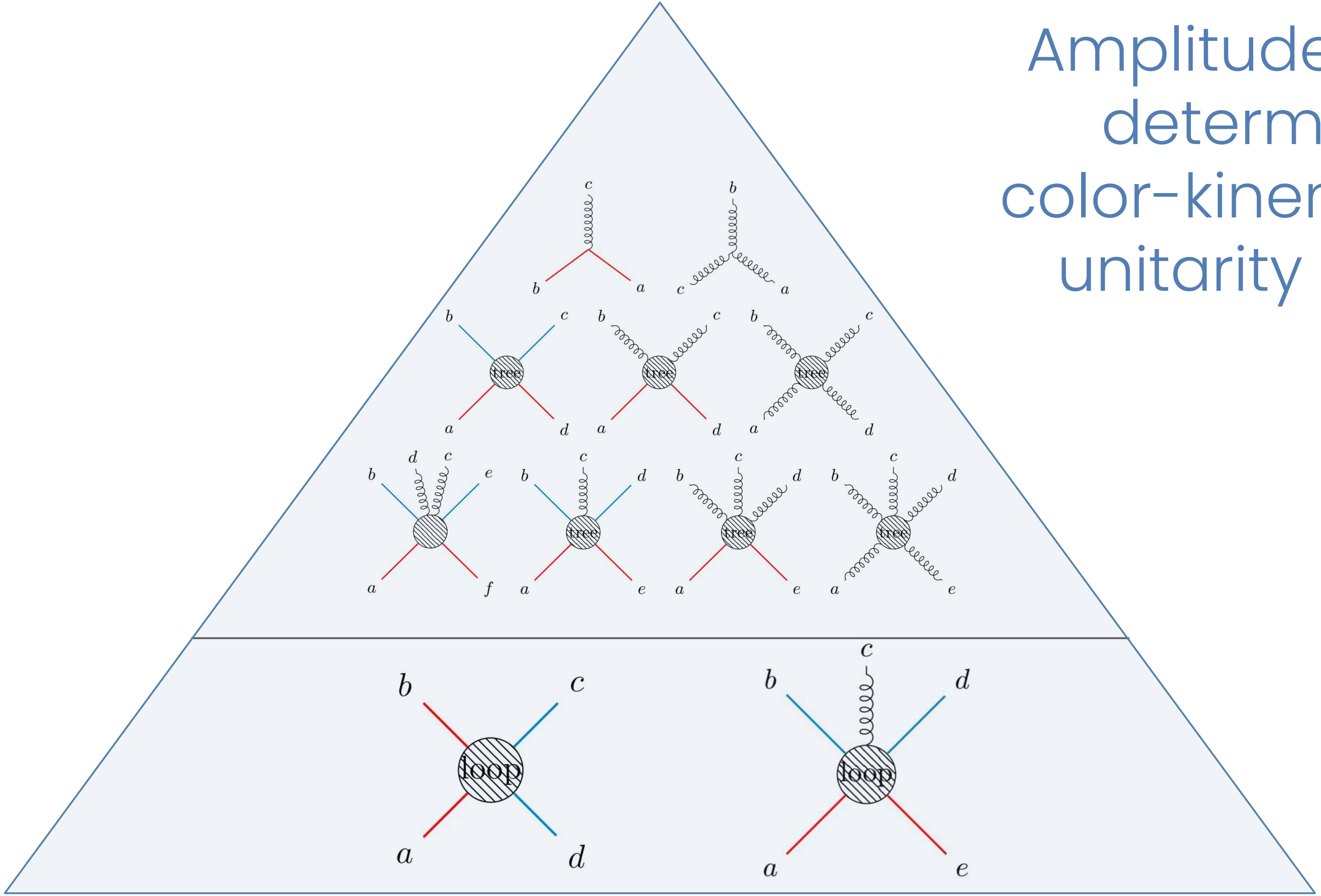
6 basis graphs

1872
parameters

Loop-level amplitudes are determined by color-kinematics and unitarity cuts

5-point one-loop





Amplitudes are fully determined by color-kinematics and unitarity methods

1 loop

Double copy to obtain N=0 supergravity predictions

$$S=\int d^D x\sqrt{-g}\left[-\frac{1}{2}R+\frac{1}{2(D-2)}\partial^\mu\phi\partial_\mu\phi+\frac{1}{6}e^{-4\phi/(D-2)}H^{\lambda\mu\nu}H_{\lambda\mu\nu}\right]$$

Scherk, Schwarz '74
Gross, Sloan '87

\+ Matter terms

Johansson, Ochirov '16,'19
Bautista, Guevara '19
Plefka, Shi, Wang '19

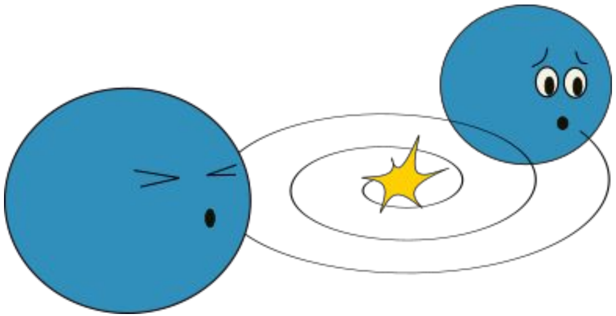
Future work includes

exploring double copies directly to pure gravity predictions,

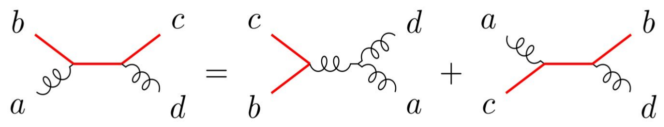
massive higher-spins in arbitrary rep,

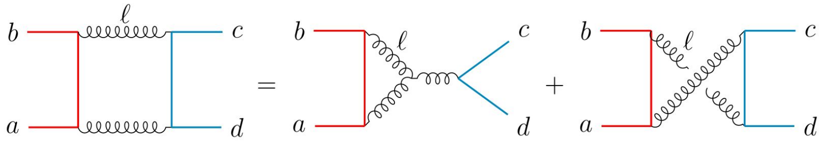
generating classical observables,

extending to higher loop order (2-loops and more)!



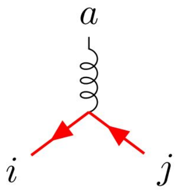
Color-kinematics and factorization are enough to build massive amplitudes





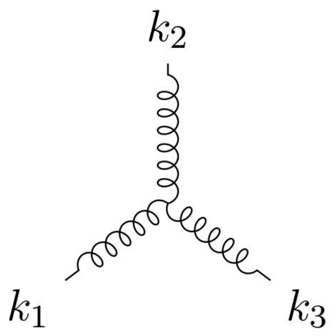
Color-kinematics duality for massive amplitudes can be manifest at loop level

Can use the same kinematic building blocks for massive scalars charged in fundamental and adjoint



Extra

3-point amplitudes are completely determined by color-kinematics and symmetries



Build the ansatz from a kinematic basis

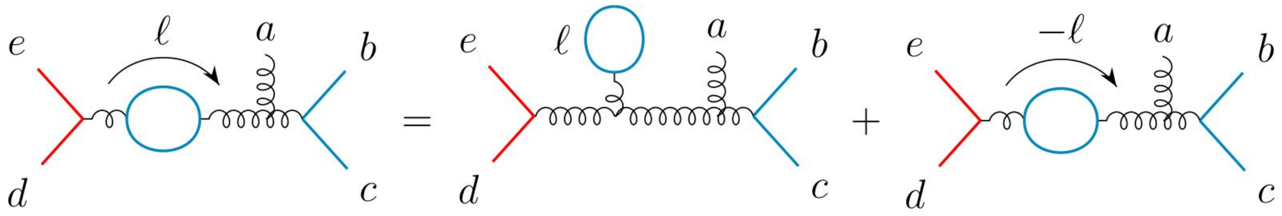
$$n_3(k_1, k_2, k_3) = \alpha_1(k_3 \cdot \epsilon_1)(\varepsilon_2 \cdot \varepsilon_3) + \alpha_2(k_3 \cdot \varepsilon_2)(\varepsilon_1 \cdot \varepsilon_3) + \alpha_3(k_2 \cdot \varepsilon_3)(\varepsilon_1 \cdot \varepsilon_2)$$

Momentum conservation $\quad k_1 + k_2 + k_3 = 0$

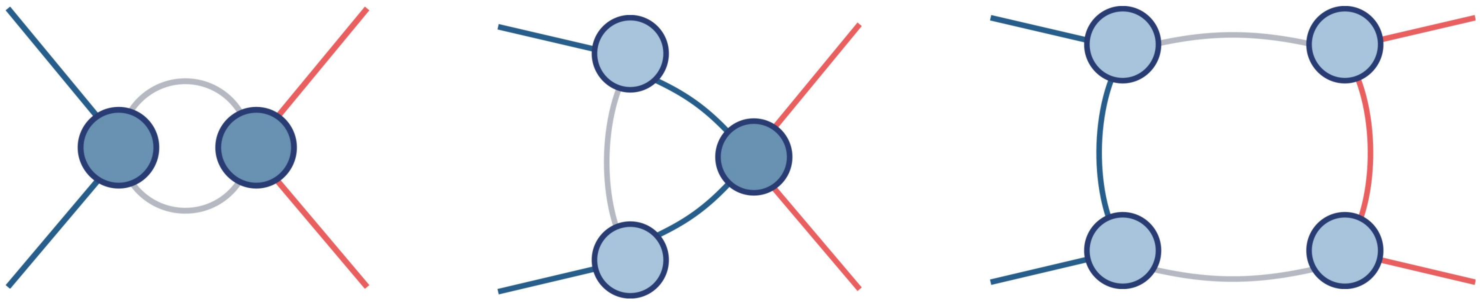
Transversality $\quad \varepsilon_i \cdot k_i = 0$

$$k_2 \cdot \varepsilon_1 = (-k_1 - k_3) \cdot \varepsilon_1 = -k_3 \cdot \varepsilon$$

Tadpoles can be reached using color-kinematics

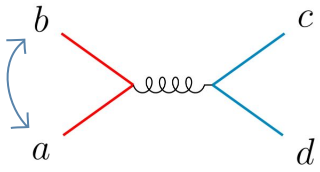


We verified on bubble, triangle and box cuts



Tree-level amplitudes are completely determined by color-kinematics and factorization

What are symmetries of the graph?



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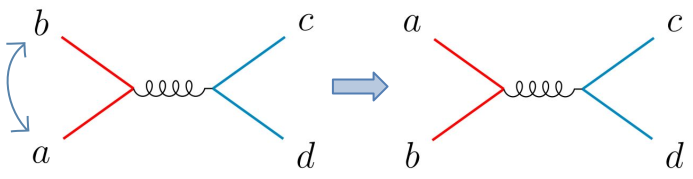
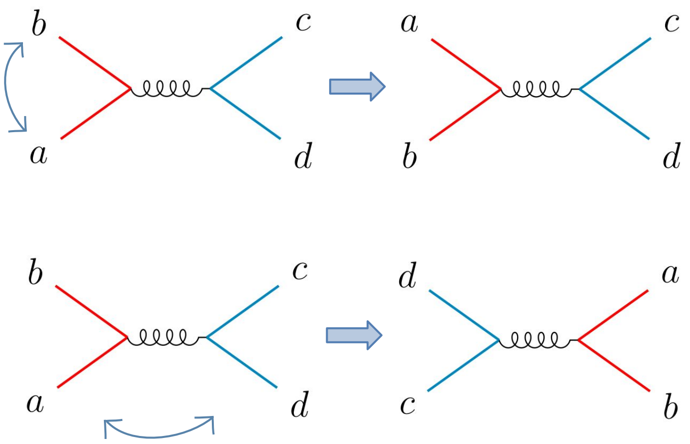


$$n_{4,2}(b,a,c,d) = -n_{4,2}(a,b,c,d)$$

Tree-level amplitudes are completely determined by color-kinematics and factorization

What are symmetries of the graph?



$$n_{4,2}(b,a,c,d) = -n_{4,2}(a,b,c,d)$$

$$n_{4,2}(c,d,a,b) = n_{4,2}(a,b,c,d)$$

$$\vdots$$