S^2 Hamiltonian from Amplitudes and EFT

QCD Meets Gravity VI

Andres Luna, Bhaumik Institute for Theoretical Physics, UCLA

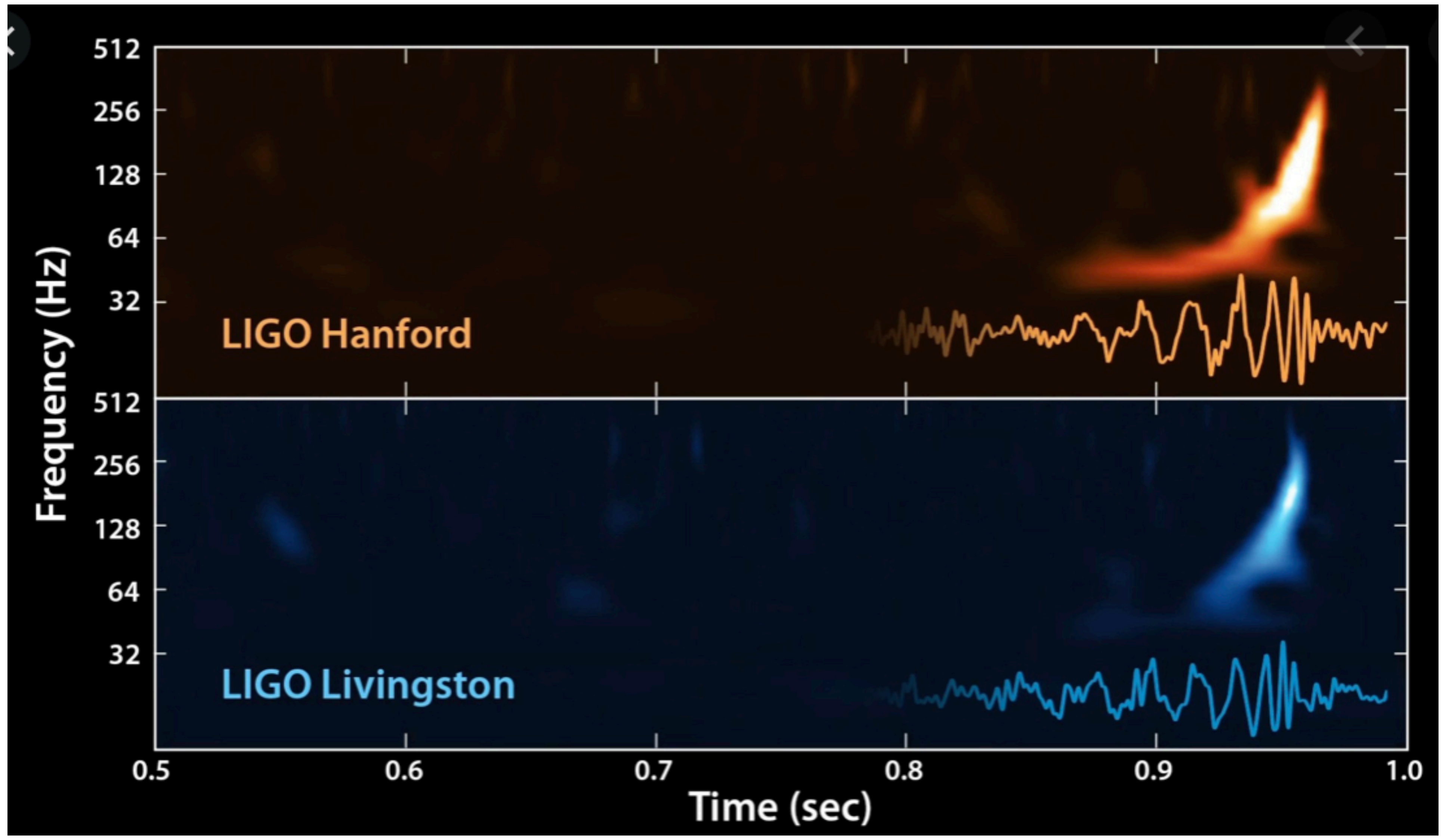
Outline

- Review of Spin Hamiltonians (based on 2005.03071 w/Bern, Roiban, Shen, Zeng)

- S_1^2 Hamiltonian (based on 2012.xxxxx w/Kosmopoulos+)

(a briefest) Introduction

(a not very needed) Motivation



An era of gravitational waves…

(EST)	Monday, Nov 30	Tuesday, Dec 1	Wednesday, Dec 2	Thursday, Dec 3	Friday, Dec 4
9:00-9:25	Free-Discussion (Euro-friendly)	Free-Discussion (Euro-friendly)	Free-Discussion (Euro-friendly)	Free-Discussion (Euro-friendly)	Free-Discussion (Euro-friendly)
9:25-9:50					
10:00-10:25		Song He		Daniel Baumann	
10:25-10:50		Gabriele Veneziano		Yvonne Geyer	
11:00-11:25	Dónal O'Connell	Michael Green	Tanja Hinderer	Rafael Porto	David Berman
11:25-11:50	Ben Maybee	Hadleigh Frost	Carlo Heissenberg	Silvia Nagy	Tim Adamo
12:00-12:25	Andrea Puhm	Yael Shadmi	Michèle Levi	Andrea Cristofoli	Ingrid Vazquez-Holm
12:25-12:50					
1:00-1:35					
1:35-2:00	Sebastian Mizera	Simon Caron-Huot	Cristian Vergu	Gregor Kälin	Agnese Bissi
2:00-2:25	Michael Ruf	Suna Zekioglu	Sebastian Poegel	Gustav Mogull	Jung-Wook Kim
2:25-2:50					
3:00-3:25	Cynthia Keeler	Julio Parra-Martinez	Shruti Paranjape	Chris White	Andres Luna
3:25-3:50	James Mangan	Oliver Schlotterer	Callum Jones	Nima Arkani-Hamed	Giulio Salvatori
4:00-4:25	Free-Discussion (West-coast-friendly)	Free-Discussion (West-coast-friendly)	Free-Discussion (West-coast-friendly)	Free-Discussion (West-coast-friendly)	Zvi Bern (3:50-4:05)
4:25-4:50					Free-Discussion (West-coast-friendly)
5:00-5:25					
5:25-5:50					

The talks related with classical gravity

Review of Spin Hamiltonians

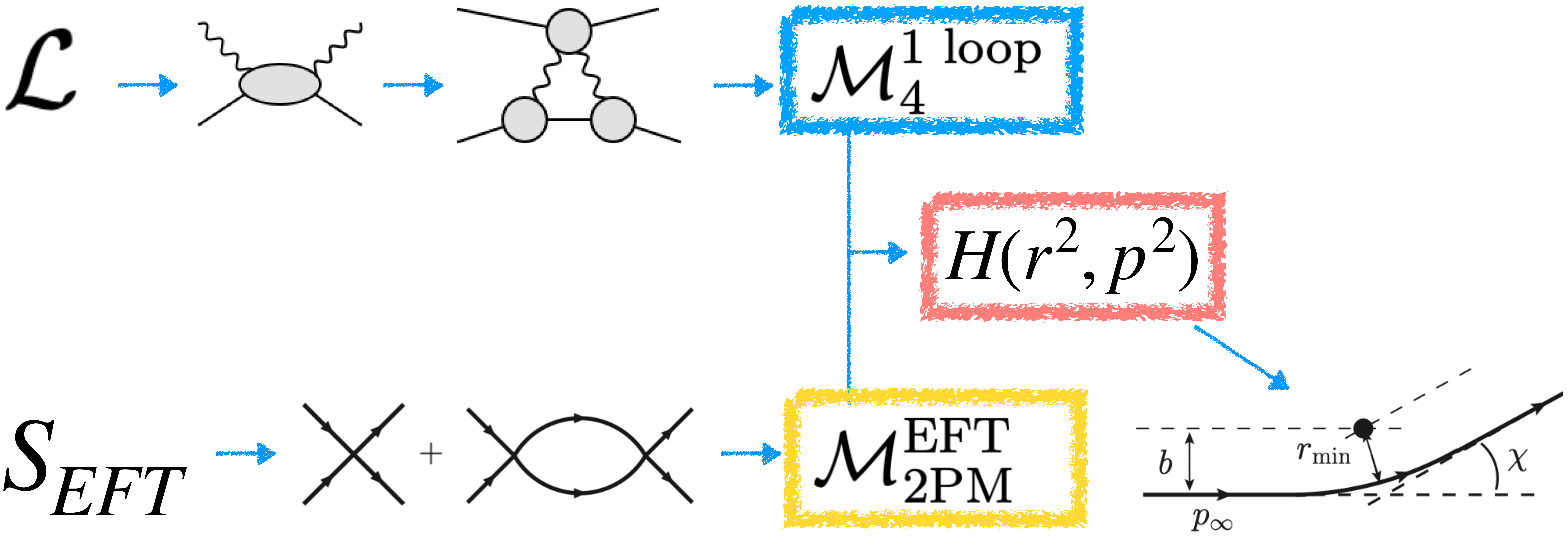
arXiv.org > hep-th > arXiv:2005.03071

High Energy Physics – Theory

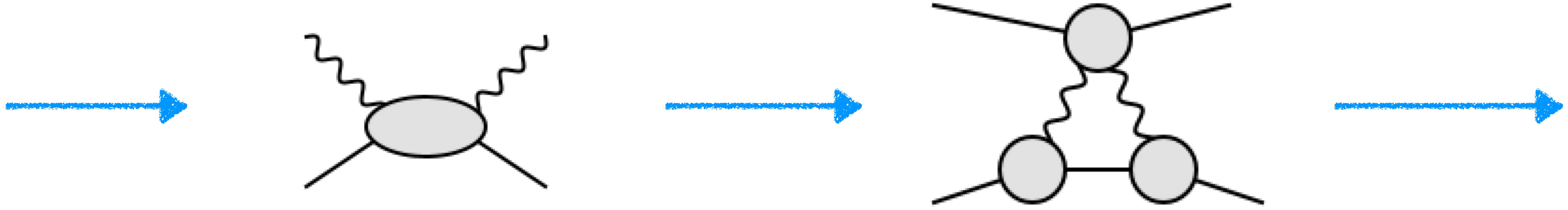
[Submitted on 6 May 2020]

Spinning Black Hole Binary Dynamics, Scattering Amplitudes and Effective Field Theory

Zvi Bern, Andres Luna, Radu Roiban, Chia-Hsien Shen, Mao Zeng



$\boxed{\mathcal{L}}$



$\mathcal{M}_4^{1\ \text{loop}}$

We consider a Lagrangian of rank-s tensor fields, minimally coupled to gravity

$$\mathcal{L}_{\text{min}} = -R(e,\omega) + \frac{1}{2} g^{\mu\nu} \boxed{\nabla(\omega)_\mu \phi_s} \nabla(\omega)_\nu \boxed{\phi_s}$$

$$\boxed{\phi_s{}^{a_1 a_2 a_3 \ldots a_s}}$$

Symmetric traceless tensor field

Covariant derivative

Lorentz Generator

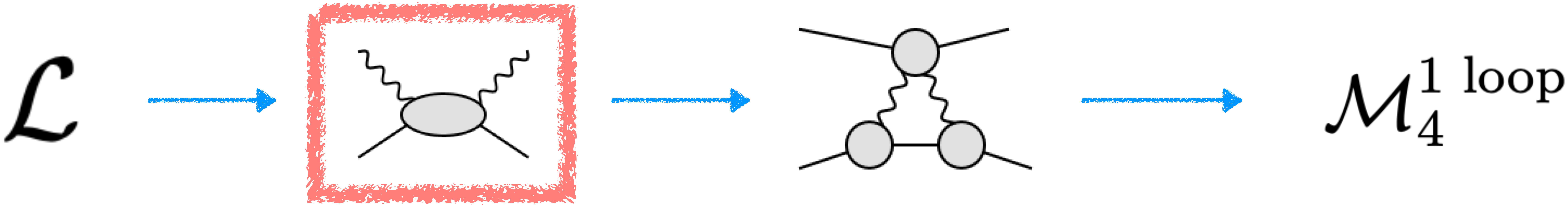
$$\boxed{\nabla(\omega)_\mu \phi_s} \equiv \partial_\mu \phi_s + \frac{i}{2} \omega_{\mu e f} \boxed{M^{ef}} \phi_s$$

Spin vector/tensor

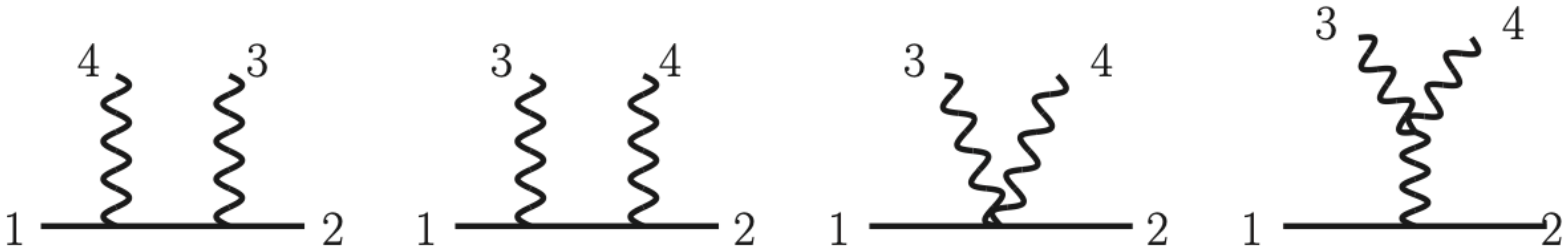
$$\boxed{\begin{aligned} \hat{S}^i &= \tfrac{1}{2} \epsilon^{ijk} M_{jk} \\ S^i &= \frac{1}{2} \epsilon^{ijk} S_{jk} \end{aligned}}$$

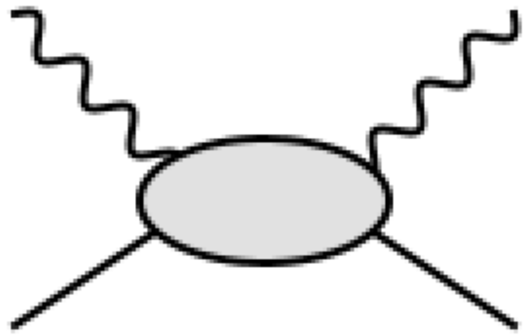
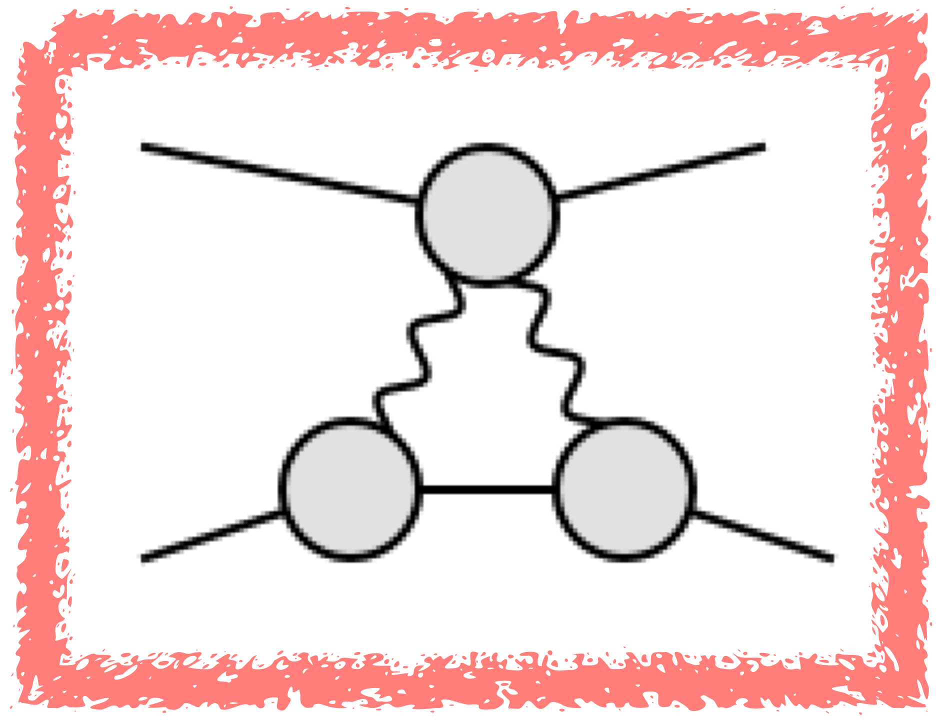
The spin tensor is obtained from the classical limit

$$\varepsilon(\boldsymbol{s}, p_1) \boxed{M^{ab}} \varepsilon(\boldsymbol{s}, p_2) = \boxed{S(p_1, \boldsymbol{S})^{ab}} \varepsilon(\boldsymbol{s}, p_1) \cdot \varepsilon(\boldsymbol{s}, p_2) + \mathcal{O}(q^0)$$



Deduce Feynman rules from Lagrangian, and compute Compton amplitude via 4 Feynman diagrams



$\mathcal{L} \longrightarrow$  $\longrightarrow$  $\longrightarrow \mathcal{M}_4^{1\text{ loop}}$

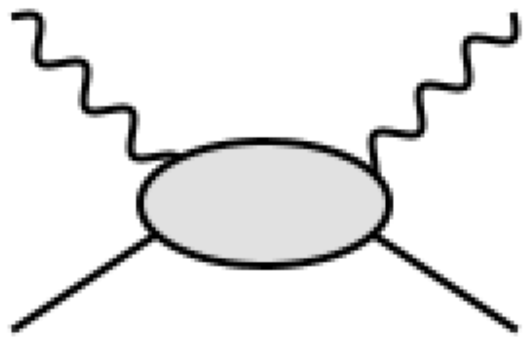
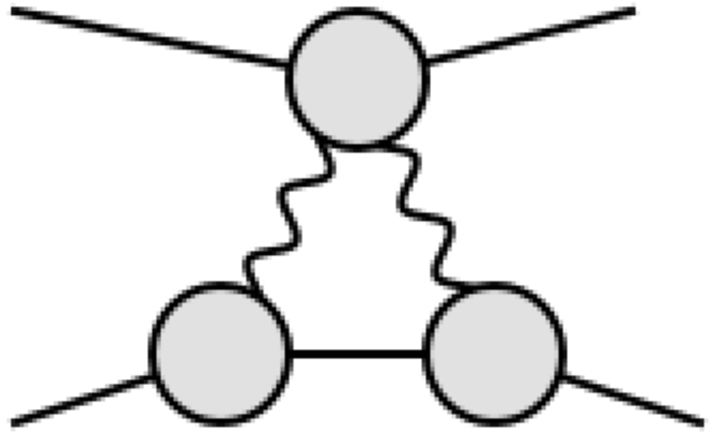
Sew Compton (and 3-point) amplitudes into unitarity (3- and 4-particle) cuts

$$\sum_\lambda \varepsilon(k)^{\mu\nu}_\lambda \varepsilon(-k)^{\alpha\beta}_{-\lambda} = \frac{1}{2}\mathcal{P}^{\mu\alpha}\mathcal{P}^{\nu\beta} + \frac{1}{2}\mathcal{P}^{\nu\alpha}\mathcal{P}^{\mu\beta} - \frac{1}{D-2}\mathcal{P}^{\mu\nu}\mathcal{P}^{\alpha\beta} \qquad \mathcal{P}^{\mu\nu}(k) = \eta^{\mu\nu} - \cancel{\frac{r^\mu k^\nu + r^\nu k^\mu}{r\cdot k}}$$

The amplitudes satisfy Generalized Ward Identities…

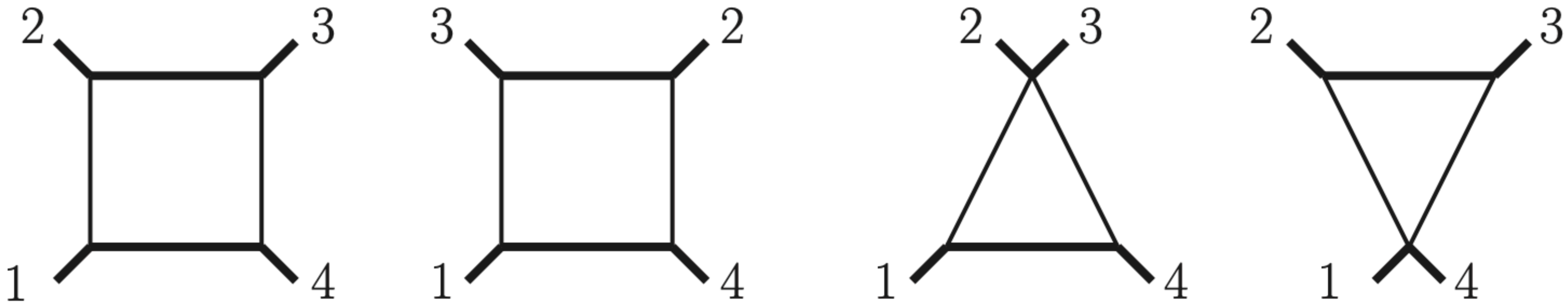
$$\sum_\lambda \varepsilon(k)^{\mu\nu}_\lambda \varepsilon(-k)^{\alpha\beta}_{-\lambda} = \frac{1}{2}\eta^{\mu\alpha}\eta^{\nu\beta} + \frac{1}{2}\eta^{\nu\alpha}\eta^{\mu\beta} - \frac{1}{D-2}\eta^{\mu\nu}\eta^{\alpha\beta}$$

…so we sew using de Donder

$\mathcal{L}$ →  →  → $\boxed{\mathcal{M}_4^{1\text{ loop}}}$

Use generalized unitarity (Forde’s method) to obtain box and triangle coefficients

$$i\mathcal{M}_4^{1\text{ loop}} = d_{\mathrm{B}}\, I_{\mathrm{B}} + d_{\overline{\mathrm{B}}}\, I_{\overline{\mathrm{B}}} + c_{\triangle}\, I_{\triangle} + c_{\bigtriangledown}\, I_{\bigtriangledown}$$



$$\boxed{S_{EFT}} \longrightarrow$$

$$\longrightarrow \mathcal{M}^{\mathrm{EFT}}_{\mathrm{2PM}}$$

Cheung, Rothstein and Solon’s EFT of non-relativistic fields…

$$S = \int_{\boldsymbol{k}} \sum_{a=1,2} \xi_a^\dagger(-\boldsymbol{k})\left(i\partial_t - \sqrt{\boldsymbol{k}^2+m_i^2}\right)\xi_a(\boldsymbol{k}) - \int_{\boldsymbol{k},\boldsymbol{k}'} \xi_1^\dagger(\boldsymbol{k}')\xi_2^\dagger(-\boldsymbol{k})\boxed{\hat V(\boldsymbol{k}',\boldsymbol{k},\hat{\boldsymbol{S}}_a)}\xi_1(\boldsymbol{k})\xi_2(-\boldsymbol{k})$$

…generalized to describe spinning fields. The potential…

$$\boxed{\hat V(\boldsymbol{k}',\boldsymbol{k},\hat{\boldsymbol{S}}_i)} = \sum_A \boxed{\hat V^A(\boldsymbol{k}',\boldsymbol{k})}\boxed{\hat{\mathbb{O}}^A}$$

$$\boxed{\hat V^A(\boldsymbol{k}',\boldsymbol{k}) = \frac{4\pi G}{\boldsymbol{q}^2} d_1^A\left(\boldsymbol{p}^2\right) + \frac{2\pi^2 G^2}{|\boldsymbol{q}|} d_2^A\left(\boldsymbol{p}^2\right) + \mathcal{O}(G^3)}$$

…considers long range interactions…

$$\hat{\mathbb{O}}^{(0)} = \mathbb{I}\,, \qquad \hat{\mathbb{O}}^{(1,1)} = \boldsymbol{L}_q \cdot \hat{\boldsymbol{S}}_1\,, \qquad \hat{\mathbb{O}}^{(1,2)} = \boldsymbol{L}_q \cdot \hat{\boldsymbol{S}}_2\,,$$

$$\hat{\mathbb{O}}^{(2,1)} = \boldsymbol{q}\cdot\hat{\boldsymbol{S}}_1\,\boldsymbol{q}\cdot\hat{\boldsymbol{S}}_2\,, \qquad \hat{\mathbb{O}}^{(2,2)} = \boldsymbol{q}^2\,\hat{\boldsymbol{S}}_1\cdot\hat{\boldsymbol{S}}_2\,, \qquad \hat{\mathbb{O}}^{(2,3)} = \boldsymbol{q}^2\,\boldsymbol{k}\cdot\hat{\boldsymbol{S}}_1\,\boldsymbol{k}\cdot\hat{\boldsymbol{S}}_2$$

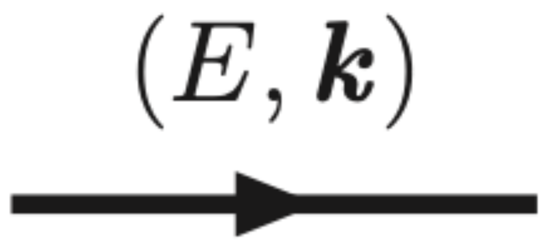
…and is organized by classical spin operators

S_{EFT} $\longrightarrow$

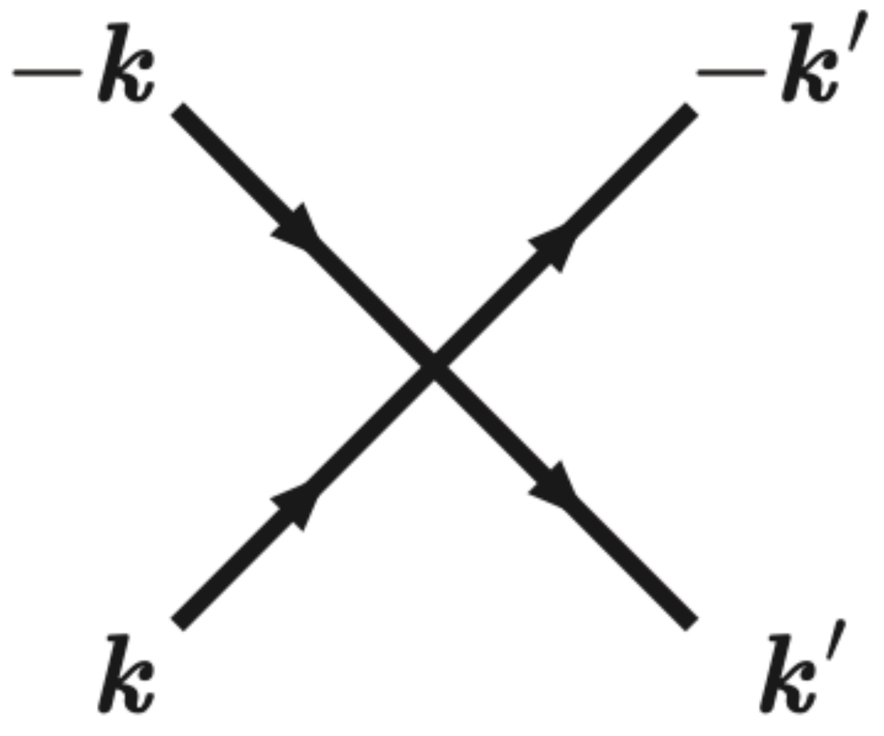
$\longrightarrow$ $\mathcal{M}^{\mathrm{EFT}}_{\mathrm{2PM}}$

$$S = \int_{\boldsymbol{k}} \sum_{a=1,2} \xi_a^\dagger(-\boldsymbol{k}) \underline{\left(i\partial_t - \sqrt{\boldsymbol{k}^2 + m_i^2} \right)} \xi_a(\boldsymbol{k}) - \int_{\boldsymbol{k},\boldsymbol{k}'} \xi_1^\dagger(\boldsymbol{k}')\xi_2^\dagger(-\boldsymbol{k}') \underline{\widehat{V}(\boldsymbol{k}', \boldsymbol{k}, \hat{\boldsymbol{S}}_a)} \xi_1(\boldsymbol{k})\xi_2(-\boldsymbol{k})$$

Deduce Feynman rules from the effective theory action…



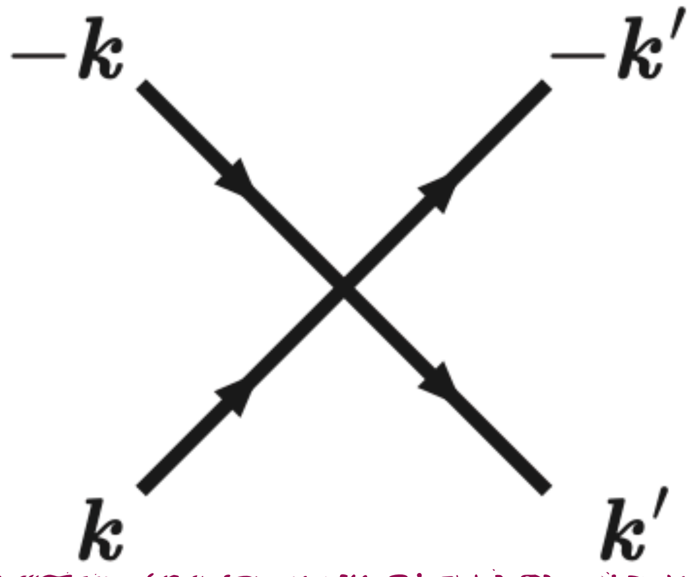
$$= \frac{i\,\mathbb{I}}{E - \sqrt{\boldsymbol{k}^2 + m^2} + i\epsilon}$$



$$= -i\hat{V}(\boldsymbol{k}', \boldsymbol{k}, \hat{\boldsymbol{S}}_i)$$

S_{EFT} →  → $\mathcal{M}^{\mathrm{EFT}}_{\mathrm{2PM}}$

$$= \frac{i\,\mathbb{I}}{E - \sqrt{\boldsymbol{k}^2 + m^2} + i\epsilon}$$



$$= -i\hat{V}(\boldsymbol{k}', \boldsymbol{k}, \hat{\boldsymbol{S}}_i)$$

…and use them to compute the amplitude

$$\widehat{\mathcal{M}}^{\mathrm{EFT}} = -\hat{V}(\boldsymbol{p}', \boldsymbol{p}) - \int_{\boldsymbol{k}} \frac{\hat{V}(\boldsymbol{p}', \boldsymbol{k})\hat{V}(\boldsymbol{k}, \boldsymbol{p})}{E_1 + E_2 - \sqrt{\boldsymbol{k}^2 + m_1^2} - \sqrt{\boldsymbol{k}^2 + m_2^2}}$$

$$S_{EFT} \longrightarrow$$

$$\longrightarrow \boxed{\mathcal{M}^{\mathrm{EFT}}_{\mathrm{2PM}}}$$

$$\boxed{\mathcal{M}^{\mathrm{EFT}}_{\mathrm{2PM}}} = \frac{2\pi^2 G^2}{|\boldsymbol{q}|}\Big[a_2^{(0)} + a_2^{(1,1)}\boldsymbol{L}_q\cdot\boldsymbol{S}_1 + a_2^{(1,2)}\boldsymbol{L}_q\cdot\boldsymbol{S}_2 + a_2^{(2,1)}\boldsymbol{q}\cdot\boldsymbol{S}_1\,\boldsymbol{q}\cdot\boldsymbol{S}_2 + \boxed{a_2^{(2,2)}}\boldsymbol{q}^2\,\boldsymbol{S}_1\cdot\boldsymbol{S}_2 + a_2^{(2,3)}\boldsymbol{q}^2\,\boldsymbol{p}\cdot\boldsymbol{S}_1\,\boldsymbol{p}\cdot\boldsymbol{S}_2\Big]$$

$$+ (4\pi G)^2\, a_{\mathrm{iter}} \int \frac{d^{D-1}\ell}{(2\pi)^{D-1}} \frac{2\xi E}{\boldsymbol{\ell}^2(\boldsymbol{\ell}+\boldsymbol{q})^2(\boldsymbol{\ell}^2+2\boldsymbol{p}\cdot\boldsymbol{\ell})}$$

$$\boxed{a_2^{(2,2)}} = -d_2^{(2,2)} - \frac{1}{2\xi E}\boxed{A_2\left[d_1^{(0)}d_1^{(2,1)}\right]} - \frac{\boldsymbol{p}^2}{\xi E}A_2\left[d_1^{(1,1)}d_1^{(1,2)}\right] + \frac{\xi E}{8}\left(d_1^{(1,1)} + d_1^{(1,2)}\right)d_1^{(2,1)}$$

$$\boxed{A_2[X]} = \left[\frac{1}{4}(1-3\xi) + \frac{\xi^2E^2}{\boldsymbol{p}^2} + \frac{1}{2}\xi^2E^2\partial\right]X$$

$$
\boxed{H} = H^{(0)}(r^2,p^2) + H^{(1,i)}(r^2,p^2)\boldsymbol{L}\cdot\boldsymbol{S}_i + \boxed{H^{(2,1)}(r^2,p^2)}\boldsymbol{r}\cdot\boldsymbol{S}_1\boldsymbol{r}\cdot\boldsymbol{S}_2 + H^{(2,2)}(r^2,p^2)\boldsymbol{S}_1\cdot\boldsymbol{S}_2 + H^{(2,3)}(r^2,p^2)\boldsymbol{p}\cdot\boldsymbol{S}_1\boldsymbol{p}\cdot\boldsymbol{S}_2 + \ldots\,,
$$

$$\mathcal{M}_4^{1\ \mathrm{loop}}$$

$$\boxed{H(r^2,p^2)}$$

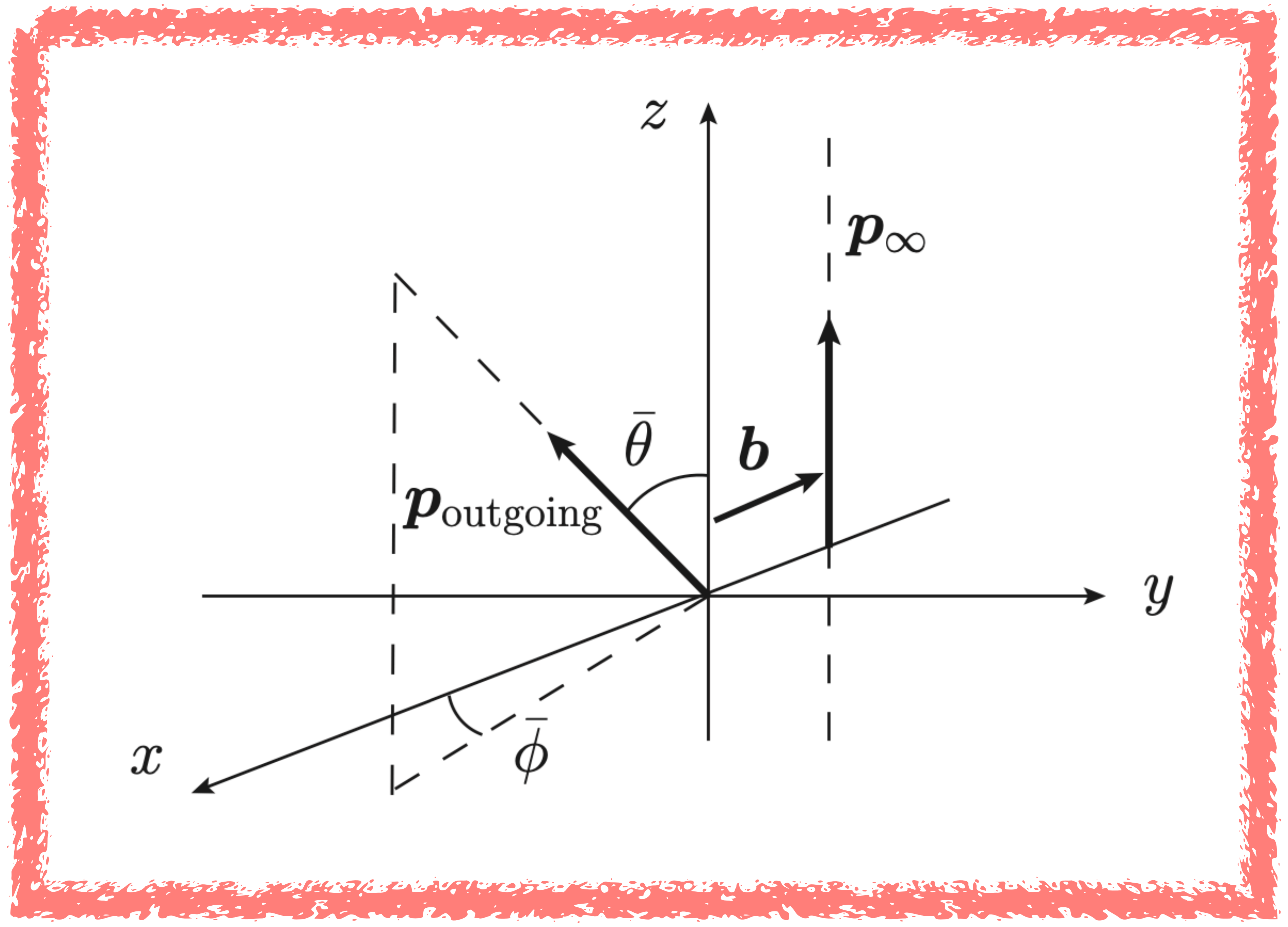
$$
\boxed{H^{(2,1)}(r^2,p^2)} = \frac{1}{r^4}\left[\frac{G}{r}\boxed{c_1^{(2,1)}(\boldsymbol{p}^2)} + \left(\frac{G}{r}\right)^2 c_2^{(2,1)}(\boldsymbol{p}^2) + \mathcal{O}(G^3)\right]
$$

$$\mathcal{M}_{2\mathrm{PM}}^{\mathrm{EFT}}$$

Determine coefficients in the Hamiltonian from Matching

$$
\underbrace{a_2^{(2,1)}}_{\text{From full theory}} = \boxed{\frac{1}{8}c_2^{(2,1)}} \underbrace{- \frac{1}{6\xi E}A_3\left[c_1^{(0)}c_1^{(2,1)}\right] + \frac{\boldsymbol{p}^2}{2\xi E}A_2\left[c_1^{(1,1)}c_1^{(1,2)}\right] + \frac{\xi E}{24}(c_1^{(1,1)} + c_1^{(1,2)})c_1^{(2,1)}}_{\text{Iteration pieces}}
$$

$$H(r^2, p^2) \longrightarrow$$



Use the Hamiltonian to determine physical observables

Write equations of motion…

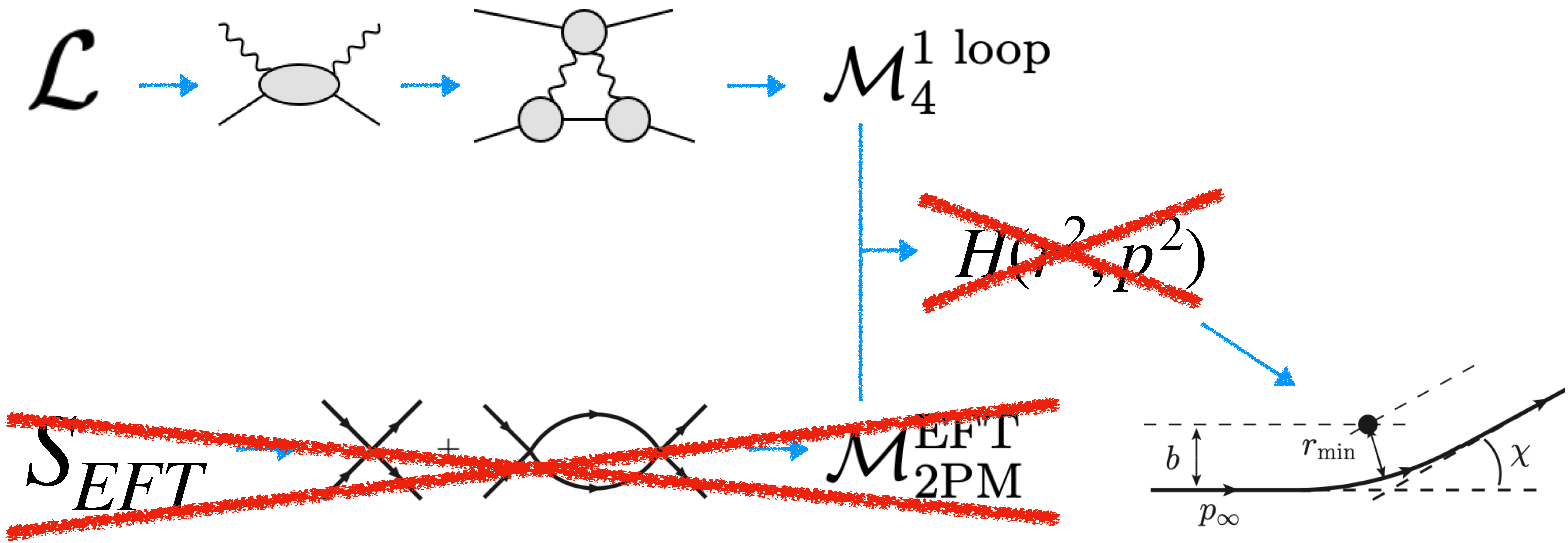
$$\dot{\boldsymbol{r}} = \frac{\partial H}{\partial \boldsymbol{p}}, \qquad \dot{\boldsymbol{p}} = -\frac{\partial H}{\partial \boldsymbol{r}}, \qquad \dot{\boldsymbol{S}}_a = -\boldsymbol{S}_a \times \frac{\partial H}{\partial \boldsymbol{S}_a}, \quad a = 1, 2$$

…and solve perturbatively

$$\Delta O_n = \int_{-\infty}^{\infty} dt \, \frac{dO_n}{dt}$$

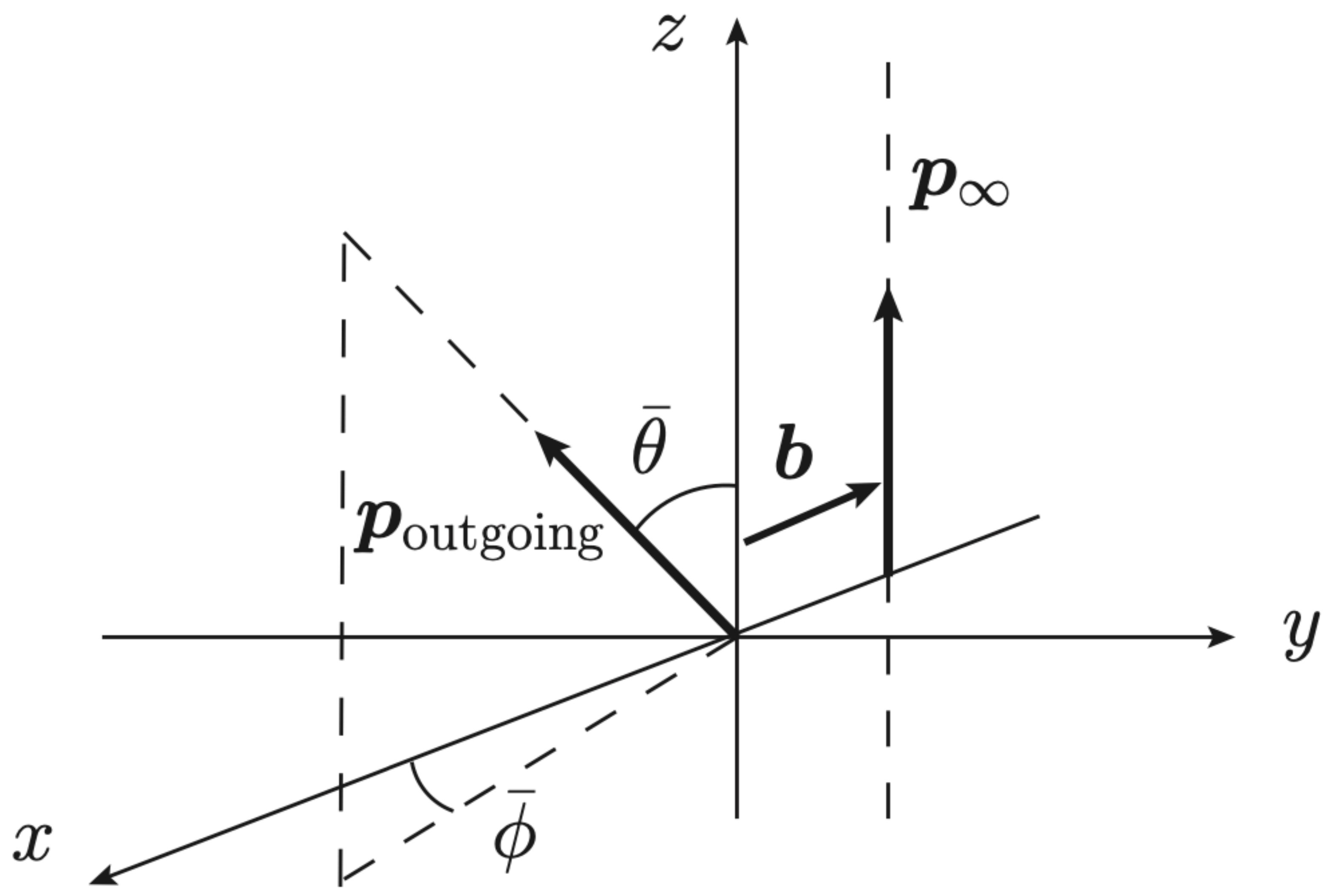
$$\boldsymbol{r}(t) = \boldsymbol{r}_0(t) + G\boldsymbol{r}_1(t) + G^2\boldsymbol{r}_2(t) + \dots \,,$$
$$\boldsymbol{p}(t) = \boldsymbol{p}_0(t) + G\boldsymbol{p}_1(t) + G^2\boldsymbol{p}_2(t) + \dots \,,$$
$$\boldsymbol{S}_a(t) = \boldsymbol{S}_{a,0}(t) + G\boldsymbol{S}_{a,1}(t) + G^2\boldsymbol{S}_{a,2}(t) + \dots$$

$$\boxed{\Delta \boldsymbol{p} \qquad \Delta \boldsymbol{S}_i}$$



Cut the middleman!

$$\boxed{\mathcal{M}_4^{1\text{ loop}}}$$

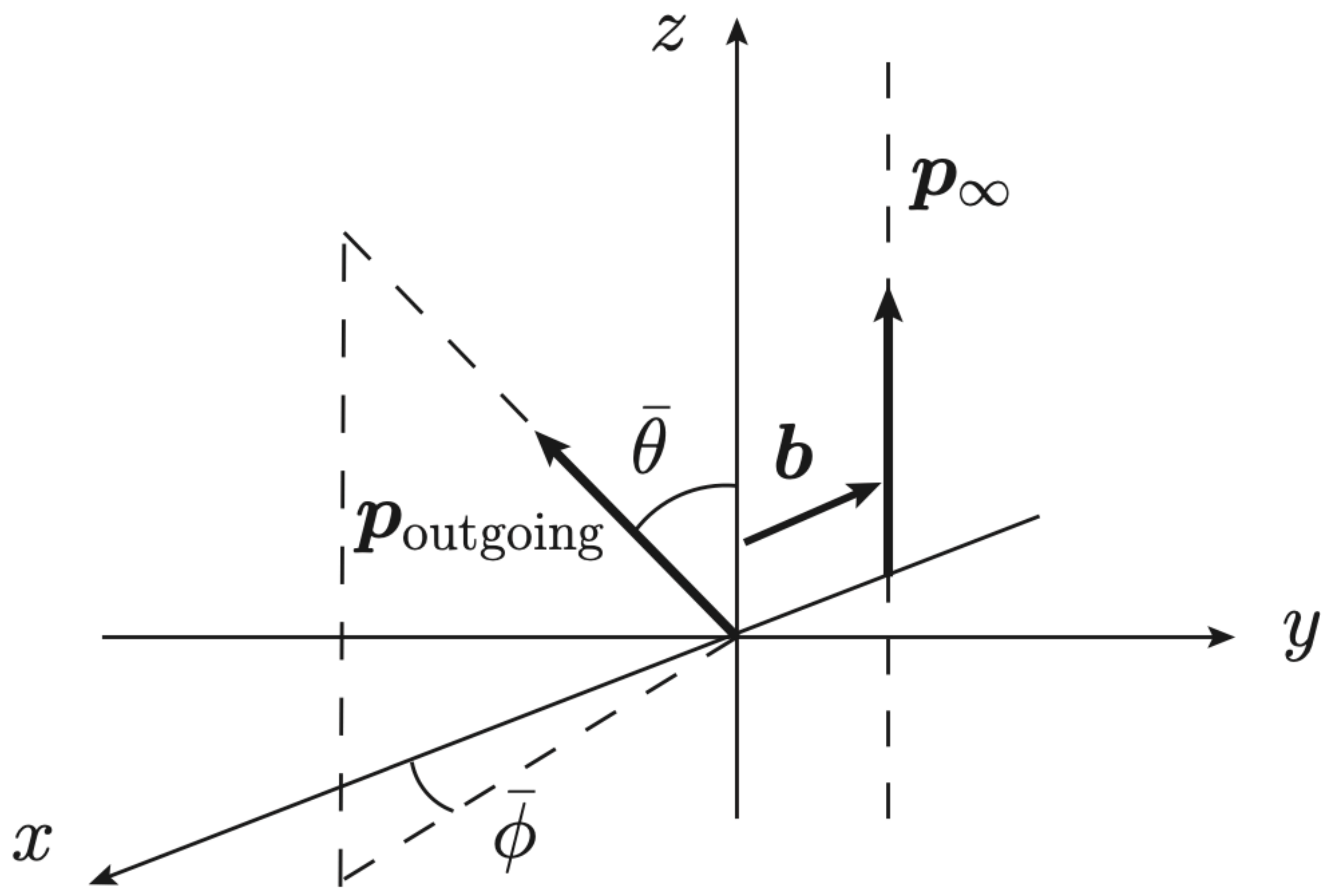


Physical observables (directly) from Amplitudes

Boundary2Bound (Kalin, Liu, Porto),
KMOC (+Vines, Cristofoli),
Eikonal (ACV, DiVecchia, Heissenberg, Russo, Bjerrum-Bohr, Damgaard, Cristofoli, Bern, Parra-Martinez, Ruf, Zeng, GOV, …)
Several other contributors (some of them in the audience(?) I apologize in advance…)
Talks by: Dónal, Ben, Gabriele, Julio, Carlo, Rafael, Andrea, Gregor, Jung-Wook

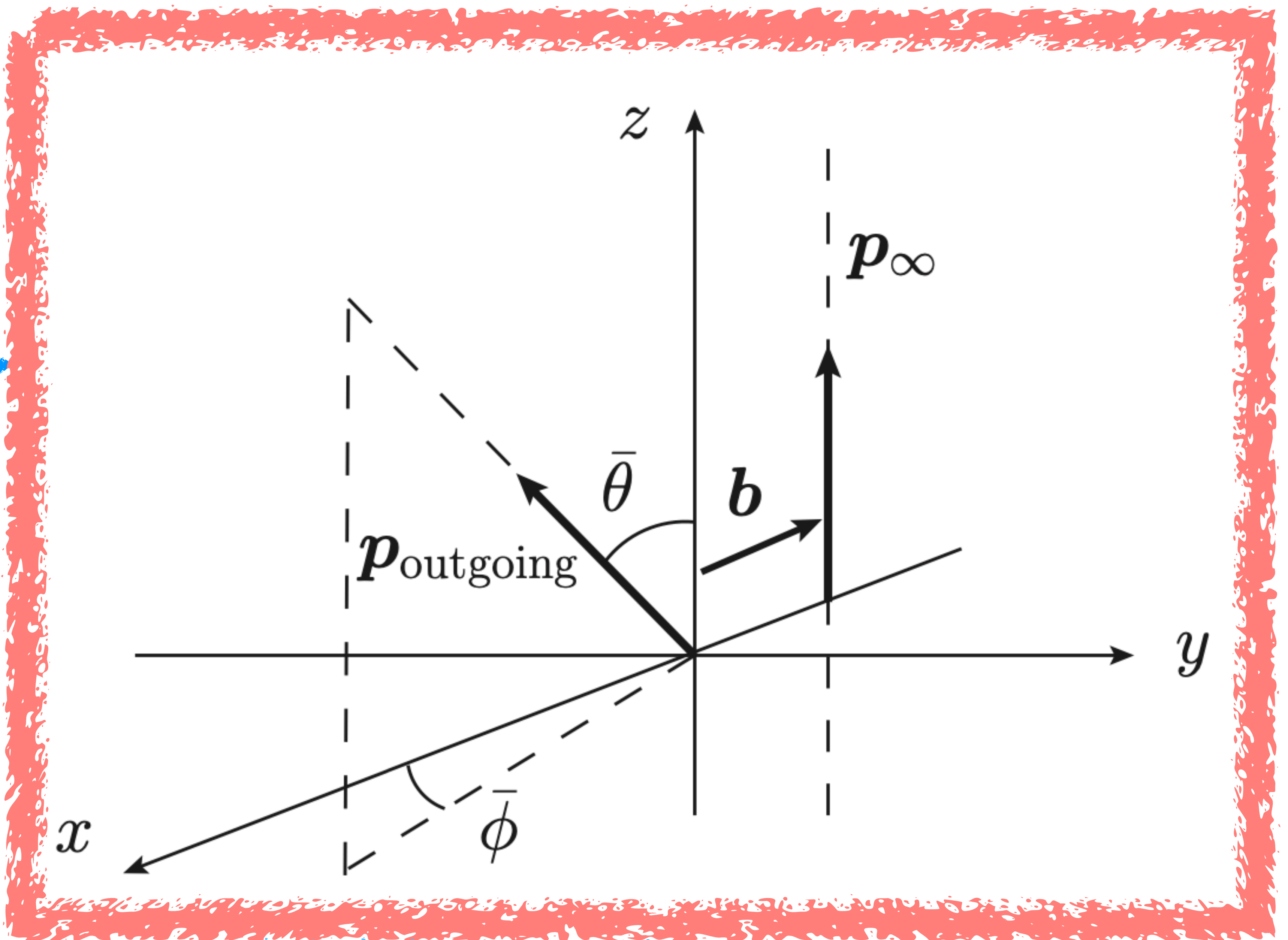
$$\boxed{\mathcal{M}_4^{1\ \mathrm{loop}}}$$

$$\chi_2 = \frac{1}{4m_1 m_2\sqrt{\sigma^2 - 1}} \int \frac{d^2\boldsymbol{q}}{(2\pi)^2} e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} \mathcal{M}^{\triangle+\bigtriangledown}(\boldsymbol{q})$$



Eikonal phase-like object. Fourier transform to impact parameter space of the (triangle part) of the amplitude.

$$\mathcal{M}_4^{1\ \mathrm{loop}}$$



$$\chi_2 = \frac{1}{4m_1 m_2\sqrt{\sigma^2-1}}\int \frac{d^2\boldsymbol{q}}{(2\pi)^2} e^{-i\boldsymbol{q}\cdot\boldsymbol{b}}\mathcal{M}^{\triangle+\nabla}(\boldsymbol{q})$$

$$\boxed{\{\boldsymbol{P}_\perp, f\}} \equiv -\nabla_{\boldsymbol{b}} f$$

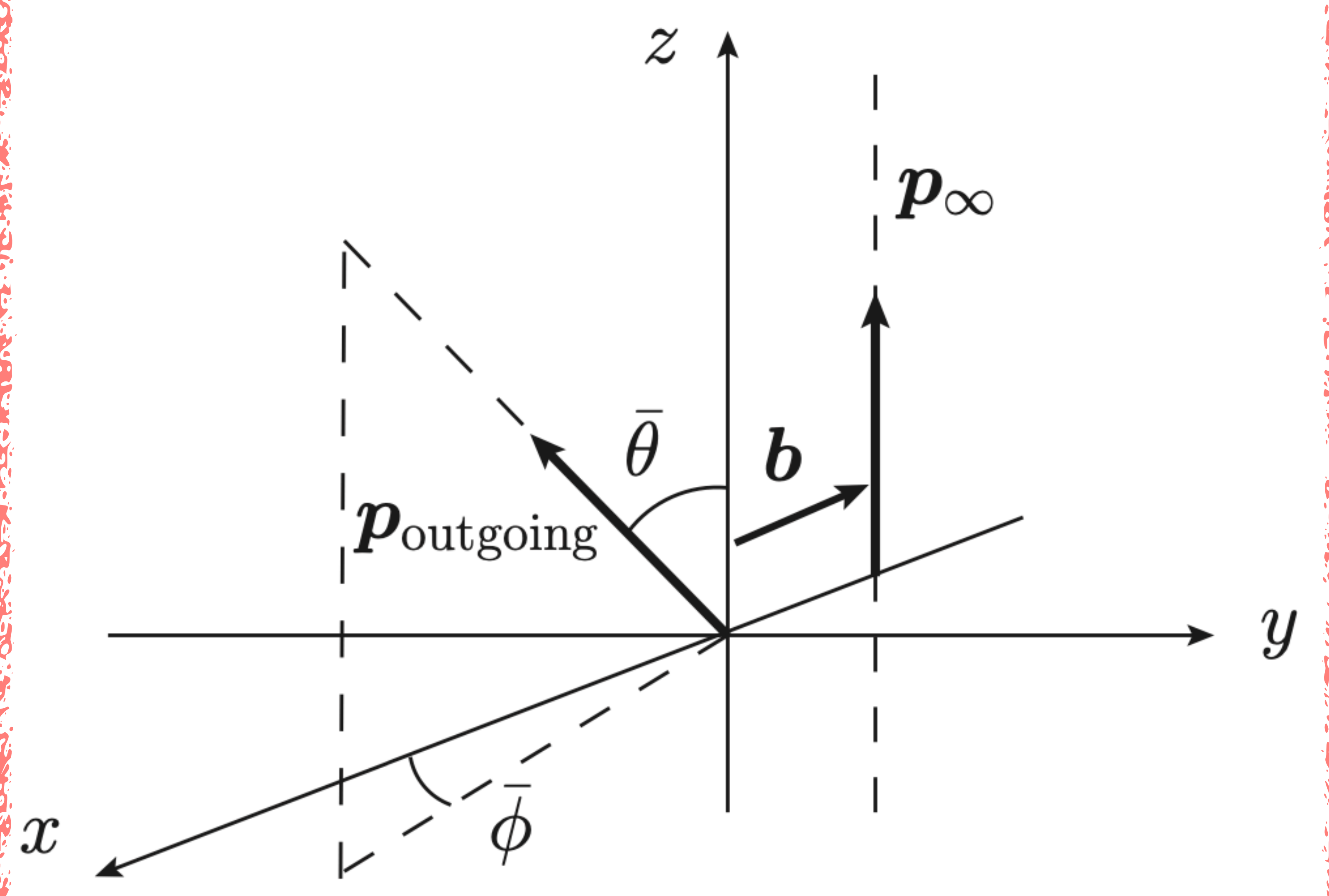
$$\boxed{\{S_a^i, f\}} \equiv \epsilon^{ijk}\frac{\partial f}{\partial S_a^j} S_a^k$$

$$\boxed{\mathcal{D}_{SL}(f,g)} \equiv -\sum_{a=1,2}\epsilon^{ijk} S_a^k \frac{\partial f}{\partial S_a^i}\frac{\partial g}{\partial L^j}$$

$$\Delta\boldsymbol{p}_\perp = -\boxed{\{\boldsymbol{P}_\perp,\chi\}} - \frac{1}{2}\{\chi,\{\boldsymbol{P}_\perp,\chi\}\} - \boxed{\mathcal{D}_{SL}(\chi,\{\boldsymbol{P}_\perp,\chi\})} + \frac{1}{2}\{\boldsymbol{P}_\perp,\mathcal{D}_{SL}(\chi,\chi)\}$$

$$\Delta\boldsymbol{S}_i = -\boxed{\{\boldsymbol{S}_i,\chi\}} - \underbrace{\frac{1}{2}\{\chi,\{\boldsymbol{S}_i,\chi\}\} - \mathcal{D}_{SL}(\chi,\{\boldsymbol{S}_i,\chi\}) + \frac{1}{2}\{\boldsymbol{S}_i,\mathcal{D}_{SL}(\chi,\chi)\}}_{\text{Iteration-like pieces}}$$

$$\mathcal{M}_4^{1\ \text{loop}}$$



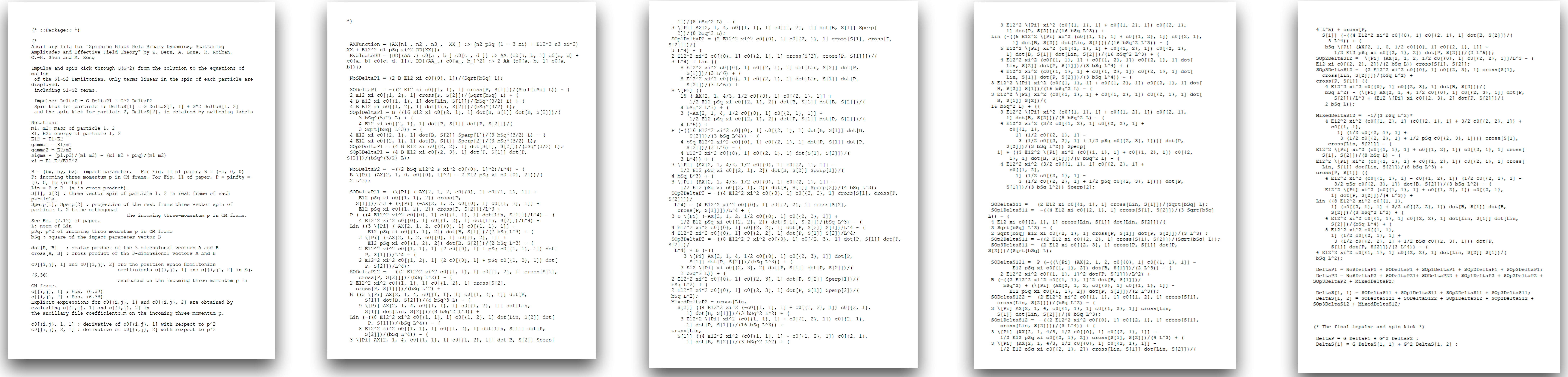
$$\chi_2 = \frac{1}{4m_1m_2\sqrt{\sigma^2-1}} \int \frac{d^2\boldsymbol{q}}{(2\pi)^2} e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} \mathcal{M}^{\triangle+\nabla}(\boldsymbol{q})$$

$$\boxed{\Delta \boldsymbol{p}_\perp} = -\{\boldsymbol{P}_\perp, \chi\} - \frac{1}{2}\{\chi, \{\boldsymbol{P}_\perp, \chi\}\} - \mathcal{D}_{SL}(\chi, \{\boldsymbol{P}_\perp, \chi\}) + \frac{1}{2}\{\boldsymbol{P}_\perp, \mathcal{D}_{SL}(\chi, \chi)\}$$

$$\boxed{\Delta \boldsymbol{S}_i} = -\{\boldsymbol{S}_i, \chi\} - \frac{1}{2}\{\chi, \{\boldsymbol{S}_i, \chi\}\} - \boxed{\mathcal{D}_{SL}}(\chi, \{\boldsymbol{S}_i, \chi\}) + \frac{1}{2}\{\boldsymbol{S}_i, \mathcal{D}_{SL}(\chi, \chi)\}$$

A conjecture for an all-orders generalization of the formula

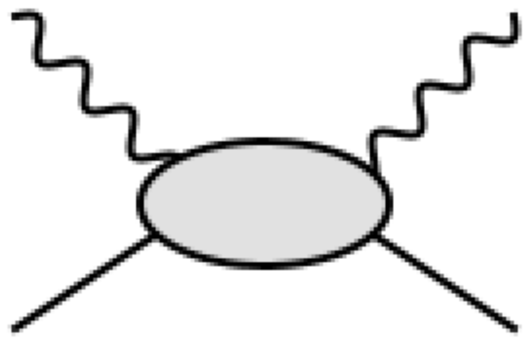
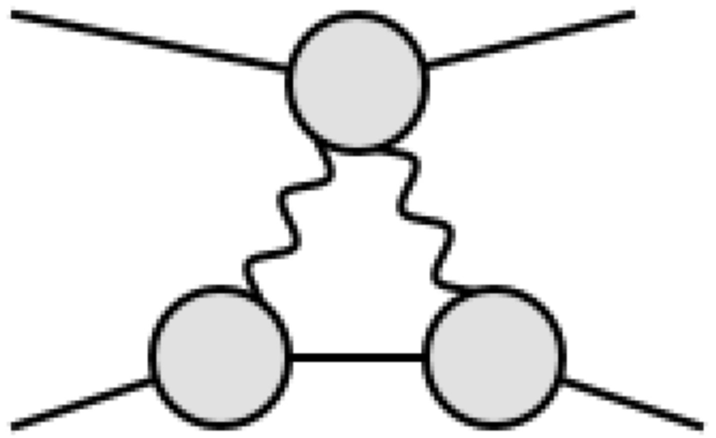
$$\boxed{\Delta\mathcal{O}} = e^{-i\chi\mathcal{D}}[\mathcal{O}, e^{i\chi\boxed{\mathcal{D}}}] \qquad \boxed{\chi\,\mathcal{D}\,g} \equiv \chi g + \boxed{D_{SL}}(\chi, g)$$



$$\boxed{\Delta\mathcal{O} = e^{-i\chi^D}\left[\mathcal{O}, e^{i\chi^D}\right]}$$

S_1^2 Hamiltonian

w/ D Kosmopoulos (UCLA)

$\boxed{\mathcal{L}} \longrightarrow$  $\longrightarrow$  $\longrightarrow \mathcal{M}_4^{1\text{ loop}}$

An effective Lagrangian encoding higher order spin interactions (Porto, Rothstein; Levi, Steinhoff). To tree-level, reproduce Kerr. What about loops?

$$\mathcal{L}_{\text{non-min}} = \sum_{\boxed{n=1}}^{\infty} \frac{(-1)^n}{(2n)!} \frac{C_{ES^{2n}}}{m^{2n}} \nabla(\omega)_{f_{2n}} \cdots \nabla(\omega)_{f_3} R_{f_1 a f_2 b} \nabla(\omega)^a \phi_s \, \mathbb{S}^{(f_1} \ldots \mathbb{S}^{f_{2n})} \nabla(\omega)^b \phi_s$$
$$- \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{C_{BS^{2n}}}{m^{2n+1}} \nabla(\omega)_{f_{2n+1}} \cdots \nabla(\omega)_{f_3} \frac{1}{2} \epsilon_{ab(c|f_1} R^{ab}{}_{|d)\, f_2} \nabla(\omega)^c \phi_s \mathbb{S}^{(f_1} \ldots \mathbb{S}^{f_{2n+1})} \nabla(\omega)^d \phi_s$$

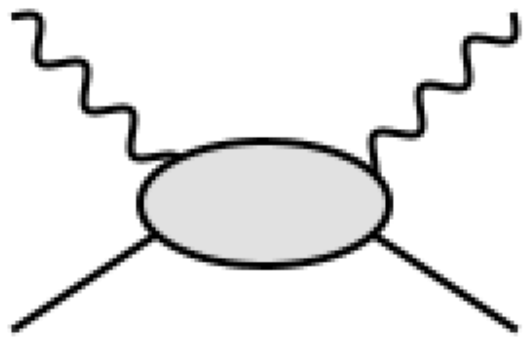
Look at the first term

$$\boxed{\mathcal{L}_{ES^2}} = -\frac{C_{ES^2}}{2m^2} R_{f_1 a f_2 b} \nabla^a \phi_s \boxed{\hat{\mathbb{S}}^{(f_1} \hat{\mathbb{S}}^{f_2)}} \nabla^b \phi_s$$

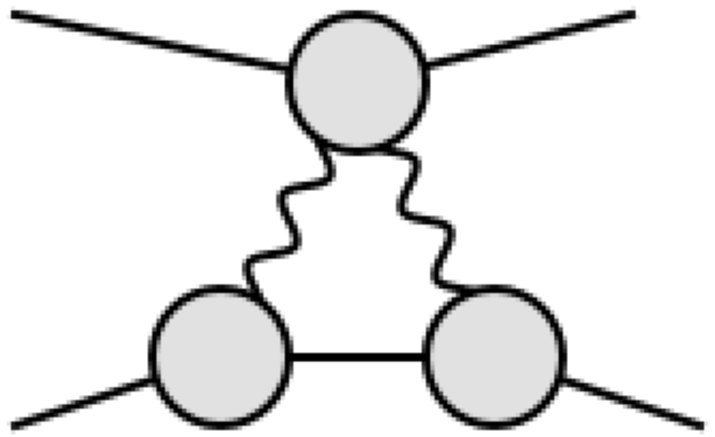
$$\boxed{\hat{\mathbb{S}}^a} \equiv \frac{-i}{2m} \boxed{\widetilde{M}^{ab}} \nabla(\omega)_b$$

$$\boxed{\widetilde{M}^{ab}} \equiv \epsilon^{abcd} M_{cd}$$

$\boxed{\mathcal{L}} \longrightarrow$



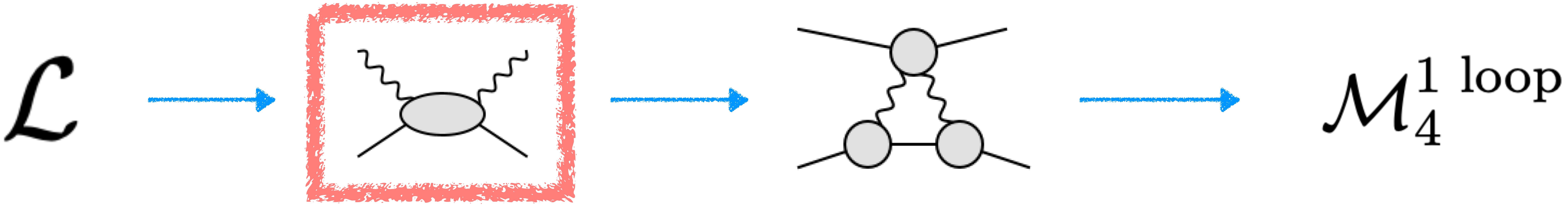
$\longrightarrow$



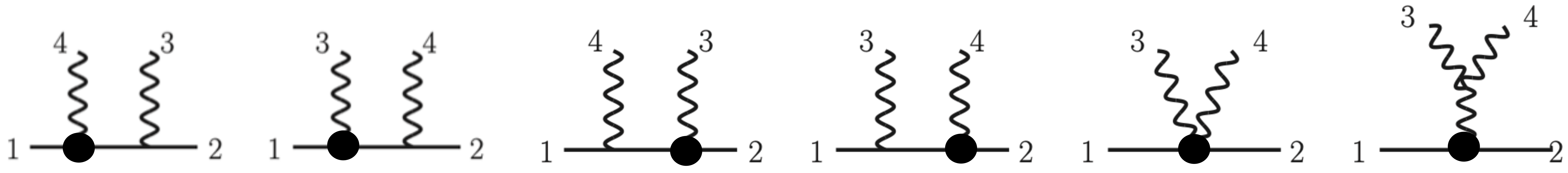
$\longrightarrow \mathcal{M}_4^{1\text{ loop}}$

$$\boxed{\mathcal{L}_{ES^2} = \frac{C_{ES^2}}{8m^4} R_{f_1 a f_2 b} \nabla^a \phi_s \, \widetilde{M}^{f_1 c_1} \widetilde{M}^{f_2 c_2} \nabla_{c_1} \nabla_{c_2} \nabla^b \phi_s .}$$

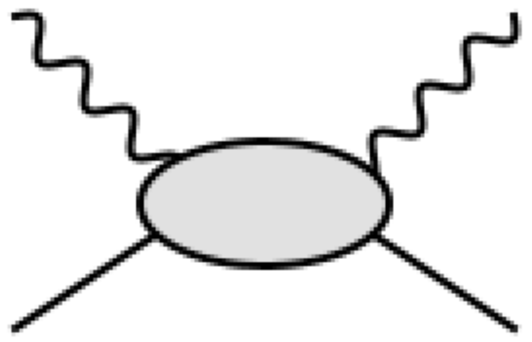
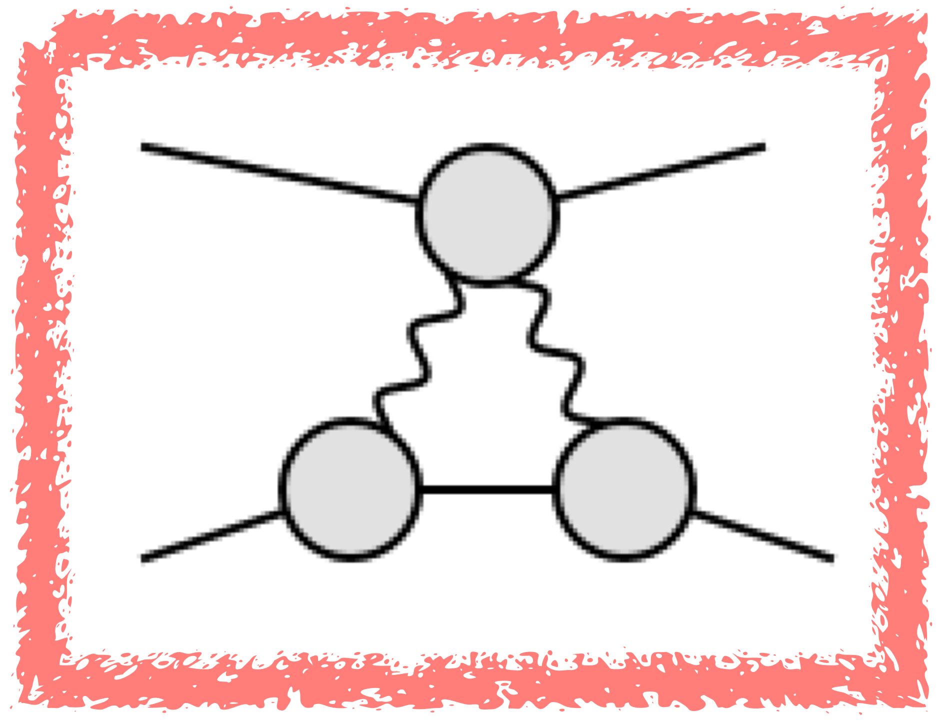
From this Lagrangian, we can deduce Feynman rules…



…and compute the Compton amplitude. For the non-minimally coupled:



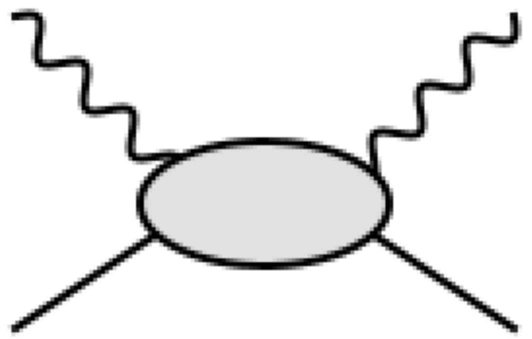
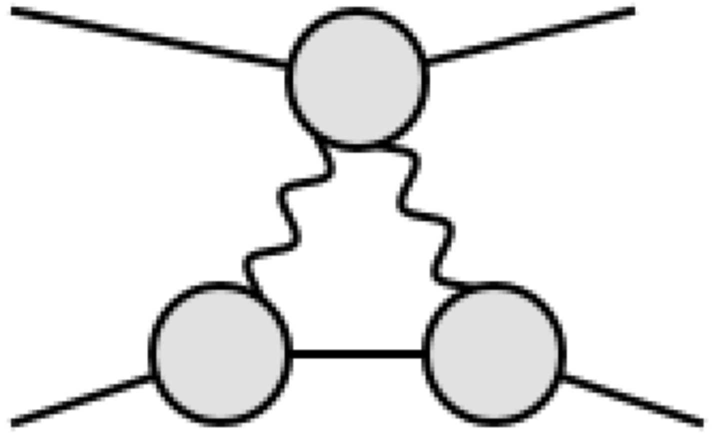
$$
\begin{aligned}
&\Big(\mathrm{i}\,\delta^{\epsilon s1\,\epsilon s4}\,\kappa^2\,C_{ES^2}\\
&\quad\Big(\big(q\cdot S_1\big)^2\Big(-m1^2\,\eta^{\gamma1\,\delta1+\gamma2\,\delta2}\,(p_1\cdot k_5)^4+4\Big(m1^2\,q\cdot q\,p_1\cdot k_5\,p_1^{\gamma1+\delta1}\Big(\eta^{\gamma2\,\delta2}\,p_1\cdot k_5+2\,q^{\delta2}\,p_1^{\gamma2}-2\,q^{\gamma2}\,p_1^{\delta2}\Big)+(q\cdot q)^2\,p_1^{\gamma1+\delta1}\Big(2\,m1^2\,\eta^{\gamma2\,\delta2}\,p_1\cdot k_5+\big(-m1^2+p_1\cdot k_5\big)\,p_1^{\gamma2+\delta2}\Big)+\\
&\qquad m1^2\,(p_1\cdot k_5)^2\Big(-q^{\delta1+\delta2}\,p_1^{\gamma1+\gamma2}+\eta^{\gamma2\,\delta2}\,p_1\cdot k_5\Big(-q^{\delta1}\,p_1^{\gamma1}+q^{\gamma1}\,p_1^{\delta1}\Big)+q^{\gamma2}\,p_1^{\delta1}\Big(2\,q^{\delta2}\,p_1^{\gamma1}-q^{\gamma1}\,p_1^{\delta2}\Big)\Big)\Big)\Big)+\\
&\quad q\cdot q\Big(-4\,(k_5\cdot S_1)^2\,p_1^{\gamma1+\delta1}\Big(-3\,m1^2\,\eta^{\gamma2\,\delta2}\,q\cdot q\,p_1\cdot k_5+2\,q\cdot q\,\big(m1^2-p_1\cdot k_5\big)\,p_1^{\gamma2+\delta2}+m1^2\,p_1\cdot k_5\Big(-q^{\delta2}\,p_1^{\gamma2}+q^{\gamma2}\,p_1^{\delta2}\Big)\Big)+k_5\cdot S_1\,p_1\cdot k_5\\
&\qquad\Big(m1^2\,\eta^{\gamma2\,\delta2}\,(-4\,q\cdot q-p_1\cdot k_5)\,p_1\cdot k_5\Big(p_1^{\delta1}\,S_1^{\gamma1}+p_1^{\gamma1}\,S_1^{\delta1}\Big)+2\,q\cdot q\,p_1^{\gamma1+\delta1}\Big(2\,m1^2\,q^{\delta2}\,S_1^{\gamma2}+\big(3\,m1^2-2\,p_1\cdot k_5\big)\,p_1^{\delta2}\,S_1^{\gamma2}+\Big(-2\,m1^2\,q^{\gamma2}+\big(3\,m1^2-2\,p_1\cdot k_5\big)\,p_1^{\gamma2}\Big)\,S_1^{\delta2}\Big)+\\
&\qquad 4\,m1^2\,p_1\cdot k_5\Big(-q^{\delta1}\,p_1^{\gamma1+\gamma2}\,S_1^{\delta2}+p_1^{\delta1}\Big(q^{\gamma1}\,p_1^{\delta2}\,S_1^{\gamma2}+p_1^{\gamma1}\Big(-q^{\delta2}\,S_1^{\gamma2}+q^{\gamma2}\,S_1^{\delta2}\Big)\Big)\Big)\Big)+\\
&\qquad p_1\cdot k_5\Big(m1^2\,(p_1\cdot k_5)^2\Big(\eta^{\gamma2\,\delta2}\,p_1\cdot k_5\,S_1^{\gamma1+\delta1}-3\Big(-q^{\delta1}\,p_1^{\gamma1}+q^{\gamma1}\,p_1^{\delta1}\Big)\,S_1^{\gamma2+\delta2}\Big)-m1^2\,(q\cdot q)^2\Big(p_1^{\gamma1+\gamma2}\,S_1^{\delta1+\delta2}+p_1^{\delta1}\,S_1^{\gamma2}\Big(p_1^{\delta2}\,S_1^{\gamma1}-2\,p_1^{\gamma1}\,S_1^{\delta2}\Big)\Big)+\\
&\qquad q\cdot q\,p_1\cdot k_5\Big(S_1^{\delta1}\Big(m1^2\,\eta^{\gamma2\,\delta2}\,p_1\cdot k_5\,S_1^{\gamma1}+p_1^{\gamma1}\Big(2\,m1^2\,q^{\gamma2}+p_1\cdot k_5\,p_1^{\gamma2}\Big)\,S_1^{\delta2}\Big)+p_1^{\delta1}\,S_1^{\gamma2}\Big(p_1\cdot k_5\,p_1^{\delta2}\,S_1^{\gamma1}-2\,m1^2\Big(q^{\delta2}\,S_1^{\gamma1}+2\,p_1^{\gamma1}\,S_1^{\delta2}\Big)\Big)\Big)\Big)\Big)-\\
&\quad q\cdot q\,q\cdot S_1\Big(-4\,k_5\cdot S_1\,p_1^{\gamma1+\delta1}\Big(-3\,m1^2\,\eta^{\gamma2\,\delta2}\,q\cdot q\,p_1\cdot k_5+2\,q\cdot q\,\big(m1^2-p_1\cdot k_5\big)\,p_1^{\gamma2+\delta2}+m1^2\,p_1\cdot k_5\Big(-q^{\delta2}\,p_1^{\gamma2}+q^{\gamma2}\,p_1^{\delta2}\Big)\Big)+\\
&\qquad p_1\cdot k_5\Big(m1^2\,p_1\cdot k_5\,p_1^{\gamma1}\Big(-\eta^{\gamma2\,\delta2}\,p_1\cdot k_5\,S_1^{\delta1}+4\Big(-q^{\delta1}\,p_1^{\gamma2}+q^{\gamma2}\,p_1^{\delta1}\Big)\,S_1^{\delta2}\Big)+q\cdot q\Big(-4\,m1^2\,q^{\gamma1}\,p_1^{\delta1+\delta2}\,S_1^{\gamma2}+\\
&\qquad\quad p_1^{\gamma1}\Big(-4\,m1^2\Big(\eta^{\gamma2\,\delta2}\,p_1\cdot k_5\,S_1^{\delta1}+q^{\delta1}\,p_1^{\gamma2}\,S_1^{\delta2}\Big)+p_1^{\delta1}\Big(-4\,p_1\cdot k_5\,p_1^{\delta2}\,S_1^{\gamma2}+2\,m1^2\Big(4\,q^{\delta2}\,S_1^{\gamma2}+\Big(2\,q^{\gamma2}+3\,p_1^{\gamma2}\Big)\,S_1^{\delta2}\Big)\Big)\Big)\Big)\Big)\Big)\Big)\Big)\Big/\Big(8\,m1^4\,q\cdot q\,p_1\cdot k_5\,(q\cdot q+p_1\cdot k_5)\Big)
\end{aligned}
$$

$\mathcal{L}$ $\longrightarrow$  $\longrightarrow$  $\longrightarrow$ $\mathcal{M}_4^{1\ \text{loop}}$

Unlike the minimal coupling, there is no GWI and the sum over states results in a projector with light cone terms.

$$\sum_\lambda \varepsilon(k)^{\mu\nu}_\lambda \varepsilon(-k)^{\alpha\beta}_{-\lambda} = \frac{1}{2}\mathcal{P}^{\mu\alpha}\mathcal{P}^{\nu\beta} + \frac{1}{2}\mathcal{P}^{\nu\alpha}\mathcal{P}^{\mu\beta} - \frac{1}{D-2}\mathcal{P}^{\mu\nu}\mathcal{P}^{\alpha\beta} \qquad \mathcal{P}^{\mu\nu}(k) = \eta^{\mu\nu} - \frac{r^\mu k^\nu + r^\nu k^\mu}{r\cdot k}$$

I pick convenient reference vectors to perform the sewing

$\mathcal{L}$ →  →  → $\boxed{\mathcal{M}_4^{1\text{ loop}}}$

$$\boxed{i\mathcal{M}_4^{1\text{ loop}}} = d_{\mathrm{B}}\, I_{\mathrm{B}} + d_{\overline{\mathrm{B}}}\, I_{\overline{\mathrm{B}}} + \boxed{c_\triangle} I_\triangle + \boxed{c_\triangledown} I_\triangledown$$

$$\frac{\boxed{c_{\triangle\,\mathrm{Min}}}}{4G^2\pi^2\mathcal{U}_1\mathcal{U}_2 m_2^2} = m_1^2(95\sigma^2-7)q^2S_1^2 - 5m_1^2(7\sigma^2-1)(q\cdot S_1)^2 + 2m_1^2(65\sigma^2-1)\frac{q^2(p_2\cdot S_1)^2}{m_2^2(\sigma^2-1)}$$

$$\frac{\boxed{c_{\triangledown\,\mathrm{Min}}}}{4G^2\pi^2\mathcal{U}_1\mathcal{U}_2 m_2^2} = 4m_2^2(15\sigma^2-2)q^2S_1^2 + 4m_2^2(q\cdot S_1)^2 + 4m_2^2(15\sigma^2+1)\frac{q^2(p_2\cdot S_1)^2}{m_2^2(\sigma^2-1)}$$

$$\frac{\boxed{c_{\triangle\,C_{ES^2}}}(\sigma^2-1)}{4G^2\pi^2\mathcal{U}_1\mathcal{U}_2 m_2^2} = m_1^2C_{ES^2}(95\sigma^4-102\sigma^2+23)q^2S_1^2 - m_1^2C_{ES^2}(155\sigma^4-174\sigma^2+35)(q\cdot S_1)^2$$
$$+ 2m_1^2C_{ES^2}(65\sigma^4-66\sigma^2+17)\frac{q^2(p_2\cdot S_1)^2}{m_2^2(\sigma^2-1)}$$

$$\frac{\boxed{c_{\triangledown\,C_{ES^2}}}(\sigma^2-1)}{4G^2\pi^2\mathcal{U}_1\mathcal{U}_2 m_2^2} = 4m_2^2C_{ES^2}(15\sigma^4-13\sigma^2+2)q^2S_1^2 - 4m_2^2C_{ES^2}(30\sigma^4-29\sigma^2+3)(q\cdot S_1)^2$$
$$+ 4m_2^2C_{ES^2}(15\sigma^4-10\sigma^2+3)\frac{q^2(p_2\cdot S_1)^2}{m_2^2(\sigma^2-1)}$$

$$\boxed{S_{EFT}} \longrightarrow$$

$$\longrightarrow \mathcal{M}^{\mathrm{EFT}}_{\mathrm{2PM}}$$

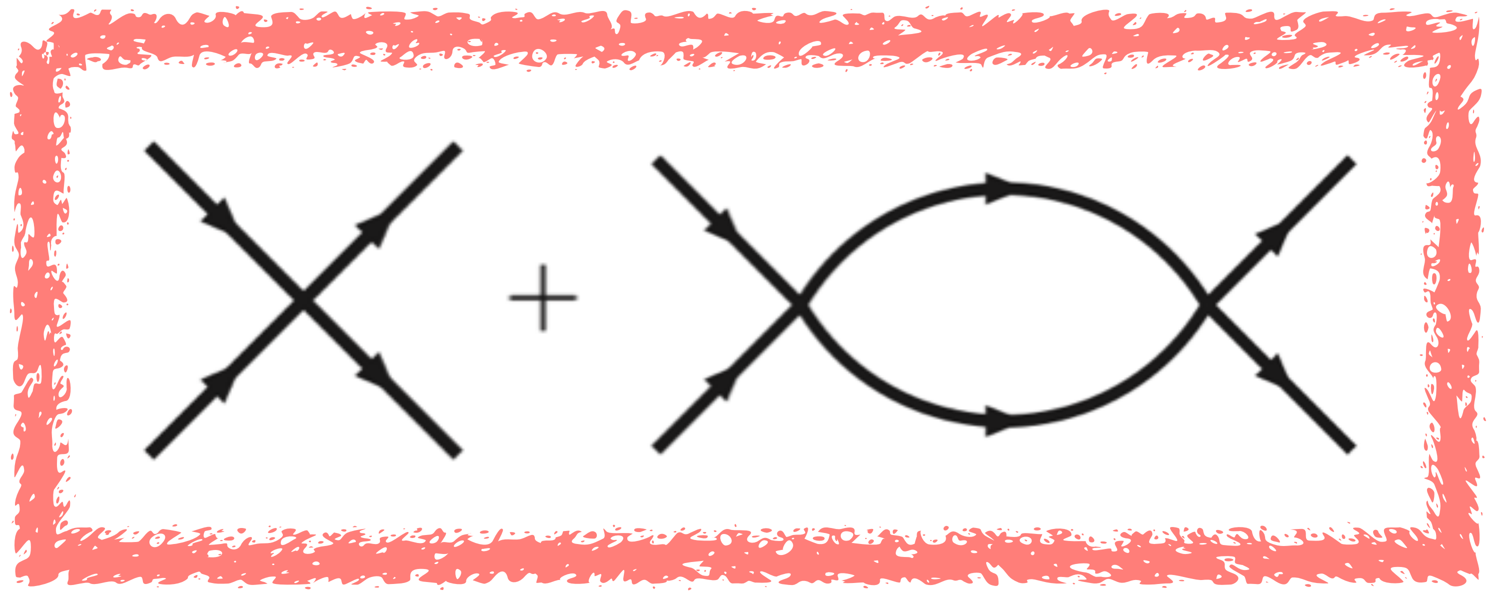
$$S = \int_{\boldsymbol{k}} \sum_{a=1,2} \xi_a^{\dagger}(-\boldsymbol{k}) \left(i\partial_t - \sqrt{\boldsymbol{k}^2 + m_i^2} \right) \xi_a(\boldsymbol{k}) - \int_{\boldsymbol{k},\boldsymbol{k}'} \xi_1^{\dagger}(\boldsymbol{k}')\xi_2^{\dagger}(-\boldsymbol{k}') \boxed{\widehat{V}(\boldsymbol{k}', \boldsymbol{k}, \hat{\boldsymbol{S}}_a)} \xi_1(\boldsymbol{k})\xi_2(-\boldsymbol{k})$$

$$\boxed{\hat{V}(\boldsymbol{k}', \boldsymbol{k}, \hat{\boldsymbol{S}}_i)} = \sum_A \boxed{\hat{V}^A(\boldsymbol{k}', \boldsymbol{k})} \boxed{\hat{\mathbb{O}}^A}$$

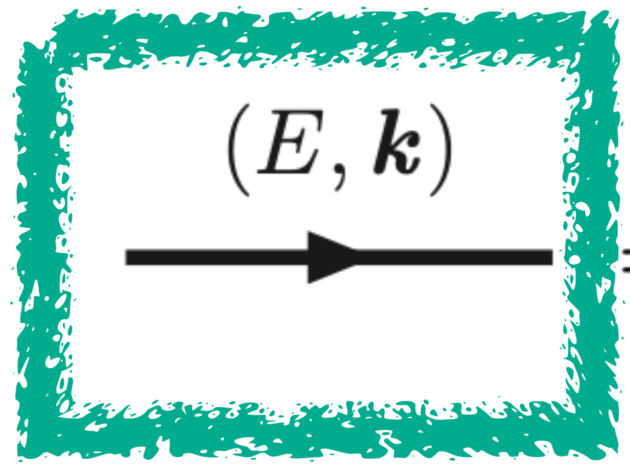
$$\boxed{\hat{V}^A(\boldsymbol{k}', \boldsymbol{k}) = \frac{4\pi G}{\boldsymbol{q}^2} d_1^A \left(\boldsymbol{p}^2\right) + \frac{2\pi^2 G^2}{|\boldsymbol{q}|} d_2^A \left(\boldsymbol{p}^2\right) + \mathcal{O}(G^3)}$$

$$\boxed{\hat{\mathbb{O}}^{(2,4)} = \mathrm{Sym}\left[\left(\boldsymbol{q} \cdot \hat{\boldsymbol{S}}_1\right)^2 \right], \qquad \hat{\mathbb{O}}^{(2,5)} = \mathrm{Sym}\left[\boldsymbol{q}^2 \hat{\boldsymbol{S}}_1^2 \right], \qquad \hat{\mathbb{O}}^{(2,6)} = \mathrm{Sym}\left[\boldsymbol{q}^2 \left(\boldsymbol{k} \cdot \hat{\boldsymbol{S}}_1\right)^2 \right]}$$

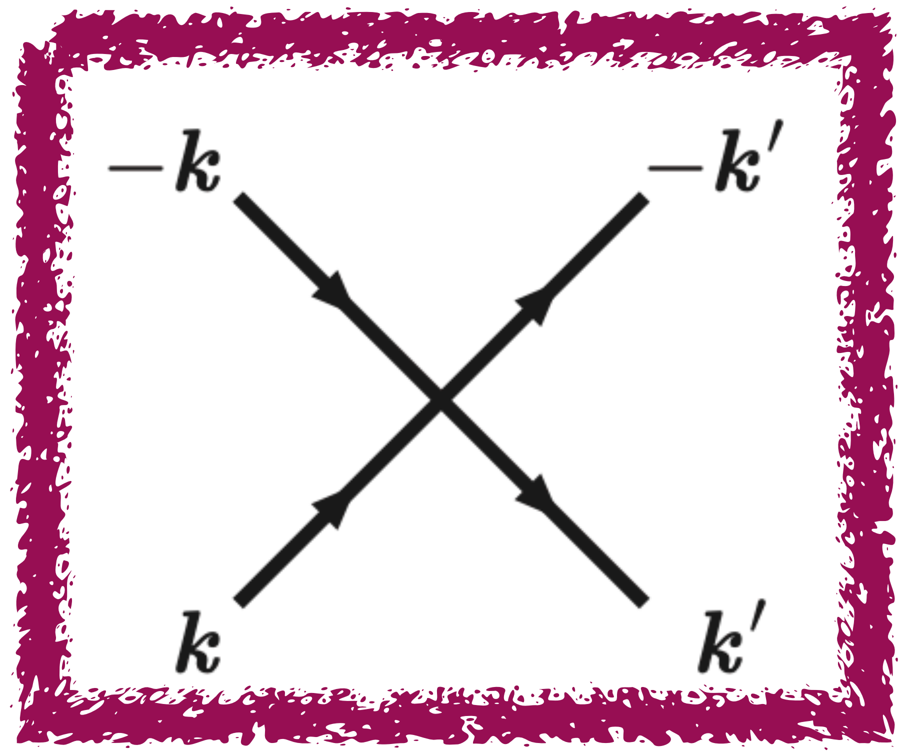
S_{EFT} $\longrightarrow$



$\longrightarrow$ $\mathcal{M}^{\mathrm{EFT}}_{\mathrm{2PM}}$



$$= \frac{i\,\mathbb{I}}{E - \sqrt{\boldsymbol{k}^2 + m^2} + i\epsilon}$$



$$= -i\hat{V}(\boldsymbol{k}', \boldsymbol{k}, \hat{\boldsymbol{S}}_i)$$

Use Feynman rules (and IBP) to compute the amplitude

$$\widehat{\mathcal{M}}^{\mathrm{EFT}} = -\hat{V}(\boldsymbol{p}', \boldsymbol{p}) - \int_{\boldsymbol{k}} \frac{\hat{V}(\boldsymbol{p}', \boldsymbol{k})\hat{V}(\boldsymbol{k}, \boldsymbol{p})}{E_1 + E_2 - \sqrt{\boldsymbol{k}^2 + m_1^2} - \sqrt{\boldsymbol{k}^2 + m_2^2}}$$

$$S_{EFT} \longrightarrow$$

$$\longrightarrow \boxed{\mathcal{M}_{2\mathrm{PM}}^{\mathrm{EFT}}}$$

$$\begin{aligned}
a_2^{(2,4)} &= \boxed{-d_2^{(2,4)}} + \frac{1}{2\xi E} A_3\left[d_1^{(0)} d_1^{(2,4)}\right] + \frac{\boldsymbol{p}^2}{4\xi E} A_2\left[\left(d_1^{(1,1)}\right)^2\right] + \frac{\xi E}{4} d_1^{(1,1)} d_1^{(2,4)}\,,\\
a_2^{(2,5)} &= \boxed{-d_2^{(2,5)}} - \frac{1}{2\xi E} A_2\left[d_1^{(0)} d_1^{(2,4)}\right] - \frac{\boldsymbol{p}^2}{2\xi E} A_2\left[\left(d_1^{(1,1)}\right)^2\right] + \frac{\xi E}{4} d_1^{(1,1)} d_1^{(2,4)}\,,\\
a_2^{(2,6)} &= \boxed{-d_2^{(2,6)}} + \frac{\xi E}{\boldsymbol{p}^4} d_1^{(0)} d_1^{(2,4)} + \frac{1}{2\xi E} A_2\left[\left(d_1^{(1,1)}\right)^2\right] - \frac{\xi E}{\boldsymbol{p}^2} d_1^{(1,1)} d_1^{(2,4)}\,,
\end{aligned}$$

$$A_2[X] = \left[\frac{1}{4}(1-3\xi) + \frac{\xi^2 E^2}{\boldsymbol{p}^2} + \frac{1}{2}\xi^2 E^2 \partial\right] X$$

$$\boxed{\mathcal{M}_{2\mathrm{PM}}^{\mathrm{EFT}}} \longrightarrow H(r^2, p^2) \longleftarrow \boxed{\mathcal{M}_4^{1\ \mathrm{loop}}}$$

$$\boxed{\hat{\mathbb{O}}^{(2,4)} = \mathrm{Sym}\left[\left(\boldsymbol{q}\cdot\hat{\boldsymbol{S}}_1\right)^2\right]} \qquad \boxed{\hat{\mathbb{O}}^{(2,5)} = \mathrm{Sym}\left[\boldsymbol{q}^2 \hat{\boldsymbol{S}}_1^2\right]} \qquad \boxed{\hat{\mathbb{O}}^{(2,6)} = \mathrm{Sym}\left[\boldsymbol{q}^2 \left(\boldsymbol{k}\cdot\hat{\boldsymbol{S}}_1\right)^2\right]}$$

Check: NNLO Hamiltonian

S^2 Hamiltonian is known up to NNLO in PN (Levi-Steinhoff '16, from EFTofPNG).

To compare with (overlapping parts of) them, we may compute Amplitudes from the Hamiltonian using EFT.

arXiv.org > gr-qc > arXiv:1607.04252

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General Relativity and Quantum Cosmology

[Submitted on 14 Jul 2016]

Complete conservative dynamics for inspiralling compact binaries with spins at fourth post-Newtonian order

Michele Levi, Jan Steinhoff

$$
\begin{aligned}
H^{\mathrm{S}^2}_{\mathrm{NNLO}} = \nu^2\Big\{\Big[&\frac{\tilde L^4}{16\tilde r^7}(11-25\nu)+\frac{\tilde L^2}{16\tilde r^6}(199+33\nu)-\frac{24}{7\tilde r^5}+\frac{\tilde L^2\tilde p_r^2}{16\tilde r^5}(7-74\nu)-\frac{9\tilde p_r^2}{16\tilde r^4}(3-5\nu)\\
&-\frac{\tilde p_r^4}{4\tilde r^3}(1+\nu)\Big]\tilde S_1^2-\Big[\frac{3\tilde L^4}{16\tilde r^7}(6-17\nu)+\frac{\tilde L^2}{16\tilde r^6}(329+61\nu)-\frac{37}{7\tilde r^5}+\frac{\tilde L^2\tilde p_r^2}{16\tilde r^5}(26-145\nu)\\
&-\frac{\tilde p_r^2}{16\tilde r^4}(3-85\nu)-\frac{\tilde p_r^4}{4\tilde r^3}(1+\nu)\Big](\vec n\cdot\vec{\tilde S}_1)^2-\Big[\frac{\tilde L^4}{16\tilde r^7}(11-25\nu)+\frac{\tilde L^2}{8\tilde r^6}(91+11\nu)\\
&-\frac{\tilde L^2\tilde p_r^2}{16\tilde r^5}(1+85\nu)\Big](\vec{\tilde S}_1\cdot\vec\lambda)^2-\Big[\frac{\tilde L^3\tilde p_r}{16\tilde r^6}(1+37\nu)-\frac{\tilde L\tilde p_r}{8\tilde r^5}(93+25\nu)\\
&-\frac{\tilde L\tilde p_r^3}{16\tilde r^4}(23-67\nu)\Big]\vec n\cdot\vec{\tilde S}_1\vec{\tilde S}_1\cdot\vec\lambda\Big\}+\nu^2C_{1(\mathrm{ES}^2)}\Big\{\Big[\frac{\tilde L^4}{4\tilde r^7}(1+3\nu)-\frac{\tilde L^2}{4\tilde r^6}(22+5\nu)+\frac{55}{14\tilde r^5}\\
&-\frac{\tilde L^2\tilde p_r^2}{4\tilde r^5}(1-6\nu)+\frac{\tilde p_r^2}{4\tilde r^4}(9-13\nu)-\frac{\tilde p_r^4}{2\tilde r^3}(1+6\nu)\Big]\tilde S_1^2+\Big[\frac{7\tilde L^2}{4\tilde r^6}(4+\nu)-\frac{95}{14\tilde r^5}+\frac{\tilde L^2\tilde p_r^2}{8\tilde r^5}(7+6\nu)\\
&+\frac{3\tilde p_r^2}{4\tilde r^4}(9+11\nu)+\frac{\tilde p_r^4}{2\tilde r^3}(1+6\nu)\Big](\vec n\cdot\vec{\tilde S}_1)^2-\Big[\frac{\tilde L^4}{4\tilde r^7}(1+3\nu)-\frac{\nu\tilde L^2}{2\tilde r^6}+\frac{\tilde L^2\tilde p_r^2}{8\tilde r^5}(5+18\nu)\Big](\vec{\tilde S}_1\cdot\vec\lambda)^2\\
&+\Big[\frac{5\tilde L^3\tilde p_r}{8\tilde r^6}+\frac{\tilde L\tilde p_r}{4\tilde r^5}(8-11\nu)-\frac{\tilde L\tilde p_r^3}{8\tilde r^4}(1-6\nu)\Big]\vec n\cdot\vec{\tilde S}_1\vec{\tilde S}_1\cdot\vec\lambda\Big\}+\frac{\nu}{q}\Big\{\Big[-\frac{\tilde L^4}{16\tilde r^7}(11-59\nu+21\nu^2)\\
&-\frac{\tilde L^2}{16\tilde r^6}(139-93\nu-33\nu^2)-\frac{1}{2\tilde r^5}(7+2\nu)-\frac{\tilde L^2\tilde p_r^2}{16\tilde r^5}(7-40\nu+60\nu^2)+\frac{3\tilde p_r^2}{16\tilde r^4}(1+63\nu+15\nu^2)\\
&+\frac{\tilde p_r^4}{16\tilde r^3}(4-4\nu-9\nu^2)\Big]\tilde S_1^2+\Big[\frac{3\tilde L^4}{16\tilde r^7}(6-37\nu+15\nu^2)+\frac{\tilde L^2}{16\tilde r^6}(269-257\nu-61\nu^2)\\
&+\frac{\tilde L^2\tilde p_r^2}{16\tilde r^5}(26-101\nu+126\nu^2)+\frac{1}{2\tilde r^5}(7+18\nu)-\frac{\tilde p_r^2}{16\tilde r^4}(3+93\nu+85\nu^2)\\
&-\frac{\tilde p_r^4}{16\tilde r^3}(4-4\nu-9\nu^2)\Big](\vec n\cdot\vec{\tilde S}_1)^2+\Big[\frac{\tilde L^4}{16\tilde r^7}(11-59\nu+21\nu^2)+\frac{\tilde L^2}{8\tilde r^6}(67-37\nu-11\nu^2)\\
&-\frac{\tilde L^2\tilde p_r^2}{16\tilde r^5}(1+35\nu-69\nu^2)\Big](\vec{\tilde S}_1\cdot\vec\lambda)^2+\Big[\frac{\tilde L^3\tilde p_r}{16\tilde r^6}(1+47\nu-33\nu^2)-\frac{\tilde L\tilde p_r}{8\tilde r^5}(69-9\nu-25\nu^2)\\
&-\frac{\tilde L\tilde p_r^3}{16\tilde r^4}(23-65\nu+57\nu^2)\Big]\vec n\cdot\vec{\tilde S}_1\vec{\tilde S}_1\cdot\vec\lambda\Big\}+\frac{\nu}{q}C_{1(\mathrm{ES}^2)}\Big\{\Big[\frac{\tilde L^4}{16\tilde r^7}(1-28\nu+9\nu^2)\\
&+\frac{\tilde L^2}{8\tilde r^6}(64+73\nu-10\nu^2)-\frac{1}{4\tilde r^5}(19+27\nu)+\frac{\tilde L^2\tilde p_r^2}{8\tilde r^5}(7-34\nu+6\nu^2)-\frac{\nu\tilde p_r^2}{8\tilde r^4}(141+26\nu)\\
&+\frac{\tilde p_r^4}{16\tilde r^3}(13+20\nu-72\nu^2)\Big]\tilde S_1^2-\Big[\frac{\tilde L^3\tilde p_r}{8\tilde r^6}(5+6\nu+9\nu^2)+\frac{\tilde L\tilde p_r}{4\tilde r^5}(16-156\nu+11\nu^2)\\
&+\frac{\tilde L\tilde p_r^3}{8\tilde r^4}(-1+24\nu+27\nu^2)\Big]\vec n\cdot\vec{\tilde S}_1\vec{\tilde S}_1\cdot\vec\lambda-\Big[\frac{3\tilde L^4}{16\tilde r^7}(5-16\nu-3\nu^2)+\frac{\tilde L^2}{4\tilde r^6}(60+11\nu-7\nu^2)\\
&-\frac{1}{4\tilde r^5}(37+21\nu)+\frac{\tilde L^2\tilde p_r^2}{8\tilde r^5}(22-66\nu-21\nu^2)+\frac{3\tilde p_r^2}{8\tilde r^4}(36-25\nu-22\nu^2)\\
&+\frac{\tilde p_r^4}{16\tilde r^3}(23-36\nu-120\nu^2)\Big](\vec n\cdot\vec{\tilde S}_1)^2+\Big[\frac{\tilde L^4}{4\tilde r^7}(1+3\nu-3\nu^2)+\frac{\tilde L^2}{8\tilde r^6}(4-97\nu+4\nu^2)\\
&+\frac{\tilde L^2\tilde p_r^2}{8\tilde r^5}(5+12\nu-15\nu^2)\Big](\vec{\tilde S}_1\cdot\vec\lambda)^2\Big\}+[1\leftrightarrow 2], \qquad (3.6)
\end{aligned}
$$

$$
\begin{aligned}
a_2{}^{\{2,4\}} = {}&\frac{C_{ES^2}\, m_1\, m_2^3}{4\, p^2\, (m_1+m_2)} + \frac{m_2\left(2\,(5+16\, C_{ES^2})\, m_1^2 + (-7+61\, C_{ES^2})\, m_1\, m_2 + (-13+29\, C_{ES^2})\, m_2^2\right)}{16\, m_1\, (m_1+m_2)} + \\
&\frac{1}{64\, m_1^3\, m_2\, (m_1+m_2)}\, p^2 \\
&\left(3\,(5+31\, C_{ES^2})\, m_1^4 + (-73+467\, C_{ES^2})\, m_1^3\, m_2 + (-361+707\, C_{ES^2})\, m_1^2\, m_2^2 + (-343+397\, C_{ES^2})\, m_1\, m_2^3 + 2\,(-41+32\, C_{ES^2})\, m_2^4\right)
\end{aligned}
$$

$$
\begin{aligned}
a_2{}^{\{2,5\}} = {}&-\frac{C_{ES^2}\, m_1\, m_2^3}{4\, p^2\, (m_1+m_2)} - \frac{m_2\left((22+20\, C_{ES^2})\, m_1^2 + (19+35\, C_{ES^2})\, m_1\, m_2 + (1+15\, C_{ES^2})\, m_2^2\right)}{16\, m_1\, (m_1+m_2)} + \\
&\frac{1}{64\, m_1^3\, m_2\, (m_1+m_2)}\, p^2 \\
&\left(-3\,(17+19\, C_{ES^2})\, m_1^4 - (115+279\, C_{ES^2})\, m_1^3\, m_2 + (53-399\, C_{ES^2})\, m_1^2\, m_2^2 + (155-209\, C_{ES^2})\, m_1\, m_2^3 + 2\,(25-16\, C_{ES^2})\, m_2^4\right)
\end{aligned}
$$

$$
\begin{aligned}
a_2{}^{\{2,6\}} = {}&\frac{C_{ES^2}\, m_1\, m_2^3}{2\, p^4\, (m_1+m_2)} + \frac{m_2\left((8+7\, C_{ES^2})\, m_1^2 + (8+11\, C_{ES^2})\, m_1\, m_2 + (1+4\, C_{ES^2})\, m_2^2\right)}{4\, p^2\, m_1\, (m_1+m_2)} + \\
&\frac{(33+39\, C_{ES^2})\, m_1^4 + (97+185\, C_{ES^2})\, m_1^3\, m_2 + (13+245\, C_{ES^2})\, m_1^2\, m_2^2 + (-73+115\, C_{ES^2})\, m_1\, m_2^3 + 4\,(-7+4\, C_{ES^2})\, m_2^4}{32\, m_1^3\, m_2\, (m_1+m_2)}
\end{aligned}
$$

Checks: Test body Hamiltonian

S^2 Hamiltonians is known to all orders in PN, in a test-body limit (Hinderer, Steinhoff, Vines '16). To compare with (overlapping parts of) it, we may compute Amplitudes using EFT.

arXiv.org > gr-qc > arXiv:1601.07529

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General Relativity and Quantum Cosmology

[Submitted on 27 Jan 2016 (v1), last revised 18 Jul 2016 (this version, v2)]

Canonical Hamiltonian for an extended test body in curved spacetime: To quadratic order in spin

Justin Vines, Daniela Kunst, Jan Steinhoff, Tanja Hinderer

$$H_{S^2} = \frac{\sqrt{w}M}{2\hat{Q}r^3}\left[\vec{S}^2 k_1(\vec{z},\vec{P}) + (\vec{n}\cdot\vec{S})^2 k_2(\vec{z},\vec{P}) + \frac{(\vec{P}\cdot\vec{S})^2}{m^2}k_3(\vec{z},\vec{P}) + \frac{\vec{n}\cdot\vec{P}\,\vec{n}\cdot\vec{S}\,\vec{P}\cdot\vec{S}}{m^2}k_4(\vec{z},\vec{P})\right]$$

$$k_1 = -C + \frac{\vec{P}^2}{m^2}\left(3(1-C) - \frac{2m^2}{(\hat{Q}+m)^2}\right) + \frac{(\vec{n}\cdot\vec{P})^2}{(\hat{Q}+m)^2}\left(3C - 1 + w + 3(C-1)\frac{\hat{Q}(\hat{Q}+2m)}{m^2}\right),$$

$$k_2 = \frac{3C\hat{Q}^2}{m^2} + \frac{3\vec{P}^2}{m^2}\left(C + \frac{\vec{P}^2 - \hat{Q}(3\hat{Q}+4m)}{(\hat{Q}+m)^2}\right) + 3(1-\sqrt{w})^2(1+2\sqrt{w}+Cw)\frac{(\vec{n}\cdot\vec{P})^4}{m^2(\hat{Q}+m)^2}$$
$$+ \frac{(\vec{n}\cdot\vec{P})^2}{(\hat{Q}+m)^2}\left(-(1-\sqrt{w})^2 + 6(w-1)\frac{\vec{P}^2}{m^2} + 6(1-\sqrt{w})(1+C\sqrt{w})\frac{\hat{Q}(\hat{Q}+m)}{m^2}\right),$$

$$k_3 = 3(C-1)\left(1 + w\frac{(\vec{n}\cdot\vec{P})^2}{(\hat{Q}+m)^2}\right) + \frac{2m^2}{(\hat{Q}+m)^2},$$

$$k_4 = 6(1-C)\frac{(1+\sqrt{w})\hat{Q}^2 + (2+\sqrt{w})m\hat{Q}}{(\hat{Q}+m)^2} - 2(3C-1+\sqrt{w})\frac{m^2}{(\hat{Q}+m)^2} + 6(1-C)(1-\sqrt{w})w\frac{(\vec{n}\cdot\vec{P})^2}{(\hat{Q}+m)^2}$$

...stay tuned for 2012.xxxxx

Moving forward…

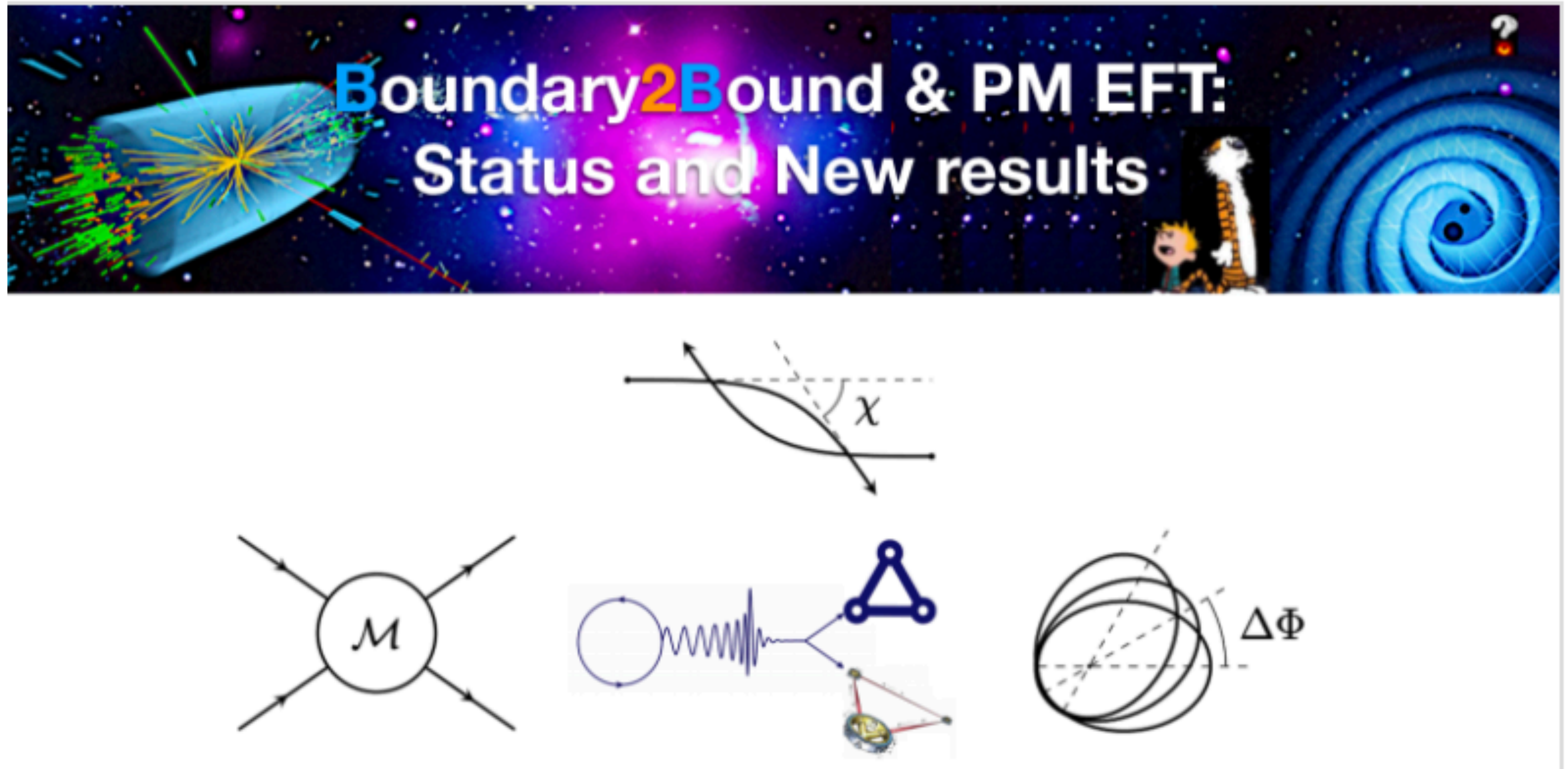
…amplitudes become increasingly cumbersome.

We need a better way to compute them:
Massive spinor helicity?
Geodesic equations?

How much can we exploit them if we aim to keep the Wilson coefficients C_{ES^2}, etc. arbitrary.

Could double copy help?

Note added: Rafael's results



Rafael 11:36 AM
@Chia-Hsien Shen @Andres Luna @Justin Vines as you can see in the slides we computed the one-loop with spin and re-derived your result (but in covariant gauge!) for the impulse (and spin kick) for spin-orbit and spin1-spin2 **generic orientation** and computed --we believe for the first time -- the spin1-spin1 finite size effects for any compact object (C_ES^2 \neq 1). We also obtained the scattering angle for aligned-spins and confirm the one-loop result for Kerr. We have extended the B2B for spin along the lines of 1911, which allows us to obtain binding energy, etc. We also confirmed the map to periastron now including finite-size effects. (footnote, the typo we thought originally in Schaefer's paper is actually much milder than we thought when we compared the Kerr case, it's just a factor of a 1/2 for generic C_ES^2). We are working in understanding how to B2B the additional angle, but I'm happy with aligned-spins for now. (Of course, we can also compute a Hamiltonian, but I rather not 😉) (edited)

Summary and Outlook

- We've developed a formalism to compute spin Hamiltonians from scattering amplitudes and Effective field theory
- We obtain an Eikonal-like formula. Need to prove/test it $\Delta\mathcal{O} = e^{-i\chi^{\mathcal{D}}}[\mathcal{O}, e^{i\chi^{\mathcal{D}}}]$
- We computed S_1^2 amplitude, Hamiltonian
- Explore relations, cross-checks with Porto+'s PM-EFT
- Outlook: More spin (S^3, comparisons with Levi(?) and GOV), more loops (2-loops seems within reach), double copy (QED? YM?)

Thank you!

QED meets Gravity

w/ T Scheopner (UCLA)

An effective action of higher spin fields coupled to photons

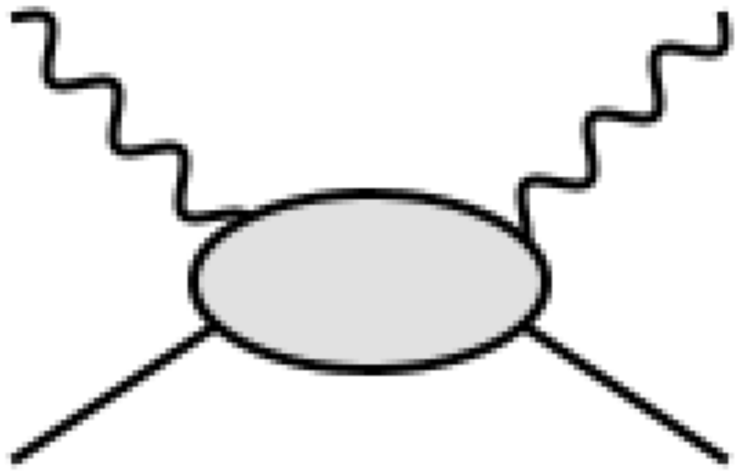
$$
\begin{aligned}
\mathcal{L} = & \frac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu} \\
& - \frac{1}{2} \sum_{n=0}^{\infty} \frac{C_n \eta_{\mu_0\nu_0}}{2^{2n} m^{4n}} \epsilon_{\mu_1\nu_1\rho_1\sigma_1} \cdots \epsilon_{\mu_{2n}\nu_{2n}\rho_{2n}\sigma_{2n}} \\
& \qquad D^{(\mu_0} D^{\mu_1} \ldots D^{\mu_{2n})} \varphi_{s_g} \mathrm{Sym}[M^{\rho_1\sigma_1}, \ldots, M^{\rho_{2n}\sigma_{2n}}] D^{(\nu_0} D^{\nu_1} \ldots D^{\nu_{2n})} \varphi_{s_g} \\
& + \frac{1}{2} m^2 \sum_{n=0}^{\infty} \frac{C_n}{2^{2n} m^{4n}} \epsilon_{\mu_1\nu_1\rho_1\sigma_1} \cdots \epsilon_{\mu_{2n}\nu_{2n}\rho_{2n}\sigma_{2n}} \\
& \qquad D^{(\mu_1} \ldots D^{\mu_{2n})} \varphi_{s_g} \mathrm{Sym}[M^{\rho_1\sigma_1}, \ldots, M^{\rho_{2n}\sigma_{2n}}] D^{(\nu_1} \ldots D^{\nu_{2n})} \varphi_{s_g} \\
& + \frac{i}{2} \sum_{n=0}^{\infty} \frac{E_n}{2^{2n} m^{4n}} \epsilon_{\mu_1\nu_1\rho_1\sigma_1} \cdots \epsilon_{\mu_{2n}\nu_{2n}\rho_{2n}\sigma_{2n}} \\
& \qquad D^{(\mu_1} \ldots D^{\mu_{2n})} \varphi_{s_g} \mathrm{Sym}[M_{\mu_0\nu_0} F^{\mu_0\nu_0}, M^{\rho_1\sigma_1}, \ldots, M^{\rho_{2n}\sigma_{2n}}] D^{(\nu_{2n}} \ldots D^{\mu_1)} \varphi_{s_g}
\end{aligned}
$$

Its three-point interaction double copies to that of the gravitationally coupled spinning particle.

It also reproduces $\sqrt{\text{Kerr}}$…

What is the relation to the effective action in Ben and Donal's talks…

What about Compton?



For minimal coupling, we know the double copy (KLT-like) relation

$$i\mathcal{M}(1^s,2^s,3^h,4^h) = -4\pi i G\,\frac{p_1\cdot p_3\, p_1\cdot p_4}{p_3\cdot p_4}A(1^0,2^0,3^A,4^A)\,A(1^s,2^s,3^A,4^A)$$

Where the amplitude comes from the Lagrangian

$$\mathcal{L}_{s,\mathrm{EM}} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + D_\mu^\dagger\bar{\phi}_s D^\mu\phi_s - m^2\bar{\phi}_s\phi_s + e(g-1)F_{\mu\nu}\bar{\phi}_s M^{\mu\nu}\phi_s$$

How do I need to adjust for the non-minimal coupling?…