S² Hamiltonian from Amplitudes and EFT **QCD Meets Gravity VI**

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Outline

Roiban, Shen, Zeng)

• S_1^2 Hamiltonian (based on 2012.xxxxx w/Kosmopoulos+)

Review of Spin Hamiltonians (based on 2005.03071 w/Bern,

(a briefest) Introduction

(a not very needed) Motivation

(EST)

9:00-9:25

9:25-9:50

12:00-12:25

12:25-12:50

1:00-1:35

1:35-2:00

2:00-2:25

2:25-2:50

3:00-3:25

3:25-3:50

4:00-4:25

4:25-4:50

5:00-5:25

5:25-5:50



The talks related with classical gravity

An era of gravitational waves...

Monday, Nov 30	Tuesday, Dec 1	Wednesday, Dec 2	Thursday, Dec 3	Friday, Dec
Free-Discussion (Euro-friendly)	Free-Discussion (Euro-friendly)	Free-Discussion (Euro-friendly)	Free-Discussion (Euro-friendly)	Free-Discussi (Euro-friendly
	Song He		Daniel Baumann	
	Gabriele Veneziano		Yvonne Geyer	
Dónal O'Connell	Michael Green	Tanja Hinderer	Rafael Porto	David Berman
Ben Maybee	Hadleigh Frost	Carlo Heissenberg	Silvia Nagy	Tim Adamo
Andrea Puhm	Yael Shadmi	Michèle Levi	Andrea Cristofoli	Ingrid Vazquez-He
Sebastian Mizera	Simon Caron-Huot	Cristian Vergu	Gregor Kälin	Agnese Bissi
Aichael Ruf	Suna Zekioglu	Sebastian Poegel	Gustav Mogull	Jung-Wook Kim
Cynthia Keeler	Julio Parra-Martinez	Shruti Paranjape	Chris White	Andres Luna
lames Mangan	Oliver Schlotterer	Callum Jones	Nima Arkani-Hamed	Giulio Salvatori
Free-Discussion West-coast-friendly)	Free-Discussion (West-coast-friendly)	Free-Discussion (West-coast-friendly)	Free-Discussion (West-coast-friendly)	Zvi Bern (3:50-4:0
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Review of Spin Hamiltonians

arXiv.org > hep-th > arXiv:2005.03071

High Energy Physics – Theory



Zvi Bern, Andres Luna, Radu Roiban, Chia-Hsien Shen, Mao Zeng















We consider a Lagrangian of rank-s tensor fields, minimally coupled to gravity

$$\mathcal{L}_{\min} = -R(e,\omega) + \frac{1}{2}g^{\mu}\nabla(\omega)_{\mu}\phi_s\nabla(\omega)_{\mu}\phi$$



The spin tensor is obtained from the classical limit











Symmetric traceless tensor field

Lorentz Generator

Spin vector/tensor





Deduce Feynman rules from Lagrangian, and compute Compton amplitude via 4 Feynman diagrams



$$\sum_{\lambda} \varepsilon(k)_{\lambda}^{\mu\nu} \varepsilon(-k)_{-\lambda}^{\alpha\beta} = \frac{1}{2} \mathcal{P}^{\mu\alpha} \mathcal{P}^{\nu\beta} + \frac{1}{2} \mathcal{P}^{\nu\alpha} \mathcal{P}^{\mu\beta} - \frac{1}{2} \mathcal{P}^{\mu\alpha} \mathcal{P}^{\mu\beta} + \frac{1}{2} \mathcal{P}^{\mu\alpha} \mathcal{P}^{\mu\beta} + \frac{1}{2} \mathcal{P}^{\mu\alpha} \mathcal{P}^{\mu\beta} - \frac{1}{2} \mathcal{P}^{\mu\alpha} \mathcal{P}^{\mu\beta} + \frac{1}{2} \mathcal{P}^{\mu\alpha} \mathcal{P}^{\mu\alpha} + \frac{1}{2} \mathcal{P}^{\mu\alpha} \mathcal{P}^{\mu\alpha} + \frac{1}{2} \mathcal{P}^{\mu\alpha} \mathcal{P}^{\mu\alpha} + \frac{1}{2} \mathcal{P}^{\mu\alpha} + \frac{1}{2$$

The amplitudes satisfy Generalized Ward Identities...

$$\sum_{\lambda} \varepsilon(k)^{\mu\nu}_{\lambda} \varepsilon(-k)^{\alpha\beta}_{-\lambda} = \frac{1}{2} \eta^{\mu\alpha} \eta^{\nu\beta} + \frac{1}{2} \eta^{\nu\alpha} \eta^{\mu\beta} - \frac{1}{2} \eta^{\mu\alpha} \eta^{\mu\beta} + \frac{1}{2} \eta^{\mu\alpha} \eta^{\mu\alpha} \eta^{\mu\alpha} + \frac{1}{2} \eta^{\mu\alpha} \eta^{\mu\alpha} \eta^{\mu\beta} + \frac{1}{2} \eta^{\mu\alpha} \eta^{\mu\alpha} \eta^{\mu\alpha} \eta^{\mu\alpha} + \frac{1}{2} \eta^{\mu\alpha} \eta^{\mu\alpha} \eta^{\mu\alpha} \eta^{\mu\alpha} + \frac{1}{2} \eta^{\mu\alpha} \eta^$$

...so we sew using de Donder

Sew Compton (and 3-point) amplitudes into unitarity (3- and 4-particle) cuts

 $\frac{1}{D-2}\eta^{\mu
u}\eta^{lphaeta}$

Use generalized unitarity (Forde's method) to obtain box and triangle coefficients

$$i\mathcal{M}_4^{1 \text{ loop}} = d_{\mathrm{B}} I_{\mathrm{B}} + d_{\overline{\mathrm{B}}} I_{\overline{\mathrm{B}}} + c_{\Delta} I_{\Delta}$$

Cheung, Rothstein and Solon's EFT of non-relativistic fields...

$$S = \int_{m{k}} \sum_{a=1,2} \xi_a^{\dagger}(-m{k}) \left(i \partial_t - \sqrt{m{k}^2 + m_i^2}
ight) \xi_a(m{k}) - \int_{m{k},m{k}'} \xi_1^{\dagger}(m{k}') \xi_2^{\dagger}(-m{k}) \left[\widehat{V}(m{k}',m{k},\hat{m{S}}_a) \right] \xi_1(m{k}) \xi_2(-m{k})$$

...generalized to describe spinning fields. The potential...

...considers long range interactions...

a - A - Britist Standing and and Road to Baca and

 $\hat{\mathbb{O}}^{(1,1)} = oldsymbol{L}_q \cdot \hat{oldsymbol{S}}_1 \, ,$ $\hat{\mathbb{O}}^{(0)} = \mathbb{I},$ $\hat{\mathbb{O}}^{(2,1)} = oldsymbol{q} \cdot \hat{oldsymbol{S}}_1 \,oldsymbol{q} \cdot \hat{oldsymbol{S}}_2\,, \qquad \hat{\mathbb{O}}^{(2,2)} = oldsymbol{q}^2 \,\hat{oldsymbol{S}}_1 \cdot \hat{oldsymbol{S}}_2\,,$ Ô

$$\hat{V}^{A}(\boldsymbol{k}',\boldsymbol{k}) = \frac{4\pi G}{\boldsymbol{q}^{2}} d_{1}^{A}\left(\boldsymbol{p}^{2}\right) + \frac{2\pi^{2}G^{2}}{|\boldsymbol{q}|} d_{2}^{A}\left(\boldsymbol{p}^{2}\right) + \mathcal{O}(G)$$

$$egin{aligned} & (1,2) = m{L}_q \cdot \hat{m{S}}_2 \,, \ & (2,3) = m{q}^2 \, m{k} \cdot \hat{m{S}}_1 \, m{k} \cdot \hat{m{S}}_2 \,. \end{aligned}$$

...and is organized by classical spin operators

$$S = \int_{\boldsymbol{k}} \sum_{a=1,2} \xi_a^{\dagger}(-\boldsymbol{k}) \left(i\partial_t - \sqrt{\boldsymbol{k}^2 + m_i^2} \right) \xi_a(\boldsymbol{k})$$

Deduce Feynman rules from the effective theory action...

$$\underbrace{(E, \mathbf{k})}_{=} = \frac{i \mathbb{I}}{E - \sqrt{\mathbf{k}^2 + m^2} + i\epsilon}$$

$m{k} = -\int_{m{k},m{k}'} \, \xi_1^\dagger(m{k}') \xi_2^\dagger(-m{k}') \, \widehat{V}(m{k}',m{k},\hat{m{S}}_a) \, \xi_1(m{k}) \xi_2(-m{k}) \, .$

...and use them to compute the amplitude

$$\widehat{\mathcal{M}}^{\text{EFT}} = - \hat{V}(\boldsymbol{p}', \boldsymbol{p}) - \int_{\boldsymbol{k}} \sum_{\boldsymbol{E}} \hat{V}(\boldsymbol{p}', \boldsymbol{p}) - \hat{V}($$

 $V(oldsymbol{p}',oldsymbol{k})$ $(oldsymbol{k},oldsymbol{p})$ $E_1 + E_2 - \sqrt{k^2 + m_1^2} - \sqrt{k^2 + m_2^2}$ Maring a service to go to see the grant of the service of the serv

S_{FFT} +

 $\mathcal{M}_{2\mathrm{PM}}^{\mathrm{EFT}} = rac{2\pi^2 G^2}{|oldsymbol{q}|} igg[a_2^{(0)} + a_2^{(1,1)} oldsymbol{L}_q \cdot oldsymbol{S}_1 + a_2^{(1,2)} oldsymbol{L}_q \cdot oldsymbol{S}_2 + a_2^{(2,1)} oldsymbol{q} \cdot oldsymbol{S}_1 \,oldsymbol{q} \cdot oldsymbol{S}_2 + a_2^{(2,2)} oldsymbol{q}^2$

$$+ (4\pi G)^2 a_{ ext{iter}} \int rac{d^{D-1}\ell}{(2\pi)^{D-1}} rac{2\xi E}{\ell^2 (\ell+q)^2 (\ell^2)}$$

 $+2\boldsymbol{p}\cdot\boldsymbol{\ell})$

$$\frac{P^2}{E}A_2\left[d_1^{(1,1)}d_1^{(1,2)}\right] + \frac{\xi E}{8}(d_1^{(1,1)} + d_1^{(1,2)})d_1^{(2,1)}$$

$$A_2[X] = \left[\frac{1}{4}(1 - 3\xi) + \frac{\xi^2 E^2}{p^2} + \frac{1}{2}\xi^2 E^2\partial\right]X$$

1 loop

 $H^{(2,1)}(r^2, p^2)$

From full theory

$$) = \frac{1}{r^4} \left[\frac{G}{r} c_1^{(2,1)}(\boldsymbol{p}^2) + \left(\frac{G}{r}\right)^2 c_2^{(2,1)}(\boldsymbol{p}^2) + \mathcal{O}(G^3) \right]$$

Determine coefficients in the Hamiltonian from Matching

$${}_{3}\left[c_{1}^{(0)}c_{1}^{(2,1)}\right] + \frac{p^{2}}{2\xi E}A_{2}\left[c_{1}^{(1,1)}c_{1}^{(1,2)}\right] + \frac{\xi E}{24}\left(c_{1}^{(1,1)} + c_{1}^{(1,2)}\right)$$

Iteration pieces

$H(r^2, p^2)$

Write equations of motion...

$$\dot{\boldsymbol{r}} = rac{\partial H}{\partial \boldsymbol{p}}, \qquad \dot{\boldsymbol{p}} = -rac{\partial H}{\partial \boldsymbol{r}},$$

...and solve perturbatively

$$m{r}(t) = m{r}_0(t) + Gm{r}_1(t) + G^2m{r}_2(t) + \dots ,$$

 $m{p}(t) = m{p}_0(t) + Gm{p}_1(t) + G^2m{p}_2(t) + \dots ,$
 $m{S}_a(t) = m{S}_{a,0}(t) + Gm{S}_{a,1}(t) + G^2m{S}_{a,2}(t) + \dots$

Use the Hamiltonian to determine physical observables

$$\dot{\boldsymbol{S}}_a = -\boldsymbol{S}_a imes \frac{\partial H}{\partial \boldsymbol{S}_a}$$
, $a = 1, 2$

$$\Delta O_n = \int_{-\infty}^{\infty} dt \, \frac{dO_n}{dt}$$

Cut the middleman!

Physical observables (directly) from Amplitudes

Boundary2Bound (Kalin, Liu, Porto), KMOC (+Vines, Cristofoli), Eikonal (ACV, DiVecchia, Heissenberg, Russo, Bjerrum-Bohr, Damgaard, Cristofoli, Bern, Parra-Martinez, Ruf, Zeng, GOV, ...) Several other contributors (some of them in the audience(?) I apologize in advance...) Talks by: Dónal, Ben, Gabriele, Julio, Carlo, Rafael, Andrea, Gregor, Jung-Wook

loop $d^2 \boldsymbol{q}$ χ_2 (2π) $4m_1m_2\sqrt{\sigma^2}$

Eikonal phase-like object. Fourier transform to impact parameter space of the (triangle part) of the amplitude.

 $\mathcal{M}^{1 \text{ loop}}_{\Lambda}$

 $\chi_2 = \frac{1}{4m_1m_2\sqrt{\sigma^2 - 1}} \int \frac{d^2 \boldsymbol{q}}{(2\pi)^2} e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} \mathcal{M}^{\Delta + \nabla}(\boldsymbol{q})$

 $\mathcal{D}_{SL}(f,g) \equiv -\sum_{a=1,2} \epsilon^{ijk} S_a^k \frac{\partial f}{\partial S_a^i} \frac{\partial g}{\partial L^j}$

Iteration-like pieces

 $\mathcal{M}^{\perp loop}$

 $\chi_2 = \frac{1}{4m_1m_2\sqrt{\sigma^2 - 1}} \int \frac{d^2\boldsymbol{q}}{(2\pi)^2} e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} \mathcal{M}^{\Delta + \nabla}(\boldsymbol{q})$

A conjecture for an all-orders generalization of the formula

(* ::Package:: *)

Ancillary file for "Spinning Black Hole Binary Dynamics, Scattering Amplitudes and Effective Field Theory" by Z. Bern, A. Luna, R. Roiban, C.-H. Shen and M. Zeng

Impulse and spin kick through $O\left(G^{2}\right)$ from the solution to the equations of motion of the S1-S2 Hamiltonian. Only terms linear in the spin of each particle are displayed, including S1-S2 terms.

Impulse: DeltaP = G DeltaP1 + G^2 DeltaP2 Spin kick for particle 1: DeltaS[1] = G DeltaS[1, 1] + G^2 DeltaS[1, 2] and the spin kick for particle 2, DeltaS[2], is obtained by switching labels

Notation: ml, m2: mass of particle 1, 2 El, E2: energy of particle 1, 2 El2 = El+E2 gammal = El/m1 gamma2 = E2/m2

sigma = (p1.p2)/(m1 m2) = (E1 E2 + pSq)/(m1 m2) xi = E1 E2/E12^2

B = {bx, by, bz} impact parameter. For Fig. 11 of paper, B = (-b, 0, 0) P: incoming three momentum p in CM frame. For Fig. 11 of paper, P = pinfty = (0, 0, 0, 0) = (-5, 0) = (-1, 0) = [0, 0, $|p_i(nty)|$ Lin = B x P (x is cross product). S[1], S[2] : three vector spin of particle 1, 2 in rest frame of each

Sperp[1], Sperp[2] : projection of the rest frame three vector spin of particle 1, 2 to be orthogonal the incoming three-momentum p in CM frame.

the incoming three See Eq. (7.13) of paper. L: norm of Lin pSq: p^2 of incoming three momentum p in CM frame bSq : square of the impact parameter vector B

dot[A, B] : scalar product of the 3-dimensional vectors A and B cross[A, B] : cross product of the 3-dimensional vectors A and B

 $\begin{array}{c} c0[\{i,j\},\ 1] \ and \ c0[\{i,j\},\ 2] \ are \ the \ position \ space \ Hamiltonian \\ coefficients \ c[\{i,j\},\ 1] \ and \ c[\{i,j\},\ 2] \ in \ Eq. \end{array}$ (6.36)

evaluated on the incoming three momentum p in $evaluated on the incoming three momentum p \\ C(M frame. \\ c[(i,j), 1]: Eqs. (6.37) \\ c[(i,j), 2]: Eqs. (6.38) \\ Explicit expressions for c0[(i,j), 1] and c0[(i,j), 2] are obtained by \\ evaluating c[(i,j), 1] and c[(i,j), 2] in \\ the ancillary file coefficients.m on the incoming three-momentum p. \\ \end{cases}$

c0[{i,j}, 1, 1] : derivative of c0[{i,j}, 1] with respect to p^2 c0[{i,j}, 2, 1] : derivative of c0[{i,j}, 2] with respect to p^2

*)

AXFunction = {AX[n1, n2, n3, XX] :> (n2 pSq (1 - 3 xi) + E12^2 n3 xi^2) XX + E12^2 n1 pSq xi^2 DD[XX]}; EvaluateDD = {DD[(AA_,) c0[a_, b_] c0[c_, d_]] :> AA (c0[a, b, 1] c0[c, d] + c0[a, b] c0[c, d, 1]), DD[(AA_.) c0[a_, b_]^2] :> 2 AA (c0[a, b, 1] c0[a, b]);

NoSDeltaP1 = (2 B E12 xi c0[{0}, 1])/(Sqrt[bSq] L);

SODeltaP1 = -((2 E12 xi c0[{1, 1}, 1] cross[P, S[1]])/(Sqrt[bSq] L)) - (
2 E12 xi c0[{1, 2}, 1] cross[P, S[2]])/(Sqrt[bSq] L) + (
4 B E12 xi c0[(1, 1), 1] dot[Lin, S[1]])/(bSq^(3/2) L) + (
4 B E12 xi c0[(1, 2), 1] dot[Lin, S[2]])/(Sqr(3/2) L);
Sop1DeltaP1 = B ((16 E12 xi c0[{2, 1}, 1] dot[B, S[1]] dot[B, S[2]])/(

Supletrar1 = B ([10 E12 x1 cu[{2, 1}, 1] dot[B, S[1]] dot[B, S[2]])/(
 3 bSq^{5(5/2)} L) + (
 4 E12 xi cu[{2, 1}, 1] dot[P, S[1]] dot[P, S[2]])/(
 3 Sqrt[bSq] L^{3}) - (
 4 E12 xi cu[{2, 1}, 1] dot[B, S[2]] Sperp[1])/(3 bSq^{3/2}) L) - (
 4 E12 xi cu[{2, 1}, 1] dot[B, S[1]] Sperp[2])/(3 bSq^{3/2}) L);
 Sop2DeltaP1 = (4 B E12 xi cu[{2, 2}, 1] dot[P, S[1]] dot[P, S[1]] dot[P,
 S[1])/(bSq^{3/2}) L); S[2]])/(bSq^(3/2) L);

NoSDeltaP2 = -({2 bSg E12^2 P xi^2 c0[{0}, 1]^2}/L^4) - (B \[Pi] (AX[2, 1, 0, c0[{0}, 1]^2] - 2 E12 pSg xi c0[{0}, 2]))/(2 L^3);

SODeltaP21 = (\[Pi] (-AX[2, 1, 2, c0[{0}, 1] c0[{1, 1}, 1]] +

SODeltaP21 = (\[Pi] (-AX[2, 1, 2, c0[{0}, 1] c0[{1, 1}, 1]] + E12 psq xi c0[{1, 1}, 2]) cross[P, S[1])/L/3 + (\[Pi] (-AX[2, 1, 2, c0[{0}, 1] c0[{1, 2}, 1]] + E12 psq xi c0[{1, 2}, 2]) cross[P, S[2]])/L^3 + P (-((4 E12^2 xi^2 c0[{0}, 1] c0[{1, 2}, 1] dot[Lin, S[1]])/L^4) - (4 E12^2 xi^2 c0[{1, 1}, 2], 1] dot[Lin, S[2]])/L^4) + Lin ((3 \[Pi] (-AX[2, 1, 2, c0[{0}, 1] c0[{1, 2}, 1] dot[Lin, S[2]])/L^4) + E12 psq xi c0[{1, 1}, 2]) dot[B, S[1])/(2 bsq L^3) + (3 \[Pi] (-AX[2, 1, 2, c0[{0}, 1] c0[{1, 2}, 1]] dot[B, S[1]])/(2 bsq L^3) - (2 E12^2 xi^2 c0[{1, 2}, 2]) dot[B, S[1])/(2 bsq L^3) - (2 E12^2 xi^2 c0[{1, 2}, 2]) dot[B, S[2]])/(2 bsq L^3) - (2 E12^2 xi^2 c0[{1, 2}, 1] (2 c0[{0}, 1] + pSq c0[{1, 2}, 1]) dot[P, S[2]])/L^4); SODeLtaP22 = - (2 E12^2 xi^2 c0[{1, 1}, 1] c0[{1, 2}, 1] cross[S[1], cross[P, S[2]]])/(bsq L^2)) - (2 E12^2 xi^2 c0[{1, 1}, 1] c0[{1, 2}, 1] cross[S[2], cross[P, S[1]])/(bsq L^2) - (9 \[Pi] AX[2, 1, 4, c0[{1, 1}, 1] c0[{1, 2}, 1]] dot[Lin, S[1]] dot[B, S[2]])/(4 bsq^2 L^3)) - (9 \[Pi] AX[2, 1, 4, c0[{1, 1}, 1] c0[{1, 2}, 1]] dot[Lin, S[1]] dot[E, S[2]])/(4 bsq^2 L^3)) + Lin (-({0 E12^2 xi^2 c0[{1, 1}, 1], 1] c0[{1, 2}, 1] dot[Lin, S[2]] dot[P, S[1]])/(bsq L^4)) - (8 E12^2 xi^2 c0[{1, 1}, 1], 1] c0[{1, 2}, 1] dot[Lin, S[2]] dot[P, S[2]])/(bsq L^4)) - (8 \[Pi] AX[2, 1, 4, c0[{1, 1}, 1] c0[{1, 2}, 1] dot[Lin, S[1]] dot[P, S[2]])/(bsq L^4)) - (9 \[Pi] AX[2, 1, 4, c0[{1, 1}, 1] c0[{1, 2}, 1]] dot[B, S[2]] Sperp[3 \[Pi] AX[2, 1, 4, c0[{1, 1}, 1] c0[{1, 2}, 1]] dot[B, S[2]] Sperp[

1])/(8 bSq^2 L) - (
3 \[P1] AX[2, 1, 4, c0[{1, 1}, 1] c0[{1, 2}, 1]] dot[B, S[1]] Sperp[
2])/(8 bSq^2 L);
SoplPeltaP2 = (2 E12^2 xi^2 c0[{0}, 1] c0[{2, 1}, 1] cross[S[1], cross[P,
construction]

SoplDeltaP2 = (2 E12^2 xi^2 c0[{0}, 1] c0[{2, 1}, 1] cross[S[1], cross]
S[2]]))/(
3 L^4) + (
2 E12^2 xi^2 c0[{0}, 1] c0[{2, 1}, 1] cross[S[2], cross[P, S[1]]))/(
3 L^4) + Lin ((
8 E12^2 xi^2 c0[{0}, 1] c0[{2, 1}, 1] dot[Lin, S[2]] dot[P,
S[1]])/(3 L^6) + (
8 E12^2 xi^2 c0[{0}, 1] c0[{2, 1}, 1] dot[Lin, S[1]] dot[P,
S[2]])/(3 L^6)) +
B \[Pi] ((
15 (-AX[2, 1, 4/3, 1/2 c0[{0}, 1] c0[{2, 1}, 1]] +
1/2 E12 pSq xi c0[{2, 1}, 2]) dot[P, S[1]] dot[B, S[2]])/(
4 L^5)) +

4 E12^2 xi^2 c0{{0}, 1} c0{{2, 1}, 1} act[s1, s1]]/(3 L^4) + (3 \[Pi] (AX[2, 1, 4/3, 1/2 c0{{0}, 1} c0{{2, 1}, 1] -1/2 E12 pSq xi c0{{2, 1}, 2] dot[B, S[2] Sperp[1])/(4 bSq L^3) + (3 \[Pi] (AX[2, 1, 4/3, 1/2 c0{{0}, 1} c0{{2, 1}, 1] -1/2 E12 pSq xi c0{{2, 1}, 2] dot[B, S[1] Sperp[2])/(4 bSq L^3); SOp2De1taP2 = -((4 E12^2 xi^2 c0{{0}, 1] c0{{2, 2}, 1] cross[S[1], cross[P, S(211))/ S[2]]]// Lr4) - (4 El2r2 xir2 co[{0}, 1] co[{2, 2}, 1] cross[S[2], cross[F, S[1]]]//Lr4 + (3 B \[Pi] (-AX[2, 1, 2, 1/2 co[{0}, 1] co[{2, 2}, 1]] + 1/2 El2 pSq xi co[{2, 2}, 2] dot[S[1], S[2]])/(bSq Lr3) - (4 El2r2 xir2 co[{0}, 1] co[{2, 2}, 1] dot[F, S[2]] S[1])/Lr4 - (4 El2r2 xir2 co[{0}, 1] co[{2, 2}, 1] dot[F, S[1]] S[2])/Lr4t; Sop3DeltaP2 = -((8 El2r2 P xir2 co[{0}, 1] co[{2, 3}, 1] dot[F, S[1]] dot[F, S[2]])/

S[2]])/ L^4) + B (-((

L^4) + B (-((3 \ [Pi] AX[2, 1, 4, 1/2 c0[{0}, 1] c0[{2, 3}, 1]] dot[P, S[1]] dot[P, S[2]])/(bSg L^3)) + (3 E12 \ [Pi] xi c0[{2, 3}, 2] dot[P, S[1]] dot[P, S[2]])/(2 bSgr2 L)) + (2 E12^2 xi^2 c0[{0}, 1] c0[{2, 3}, 1] dot[P, S[2]] Sperp[1])/(ber L^2) + (

bsq L^2); MixedDeLtaP2 = cross[Lin, S[2]] ((4 El2^2 xi^2 (-c0[{1, 1}, 1] + c0[{1, 2}, 1]) c0[{2, 1}, 1] dot[B, S[1]])/(3 bsq^2 L^2) + (3 El2^2 \[Pi] xi^2 (c0[{1, 1}, 1] + c0[{1, 2}, 1]) c0[{2, 1}, 1] dot[P, S[1]])/(16 bsq L^3)) + cros S[1] ((4 El2^2 xi^2 (c0[{1, 1}, 1] - c0[{1, 2}, 1]) c0[{2, 1}, 1] dot[B, S[2]])/(3 bSq^2 L^2) + (

bSq L^2) + (2 E12^2 xi^2 c0[{0}, 1] c0[{2, 3}, 1] dot[P, S[1]] Sperp[2])/(

- 3 E12^2 \[Pi] xi^2 (c0[{1, 1}, 1] + c0[{1, 2}, 1]) c0[{2, 1}, 1] dot[P, S[2]])/(16 bSg L^3)) + Lin (-((5 E12^2 \[Pi] xi^2 (c0[1, 1], 1] + c0[{1, 2}, 1]) c0[{2, 1}, 1] dot[B, S[2]] dot[Lin, S[1]])/(16 bSg^2 L^3)) (5 E12^2 \[Pi] xi^2 (c0[{1, 1}, 1] + c0[{1, 2}, 1]) c0[{2, 1}, 1] dot[B, S[1]] dot[Lin, S[2]])/(16 bSg^2 L^3) + (4 E12^2 xi^2 (c0[{1, 1}, 1] + c0[{1, 2}, 1]) c0[{2, 1}, 1] dot[Lin, S[2]] dot[P, S[1]])/(3 bSg L^4) + (4 E12^2 xi^2 (c0[{1, 1}, 1] + c0[{1, 2}, 1]) c0[{2, 1}, 1] dot[Lin, S[1]] dot[P, S[2]])/(3 bSg L^4) (3 E12^2 (Pi] xi^2 (c0[{1, 1}, 1] + c0[{1, 2}, 1]) c0[{2, 1}, 1] dot[B, S[2]] S[1])/(16 bSg^2 L) (3 E12^2 \[Pi] xi^2 (c0[{1, 1}, 1] + c0[{1, 2}, 1]) c0[{2, 1}, 1] dot[B, S[1]] S[1])/(1

- t L2 2 k1 2 (3/2 co[(1, 2], 1] + co[(1, 1), 1] (1/2 co[(2, 1], 1] 3 (1/2 co[(2, 2], 1] + 1/2 pSq co[(2, 3], 1]))) dot[P, S[2]])/(3 bSq L^2)) Sperp[1] + ((3 E12² \[P1] x1² (co[(1, 1], 1] + co[(1, 2], 1]) co[(2, 1), 1] dot[B, S[1]])/(8 bSq² L) (4 E12² 2 x1² (3/2 co[(1, 1], 1] co[(2, 2), 1] + co[(1, 2])
- c0[[1, 2], 1] (1/2 c0[[2, 1], 1] -3 (1/2 c0[[2, 2], 1] + 1/2 pSq c0[[2, 3], 1]))) dot[P, S[1]])/(3 bSq L^2)) Sperp[2];

SODeltaS11 = (2 E12 xi c0[{1, 1}, 1] cross[Lin, S[1]])/(Sqrt[bSq] L); SOpiDeltaS11 = -((4 E12 xi c0[{2, 1}, 1] cross[S[1], S[2]])/(3 Sqrt[bSq] 4 E12 xi c0[{2, 1}, 1] cross[Lin, S[1]] dot[Lin, S[2]])/(

3 Sqrt[bSq] L1%) - (
2 Sqrt[bSq] E1% = co[{2, 1}, 1] cross[P, S[1]] dot[P, S[2]])/(3 L^3);
Sop2DeltaSi1 = -((2 E12 xi co[{2, 2}, 1] cross[S[1], S[2]])/(Sqrt[bSq] L));
Sop3DeltaSi1 = (2 E12 xi co[{2, 3}, 1] cross[P, S[1]] dot[P,
S[2]])/(Sqrt[bSq] L);

- SoDeltaSl21 = P (-((\[Pi] (AX[2, 1, 2, c0[{0}, 1] c0[{1, 1}, 1]) -El2 pSq xi c0[{1, 1}, 2]) dot[B, S[1]])/(2 L^3)) (2 El2^2 xi^2 c0[{1, 1}, 1]^2 dot[P, S[1]])/(L^2) + B (-((2 El2^2 xi^2 c0[{1, 1}, 1], 1]^2 dot[B, S[1]])/ bSq^2) + (\[Pi] (AX[2, 1, 2, c0[{0}, 1] c0[{1, 1}, 1]] -El2 pSq xi c0[{1, 1}, 2]) dot[P, S[1]])/(2 L^3)); SODeltaSl22 = (2 El2^2 xi^2 c0[{1, 1}, 1] c0[{1, 2}, 1] cross[S1], cross[Lin, S[2]]])/(bSq L^2) (3 \[Pi] AX[2, 1, 4, c0[{1, 1}, 1] c0[{1, 2}, 1]] cross[Lin, S[1]] dot[Lin, S[2]])/(B Sq L^3); SOpiDeltaSl2 = -((2 El2^2 xi^2 c0[{0}, 1] c0[{2, 1}, 1] cross[S1], cross[Lin, S[2]])/(3 L^4)) + (3 \[Pi] (AX[2, 1, 4/3, 1/2 c0[{0}, 1] c0[{2, 1}, 1]] -

- 3 \[\Fi] \[\Ki [2, 1, 4/3, 1/2 c0[(0, 1] c0[{2, 1}, 1]] -1/2 El2 pSq xi c0[{2, 1}, 2]) cross[S[1], S[2]])/(4 L^3) + (3 \[Fi] (Ki [2, 1, 4/3, 1/2 c0[(0, 1] c0[[2, 1], 1]] -1/2 El2 pSq xi c0[{2, 1}, 2]) cross[Lin, S[1]] dot[Lin, S[2]])/(

4 L^5) + cross[P, S[1]] (-((4 E12^2 xi^2 c0[{0}, 1] c0[{2, 1}, 1] dot[B, S[2]])/(3 L^4)) + (bSq \{Pi1 (AX[2, 1, 0, 1/2 c0[{0}, 1] c0[{2, 1}, 1]] -1/2 E12 pSq xi c0[{2, 1}, 2]) dot[P, S[2]])/(2 L^5)); S0p2De1xa512 = \{Pi1 (AX[2, 1, 2, 1/2 c0[{0}, 1] c0[{2, 2}, 1]]/L^3 - (E12 xi c0[{2, 2}, 2])/(2 bSq L)) cross[S[1], S[2]]; S0p3De1xa512 = (2 E12^2 xi^2 c0[{0}, 1] c0[{2, 3}, 1] cross[S[1], cross[Lin, S[2]])/(bSq L^2) + cross[P, S[1]] ((4 E12^2 xi^2 c0[{0}, 1] c0[{2, 3}, 1] dot[B, S[2]])/(bSq L^2) - (\{Pi1 AX[2, 1, 4, 1/2 c0[{0}, 1] c0[{2, 3}, 1]] dot[P, s[2]])/L^3 + (E12 \{Pi1 xi c0[{2, 3}, 2] dot[P, S[2]])/(2 bSq L)); 2 bSq L)); MixedDeltaS12 = -1/(3 bSg L^2)*
4 E12^2 xi^2 (c0[{1, 2}, 1] (c0[{2, 1}, 1] + 3/2 c0[{2, 2}, 1]) + c0[11, 1], 1] (1/2 c0[{2, 1}, 1] + 3 (1/2 c0[{2, 2}, 1] + 1/2 pSq c0[{2, 3}, 1]))) cross[S[1], cross[Lin, S[2]]] - (E12^2 [Pi] xi² (c0[1, 1], 1] + c0[{1, 2}, 1]) c0[{2, 1}, 1] cross[S[1], S[2]])/(8 bSq L) - (El2^2 \[Pi] xi^2 (col{1, 1}, 1] + col[1, 2}, 1]) col[2, 1], 1] cross[Lin, S[1]] dot[Lin, S[2]])/(8 bSq L^3) + Lin, S[1]] dot[Lin, S[2]])/(8 bsg L^3) + cross[P, S[1]] ((4 E12^2 xi^2 (c0[[1, 1], 1] - c0[[1, 2], 1]) (1/2 c0[[2, 1], 1] -3/2 pSg c0[[2, 3], 1]) dot[P, S[2]])/(3 bSg L^2) - (E12^2 \[Pi] xi^2 (c0[[1, 1], 1] + c0[[1, 2], 1]) c0[[2, 1], 1] dot[P, S[2]])/(4 L^3)) + Lin ((8 E12^2 xi^2 c0[[1, 1], Lin ((8 E12^2 xi^2 c0[[2, 2], 1]) dot[P, S[1]] dot[P, S[1]]) dot[P, S[2]])/(4 L^3)) + Lin ((1 dot[2] xi^2 c0[[2, 2], 1]) dot[P, S[1]] dot[P, S[1]]) dot[P, S[1]] 1] (c0[[2, 1], 1] + 3/2 c0[[2, 2], 1]) dot[B, 5[1]] dot[B, 5[2]])/(3 b5q^2 L^2) + (4 E12^2 xi^2 c0[{1, 1}, 1] c0[{2, 2}, 1] dot[Lin, S[1]] dot[Lin,])/(bSq L^4) + (8 E12^2 xi^2 c0[{1, 1} 1] (1/2 co[{2, 1}, 1] +
3 (1/2 co[{2, 2}, 1] + 1/2 pSq co[{2, 3}, 1])) dot[P,
s[1]] dot[P, S[2])/(3 L^4)) - (
4 El2^2 xi^2 co[{1, 1}, 1] co[{2, 2}, 1] dot[Lin, S[2]] S[1])/(
beg L^2); bSq L^2); DeltaP1 = NoSDeltaP1 + SODeltaP1 + SOp1DeltaP1 + SOp2DeltaP1 + SOp3DeltaP1; DeltaP2 = NoSDeltaP2 + SODeltaP21 + SODeltaP22 + SOp1DeltaP2 + SOp3DeltaP2 + MixedDeltaP2; DeltaS[1, 1] = SODeltaS11 + SOp1DeltaS11 + SOp2DeltaS11 + SOp3DeltaS11; DeltaS[1, 2] = SODeltaS121 + SODeltaS122 + SOp1DeltaS12 + SOp3DeltaS12 + MixedDeltaS12;

(* The final impulse and spin kick *) $DeltaP = G DeltaP1 + G^2 DeltaP2 ; \\ DeltaS[1] = G DeltaS[1, 1] + G^2 DeltaS[1, 2] ;$

S²₁ Hamiltonian w/ D Kosmopoulos (UCLA)

An effective Lagrangian encoding higher order spin interactions (Porto, Rothstein; Levi, Steinhoff). To tree-level, reproduce Kerr. What about loops?

$$\mathcal{L}_{\text{non-min}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!} \frac{C_{ES^{2n}}}{m^{2n}} \nabla(\omega)_{f_{2n}} \cdots \nabla(\omega)_{f_{2n+1}} \\ -\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{C_{BS^{2n}}}{m^{2n+1}} \nabla(\omega)_{f_{2n+1}} \cdots \nabla(\omega)_{f_{$$

Look at the first term

 $\int_{f_3} R_{f_1 a f_2 b} \nabla(\omega)^a \phi_s \mathbb{S}^{(f_1} \dots \mathbb{S}^{f_{2n})} \nabla(\omega)^b \phi_s$

 $\nabla(\omega)_{f_3} \frac{1}{2} \epsilon_{ab(c|f_1} R^{ab}_{|d)f_2} \nabla(\omega)^c \phi_s \mathbb{S}^{(f_1} \dots \mathbb{S}^{f_{2n+1})} \nabla(\omega)^d \phi_s$

 $=\frac{C_{ES^2}}{8m^4}R_{f_1af_2b}\nabla^a\phi_s\,\widetilde{M}^{f_1c_1}\widetilde{M}^{f_2c_2}\nabla_{c_1}\nabla_{c_2}\nabla^b\phi_s$ \mathcal{L}_{ES^2} a cush contraction with a long of the second o

From this Lagrangian, we can deduce Feynman rules...

L

...and compute the Compton amplitude. For the non-minimally coupled:

$$\begin{split} i\,\delta^{esles4}\,\kappa^{2}\,C_{ES^{2}} \\ & \left(\left(q\cdot S_{1}\right)^{2}\left(-ml^{2}\,\eta^{\gamma 1\,\delta 1+\gamma 2\,\delta 2}\left(p_{1}\cdot k_{5}\right)^{4}+4\left(ml^{2}\,q\cdot q\,p_{1}\cdot k_{5}\,p_{1}^{\gamma 1+\delta 1}\left(\eta^{\gamma 2\,\delta 2}\,p_{1}\cdot k_{5}+2\,q^{\delta 2}\,p_{1}^{\gamma 2}-2\,q^{\gamma 2}\,p_{1}^{\delta 2}\right)+\left(q\cdot q\right)^{2}\,p_{1}^{\gamma 1+\delta 1}\left(2\,ml^{2}\,\eta^{\gamma 2\,\delta 2}\,p_{1}\cdot k_{5}+\left(-ml^{2}+p_{1}\cdot k_{5}\right)\,p_{1}^{\gamma 2+\delta 2}\right)+ml^{2}\left(p_{1}\cdot k_{5}\right)^{2}\left(-q^{\delta 1+\delta 2}\,p_{1}^{\gamma 1+\gamma 2}+\eta^{\gamma 2\,\delta 2}\,p_{1}\cdot k_{5}\left(-q^{\delta 1}\,p_{1}^{\gamma 1}+q^{\gamma 1}\,p_{1}^{\delta 1}\right)+q^{\gamma 2}\,p_{1}^{\delta 1}\left(2\,q^{\delta 2}\,p_{1}^{\gamma 1}-q^{\gamma 1}\,p_{1}^{\delta 2}\right)\right)\right)\right)+\\ & q\cdot q\left(-4\,\left(k_{5}\,\cdot S_{1}\right)^{2}\,p_{1}^{\gamma 1+\delta 1}\left(-3\,ml^{2}\,\eta^{\gamma 2\,\delta 2}\,q\cdot q\,p_{1}\cdot k_{5}+2\,q\cdot q\,q\left(ml^{2}-p_{1}\cdot k_{5}\right)\,p_{1}^{\gamma 2+\delta 2}+ml^{2}\,p_{1}\cdot k_{5}\left(-q^{\delta 2}\,p_{1}^{\gamma 2}+q^{\gamma 2}\,p_{1}^{\delta 2}\right)\right)\right)+k_{5}\,\cdot S_{1}\,p_{1}\cdot k_{5}\left(ml^{2}\,q^{\gamma 2}\,p_{1}\cdot k_{5}\right)\,p_{1}\cdot k_{5}\left(p_{1}^{\delta 1}\,s_{1}^{\gamma 1}+p_{1}^{\gamma 1}\,s_{1}^{\delta 1}\right)+2\,q\cdot q\,p_{1}^{\gamma 1+\delta 1}\left(2\,ml^{2}\,q^{\delta 2}\,p_{1}^{\gamma 2}-2\,p_{1}\cdot k_{5}\right)\,p_{1}^{\delta 2}\,s_{1}^{\gamma 2}+\left(-2\,ml^{2}\,q^{\gamma 2}+\left(3\,ml^{2}-2\,p_{1}\cdot k_{5}\right)\,p_{1}^{\gamma 2}\,s_{1}^{\delta 2}\right)+k_{5}\,\left(ml^{2}\,q^{\gamma 2}\,q^{2}\,(-4\,q\cdot q-p_{1}\cdot k_{5})\,p_{1}^{\delta 1}\,s_{1}^{\gamma 1}+p_{1}^{\gamma 1}\,s_{1}^{\delta 1}\right)+2\,q\cdot q\,q\,p_{1}^{\gamma 1+\delta 1}\left(2\,ml^{2}\,q^{\delta 2}\,s_{1}^{\gamma 2}+\left(-2\,ml^{2}\,q^{\gamma 2}\,q^{\gamma 2}+\left(3\,ml^{2}-2\,p_{1}\cdot k_{5}\right)\,p_{1}^{\gamma 2}\,s_{1}^{\delta 2}\right)+k_{5}\,\left(ml^{2}\,q^{\gamma 2}\,q^{\gamma 2}\,q^{2}\,q$$

Unlike the minimal coupling, there is no GWI and the sum over states results in a projector with light cone terms.

$$\sum_{\lambda} \varepsilon(k)_{\lambda}^{\mu\nu} \varepsilon(-k)_{-\lambda}^{\alpha\beta} = \frac{1}{2} \mathcal{P}^{\mu\alpha} \mathcal{P}^{\nu\beta} + \frac{1}{2} \mathcal{P}^{\nu\alpha} \mathcal{P}^{\mu\beta} - \frac{1}{D-2} \mathcal{P}^{\mu\nu} \mathcal{P}^{\alpha\beta} \qquad \qquad \mathcal{P}^{\mu\nu}(k) = \eta^{\mu\nu} - \frac{r^{\mu}k^{\nu} + r^{\nu}k^{\mu}}{r \cdot k}$$

I pick convenient reference vectors to perform the sewing

$$i\mathcal{M}_{4}^{1 \text{ loop}} = d_{B} I_{B} + d_{\overline{B}} I_{\overline{B}} + c_{\Delta} I_{\Delta} + c_{\nabla} I_{\nabla}$$

$$\stackrel{i\mathcal{C} \land \text{Min}}{4G^{2}\pi^{2}\mathcal{U}_{1}\mathcal{U}_{2}m_{2}^{2}} = m_{1}^{2}(95\sigma^{2} - 7)q^{2}S_{1}^{2} - 5m_{1}^{2}(7\sigma^{2} - 1)(q \cdot S_{1})^{2} + 2m_{1}^{2}(65\sigma^{2} - 1)\frac{q^{2}(p_{2} \cdot S_{1})^{2}}{m_{2}^{2}(\sigma^{2} - 1)}$$

$$\stackrel{c_{\nabla} \text{Min}}{4G^{2}\pi^{2}\mathcal{U}_{1}\mathcal{U}_{2}m_{2}^{2}} = 4m_{2}^{2}(15\sigma^{2} - 2)q^{2}S_{1}^{2} + 4m_{2}^{2}(q \cdot S_{1})^{2} + 4m_{2}^{2}(15\sigma^{2} + 1)\frac{q^{2}(p_{2} \cdot S_{1})^{2}}{m_{2}^{2}(\sigma^{2} - 1)}$$

$$\stackrel{c_{\Delta} C}{=} s^{2}}{s^{2}} \frac{r^{2} - 1}{\mathcal{U}_{1}\mathcal{U}_{2}m_{2}^{2}} = m_{1}^{2}C_{ES^{2}}(95\sigma^{4} - 102\sigma^{2} + 23)q^{2}S_{1}^{2} - m_{1}^{2}C_{ES^{2}}(155\sigma^{4} - 174\sigma^{2} + 35)(q \cdot S_{1})^{2}$$

+2i

+4n

$$m_1^2 C_{ES^2} (65\sigma^4 - 66\sigma^2 + 17) \frac{q^2 (p_2 \cdot S_1)^2}{m_2^2 (\sigma^2 - 1)}$$

$${}_{2}^{2}C_{ES^{2}}(30\sigma^{4}-29\sigma^{2}+3)(q\cdot S_{1})^{2}$$

$$m_2^2 C_{ES^2} (15\sigma^4 - 10\sigma^2 + 3) \frac{q^2 (p_2 \cdot S_1)^2}{m_2^2 (\sigma^2 - 1)}$$

$$S = \int_{\boldsymbol{k}} \sum_{a=1,2} \xi_a^{\dagger}(-\boldsymbol{k}) \left(i\partial_t - \sqrt{\boldsymbol{k}^2 + m_i^2} \right) \xi_a(\boldsymbol{k})$$

 $\xi(x) - \int_{m{k},m{k}'} \xi_1^\dagger(m{k}') \xi_2^\dagger(-m{k}') \, \widehat{V}(m{k}',m{k},\hat{m{S}}_a) \, \xi_1(m{k}) \xi_2(-m{k}) \, .$

THE STATE TO A PROPERTY AND THE STATE SET TO A PROPERTY AND THE PROPERTY A $\hat{V}^{A}(\boldsymbol{k}',\boldsymbol{k}) = \frac{4\pi G}{\boldsymbol{q}^{2}} d_{1}^{A}\left(\boldsymbol{p}^{2}\right) + \frac{2\pi^{2}G^{2}}{|\boldsymbol{q}|} d_{2}^{A}\left(\boldsymbol{p}^{2}\right) + \mathcal{O}(G^{3})$ and the state of the second state of the secon

 $\mathrm{Sym} \left[oldsymbol{q}^2 oldsymbol{\hat{S}}_1^2
ight], \qquad \hat{\mathbb{O}}^{(2,6)} = \mathrm{Sym} \left[oldsymbol{q}^2 oldsymbol{(k} \cdot oldsymbol{\hat{S}}_1ig)^2
ight]$

Use Feynman rules (and IBP) to compute the amplitude

$$\widehat{\mathcal{M}}^{\text{EFT}} = - \frac{\widehat{V}(\mathbf{p}', \mathbf{p})}{V(\mathbf{p}', \mathbf{p})} - \int_{\mathbf{k}} \frac{\widehat{V}(\mathbf{p}', \mathbf{k}) \widehat{V}(\mathbf{k}, \mathbf{p})}{E_1 + E_2 - \sqrt{\mathbf{k}^2 + m_1^2} - \sqrt{\mathbf{k}^2 + m_2^2}}$$

 $H(r^2, p^2)$

$\hat{\mathbb{O}}^{(2,4)} = \text{Sym}$	$\left[(oldsymbol{q}\cdotoldsymbol{\hat{S}}_1)^2 ight]^2$
and the second	

 $A_2[X] = \left[\frac{1}{4}(1 - 3\xi) + \frac{\xi^2 E^2}{p^2} + \frac{1}{2}\xi^2 E^2\partial\right]X$

Check: NNLO Hamiltonian

arXiv.org > gr-qc > arXiv:1607.04252

General Relativity and Quantum Cosmology

[Submitted on 14 Jul 2016]

Complete conservative dynamics for inspiralling compact binaries with spins at fourth post-Newtonian order

Michele Levi, Jan Steinhoff

$$\begin{split} H^{3}_{\rm NLO} &= \nu^2 \bigg\{ \bigg[\frac{\tilde{L}^4}{16\tilde{r}^7} (11-25\nu) + \frac{\tilde{L}^2}{16\tilde{r}^6} (199+33\nu) - \frac{24}{7\tilde{r}^5} + \frac{\tilde{L}^2}{16\tilde{r}^5} (7-74\nu) - \frac{9\tilde{r}^2}{16\tilde{r}^6} (3-5\nu) \\ &- \frac{\tilde{p}_1^4}{4\tilde{r}^3} (1+\nu) \bigg] \tilde{S}_1^2 - \bigg[\frac{3\tilde{L}^4}{16\tilde{r}^7} (6-17\nu) + \frac{\tilde{L}^2}{16\tilde{r}^6} (329+61\nu) - \frac{37}{7\tilde{r}^5} + \frac{\tilde{L}^2}{16\tilde{r}^5} (26-145\nu) \\ &- \frac{\tilde{p}_1^2}{16\tilde{r}^4} (3-85\nu) - \frac{\tilde{p}_1^4}{4\tilde{r}^3} (1+\nu) \bigg] (\tilde{n} \cdot \tilde{S}_1)^2 - \bigg[\frac{\tilde{L}^3}{16\tilde{r}^6} (1+37\nu) - \frac{\tilde{L}^2}{8\tilde{r}^5} (93+25\nu) \\ &- \frac{\tilde{L}^2}{16\tilde{r}^4} (23-67\nu) \bigg] (\tilde{S}_1 \cdot \tilde{\lambda})^2 - \bigg[\frac{\tilde{L}^3}{16\tilde{r}^6} (1+37\nu) - \frac{\tilde{L}^2}{8\tilde{r}^5} (93+25\nu) \\ &- \frac{\tilde{L}^2}{16\tilde{r}^4} (23-67\nu) \bigg] (\tilde{n} \cdot \tilde{S}_1 \tilde{S}_1 \cdot \tilde{\lambda} \bigg\} + \nu^2 C_{1(\rm ES^3)} \bigg\{ \bigg[\frac{\tilde{L}}{4\tilde{r}^7} (1+3\nu) - \frac{\tilde{L}^2}{4\tilde{r}^6} (22+5\nu) + \frac{55}{14\tilde{r}^5} \bigg] \\ &- \frac{\tilde{L}^2}{4\tilde{r}^5} (1-6\nu) + \frac{\tilde{p}_1^2}{4\tilde{r}^4} (9-13\nu) - \frac{\tilde{p}_1^4}{2\tilde{r}^4} (1-6\nu) \bigg] \tilde{S}_1^2 + \bigg[\frac{\tilde{T}\tilde{L}^2}{4\tilde{r}^6} (4+\nu) - \frac{95}{14\tilde{r}^5} + \frac{\tilde{L}^2\tilde{p}_1^2}{4\tilde{r}^5} (7+6\nu) \bigg] \\ &+ \frac{3\tilde{p}_1^2}{4\tilde{r}^4} (1-6\nu) + \frac{\tilde{p}_1^4}{4\tilde{r}^4} (1-6\nu) \bigg] (\vec{n} \cdot \tilde{S}_1)^2 - \bigg[\frac{\tilde{L}^4}{4\tilde{r}^7} (1+3\nu) - \frac{\nu\tilde{L}^2}{2\tilde{r}^6} + \frac{\tilde{L}^2\tilde{p}_1^2}{4\tilde{r}^5} (1+6\nu) \bigg] (\tilde{S}_1 \cdot \tilde{\lambda})^2 \\ &+ \bigg[\frac{\tilde{5}\tilde{L}^3\tilde{p}_1}{8\tilde{r}^5} + \frac{\tilde{L}\tilde{p}_1}{4\tilde{r}^5} (8-11\nu) - \frac{\tilde{L}\tilde{p}_1^3}{8\tilde{r}^4} (1-6\nu) \bigg] \vec{n} \cdot \tilde{S}_1 \tilde{S}_1 \cdot \tilde{\lambda} \bigg\} + \frac{\nu}{q} \bigg\{ \bigg[- \frac{\tilde{L}^4}{16\tilde{r}^7} (11-59\nu+21\nu^2) \bigg] \\ &- \frac{\tilde{L}^2\tilde{p}_1^2}{16\tilde{r}^6} (139-93\nu-33\nu^2) - \frac{1}{2\tilde{r}^5} (7+18\nu) - \frac{\tilde{L}^2\tilde{p}_1^2}{16\tilde{r}^6} (7-40\nu+60\nu^2) + \frac{3\tilde{p}_1^2}{16\tilde{r}^6} (1+63\nu+15\nu^2) \\ &+ \frac{\tilde{L}^2\tilde{p}_1^2}{16\tilde{r}^6} (26-101\nu+126\nu^2) + \frac{1}{2\tilde{r}^5} (7+18\nu) - \frac{\tilde{p}_1^2}{16\tilde{r}^6} (39-37\nu-61\nu^2) \bigg\} \\ &- \frac{\tilde{L}^2\tilde{p}_1^2}{16\tilde{r}^6} (1-4\nu-9\nu-9\nu^2) \bigg] (\tilde{S}_1 \cdot \tilde{\lambda})^2 + \bigg[\frac{\tilde{L}^3\tilde{p}_1}{16\tilde{r}^6} (1+47\nu-33\nu^2) - \frac{\tilde{L}^2}{8\tilde{r}^6} (67-37\nu-11\nu^2) \\ &- \frac{\tilde{L}^2\tilde{p}_1^2}{16\tilde{r}^4} (1-4\nu-9\nu)^2) \bigg] (\tilde{S}_1 \cdot \tilde{\lambda})^2 + \bigg[\frac{\tilde{L}^3\tilde{p}_1}{16\tilde{r}^6} (1+47\nu-33\nu^2) - \frac{\tilde{L}^2}{8\tilde{r}^6} (67-37\nu-11\nu^2) \\ &- \frac{\tilde{L}^2\tilde{p}_1^2}{16\tilde{r}^4} (23-65\nu+57\nu^2) \bigg] \tilde{n} \cdot \tilde{S}_1\tilde{S}_1} \tilde{\lambda} \right\} + \frac{\nu}{q} C_{1(\rm ES^3} (\tilde{\tau})^2 + \bigg[\frac{\tilde{L}^3\tilde{p}$$

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$$a_{2}^{\{2,4\}} = \frac{C_{ES^{2}} m_{1} m_{2}^{3}}{4 p^{2} (m_{1} + m_{2})} + \frac{m_{2} \left(2 \left(5 + 16 C_{ES^{2}}\right) m_{1}^{2} + \left(-7 + 61 C_{ES^{2}}\right) m_{1} m_{2} + \left(-13 + 29 C_{ES^{2}}\right) m_{2}^{2}\right)}{16 m_{1} (m_{1} + m_{2})} + \frac{1}{64 m_{1}^{3} m_{2} (m_{1} + m_{2})} p^{2}}{\left(3 \left(5 + 31 C_{ES^{2}}\right) m_{1}^{4} + \left(-73 + 467 C_{ES^{2}}\right) m_{1}^{3} m_{2} + \left(-361 + 707 C_{ES^{2}}\right) m_{1}^{2} m_{2}^{2} + \left(-343 + 397 C_{ES^{2}}\right) m_{1} m_{2}^{3} + 2 \left(-41 + 32 C_{ES^{2}}\right) m_{2}^{4}\right)}$$

$$a_{2}^{\{2,5\}} = -\frac{C_{ES^{2}} m_{1} m_{2}^{3}}{4 p^{2} (m_{1} + m_{2})} - \frac{m_{2} \left(\left(22 + 20 C_{ES^{2}}\right) m_{1}^{2} + \left(19 + 35 C_{ES^{2}}\right) m_{1} m_{2} + \left(1 + 15 C_{ES^{2}}\right) m_{2}^{2}\right)}{16 m_{1} (m_{1} + m_{2})} + \frac{1}{64 m_{1}^{3} m_{2} (m_{1} + m_{2})} p^{2} \left(-3 \left(17 + 19 C_{ES^{2}}\right) m_{1}^{4} - \left(115 + 279 C_{ES^{2}}\right) m_{1}^{3} m_{2} + \left(53 - 399 C_{ES^{2}}\right) m_{1}^{2} m_{2}^{2} + \left(155 - 209 C_{ES^{2}}\right) m_{1}^{2} m_{2}^{2}}{c_{ES^{2}} m_{1} m_{2}^{3}} m_{2} \left(\left(8 + 7 C_{ES^{2}}\right) m_{1}^{2} + \left(8 + 11 C_{ES^{2}}\right) m_{1} m_{2} + \left(1 + 4 C_{ES^{2}}\right) m_{2}^{2}\right)}$$

$$\left(-3 \left(17 + 19 C_{ES^{2}}\right) m_{1}^{4} - \left(115 + 279 C_{ES^{2}}\right) m_{1}^{3} m_{2} + \left(53 - 399 C_{ES^{2}}\right) m_{1}^{2} m_{2}^{2} + \left(155 - 209 C_{ES^{2}}\right) m_{1} m_{2}^{3} + 2 \left(25 - 16 C_{ES^{2}}\right) m_{2}^{4} \right) m_{2}^{4} \right)$$

$$a_{2}^{\{2,6\}} = \frac{C_{ES^{2}} m_{1} m_{2}^{3}}{2 p^{4} (m_{1} + m_{2})} + \frac{m_{2} \left(\left(8 + 7 C_{ES^{2}}\right) m_{1}^{2} + \left(8 + 11 C_{ES^{2}}\right) m_{1} m_{2} + \left(1 + 4 C_{ES^{2}}\right) m_{2}^{2}\right)}{4 p^{2} m_{1} (m_{1} + m_{2})} + \frac{m_{2} \left(\left(8 + 7 C_{ES^{2}}\right) m_{1}^{3} m_{2} + \left(13 + 245 C_{ES^{2}}\right) m_{1}^{2} m_{2}^{2} + \left(-73 + 115 C_{ES^{2}}\right) m_{1} m_{2}^{3} + 4 \left(-7 + 4 C_{ES^{2}}\right) m_{2}^{4}}{32 m_{1}^{3} m_{2} (m_{1} + m_{2})}$$

 S^2 Hamiltonian is known up to NNLO in PN (Levi-Steinhoff '16, from EFTofPNG). To compare with (overlapping parts of) them, we may compute Amplitudes from the Hamiltonian using EFT.

Checks: Test body Hamiltonian

 S^2 Hamiltonians is known to all orders in PN, in a test-body limit (Hinderer, Steinhoff, Vines '16). To compare with (overlapping parts of) it, we may compute Amplitudes using EFT.

General Relat [Submitted on 27 J. Canonica

Canonical Hamiltonian for an extended test body in curved spacetime: To quadratic order in spin Justin Vines, Daniela Kunst, Jan Steinhoff, Tanja Hinderer

 $H_{S^2} =$

 $k_1 = -0$

 $k_2 = \frac{3C}{r}$

+

 $k_3 = 3(0)$

 $k_4 = 6($

...stay tuned for 2012.xxxxx

arXiv.org > gr-qc > arXiv:1601.07529

General Relativity and Quantum Cosmology

[Submitted on 27 Jan 2016 (v1), last revised 18 Jul 2016 (this version, v2)]

$$\begin{split} \frac{\sqrt{w}M}{2\hat{Q}r^3} \left[\vec{S}^2 k_1(\vec{z},\vec{P}) + (\vec{n}\cdot\vec{S})^2 k_2(\vec{z},\vec{P}) + \frac{(\vec{P}\cdot\vec{S})^2}{m^2} k_3(\vec{z},\vec{P}) + \frac{\vec{n}\cdot\vec{P}\cdot\vec{n}\cdot\vec{S}\cdot\vec{P}\cdot\vec{S}}{m^2} k_4(\vec{z},\vec{P}) \right] \\ C + \frac{\vec{P}^2}{m^2} \left(3(1-C) - \frac{2m^2}{(\hat{Q}+m)^2} \right) + \frac{(\vec{n}\cdot\vec{P})^2}{(\hat{Q}+m)^2} \left(3C - 1 + w + 3(C-1)\frac{\hat{Q}(\hat{Q}+2m)}{m^2} \right), \\ \frac{C\hat{Q}^2}{m^2} + \frac{3\vec{P}^2}{m^2} \left(C + \frac{\vec{P}^2 - \hat{Q}(3\hat{Q}+4m)}{(\hat{Q}+m)^2} \right) + 3(1 - \sqrt{w})^2 (1 + 2\sqrt{w} + Cw) \frac{(\vec{n}\cdot\vec{P})^4}{m^2(\hat{Q}+m)^2} \\ \frac{(\vec{n}\cdot\vec{P})^2}{(\hat{Q}+m)^2} \left(-(1 - \sqrt{w})^2 + 6(w-1)\frac{\vec{P}^2}{m^2} + 6(1 - \sqrt{w})(1 + C\sqrt{w})\frac{\hat{Q}(\hat{Q}+m)}{m^2} \right), \\ C - 1) \left(1 + w\frac{(\vec{n}\cdot\vec{P})^2}{(\hat{Q}+m)^2} \right) + \frac{2m^2}{(\hat{Q}+m)^2}, \\ 1 - C)\frac{(1 + \sqrt{w})\hat{Q}^2 + (2 + \sqrt{w})m\hat{Q}}{(\hat{Q}+m)^2} - 2(3C - 1 + \sqrt{w})\frac{m^2}{(\hat{Q}+m)^2} + 6(1 - C)(1 - \sqrt{w})w\frac{(\vec{n}\cdot\vec{P})^2}{(\hat{Q}+m)^2} \end{split}$$

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Moving forward...

...amplitudes become increasingly cumbersome.

We need a better way to compute them: Massive spinor helicity? Geodesic equations?

How much can we exploit them if we aim to keep the Wilson coefficients C_{ES^2} , etc. arbitrary.

Could double copy help?

Note added: Rafael's results

Rafael 11:36 AM

@Chia-Hsien Shen @Andres Luna @Justin Vines as you can see in the slides we computed the one-loop with spin and re-derived your result (but in covariant gauge!) for the impulse (and spin kick) for spin-orbit and spin1-spin2 generic orientation and computed --we believe for the first time -- the spin1-spin1 finite size effects for any compact object (C_ES^2 \neq 1). We also obtained the scattering angle for alignedspins and confirm the one-loop result for Kerr. We have extended the B2B for spin along the lines of 1911, which allows us to obtain binding energy, etc. We also confirmed the map to periastron now including finite-size effects. (footnote, the typo we thought originally in Schaefer's paper is actually much milder than we thought when we compared the Kerr case, it's just a factor of a 1/2 for generic C_ES^2). We are working in understanding how to B2B the additional angle, but I'm happy with aligned-spins for now. (Of course, we can also compute a Hamiltonian, but I rather not 😏) (edited)

Summary and Outlook

- We've developed a formalism to compute spin Hamiltonians from scattering amplitudes and Effective field theory
- We obtain an Eikonal-like formula. Need to prove/test it $\Delta \mathcal{O} = e^{-i\chi \mathcal{D}}[\mathcal{O}, e^{i\chi \mathcal{D}}]$
- We computed S_1^2 amplitude, Hamiltonian
- Explore relations, cross-checks with Porto+'s PM-EFT
- Outlook: More spin (S^3 , comparisons with Levi(?) and GOV), more loops (2-loops seems within reach), double copy (QED? YM?)

Thank you!

QED meets Gravity w/ T Scheopner (UCLA)

An effective action of higher spin fields coupled to photons

$$\begin{split} \mathcal{L} &= \frac{1}{4} F_{\mu\nu}^{a} F^{a,\mu\nu} \\ &- \frac{1}{2} \sum_{n=0}^{\infty} \frac{C_{n} \eta_{\mu_{0}\nu_{0}}}{2^{2n} m^{4n}} \epsilon_{\mu_{1}\nu_{1}\rho_{1}\sigma_{1}} \dots \epsilon_{\mu_{2n}\nu_{2n}\rho_{2n}\sigma_{2n}} \\ &\quad D^{(\mu_{0}} D^{\mu_{1}} \dots D^{\mu_{2n})} \varphi_{s_{g}} \mathrm{Sym}[M^{\rho_{1}\sigma_{1}}, \dots, M^{\rho_{2n}\sigma_{2n}}] D^{(\nu_{0}} D^{\nu_{1}} \dots D^{\nu_{2n})} \varphi_{s_{g}} \\ &+ \frac{1}{2} m^{2} \sum_{n=0}^{\infty} \frac{C_{n}}{2^{2n} m^{4n}} \epsilon_{\mu_{1}\nu_{1}\rho_{1}\sigma_{1}} \dots \epsilon_{\mu_{2n}\nu_{2n}\rho_{2n}\sigma_{2n}} \\ &\quad D^{(\mu_{1}} \dots D^{\mu_{2n})} \varphi_{s_{g}} \mathrm{Sym}[M^{\rho_{1}\sigma_{1}}, \dots, M^{\rho_{2n}\sigma_{2n}}] D^{(\nu_{1}} \dots D^{\nu_{2n})} \varphi_{s_{g}} \\ &+ \frac{i}{2} \sum_{n=0}^{\infty} \frac{E_{n}}{2^{2n} m^{4n}} \epsilon_{\mu_{1}\nu_{1}\rho_{1}\sigma_{1}} \dots \epsilon_{\mu_{2n}\nu_{2n}\rho_{2n}\sigma_{2n}} \\ &\quad D^{(\mu_{1}} \dots D^{\mu_{2n})} \varphi_{s_{g}} \mathrm{Sym}[M_{\mu_{0}\nu_{0}} F^{\mu_{0}\nu_{0}}, M^{\rho_{1}\sigma_{1}}, \dots, M^{\rho_{2n}\sigma_{2n}}] D^{(\nu_{2n}} \dots D^{\mu_{1})} \varphi_{s_{g}} \end{split}$$

Its three-point interaction double copies to that of the gravitationally coupled spinning particle.

It also reproduces $\sqrt{\text{Kerr...}}$

What is the relation to the effective action in Ben and Donal's talks...

For minimal coupling, we know the double copy (KLT-like) relation

$$i\mathcal{M}(1^s, 2^s, 3^h, 4^h) = -4\pi i G \frac{p_1 \cdot p_3 p_1 \cdot p_3 p_1 \cdot p_3 \cdot p_4}{p_3 \cdot p_4}$$

Where the amplitude comes from the Lagrangian

$$\mathcal{L}_{s,\text{EM}} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + D^{\dagger}_{\mu} \bar{\phi}_s D^{\mu} \phi_s - m^2 \bar{\phi}_s \phi_s + e(g-1) F_{\mu\nu} \bar{\phi}_s M^{\mu\nu} \phi_s$$

How do I need to adjust for the non-minimal coupling?...

What about Compton?

- $\frac{p_4}{-}A(1^0, 2^0, 3^A, 4^A) A(1^s, 2^s, 3^A, 4^A)$