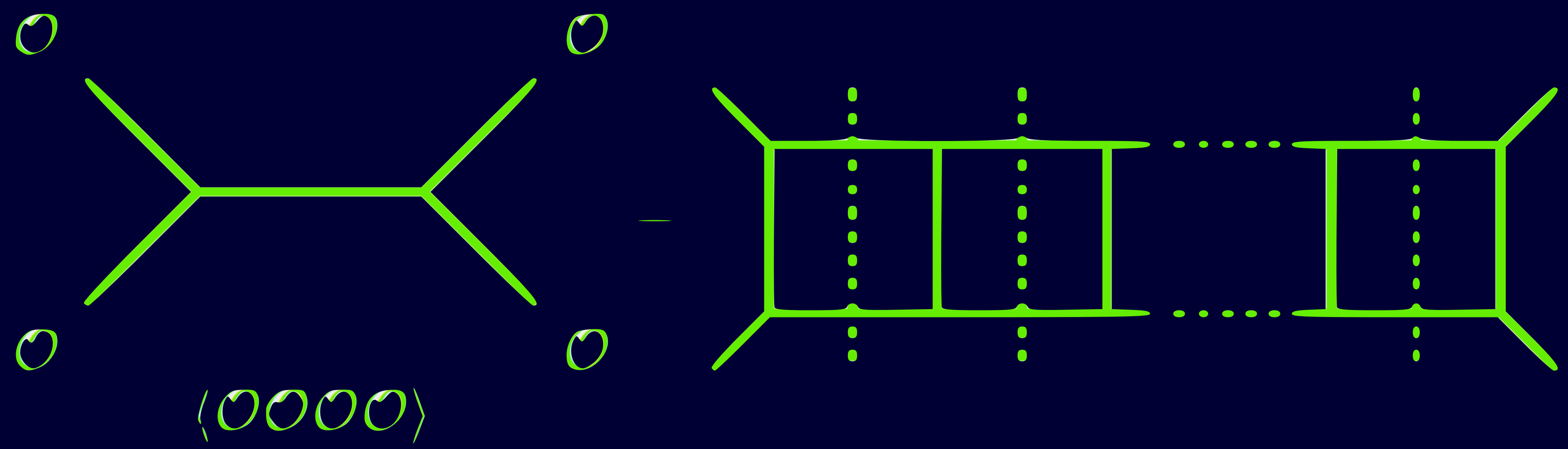


All loop Structures in Supergravity

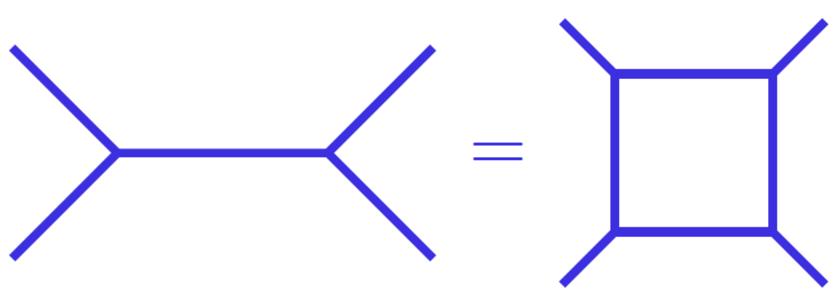
AGNESE BISSI, UPPSALA UNIVERSITY, DEC 4TH 2020– QCD MEETS GRAVITY 2020

based mostly on

- A.B., G. Fardelli and A. Georgoudis, arXiv: 2010.12557
- A.B., G. Fardelli and A. Georgoudis, arXiv: 2002.04604

Goals

- Understand better the unitarity structure of $AdS_5 \times S^5$ supergravity amplitudes, at all loops in G_N.
- Apply CFT bootstrap techniques to get information on amplitudes in curved space-times, which are hard to access with more conventional amplitude techniques.
- This method has been very successful at one loop $\longrightarrow$ full amplitude



Alday, AB
Aprile, Drummond, Heslop, Paul
Alday, Caron-Huot

- What can we say about all loops?

AdS/CFT correspondence

- 4 dimensional SU(N) $\mathcal{N} = 4$ Super Yang Mills (SYM) at large N and large 't Hooft coupling $\lambda = g_{YM}^2 N$

$$\lambda = \frac{L^4}{\ell_s^4} \qquad\qquad \frac{g_{YM}^2}{4\pi} = g_s$$

- type IIB supergravity on $AdS_5 \times S^5$, with coupling g_s

Observables

- correlation functions in the SCFT

- scattering amplitudes in $AdS_5 \times S^5$

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- correlation functions in the SCFT

$\mathcal{O}_p$ → Half-BPS operators, of dimension p and transforming under the [0,p,0] of SU(4)

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Kaluza Klein modes

Observables

- correlation functions in the SCFT

$\mathcal{O}_p \longrightarrow$ Half-BPS operators, of dimension p and transforming under the [0,p,0] of SU(4)

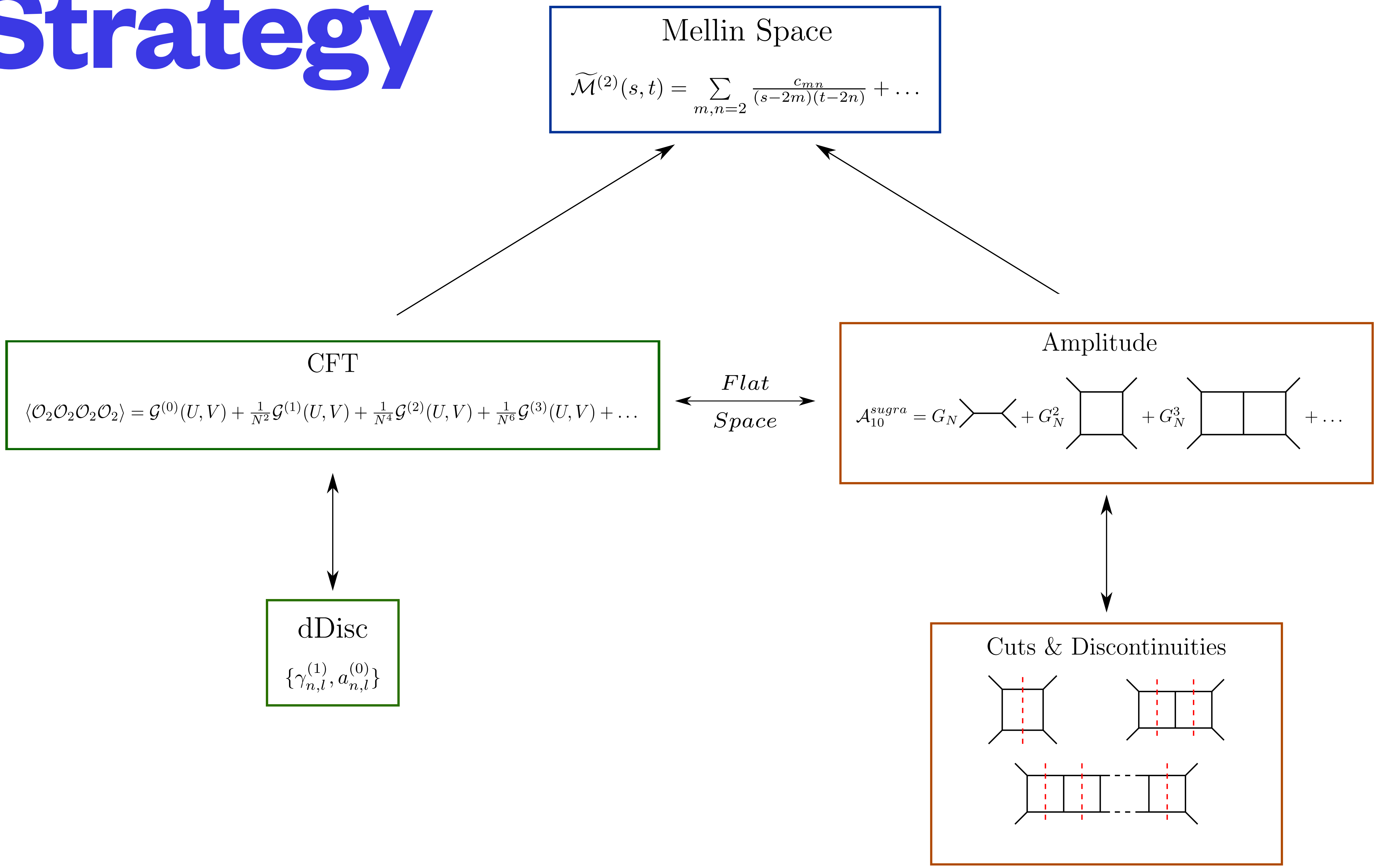
$\mathcal{O}_2 \longrightarrow$ superconformal primary operator of the stress tensor supermultiplet, $\Delta = 2$

- scattering amplitudes in $AdS_5 \times S^5$

Kaluza Klein modes

Graviton supermultiplet

Strategy


Mellin Space
$\widetilde{\mathcal{M}}^{(2)}(s,t) = \sum_{m,n=2} \frac{c_{mn}}{(s-2m)(t-2n)} + \dots$
CFT
$\langle \mathcal{O}_2\mathcal{O}_2\mathcal{O}_2\mathcal{O}_2 \rangle = \mathcal{G}^{(0)}(U,V) + \frac{1}{N^2}\mathcal{G}^{(1)}(U,V) + \frac{1}{N^4}\mathcal{G}^{(2)}(U,V) + \frac{1}{N^6}\mathcal{G}^{(3)}(U,V) + \dots$
Flat
Space
Amplitude
$\mathcal{A}_{10}^{sugra} = G_N$
$+ G_N^2$
$+ G_N^3$
$+ \dots$
dDisc
$\{\gamma_{n,l}^{(1)}, a_{n,l}^{(0)}\}$
Cuts & Discontinuities

Four point function

$$\langle \mathcal{O}_2(x_1)\mathcal{O}_2(x_2)\mathcal{O}_2(x_3)\mathcal{O}_2(x_4)\rangle = \frac{G(u,v)}{x_{12}^4 x_{34}^4} = \frac{\sum_R G^{(R)}(u,v)}{x_{12}^4 x_{34}^4}$$

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$[0,2,0] \times [0,2,0] =$ six representations

- Superconformal Ward identities: 1) algebraic relations among the different six $G^{(R)}(u,v)$, 2) all the six functions can be written in terms of a single function $F(u,v)$

Block decomposition

$$F(u,v) = \sum_{\Delta,\ell} a_{\Delta,\ell} u^{\frac{\Delta-\ell}{2}} g_{\Delta,\ell}(u,v)$$

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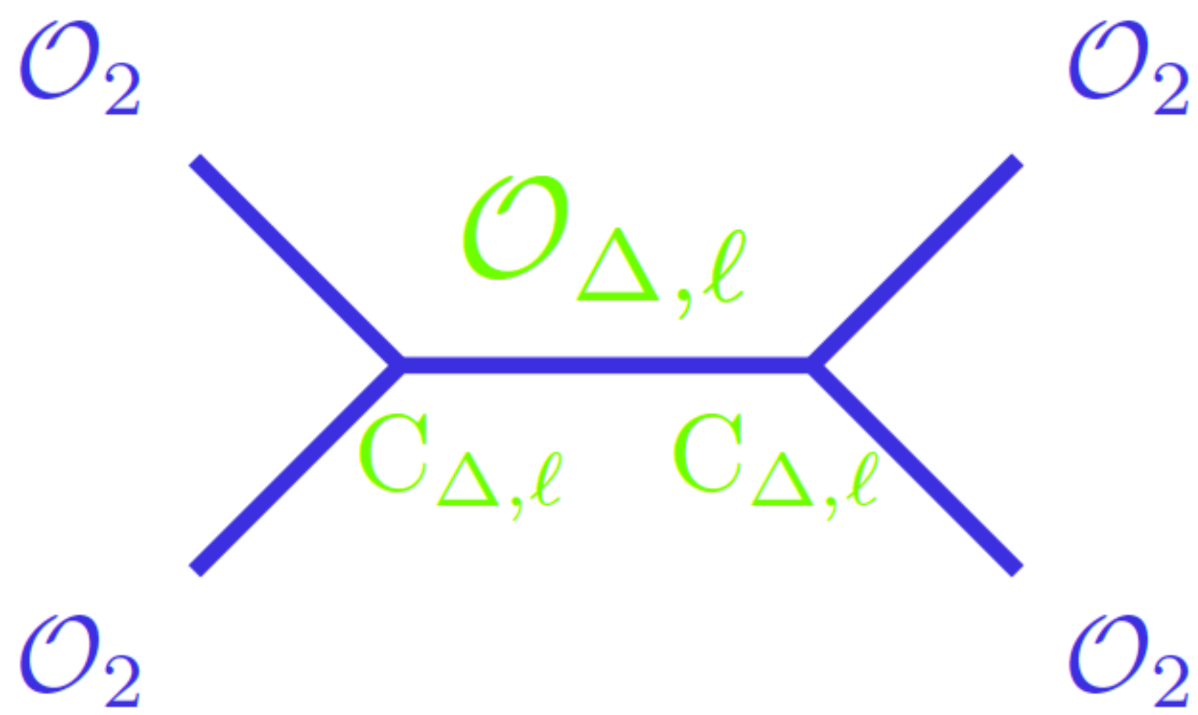

SUSY constraints

$\mathcal{O}_{\Delta,\ell}$ $\Bigg\{$

operators belonging to short and semishort multiplets

$a_{\Delta,\ell} \sim \langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_{\Delta,\ell} \rangle^2$ and Δ protected

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$$F(u,v) = G^{\text{short}}(u,v,N) + \mathcal{H}(u,v,N,\lambda)$$

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$$\mathcal{H}(u,v) = \sum_{\Delta,\ell} \tilde{a}_{\Delta,\ell} u^{\frac{\Delta-\ell}{2}} \tilde{g}_{\Delta,\ell}(u,v)$$

LONG SUPERCONFORMAL PRIMARIES

Expansions

- Expand $\mathscr{H}(u,v)$, $\tilde{a}_{\Delta,\ell}$ and Δ for large N and large λ.

all orders

leading

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INTERMEDIATE OPERATORS:
double trace operators at leading order $\mathcal{O}(N^{-4})$
higher trace at higher orders

One loop example

LEADING ORDER CAVEAT & FEATURES

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- Mixing problem: different operators with same ‘’bare’’ dimension

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inversion formula   large spin perturbation theory

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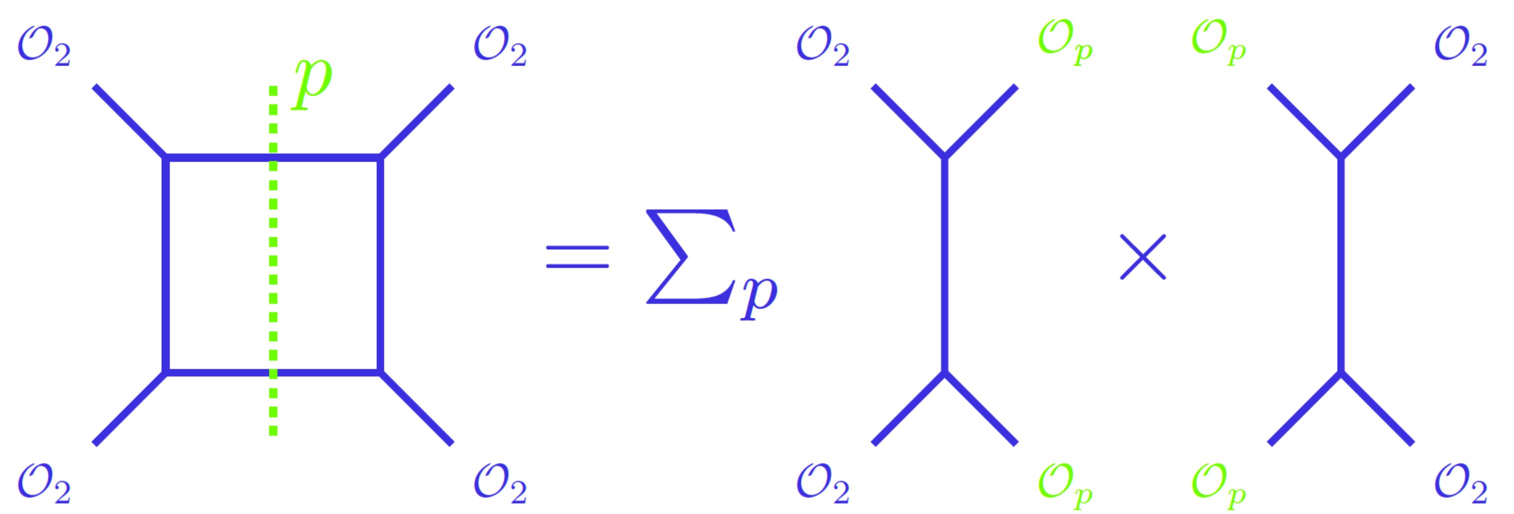

Highest trascendental at all loops

- At any loop order $N^{-2\kappa}$ there is a term of the form:

$$\frac{\log^\kappa u}{2^\kappa \kappa!} \sum_{I,n,\ell} u^{n+2} \tilde{a}^{(0)}_{n,\ell,I} \left(\gamma^{(1)}_{n,\ell,I}\right)^\kappa \tilde{g}_{2n+\ell,\ell}(u,v)$$

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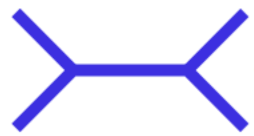
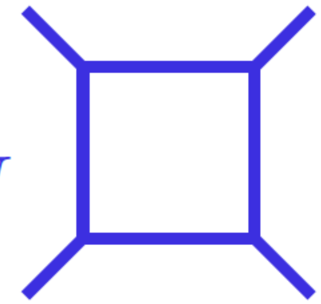
To which term in the dual amplitude it corresponds to?

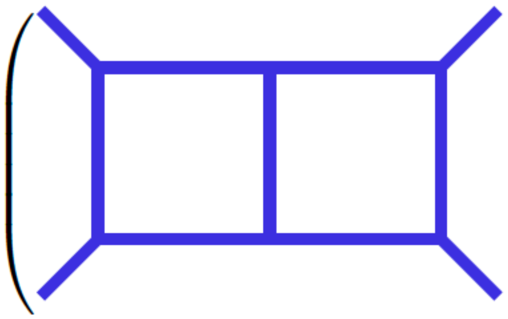
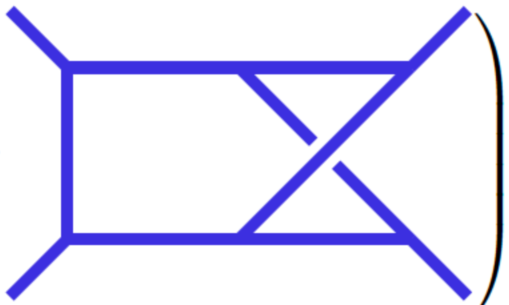
Idea

- Take a limit in the correlator which corresponds to the flat space limit (bulk point limit)
- The sums of the previous slide are much simpler!
- Compare the result with the 10d four graviton supergravity amplitude in flat space.

Flat space amplitude

Flat space amplitude

$\mathcal{A}_{10}^{sugra} = G_N$  $+G_N^2$ 

$+G_N^3 \Bigg($  $+$  $\Bigg) + \ldots$

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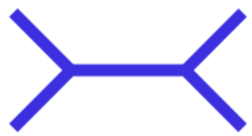
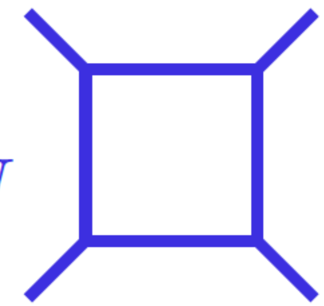
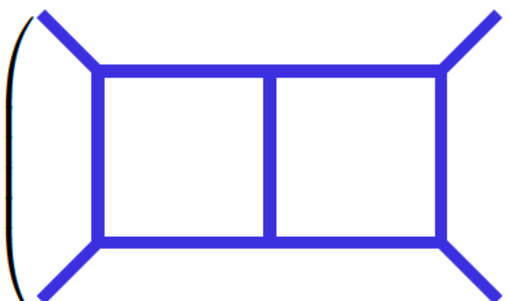
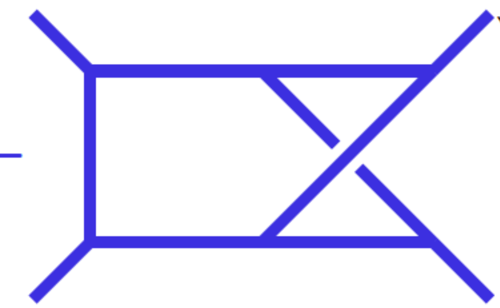
$\mathcal{A}_{10}^{sugra} = G_N$  $+G_N^2$ 

$+G_N^3 \Big($  $+$  $\Big) + \dots$

non planar

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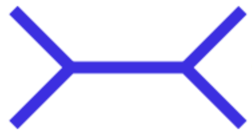
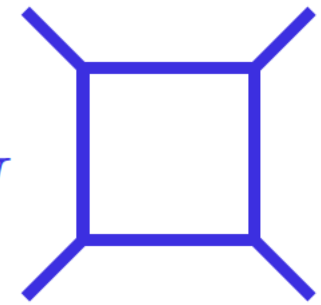
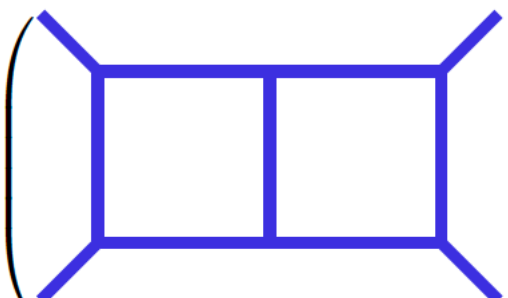
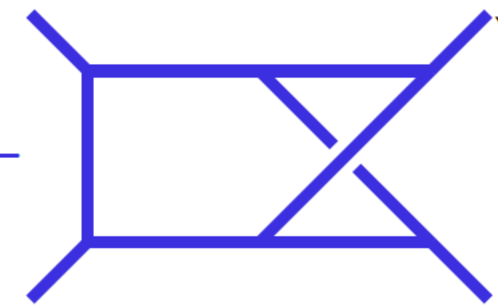
$\mathcal{A}_{10}^{sugra} = G_N$  $+ G_N^2$ 

$G_N = \dfrac{\pi^4 L^8}{2(N^2-1)}$

$+ G_N^3 \Big($  $+$  $\Big) + \ldots$

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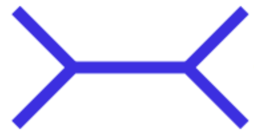
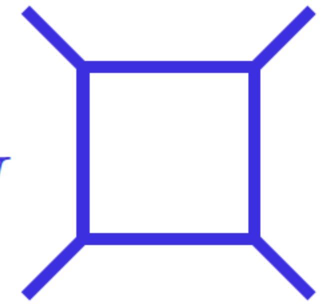
$\mathcal{A}_{10}^{sugra} = G_N$  $+G_N^2$ 

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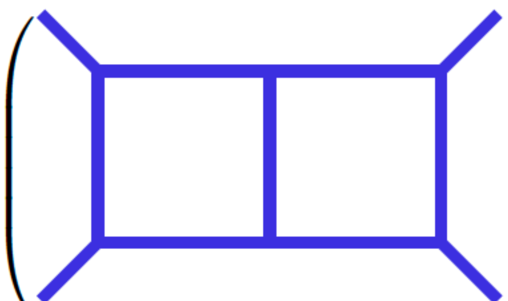
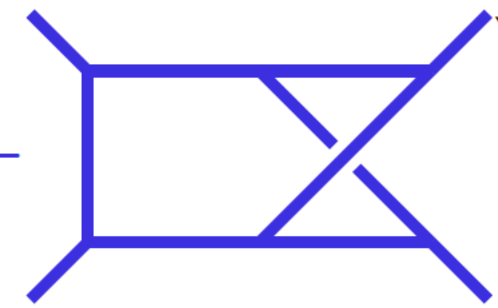
$+G_N^3 \Big($  $+$  $\Big) + \ldots$

non planar

- Reduce to 5 dimensions, by computing discontinuities, for ex. at two loops

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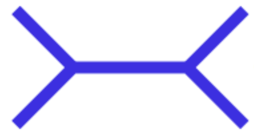
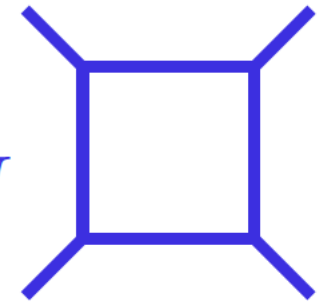
$\mathcal{A}_{10}^{sugra} = G_N$  $+G_N^2$ 

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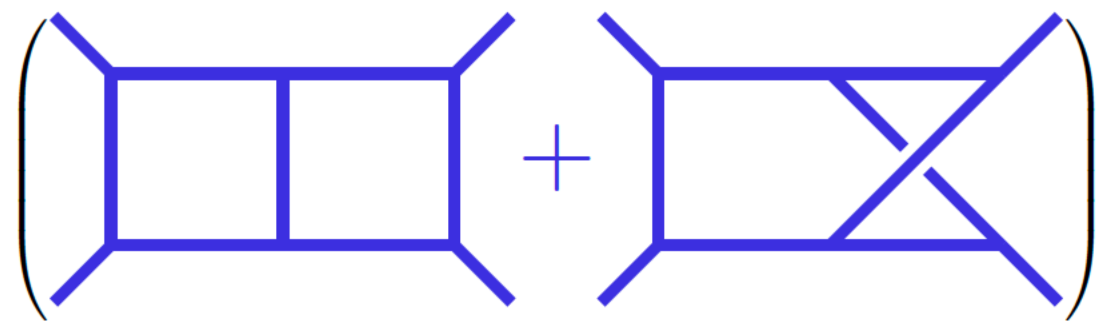
$+G_N^3$  $+\ldots$

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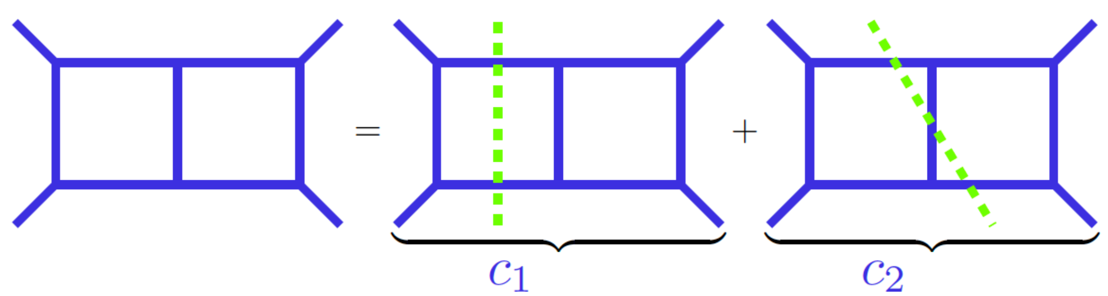
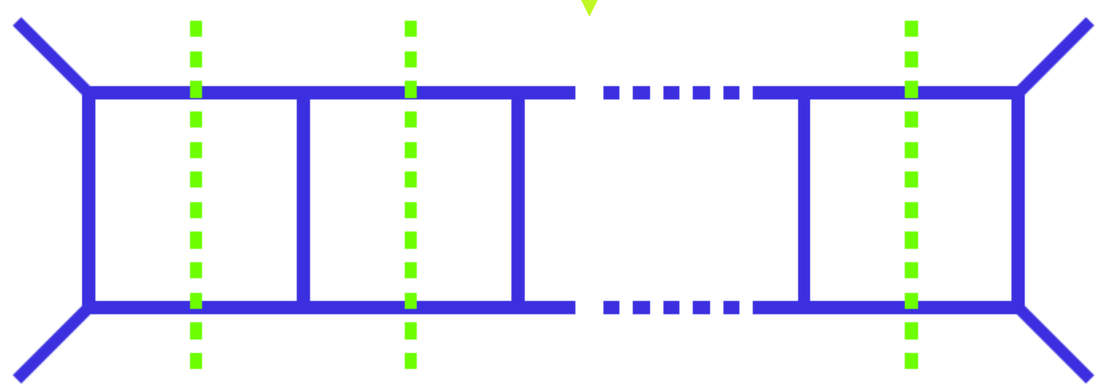
$2\pi i \; \mathrm{disc}_s$ 

Result

$$\frac{\log^{\kappa} u}{2^{\kappa}\kappa!}\sum_{I,n,\ell} u^{n+2}\tilde{a}^{(0)}_{n,\ell,I}\left(\gamma^{(1)}_{n,\ell,I}\right)^{\kappa}\tilde{g}_{2n+\ell,\ell}(u,v)$$

FLAT SPACE LIMIT



ITERATED s-CUTS

Results

- All loop structure in the CFT dual to iterated s-cuts in the amplitude
- We believe that this is valid also in curved space

Open problems

- Extend this result to include stringy corrections
- Resum the contribution coming from all the cuts
- Understand better the singularity structure
- Try to match it away from flat space