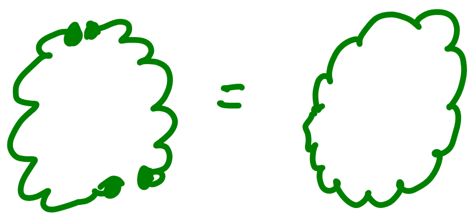
Loops of Loops

and

Non-Perturbative Negative Geometries

w/ J. Henn, J. Trnka

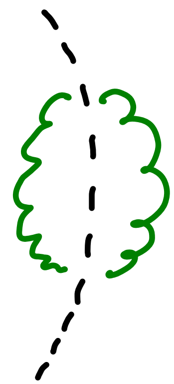
"Closed = Open^2"



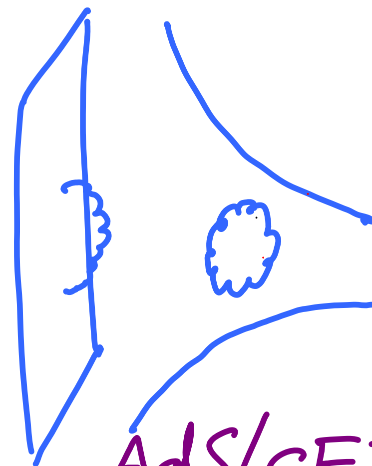
KLT

$\downarrow$

BCJ



BH's

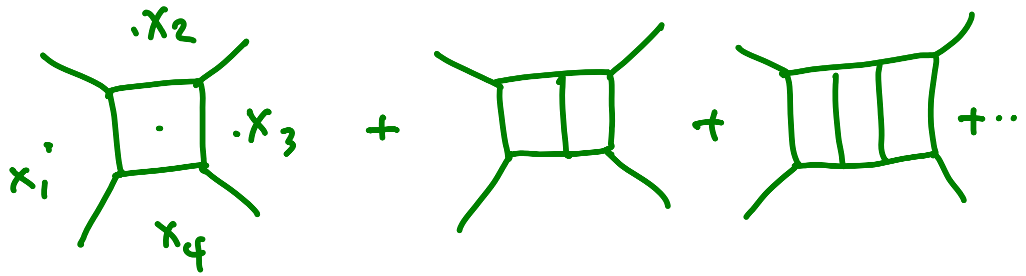


AdS/CFT

$\downarrow$

$$\underset{g^2N \ll 1}{\mathcal{N}=4} \longrightarrow \underset{g^2N \gg 1}{\text{AdS}}$$

Fantasy: can we just sum all diagrams?

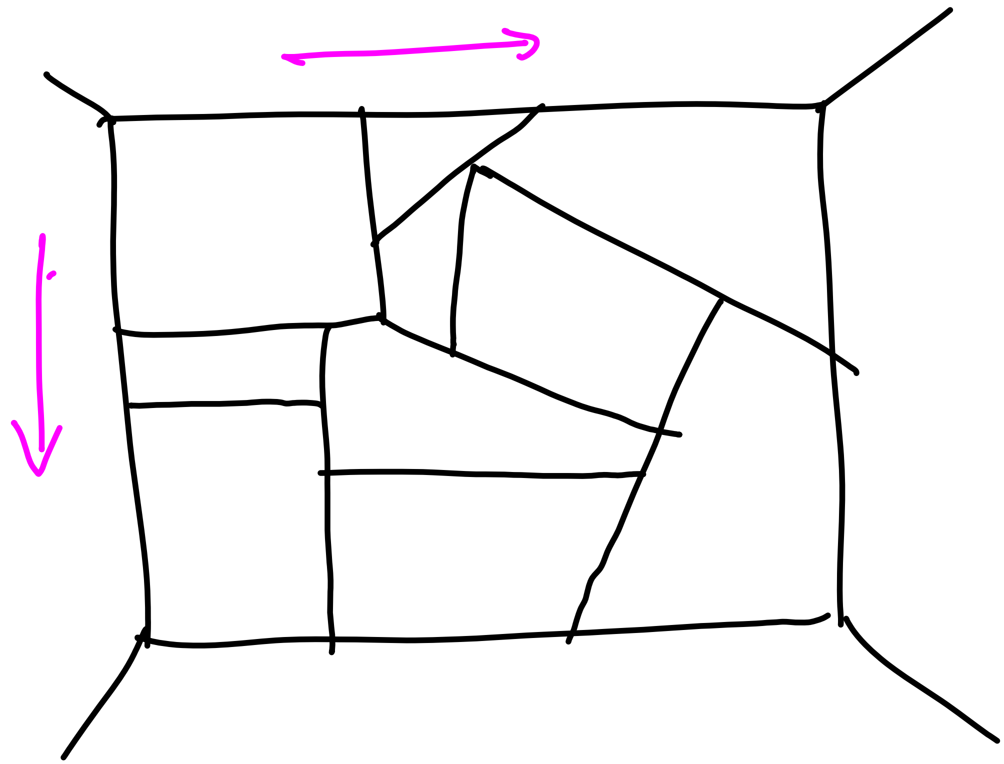
Toy ex: 

"Boxing" $\Box_{x_1} F^{(1)} = 1 \equiv F^{(0)}$, $\Box_x F^{(i+1)} = F^{(i)}$

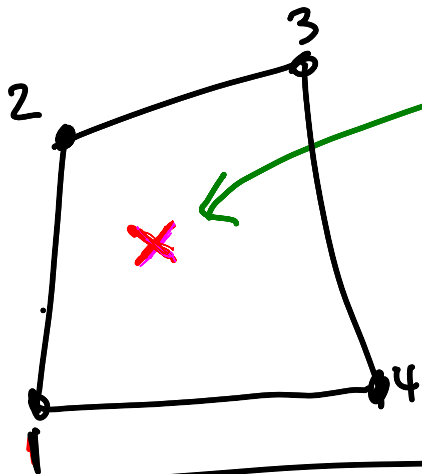
$$F(g) = \sum (-g^2)^L F_L ,$$

$(\Delta_{z,\bar{z}} + g^L) F = 0 \longrightarrow$ Solve + take to Strong Coupling!

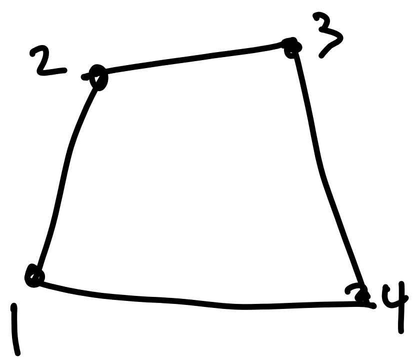
But we need "some other direction" too...



Simplest Finite Observable



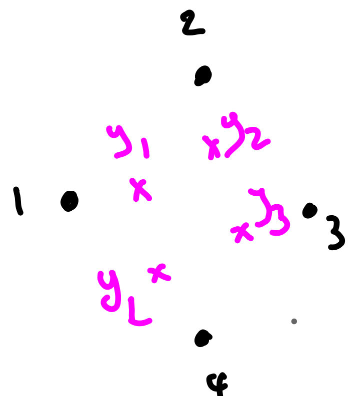
$$= \int_{\text{(All but 1-loop var)}} \log M$$

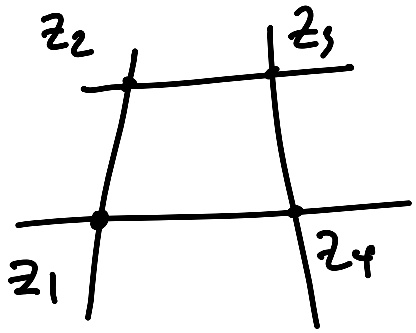


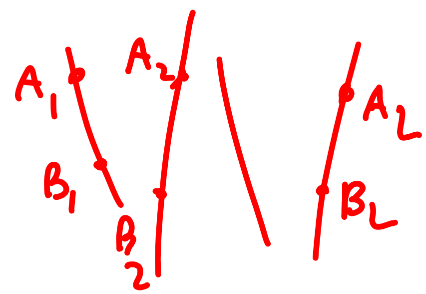
$$= \mathcal{F}\left[g^2_{th}, z\right]$$

single kin. Var!

4 pt L-loop Amplituhedron







* $\langle AB\, i\, j \rangle > 0$

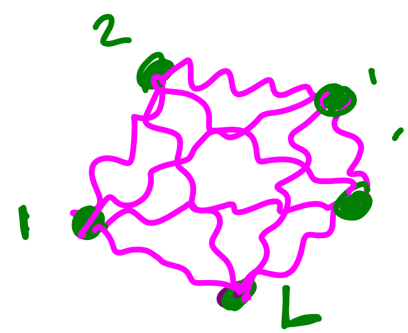
"external positivity"

cyclic, external ordering

$\langle AB_\alpha\, AB_\beta \rangle > 0$

"Mutual Positivity"

Permutation Invariant!

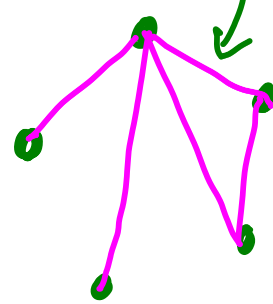


This Talk

(1) From Amplituhedron, very natural to discover ($\log M$) as sum over "negative" geometries

$$\log M = \sum_{\substack{\text{connected} \\ \text{graphs}}} (-g^2)^L$$

$\langle AB_\alpha AB_\beta \rangle < 0$



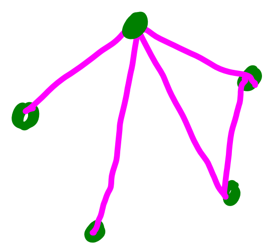
Each term gives finite $F_{G,L}(z)$

Pos. Geometry $\longrightarrow$ Polylogs

This Talk

(2) Expansion natural from Amplituhedron, no interpretation as Feynman Diagram. Deform:

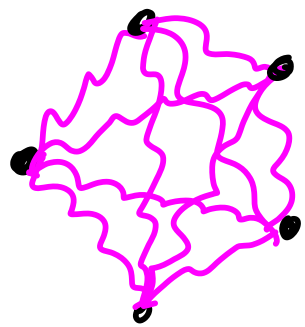
$$F[g,z;y] = \sum_{G,\ell}$$

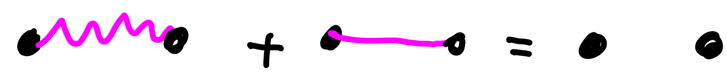


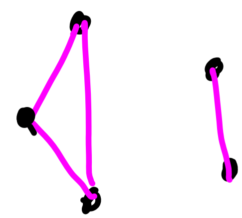
$$(-g^2)^L\, y^{\ell}$$

We will solve this "loops of loops"* theory @ "tree-level" [smally]; BUT NONPERTURBATIVELY IN g.

$\log M \longleftrightarrow$ "Negative" Geometry

$M =$ 

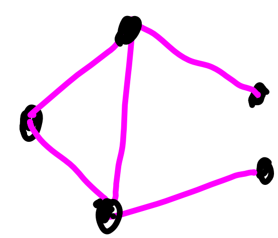


$= \sum_{\text{all } G} (-1)^{E(G)}$ 

No FD int!

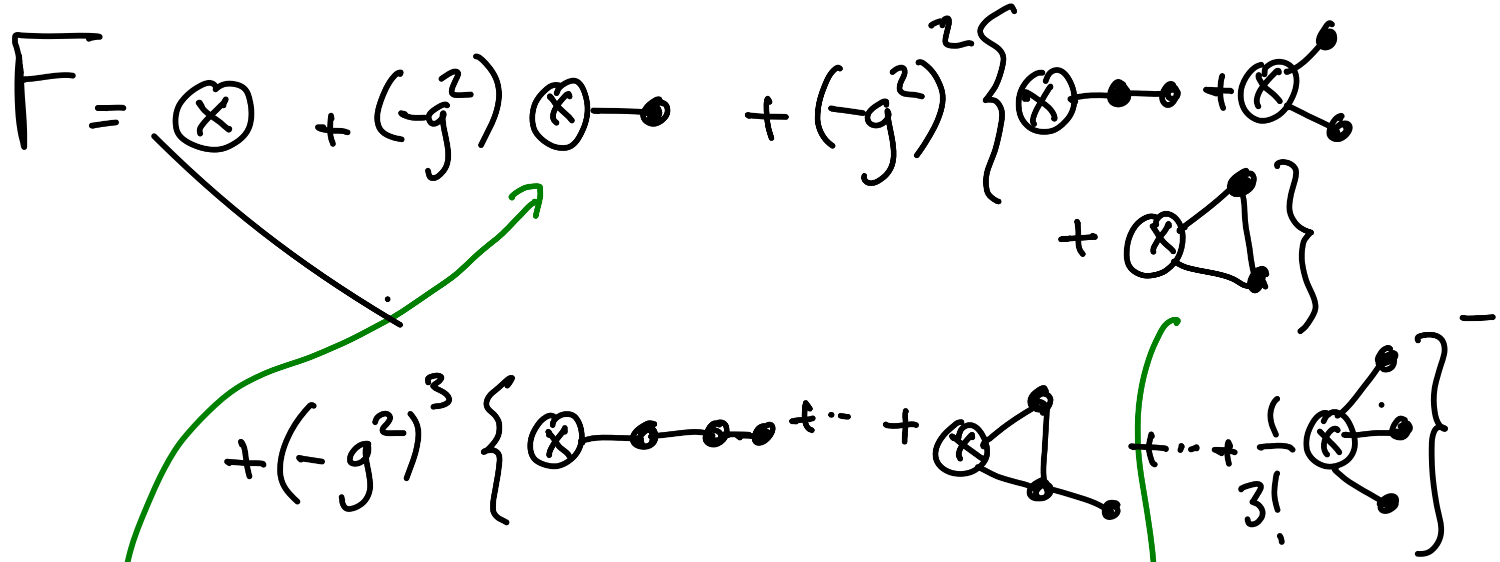
$\Downarrow$

$$\boxed{\log M = \sum_{\substack{\text{all connected} \\ G}} (-1)^{E(G)}}$$



But each $\log^2$ div.

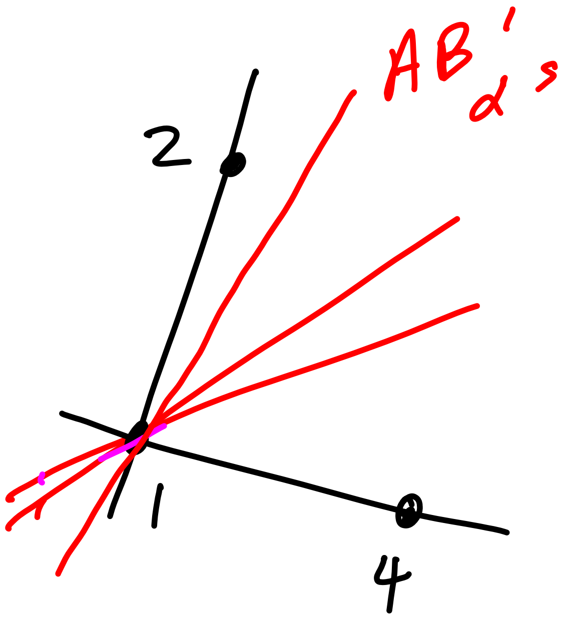
Fixing one loop ⊗



$(\pi^2 + \log^2 z)$

$H_{-1,-1,0,0}(z) + \cdots - \frac{13}{5760}\pi^4$

Pos/Neg + Finiteness



AB through 1,
in plane 412
= soft/collinear

$A = 1,\ B = 4 + x2$

$\langle AB_\alpha\, i\, i{+}1 \rangle > 0 \Rightarrow x > 0$

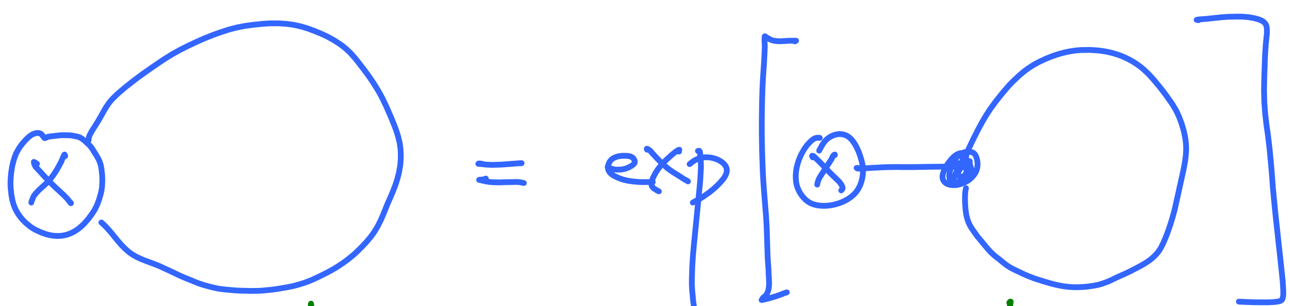
$\langle AB_\alpha AB_\beta \rangle < 0 \Rightarrow x < 0$!

NO Residue
in soft-coll.
regime!

Summing "Trees"

Define $F^{tree}(z)$ =

Also useful $H^{tree}(z)$ =

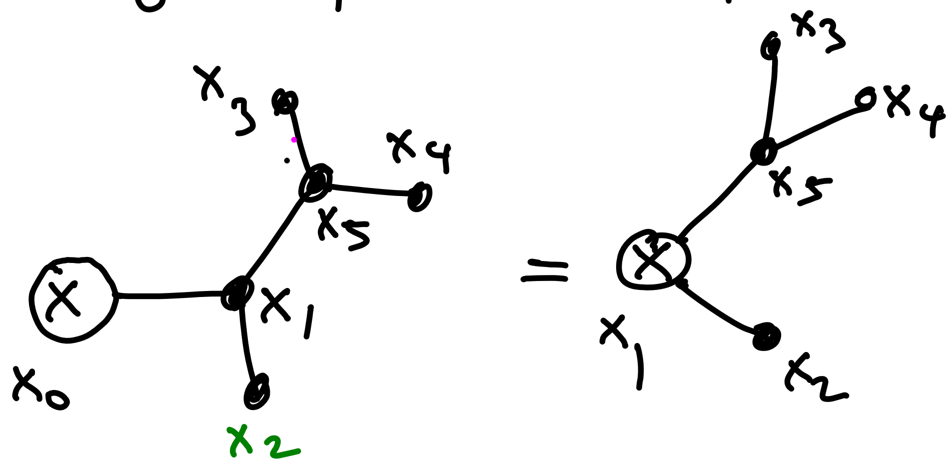


$$F^{tree} = \exp H^{tree}$$

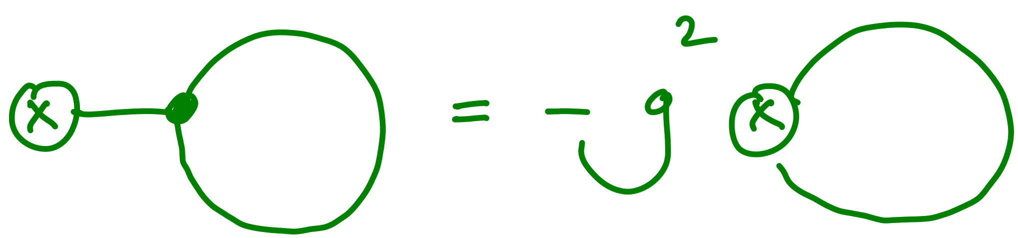
"Boxing"

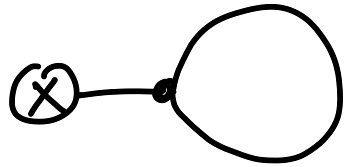
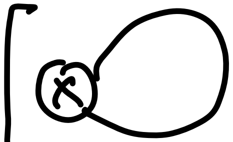
$\Box_{x_0}$

$\Box_{x_0}$



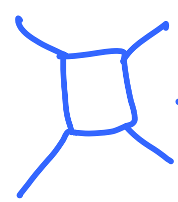
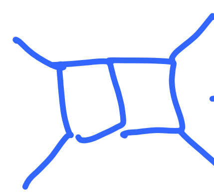
$\Box_{x_0}$



... Using  = log [] ...

$$\boxed{(z\partial_z)^2 H^{\text{tree}} + g^2 e^{H^{\text{tree}}} = 0}$$

* Liouville! Brdy condition $H(z=-1) = 0$

* <u>Non-linear</u> vs.  +  + ...

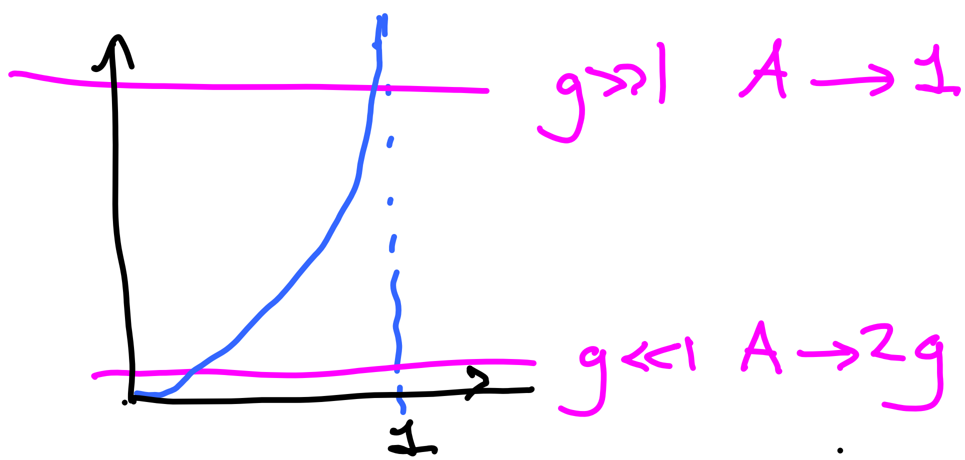
"Extra direction"

$$(\Delta + g^2) F = 0$$

Solution

$$F^{\text{tree}}(g,z) = e^{H^{\text{tree}}(g,z)} = \frac{A^2}{g^2}\,\frac{z^A}{(z^A+1)^2}$$

where $\dfrac{A}{2g\cos\frac{\pi A}{2}} = 1$



$$g \ll 1$$

$$\Gamma^{\text{tree}}(g,z) = 1 - g^2(\log^2 z + \pi^2) + \ldots$$

$$g \gg 1$$

$$\Gamma^{\text{tree}}(g,z) = \frac{1}{g^2}\frac{z}{(1+z)^2} + \left(\frac{z^2\log z - z\log z - 2z(1-z)}{(1+z)^3}\right)\left(\frac{1}{g^2}\right)^2 + \ldots$$

γ_{cusp}

$$\int d^4x_0 \; F(g,z,\dots) = (\log^2) \times \gamma$$

$$\boxed{\gamma^{tree}(g) = \frac{A}{2\pi g^2} \tan\frac{\pi A}{2}, \quad \frac{A}{2g\cos\frac{\pi A}{2}} = 1}$$

$1 - \frac{2}{3}\pi^2 g^2 + \dots$

$g \ll 1$

exp. in g^2

$\frac{1}{\pi}\frac{1}{g} + \dots\dots$

$g \gg 1$

expansion in $\frac{1}{\sqrt{g^2}}$

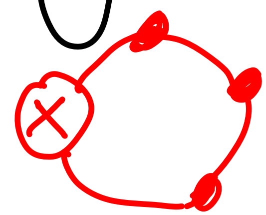
AS IN ADS!

* We now have a direct connection between Positive Geometries and Finite (Polylog...) Amplitudes

* Is there a "loop of loops theory"?

Non-p in g; deformed by y.

We have "classical limit" $y \to 0$. Need $y \to 1$ for "Real Amplitude".

* To move beyond "Boxing trick", need to understand neg. geometries more systematically, e.g. 

* $\Delta \mathcal{F}_G = \sum_{G'} c_{G'} \mathcal{F}_{G'}$?

"other direction"

Positive Geometries $\longrightarrow$ Non-pert Amps

Amplituhedron $\longrightarrow$ AdS

$$\begin{tikzcd}
\text{Gluons} \arrow[d, leftrightarrow] & \text{Strings in AdS} \arrow[d, leftrightarrow] \\
\text{Amplituhedron} \arrow[r, leftrightarrow] & \text{Dual Amplituhedron}
\end{tikzcd}$$